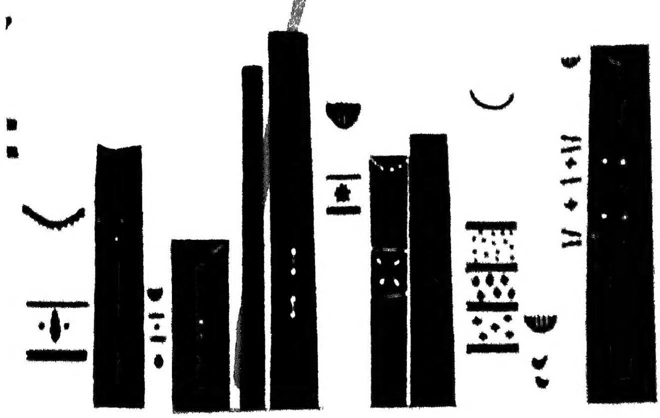

THE SCHILLINGER SYSTEM OF MUSICAL COMPOSITION

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

IN TWO VOLUMES

Volume I: Books I-VII

Volume II: Books VIII-XII

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Mrs. Joseph Schillinger wishes to acknowledge gratefully the editorial contribution of Lyle Dowling and Arnold Shaw, co-editors of *The Schillinger System of Musical Composition*. The manuscript was originally in the form of correspondence lessons. The work of editing the manuscript was begun by Lyle Dowling, a friend of Schillinger's. Because of his induction into the armed forces, the task was continued by Arnold Shaw, who is responsible for the bulk of the work and who saw the manuscript through the various stages of production. It is appropriate also to note here that Arnold Shaw, executive director of the Schillinger Society, conducted the first lecture course—Introduction to the Schillinger System—at the Juilliard School of Music during the summer of 1945.

VOLUME I

CONTENTS

OVERTURE TO THE SCHILLINGER SYSTEM

by HENRY COWELL

The Schillinger System makes a positive approach to the theory of musical composition by offering *possibilities* for choice and development by the student, instead of the rules hedged round with prohibitions, limitations and exceptions, which have characterized conventional studies.

If a creative musician has something of importance to say, his need for studying the materials with which he must say it is acknowledged as a matter of course. No great composer has ever omitted the study of techniques. Musical theory as traditionally taught, however, has always been a profound disappointment to truly creative individuals. Such men have invariably added to the body of musical theory with researches of their own. Invariably, also, they have not followed the "rules" laid down in conventional text-books with any consistency. If these rules had been based on something inevitable in the nature of music, composers would have had no reason to disregard them.

Actually, musical theory has dealt with no more than a small part of the potential musical materials; its assumptions concerning the science of sound have often been based on misapprehension, and the rules it lays down often reflect the personal taste of a certain theorist, or they may be based on the study of a single composer or of some one historical period. The resulting generalizations are far from being objective, but they are nonetheless imposed upon the student in the form of "rules". Writers on theory have not been scientists, and no scientist has tried to make a complete and co-ordinated system of musical possibilities.

Joseph Schillinger is the single exception: he was superbly competent in the two fields of musical composition and science. His monumental *System of Musical Composition* represents a lifetime of work in research, co-ordination and creative discovery. The synthesis he achieved has resulted in an entirely new point of view about the function of theory studies.

In the course of the research which led to the formulation of his system of musical composition, Schillinger took all known facts concerning the nature of musical materials from conventional theory studies, and added to the discoveries and speculations of modern and less conventional theorists such as Schoenberg, Conus and myself. By applying the laws of mathematical logic as developed by modern science, he found that he could co-ordinate all of the seemingly diverse factors. He found also that he could open further untried possibilities for the development of new materials. A glance at his Table of Contents will show an extraordinary number of aspects of music here organized for the first time for inclusion in the theoretical approach to the study of composition.

The idea behind the Schillinger System is simple and inevitable: it undertakes the application of mathematical logic to all the materials of music and to their functions, so that the student may know the unifying principles behind these functions, may grasp the method of analyzing and synthesizing any musical

materials that he may find anywhere or may discover for himself, and may perceive how to develop new materials as he feels the need for them. Thus the Schillinger System offers possibilities, not limitations; it is a positive, not a negative approach to the choice of musical materials. Because of the universality of the esthetic concepts underlying it, the System applies equally to old and new styles in music and to "popular" and "serious" composition.

Schillinger is sometimes criticized on the basis that his system reduces everything to mathematics and that musical intuition and the subjective side of creativity are neglected. I have never been able to understand this criticism. The currently taught rules of harmony, counterpoint, and orchestration certainly do not suggest to the student materials adapted to his own expressive desires. Instead he is given a small and circumscribed set of materials, already much used, together with a set of prohibitions to apply to them, and then he is asked to express himself only within these limitations. It has been the constant complaint from students of composition that their teachers fail to make clear the distinction between the objective and subjective factors in music. A young composer is constrained, as things are now, to spend several years following rules deduced or assumed from the works of his predecessors, but as soon as his works begin to be heard he is reproached, and rightly so, if they sound like somebody else's. He has not been shown what possibilities there really are in music in any objective, scientific way, nor has he been trained in the manner best calculated to develop an original talent, by exercising his own taste and judgment in choosing from among those possibilities the materials best suited to his musical intention.

Whether or not one agrees with Schillinger's great personal interest in the scientific realities of music, it is nevertheless true that no composer is well equipped to express himself subjectively until he has so profound a knowledge of musical materials and their relationships that, consciously or unconsciously, he seizes on just the right ones to use for whatever he wishes to say in music. He can be trained to do this if he will subject himself to the disciplines inherent in musical materials themselves, as they are set before him by the Schillinger System.

INTRODUCTION

by ARNOLD SHAW and LYLE DOWLING

Co-editors *Schillinger System of Musical Composition*

The Schillinger System is a synthesis of musical theory and the most recent discoveries of modern physics, psychology and mathematics. Historically, it represents the first successful effort to classify scientifically the resources of our musical system. In view of the highly original character of Schillinger's approach, a brief description of his methodology and underlying ideas seems desirable.*

1. *Music and Science*

Efforts to establish musical theory on a scientific basis date back to antiquity. Among the Greeks, Pythagoras and his followers investigated the mathematical ratios underlying harmonic intervals. Down the ages the fabrication of musical instruments and the theory of instrumentation have been correlated with developments in physics and mathematics. Within the past 200 years, two of the greatest musical theorists based their work on scientific data. In Jean-Philippe Rameau's *Treatise on Harmony* (1722), we have the beginnings of a school of thought, recently given new impetus, that makes acoustics the foundation of musical theory. In *Sensations of Tone* (1863) Helmholtz developed his theories on the basis of the findings of physiology and psychology. Zarlino, Kircher, Taneiev and others have evolved theories employing data of the various sciences. In *The Craft of Musical Composition*, Paul Hindemith, commenting on the pleasure derived from hearing vibration-combinations in simple ratios, writes: "This basic fact of our hearing process reveals to us how closely related are number and beauty, mathematics and art".

Despite this history, the idea of mixing music with mathematics is a disquieting one to some. Since music is generally portrayed as the most evanescent of the arts, to wed it to the most exacting and the most rigid of the sciences seems to produce an ill-conceived union. In part the feeling of uneasiness may stem also from an unconscious desire on the part of some composers to keep their craft in the realm of the cabalistic mysteries.

From the historical point of view, the clash between the arts and sciences is of comparatively recent origin—for us, largely the result of the romantic movement. We are heirs of the tradition that swept across Europe and the United States toward the end of the eighteenth century and that revived Plato's view of the artist as an inspired madman. As a result both critical and lay musical circles tend to exalt inspiration, genius and intuition over knowledge of resources, mastery of technique, craftsmanship, etc. Viewed practically, the dichotomy between learning and genius—between science and art—is not merely a product of romanticism. It has also been the result of the limitations of musical

*The material in this Introduction is based in part on lectures delivered by Arnold Shaw at the Julliard School of Music during the summer of 1945.

theory and pedagogy. "If you cannot define and explain certain aspects of musical composition, you fall back, of necessity, on that vague and indefinable thing called "genius" or "inspiration". Schillinger's own experience as a student at the St. Petersburg Conservatory† led him to embark on the investigation that yielded, after twenty-five years of work, the theory now known as the Schillinger System.

Schillinger's voyage of intellectual discovery began in 1914 while he was a student at the St. Petersburg Conservatory. It continued during a period of ten years (1918-1928) while he held various teaching posts in his native Russia. On coming to the United States in 1928, it took on new impetus as a result of his collaborating with Leon Theremin on electro-magnetic musical experiments and inventions. From 1932 on, when much of the system had taken form, Schillinger had opportunities to test it at various American schools and colleges. He was either a lecturer or an instructor at the David Berend School of Music, the Florence Cane School of Art, the New School for Social Research, and New York University. The reaches of his theory, embracing as it did mathematics, music and the spatial arts, were afforded varied expression at Columbia University where he gave courses or lectures in three departments: the Mathematics, Fine Arts and Music departments of Teachers College. When Schillinger was convinced of the practical nature of his discoveries, he turned also to private instruction. So successful were his students as composers and arrangers for radio, the motion pictures and the theatre that Schillinger attracted to his studio many of America's best known musicans.‡ By the time of his sudden death in March 1943, Schillinger regarded his theories as sufficiently formulated to have incorporated them in two significant works: *Mathematical Basis of the Arts* and the present publication.

2. *Mathematics of Voice Leading*

A valuable clue to Schillinger's approach to music is offered by him in his introduction to Book V, *Special Theory of Harmony*. "The main defect of existing theories of harmonies", he writes, "is in the use of the *descriptive method*. Each case is analyzed apart from all other cases and without yielding any general underlying principles. But the mathematical treatment of the subject discloses general properties of the positions and movements of the voices in terms of the

*One of our most enlightened contemporary theorists and composers, Walter Piston of Harvard University, describes musical theory in his book on *Harmony* as follows: "If we reflect that theory must follow practice, rarely preceding it except by chance, we must realize that musical theory is not a set of directions for composing music. It is rather the collected and systematized deductions gathered by observing the practice of composers over a long time, and it attempts to set forth what is or has been the common practice. It tells not how music will be written in the future, but how music has been written in the past."

†Schillinger (1895-1943) entered the St. Petersburg Imperial Conservatory in 1914 and was graduated in 1918. A biographical sketch is included at the end of Volume II of the present work.

‡A partial list of Schillinger students would include: George Gershwin, who studied for more than four years; Oscar Levant, Glenn Miller, Benny Goodman, Paul Lavalle, Nathan Van Cleave, Lyn Murray, Charles Previn, Will Bradley, Jesse Crawford, Carmine Coppola, Lennie Hayton, Joseph Lilley, Jeff Alexander, Franklyn Marks, Jack Miller, Edward Powell, Alvino Rey, Ted Royal, Frank Skinner, Herbert Spencer, Paul Sterrett, Leith Stevens, Mme Koshetz, Lazar Weiner; also Dr. Myron Schaeffer, formerly head of the music department of the University of Panama; and Edwin Gerschefski, Dean of the School of Music of Converse College.

transformation of chordal functions." Schillinger thereupon proceeds on the assumption that any chord is an assemblage of pitch-units, or to use mathematical terminology, a group of conjugate elements. In three-part structures (S = 3p), the voices or functions may be designated as a, b, and c, or 1, 3, 5.

It is to be observed that clockwise structures (reading downwards) are traditionally known as *open* positions,

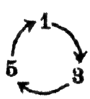
$\begin{bmatrix}1\\3\\5\end{bmatrix} \begin{bmatrix}3\\5\\1\end{bmatrix} \begin{bmatrix}5\\1\\3\end{bmatrix}$

and counterclockwise structures (reading downwards) as *close.*

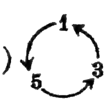
$\begin{bmatrix}1\\5\\3\end{bmatrix} \begin{bmatrix}5\\3\\1\end{bmatrix} \begin{bmatrix}3\\1\\5\end{bmatrix}$

Insofar as voice-leading is concerned, these voices behave, Schillinger demonstrates, not through any musical specifications but through *general forms of transformation.* They move in a clockwise direction, a counterclockwise direction, or in some variation of these when one voice is constant.

Clockwise



Counterclockwise



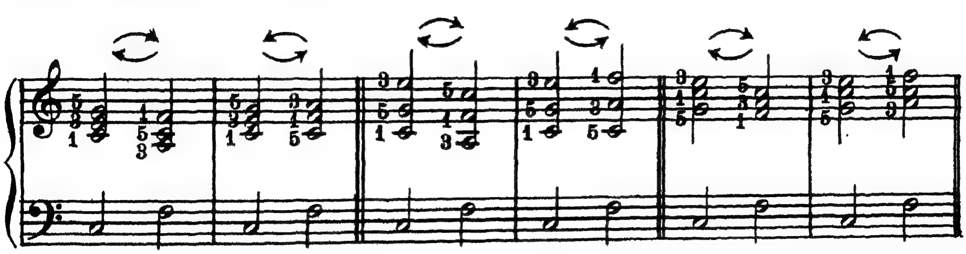
Constant 3

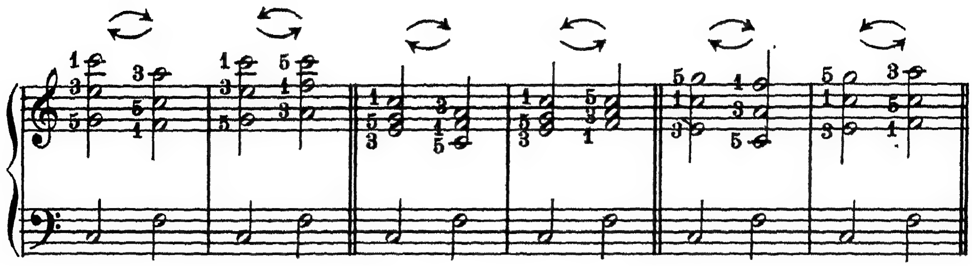
First chord		*Next chord*
Voice 1	→	Voice 3
" 3	→	" 5
" 5	→	" 1

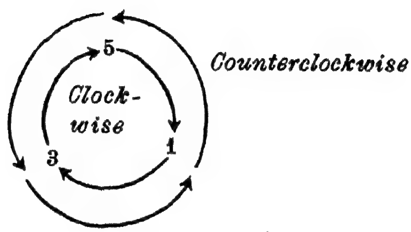
First chord		*Next chord*
Voice 1	→	Voice 5
" 3	→	" 1
" 5	→	" 3

First chord		*Next chord*
Voice 1	→	Voice 5
" 3	→	" 3
" 5	→	" 1

Without developing the theory, which is presented in detail in Book V, but simply to illustrate—suppose we have a triad on C followed by a triad on F. From a mathematical standpoint—and we soon discover, also from a musical standpoint, depending on sequence and the effect desired—here is what may happen according to the general forms of transformation:







Additional possibilities may be developed if one of the voices is constant, i.e., $3 \rightarrow 3$, $1 \rightarrow 1$ or $5 \rightarrow 5$.

3. *Schillinger's Major Aims*

On the basis of these illustrations, certain inferences may be drawn concerning Schillinger's aims. First, that he is concerned with discovering ***general underlying principles*** of the behavior of tonal phenomena. Unlike most theoreticians who have preceded him, his generalizations are not based on the practices of selected composers, or selected schools of music. He is not interested in dogmatic rules, based on the achievements of given composers, or in countless exceptions to such rules, coined to explain practices characteristic of other composers. Schillinger is interested in generalizations based on the properties of tonal materials themselves and on the possible combinations, permutations and structural relations of such materials.

Secondly, he is interested in uncovering and classifying ***all of the available resources of our tonal system.*** To be sure, this is a gargantuan task. Yet in a sense, it has been the expressed or unexpressed goal of all musical theorists. Some approached it by way of musical usage. Along this road, success was attainable provided analysis was not limited to given composers, given schools, or given musical civilizations—and then this procedure could not encompass the future. Some theorists approached the goal of exhaustiveness by way of tonal materials themselves, which could chart the future. Unfortunately, no theorist prior to Schillinger adopted a methodology of sufficient scope to achieve the desired result. This is another way of saying that no theorist adopted the method of mathematics.

For it must be evident that mathematics as the general science of number, sequence, combination and structure presents the necessary and most practical tool for achieving complete classification.

Beyond these two purposes, Schillinger set himself a third and larger goal, but a necessary corollary of the first two. From the standpoint of science, analysis implies synthesis. Success in reducing a process or a substance to its component elements implies the ability to reconstruct the object or process through synthesizing such elements. The ultimate in science is not attained when we discover that atomic energy is the source of the sun's power. We achieve the ultimate when we can reproduce such power in the atomic bomb. From Schillinger's standpoint, therefore, scientific analysis of the tonal art implied scientific synthesis as a corollary. If one could take a composition and reduce it through computation to its component elements, then mastery of the process implied an ability to compose through computation. To be sure, this is the juncture at which Schillinger would run into conflict most sharply with tradition. Insofar as he succeeded in elaborating a method of composing through the preselection and synthesis of component tonal elements, he could expect strong opposition from certain quarters. But this was unavoidable.

From a scientific standpoint Schillinger's achievement of his first two aims [1) generalizing underlying principles and 2) classifying tonal resources] is demonstrated by his success in applying the methodology to composition itself. Earlier scientific approaches to musical theory were partial in character. Their generalizations were applicable to given aspects, not to the art of composition as a whole. In Schillinger we encounter for the first time a comprehensive application of scientific method to all components of the tonal art, to problems of rhythm, melody, harmony, counterpoint, instrumentation, etc., and to the fundamental problem of all—composition itself. The individual techniques evolved in the different books—(1) temporal organization (Book I); (2) development of pitch-scales (Book II); (3) composition of melody (Books III and IV); (4) formation of chord structures and progressions (Books V and IX); (5) melodization of harmony (Book VI); and (6) correlation of melodies—counterpoint and melodic forms (Book VII)—these individual techniques are all integrated in Book XI, *Theory of Composition.* As Schillinger phrases it, his complete method involves "the prefabrication and the essembly of components according to a preconceived design of the whole."

4. *Tools and Concepts*

It is to be noted that the tools of analysis employed by Schillinger were available to musical theorists long before he used them. The graph method, a commonplace in our daily stock reports, is almost three hundred years old. Trigonometry and algebra are of ancient origin. The marriage of music and mathematics was not achieved because the basic concepts to make it possible were lacking. These concepts are of comparatively recent origin although the tools are not. These concepts stem from the findings of modern physics, modern psychology and modern mathematics. What are they?

Modern physics suggested to Schillinger his own theory of interference. Modern psychology offered usable ideas in the Weber-Fechner law of sensation and other discoveries concerning the correlation of stimulus configurations and reaction configurations. Modern mathematics provided the highly valuable ideas of relativity and mathematical logic. It is not within the scope of this brief essay to trace in detail the relation of these ideas to Schillinger's system; but some description will be useful in guiding future research.

5. *Theory of Rhythm*

The foundation of Schillinger's work is the theory of rhythm developed in Book I. Viewed simply from the standpoint of problems of duration, meter and accent, this theory provides the means for evolving all conceivable rhythms of the past, present and future—a remarkable achievement considering that most musical pedagogy has had no comprehensive theory of rhythm.*

Schillinger's theory is based on the phenomena of interference as revealed in the physics of wave motion. We find that *non-uniform* durations may be produced by combining or causing interference between *two uniform series* of durations. Thus, the rhythm

or

may be evolved by causing a uniform series of durations of numerical value 4 to interfere with a series of durations of numerical value 3. The student may perform this operation by beating one series with the right hand, the other with the left hand, and listening to the result or resultant. Pursuing the process methodically and using different durations or number values [2 ÷ 1, 3 ÷ 2, 4 ÷ 3, etc.], the student may evolve for himself every conceivable type of rhythm. Thereafter, he may apply mathematical combinations and permutations to discover thousands of possible variant patterns.

Schillinger's theory of rhythm, of inestimable value to the student and teacher of music, is more basic and comprehensive than the foregoing would suggest. The process of producing rhythmic resultants is based on numbers and the resultants are a series of numbers. The number values underlying the two musical illustrations (presented in the preceding paragraph) are, for example, 3+1+2+ +2+1+3. As Schillinger turns from rhythm to other aspects of the tonal art, these series or patterns become *coëfficients of recurrence*, and are applied to pitch-units, to pitch-scales, to harmonic cycles, to correlated melodies, to densities, etc. Applied to scales, these coefficients of recurrence yield melodies; applied to two or more melodies, they produce counterpoint; applied to tonal cycles, they produce harmonic patterns; applied to harmonic patterns, they produce different styles, etc.

*In *The Craft of Musical Composition* Paul Hindemith writes: "The domain of harmony has been explored from end to end, while rhythm, as I have previously said, has escaped all attempts to study it systematically."

6. *Patterns of Music and Nature*

Schillinger designates his theory of rhythm, the ***theory of regularity and coordination****; it represents an effort to set down the basis of all pattern-making in the universe. Musical patterns, viewed in the universe of biological, physical and esthetic objects, are really only special cases of the general process of pattern-making. Before man appeared to produce physical objects and to create art forms, there were obviously natural objects. When man appeared, some of these objects excited esthetic reactions. Painting and sculpture sought to reproduce the spatial forms of nature that appealed to man's senses. Whether the artist realized it or not, he was abstracting quantities, forms and structures and reproducing them in the hope of exciting sensations similar to those provoked by the natural objects. Recent researches in the field of psychology have supplied concrete data to validate this procedure. In studying sensation, modern psychology has found that similar configurations, whether in nature, on a canvas, or in a piece of music, excite similar sensations. Thus, just as a series of jagged rocks produce an impression of imbalance and tension, so a melody having a similar configuration will produce like sensations. Operating on this assumption, Schillinger became interested in discovering the basic patterns of growth, motion and evolution in the universe. This is what the theory of regularity and coordination represents. This is the theory of rhythm in the Schillinger System.

In addition to the resultants of interference, the basic patterns of growth, motion and evolution involve various number series, especially the ***series of acceleration***. These include the natural series 1, 2, 3, 4, 5, 6, 7, 8, 9; the harmonic series 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9; arithmetic progressions 1, 3, 5, 7, 9; geometric progressions 1, 2, 4, 8, 16, etc., including various power series 3, 9, 27, 81, 243, etc.; prime number series 1, 3, 5, 7, 11, 13, etc.; and various summation series 1, 2, 3, 5, 8, 13, 21, etc.—all of which are fully discussed in Book I. The most interesting phase of the theory of rhythm is that the resultant patterns have a universality which is breathtaking. Time and again it has been found that patterns repeat themselves in phenomena as diverse as the division and multiplication of cells, the formation of crystals, the ratios of curvature of celestial trajectories, the tangent trajectories of planetary motion, etc. Beginning with the assumption that great music might also make use of these basic patterns, Schillinger subjected the works of Bach, Beethoven, Brahms, Wagner and other immortal composers to intensive analysis. After such tests, he concluded that these great musicians had intuitively employed these patterns in their works. Through their sensuous experience, he concluded, they had realized the mathematical logic of structure.

"The patterns of growth," Schillinger writes in his *Theory of Melody*, "stimulate in human beings a response which is more powerful than many other similar but casual formations. Thus, we see that the forms of organic growth

*Schillinger employs this term in *Mathematical Basis of the Arts*, in which he evolves the fundamental theory and practice of scientific art production with regard to all of the arts. The present publication represents only one phase (musical) of Schillinger's discoveries just as the theory of rhythm in music is to be regarded as one phase of pattern-making in the universe.

associated with life, well-being, self-preservation and evolution appeal to us as forms of beauty when expressed through the art medium. Intuitive artists of great merit are usually endowed with great sensitiveness and intuitive knowledge of the underlying scheme of things. This is why a composer like Wagner is capable of projecting spiral formations . . . without any analytical knowledge of the process involved."

7. *Aristotle, Einstein and Geometry*

While the influence of modern physics in Schillinger's thinking is methodological, the impact of modern mathematics is conceptual. Schillinger makes direct use of the theory of interference. He does not use any of the special techniques of the mathematics of relativity. The impact of relativity is through its underlying ideas.

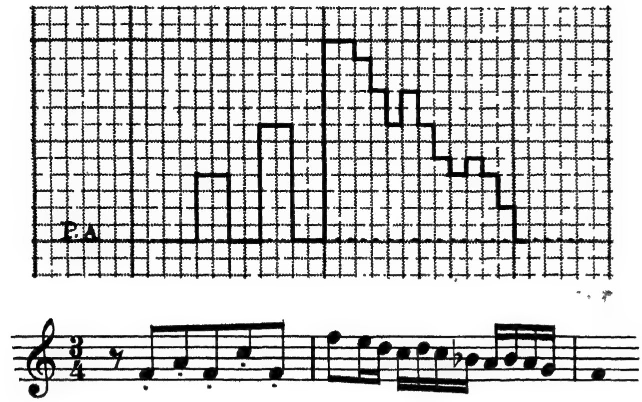
Schillinger arrived at the esthetic concept that great music and all great art reproduce the laws of development of the universe—not the appearance of this development, but the actual processes themselves. This suggests an Aristotelian idea. "Art imitates nature", Aristotle wrote, and meant that the artist did not copy appearances, but reproduced and perfected nature's forms and processes. Now Schillinger made the interrelation of music and nature (of musical forms and natural forms) basic in his thinking. He was continually at pains to point out that various musical forms were not purely esthetic developments. He reminds us, for example, that the canon is to be found in nature in the echo. Music imitates nature, and particularly the forms of motion in the universe, i.e. the growth and evolution of natural forms. Living fully and conceptually in our mathematical universe, Schillinger may be said to have given to the Aristotelian idea a materialist interpretation running far beyond the ancient concept and bulwarked by extensive musical research.

Now, what relation could be established between musical forms (tonal assemblages) and natural forms? Patently, a congruence of structure . . . a similarity of inner relations . . . an equivalence of structural quantities. It was relativity that provided foundation ideas for developing this geometrical interrelationship.

For our purposes, the significant aspect of relativity is its new treatment of time and space. Einstein demonstrated that measurements of space and time are not independent and absolute properties of the object measured—but properties of the relation between observer and object. Having "relatived" all measurements by projecting the observer into them, Einstein searched for invariant measurements not dependent on the observer. These he discovered by emphasizing the fourth dimension (time) in relation to space. Thus, we no longer talk of time and space, but of space-time, and we designate events through space-time coordinates. Now, Einstein demonstrated that such space-time coordinates express both the *metric* properties of space and the *physical* properties of the natural universe. Nature (including music) was grasped as measure relations. In other words, it became possible to study natural phenomena—or, as Schillinger concluded, artistic phenomena—through analyzing the coincidence and correspondence of their space-time coordinates.

8. *The Graph Method: Music and Motion*

Schillinger's idea was to transform musical qualities into time-space structures, i.e., into the geometric relations of their components. How? At this juncture the graph method came to Schillinger's aid. Through graphs, music could be projected into space, and sonic symbols converted into linear configurations. Here, for example, is a graph representation of Bach's *Two-Part Invention No. 8.*



(Each horizontal square represents an eighth-note and each vertical square represents a semitone).

" 'Musical motion', when projected into spatial configuration," Schillinger writes in Book XI, "possesses characteristics similiar to that of motion, action, growth or other eventual processes. It particularly resembles mechanical trajectories and projections of periodic phenomena, i.e., processes which are characterized by a high degree of regularity. As mechanical trajectories are the inherent patterns of 'musical motion', music is capable of expressing everything which can be translated into a form of motion."

Schillinger's statement and the graph vivify one aspect of the relationship between music and motion. It is quite evident that just as music may be projected into a form of motion, so forms of motion may be converted into music. The graph method of notation, i.e., the rectangular projection of music, offers a device for securing congruence of quantity and structure. It offers a new, vivid approach to such problems as variation through inversion, the analysis of melody, the modernization and "antiquation" of music, the variation of density in orchestration, the correlation of music and emotion, and other phases of composition. To Schillinger, the problems of musical components therefore presented these general aspects: 1) substitution of a scientific method of recording a musical composition for inadequate systems of notation; 2) modification of a musical work through variation of its geometric properties; and 3) production of music from a system of number values translated into geometric relations and thereafter into corresponding components of the tonal art.

9. *Schillinger's Masterful Pedagogy*

One other aspect of Schillinger's method requires discussion. This relates to the masterful pedagogy underlying his system. Discussion of the concepts basic to it may give the impression that mastery of the material is difficult. This is not true. A student does not need to be aware of the influence of modern mathematics and psychology on Schillinger's System in order to study his work. Insofar as mathematics is concerned, he does not need to know more than ordinary arithmetic. Many students studied with Schillinger by correspondence without having more than a knowledge of elementary mathematics. Beginning simply with an understanding of musical notation, students learned all phases of composition, and completed the course equipped to compose in the larger forms to orchestrate their work, and to compose directly for orchestral groups.*

Schillinger's technique is one of building complex units from simple ones. In pitch-scales, for example, he begins with a one-unit scale and proceeds numerically to construct scales of a larger and larger number of units. Mathematical theories of combination and permutation serve to reveal all of the possible variations within certain scale patterns. From scales built on units of the diatonic system, Schillinger proceeds to expanded scales, and finally to scales based on symmetric intervals, a type unknown in traditional theory. By the time the student has concluded his study of Book II, he has become aware of all the scales which may be built on the 12 semitones of the equal temperament system. The possibilities revealed by mathematical analysis are correlated with the practices of different composers, for Schillinger had an encyclopedic knowledge of musical usage and history. Copious illustration, including original compositions produced by his techniques, vivifies every step of the presentation.

A similar comprehensiveness and exhaustiveness marks the other phases of the Schillinger System. Previous theorists had systematized and classified certain areas of the tonal art.† It is Schillinger's achievement—through the methods of mathematics—that he systematizes, classifies and analyzes all the resources of the tonal art. This is not to imply that music theory stops with Schillinger. Rather it suggests that he has opened the door to the most exacting and the most scientific explorations of the art. As Nicolas Slonimsky has phrased it, Schillinger has done for music what Mendeleyeff did for chemistry: he has provided an exhaustive periodic chart of all its elements making possible the discovery of those that are not now known.

*Each branch of Schillinger's work contains novel ideas and techniques as practical as they are daring. To be of use, a summary description of these would require more space than this introductory essay will permit. The reader is referred instead to the following materials in the text itself: factorial continuity (I, 12), distributive powers (I, 12), displacement (II, 3), tonal expansion (II, 5), symmetric tonics (II, 6), geometrical expansion (III, 2), primary and secondary axes of melody (IV, 3), psychological dial (IV, 4), forms of trajectorial motion (IV, 5), symmetric harmony (V, 3 and 5), theory of indirect modulation (V, 14), sigma (Σ) concept (V, 22 and VI, 2), melodization (VI, 1), harmonization of two-part counterpoint (VII, 11), and strata harmony (Book IX). Roman numerals refer to the Books and Arabic numerals to the Chapters.

†In his book on *Harmony*, for example, Walter Piston describes his purpose as follows: "The aim of this book is to present as concisely as possible the harmonic common practice of composers of the eighteenth and nineteenth centuries. Rules are announced as observations reported, without attempt at their justification on esthetic grounds or as laws of nature."

10. *Summary of Theoretical Foundations*

By way of summary, the theoretical foundations of the Schillinger System may be described as follows. Viewing music as a space-time entity capable of graphic projection into space, Schillinger arrived at these fundamental ideas:

1. That music is determined as a logical system in the Cartesian or Einsteinian manner, i.e., that it consists of a system of correlated variables.
2. The esthetic qualities of music may be analyzed into the geometric relation of its components: rhythm, melody, harmony.
3. Variation may be achieved through modification of the inherent geometric relations.
4. Music may be composed by taking a system of number values, transforming them into geometric relations, and thereafter into corresponding components of rhythm, melody and harmony.
5. Just as the understanding of natural and biological forms requires an understanding of the laws of their growth—i.e., the forms of regularity and evolution, so an insight into music necessitates discovery of the patterns of regularity and evolution. This is what rhythm is.
6. Musical patterns, viewed in the universe of physical, biological and esthetic objects, are only special cases of the general scheme of pattern-making.
7. Schemes of pattern-making take their origin in natural and biological objects—the ratios of curvature of celestial trajectories; the formation of crystals; the division and multiplication of cells in growing things etc. When they are analyzed quantitatively, such patterns yield various number series.
8. These number series or quantities projected into music excite the same cerebral centers as were stimulated by animate beauty.
9. Thus, every great work of art, every great musical composition, realizes a certain mathematical logic. The creations of the non-mathematical musician involve such logic regardless of whether he is conscious of it or not. The esthetic harmony embodied in all great musical compositions may be discovered through the application of mathematical techniques of analysis.

In terms of the history of music and art, Schillinger has summarized his basic ideas as follows:

1. Nature produces physical phenomena which reveal an esthetic harmony to us; this harmony is due to periodic and combinatory processes; esthetic realities embody mathematical logic.
2. Man recreates esthetic realities by reproducing the appearance of the physical realities through his own body or through a material at his command; this process involves mathematical logic regardless of whether the artist is conscious of it or not. Imitating nature in an artistic medium, the artist achieves the laws of mathematical logic through his sensuous experience. This is the intuitive period of art creation.

3. Becoming more and more conscious in the course of his evolution, man begins to produce directly from the laws themselves. With the development of the technique of handling the materials of the art medium (special components) as well as rhythm of the composition as a whole (general components: space, time), man is enabled to select the desired product and the machine does the rest; this is the period of rational art creation. Thus the evolution of art falls into a closed system. An esthetic reality may be either a natural product, a product of human creative intuition, or a product of composition, realized through computation by mathematical logic.

These ideas are developed in Schillinger's masterwork *Mathematical Basis of the Arts*,* a work of world-shaking importance and revolutionary implications in the field of esthetics, in which he formulates the general laws of mathematical logic underlying all art structures.

11. *Achievements of the Schillinger System*

Percy Scholes, the well-known writer on music, has recently said: "In every other age, the rules have been based more or less upon the music of the time . . . We are still teaching on the basis of these (traditional) rules, as every published harmony textbook shows—even Schoenberg's. Yet not merely the idiom but the very principles of the art have changed . . ." The Schillinger System is the culmination, not only of century-old efforts to approach music scientifically, but of the practices of modern composers for the past century who broke away from long-established traditions and limitations. Schillinger's work is comprehensive enough to rationalize, not only the practices of the great composers of the past, but the new usages of composers of the present. For the first time, the materials of contemporary music, both its polychords and polyrhythms—including, for example, the tonal resources of Hindemith and Schoenberg—are organized into a unified system.

In addition, the Schillinger System achieves the following:

1. It establishes general laws, true in any special instance.
2. It offers techniques for composing an incredible number of variations.
3. It develops the first rational theory of melody†, removing this aspect of the tonal art from the realm of the inexplicable.
4. It makes possible the modernization or antiquation of music.
5. It marshals exhaustively all the rhythmic patterns of past, present and future.
6. It presents techniques for harmony involving as many as 576 voices.
7. It evolves objective methods for analyzing music, making it possible to compose in any selected style.
8. It makes the processes of composition available for the first time to all persons regardless of inborn ability. It makes it possible for all reason-

*To be published shortly.

†"It is an astounding fact," writes Paul Hindemith in *The Craft of Musical Composition*, "that instruction in composition has never developed a theory of melody".

ably intelligent people to master the art of composition.

9. It provides the foundation for a more objective criticism of music.
10. It establishes an objective and verifiable relation between musical works of definite form and variations (as generating excitors) and desired emotional reactions—simplifying and effectualizing the efforts of those who write background music for radio, motion pictures or television.
11. It substantially shortens the period required for mastery of musical materials and form.
12. It does not circumscribe the freedom of the individual composer but merely releases him from vagueness by giving him an exhaustive knowledge of his material and by making available prodigious resources to satisfy requirements set by any problem.

12. *Conclusion*

The monumental character of Schillinger's work must be evident even from this cursory description of its methodology and scope. Apart from the highly original nature of its approach, the work will stand as one of the most comprehensive studies ever written of the materials and processes of musical composition. Schillinger had an encyclopedic mind, was a master of musical history, including the tonal material of primitive societies, and knew intimately the scores, styles and methods of the outstanding composers of Europe, Asia and America. To this musical erudition, he added a thorough and practical knowledge of the most recent discoveries in the sciences. An accomplished composer himself, he was a student of the other arts as well and developed an esthetic based on a knowledge of the different art forms. Basic to Schillinger's approach is the scientific method, the correlation of music with the forms of motion, and the conception of the tonal art as space-time patterns translatable into sound. The Schillinger System establishes its creator as one of the great musical theorists of all times, for Schillinger achieved a synthesis as rare in cultural history as Leonardo da Vinci's—a synthesis of science and art.

New York, December 1945.

To the Reader

The reader's attention is called to the Glossary, printed at the end of Volume II. Schillinger sometimes uses conventional terms in special senses. It will facilitate the study of certain passages if the student bears in mind that explanations are available there. It is felt that no table of abbreviations is needed since the significance of each symbol (which sometimes recurs in varying senses in different parts of the book) is always made clear in its context.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK I

THEORY OF RHYTHM

BOOK I

THEORY OF RHYTHM

PRELIMINARY REMARKS ON THE THEORY OF RHYTHM

The Theory of Rhythm is the foundation of Schillinger's system. But for him, rhythm is not simply a matter of time-rhythm, which is what is ordinarily meant by the term. Schillinger begins by applying rhythm to time durations, and then extends it to all other phases of composition—the way in which block-harmonies change, intervals in scales and melody, entrances of counterthemes in counterpoint, distribution of parts through a score, and other processes of composition. Schillinger's statements are clear provided the reader takes the trouble to work them out, rather than merely read them. It must be borne in mind at this stage that the individual processes worked out in this book are *all to be used* in the actual composition of music.

The Schillinger System of Musical Composition has the integrated construction of a closely reasoned work of science or mathematics. Beginning with Book I, *Theory of Rhythm*, Schillinger successively presents techniques relating to the various phases of composition. Book II develops the *Theory of Pitch Scales*; Book IV, *Melody*; Book V, *Harmony*; Book VI, *Correlation of Melody and Harmony*; Book VII, *Counterpoint*; etc.

Mastery of the materials of any one of these books will provide the student with undreamed-of new resources. However, the Schillinger System places its emphasis on *composition*, that is, on the procedure for integrating elements and structures, and not on the detached and uncoordinated techniques. The method for integrating the individual techniques is presented in Book XI, *Theory of Composition*, which is the crowning summit of this work, as the Theory of Rhythm is its foundation.

It should be emphasized that study of the Theory of Rhythm is the pre-requisite to any real understanding of the entire work. Each of the succeeding books employs devices initially presented in the Theory of Rhythm, so that the student who skips ahead in an effort to cover ground quickly will find it necessary to retrace his steps. Thereafter, each book in turn requires a thorough understanding of preceding books.

Readers who are interested in knowing how Schillinger came to devise the system of notation he employs are referred to Chapters 1 and 2 of Book IV *Theory of Melody*. In the first chapter Schillinger presents an engrossing analysis of the physical components of music. In the second chapter he traces the history of musical notation and demonstrates the inadequacy which caused him to search for a new and more exact system of notation. Both these chapters contain insights which will assist the reader in understanding details of the Schillinger system. (Ed.)

CHAPTER I

NOTATION SYSTEM

THE CUSTOMARY method of musical notation, which is a product of the "trial and error" method, is inadequate for the analysis and study of rhythmic patterns. It offers no common basis for computations. The history of creative experience in music shows that even the greatest composers have been unnecessarily limited in their rhythmic patterns because they thought in terms of ordinary musical notation.*

The arrangement of time-durations, which constitutes the theory of rhythm, may be studied through three parallel systems of notation: (1) numbers, (2) graphs, (3) musical notes.

Understanding the nature of these group formations helps us to compose, to arrange any given musical material, and to play the most involved rhythmic patterns.

Number values will be used in this system in their normal mathematical operations (such as the four actions—addition, subtraction, multiplication, and division—, raising to powers, extracting roots, permutations, etc.)**

A. Graphing Music

The graph method used in this system is the general method of graphs, *i.e.*, a record of variation of special components, such as pitch or intensity in music, stocks in finance , diseases in medicine, etc., during a given time-period. In our theory of rhythm we shall deal with time only. The horizontal coordinate (known as *abscissa*) reads always from left to right. Here it will express *time*. The vertical coordinate (known as *ordinate*) will express the recurrence of a phase, *i.e.*, the moment of attack. This graph method is a general method and therefore objective.

Any wave motion records itself automatically. Let the pendulum of a clock swing uniformly over a strip of paper while the latter is being moved uniformly—and in a direction perpendicular to the movements of the pendulum itself.

Such record will have approximately this appearance:

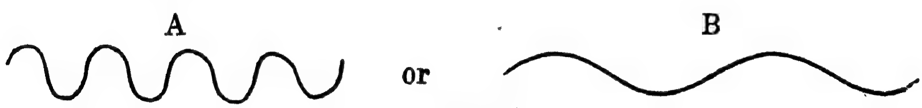


Figure 1.

* If, from experience outside the musical field, you already know how graphs are used, it will be sufficient to say at this point that (a) music can be graphed by allowing the lengths of a number of horizontal lines to stand for the *durations* of tones, and causing the distance up or down (ordinate) to stand for the *pitch* levels of the tones; and (b) when graphing duration only, as in these studies, the end of one duration and beginning of the next may be indicated by a 'turn' (phase change) in the line, as shown in Figure 47. (Ed.)

**Although Schillinger makes much use of mathematics in this work, the reader is not presumed to be a student of mathematics. Each mathematical operation is carefully explained so that those who possess the most elementary knowledge of mathematics will not encounter difficulty either in understanding the text or in performing the necessary operations. (Ed.)

depending on the speed with which the strip of paper is moving. In case A (see Figure 1) the speed is less than in case B.

Similar configurations of curves of different degrees of complexity may be observed in the projected oscillograms of sound waves. The complexity of a wave depends upon the number of components in such a wave. The simplest wave has the form which is shown in Figure 1. All clock mechanisms produce such waves (pendulum, sewing machine, etc.). In frequencies which produce musical pitch, the simplest wave may be found in the sound of tuning forks and of the flute-stops of a pipe organ.

The general form of the analysis of wave-motion is the Fourier method which Fourier developed in 1822[1] for the purpose of analyzing *heat*-waves. This method is very precise. It is used in all fields dealing with oscillatory phenomena. Yet it is a very complicated method to use for the purpose of analyzing the music of human performers. It takes about twelve hours to analyze a wave of thirty components. Machines known as harmonic analyzers have been devised. These machines perform the work of an expert mathematician in about ten minutes without any possibility of error. They are used in various fields of physics and in meteorological departments, mainly to predict tidal variations.

The simplest (i.e., one-component) wave of *one period* (recurrence group) has this appearance:

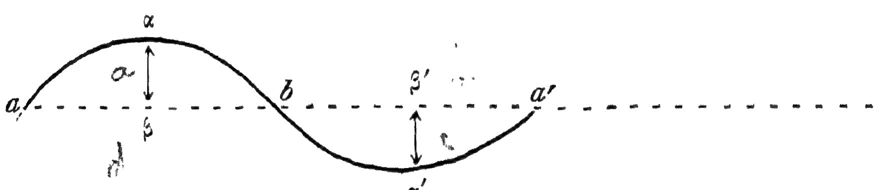


Figure 2.

The distances, a α b and b α′ a′, are equal. These curves are *phases* of the wave. Two phases constitute a period. For the purpose of studying periodic groups and their recurrences, we shall use *phases* as units of measurement. In continuous sequence they constitute the *periodicity of phases.*

The distances, αβ and α′ β′, are equal, and constitute *amplitudes.* The latter are physical expressions of *intensity.*

We shall consider intensity in the study of durations in reference to accents only. The coincidence of phases of two different periodicities intensifies the attack. The recurrence of intensified attacks ("accents") will constitute musical measures ("bars"). The reality of "bars" depends actually on the placement of attacks, not on the placement of bar lines on music paper.

By assuming that the arrangement of durations does not necessitate the expression of amplitudes, we shall use rhythm graphs in the following form:

Figure 3.

Here the horizontal lines are a simplification of the general curve; they express time. The vertical segments express the moment of attack. In the following graphs the forms of attack will be constant, and the time durations will assume various values.

B. Forms of Periodicity

Continuous recurrence of a group constitutes *periodicity*. Periodicity in which all groups are identical constitutes *uniform periodicity*. The difference between various forms of uniform periodicity may be distinguished by the number of *terms* (places) in a recurring group.

Groups with one term (a *monom*) constitute *monomial periodicity*.

The algebraic expression for monomial periodicity is:

$$at_1 + at_2 + at_3 + \ . \ . \ . + at_n$$

where a is the recurring monom and where t_1, t_2 . . . are the consecutive time moments; a may assume different values. In the field of musical time durations these values are integers; a may equal 1, 2, 3, . . . n.

When the forms of such periodicities are expressed in number-values, they have this appearance:

$1 + 1 + 1 + 1 + \ldots$
$2 + 2 + 2 + 2 + \ldots$
$3 + 3 + 3 + \ldots$
$n + n + n + \ldots$

Their graph expression is –

Figure 4.

—where each rectilinear segment represents a time-unit expressed in some space unit (inches, centimeters, etc.).

When a unit is defined, the respective values of units in different monomial periodicities will be:

$1 + 1 + 1 + \ldots$ 12

$2 + 2 + 2 + \ldots$ 6

$3 + 3 + 3 + \ldots$ 4

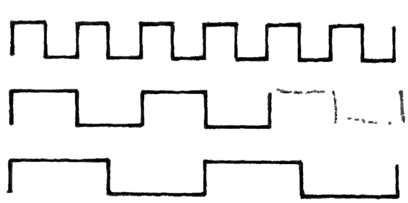


Figure 5.

Musical notation will serve as a *final* form into which number and graph expressions will be translated.

Thus, if 1 represents ♩ (or 1 = ♩), 2 = 𝅗𝅥 , 3 = 𝅗𝅥. , 4 = 𝅝 , etc.

CHAPTER 2

INTERFERENCES OF PERIODICITIES

WE ARE now concerned with what may technically be called the "generation of resultant rhythmic groups as produced by the interference of two synchronized monomial periodicities"--that is to say, the way in which one monomial periodicity (say, 3, 3, 3, 3) may be combined with another (say, 4, 4, 4, 4) so as to produce still another rhythm.

A periodicity consisting of *greater* number values will be denoted by the term, "major generator"; the smaller of the two will be called, "minor generator." The way in which we will express two synchronized generators producing one interference-group is $a \div b$.* The expression for the *resultant* of interference is $r_{a \div b}$.

A. Binary Synchronization

To synchronize two monomial periodicities it is necessary:

(1) to find *common product* or common *denominator* (c.p. or c.d.)

(2) to find *complementary factors* of both generators; the complementary factor of *a* is $\frac{ab}{a} = b$, and the complementary factor of *b* is $\frac{ab}{b} = a$.

After this is completed, it is necessary to draw a graph of both generators in their synchronization. To find the resultant (r), drop perpendiculars from all points of attack on both generators. The resultant is discovered by drawing lines through these points. The common product is then added to the diagram, and the number-values of the resultant are indicated. The entire diagram is then translated into musical notation.

When *a* equals *any number-value*, and *b* equals one, the resultant expresses a musical "bar," whether or not this bar-line would actually be drawn on music paper. Thus, a formula for a musical bar (or measure) is:

$$T = r_{a \div 1}$$

(*read:* musical bar (T) is the resultant of *a* to *one.*)

First Case

$2 \div 1$

Find the resultant, $r_{2 \div 1}$

Common product (c.p.) $2 \times 1 = 2$

Complementary factor of *a* $\frac{2}{2} = 1$ $1(2)$

Complementary factor of *b* $\frac{2}{2} = 2$ $2(1)$

a consists of two's

b consists of one's

*Although Schillinger here and elsewhere uses the division sign to indicate the a and b relationship ($a \div b$), it should be noted that Schillinger on other occasions uses a *colon* in place of the division sign (a:b).

Neither the colon nor the division sign is employed as in ordinary arithmetic. 4:3 means, in ordinary arithmetic, 4 divided by 3 or 4/3—and $4 \div$ by 3 means the same. In Schillinger's use of these signs, neither a ratio nor division is meant. He meant *interference* by them, the full mathematical formula for which is: $a:b = b + (a - b) + (2a - 2b) + (2a - 2b) + (2a - 2b) + (a - b) + b \ldots$ In arithmetic, $4:3 = 1\frac{1}{3}$. In Schillinger, $4:3 = \frac{3 + 1 + 2 + 2 + 1 + 3}{12} = \frac{12}{12} = 1$. (Ed.)

Here is the operation expressed in numbers, in graph, and in musical notation:

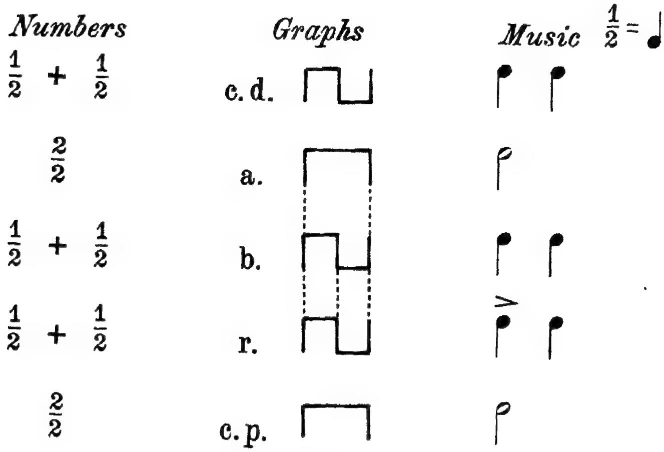


Figure 6.

The resultant differs from *b* with respect to accent (which results from the coincidence of attacks of both generators).

Musically, the *first case* establishes a bar in which the musical numerator is 2, i.e., $\frac{2}{2}$ (₵), $\frac{2}{4}$, $\frac{2}{8}$. When the bar is $\frac{2}{2}$, $\frac{1}{2}$ = 𝅗𝅥 ; when the bar is $\frac{2}{4}$, $\frac{1}{2}$ = ♩ when the bar is $\frac{2}{8}$, $\frac{1}{2}$ = ♪

Second Case

$3 \div 1$

Find the resultant, $r_{3 \div 1}$

$3 \times 1 = 3$

1 (3)

3 (1)

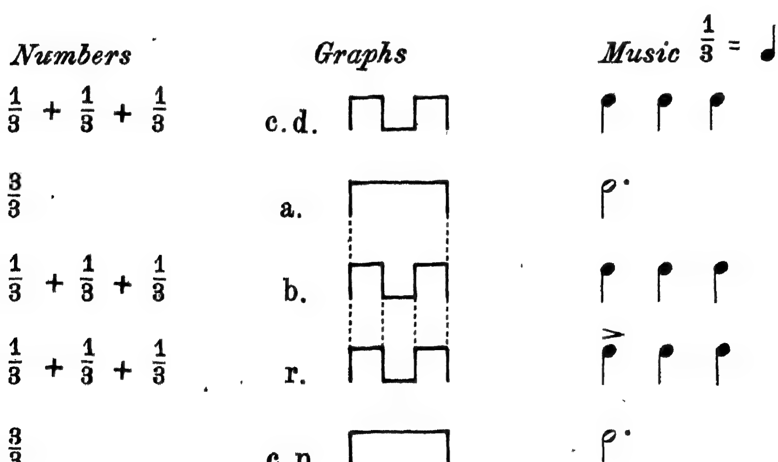


Figure 7

In this case, using a and b we hear the resultant, i.e., three uniform durations, with the accent on the first. This produces all bars with musical numerator three, i. e., $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$.

Figure 8.

Third Case

$4 \div 1$

Find the resultant, $r_{4 \div 1}$

4×1

1 (4)

4 (1)

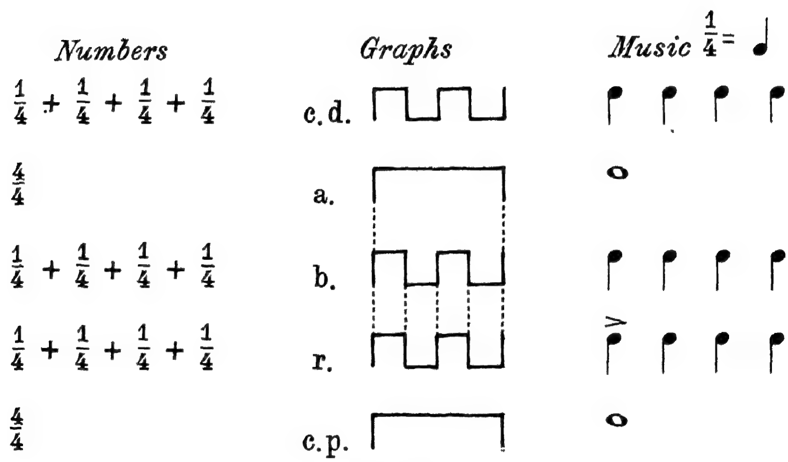


Figure 9.

The importance of this procedure lies in the fact that even the most noted composers of today do not seem to know that to express a bar before a non-uniform group is offered *is to represent the scheme of uniformity with respect to the periodicity of accents.* This means that an accent should not be forced but should result from superimposition of a on b.

When it comes to the application of higher numerators—such as 5, 7, or 11—the entire music becomes incomprehensible to the average listener, and the composer is the one to blame. When it comes to the shifting of accents which are not correctly expressed (i.e., through the use of a and b), the performance is never adequate; the performer suffers (for example, hear Stokowski in Stravinsky's *Rites of Spring*), and the listener wonders what it is all about.

Non-uniform rhythmic resultants occur when $b \neq 1$. Through the procedure described above, one may obtain *all* the rhythmic patterns of the past, present and future, including all the possible rhythms of the Orient or of the primitives.

$3 \div 2$

Find the resultant, $r_{3 \div 2}$

$3 \times 2 = 6$

2 (3)

3 (2)

Numbers

$$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$$

$$\frac{3}{6}+\frac{3}{6}$$

$$\frac{2}{6}+\frac{2}{6}+\frac{2}{6}$$

$$\frac{2}{6}+\frac{1}{6}+\frac{1}{6}+\frac{2}{6}$$

$$\frac{6}{6}$$

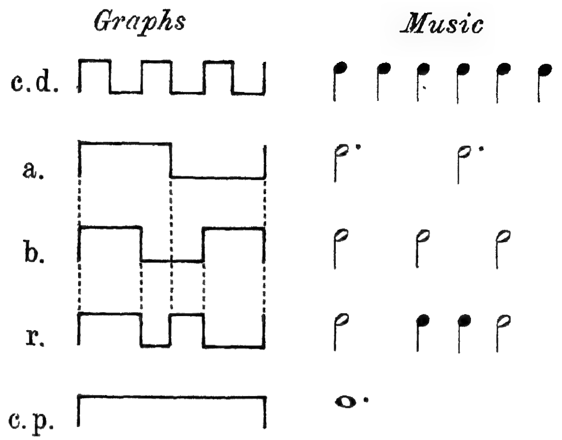


Figure 10.

B. Grouping

Three forms of grouping are available.*

(1) **Grouping by c.p.** In this case, c.p. = 6, which may express musical quarters or eighths.

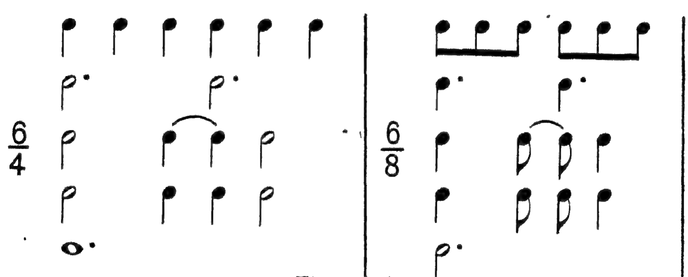


Figure 11.

Six may also express six units in $\frac{3}{2}$ or $\frac{3}{4}$ time, then:

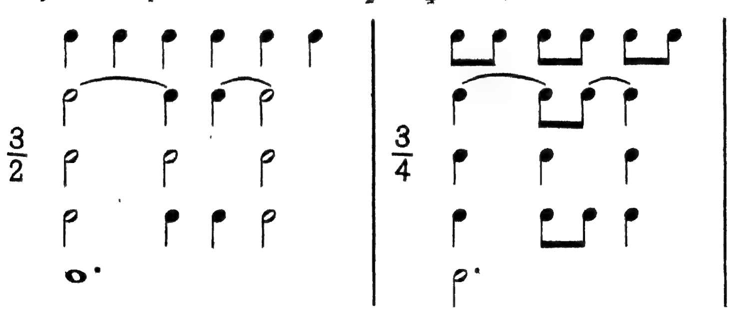


Figure 12.

*The technique and theory of grouping are described in detail in Chapter 3. (Ed.)

(2) **Superimposition of a.** a = 3. In order to get the reality of such superimposition, c.p. must be excluded and b becomes merely an optional component.

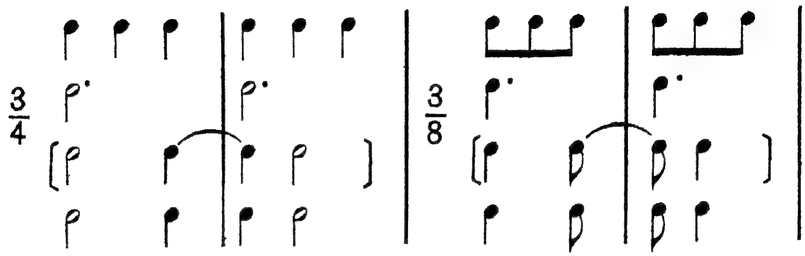


Figure 13.

(3) **Superimposition of b.** b = 2; c.p. is excluded; a becomes optional.

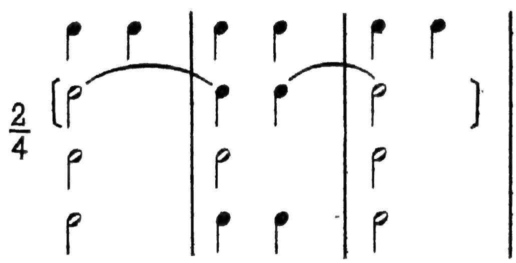


Figure 14.

4 ÷ 3

Find the resultant, $r_{4 \div 3}$

3 (4)

4 (3)

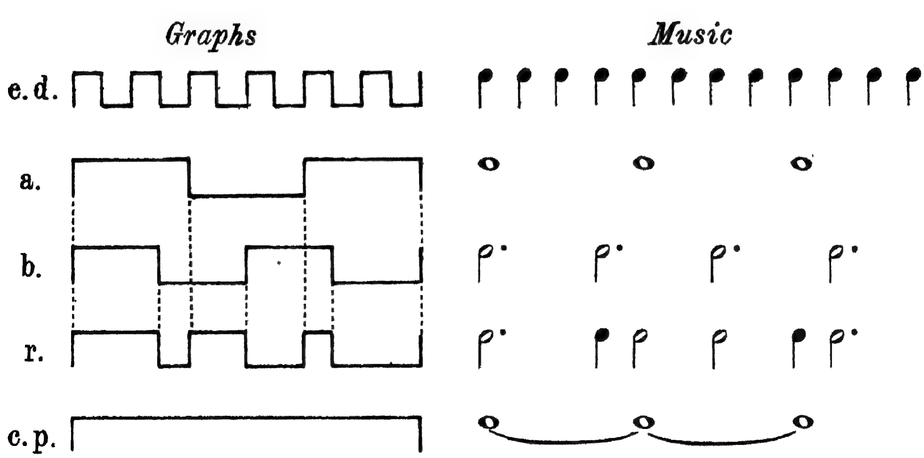


Figure 15.

$\frac{1}{12} =$ 𝅘𝅥𝅯 superimposition of c.p.

Figure 16.

$\frac{1}{12} =$ ♩ superimposition of a.

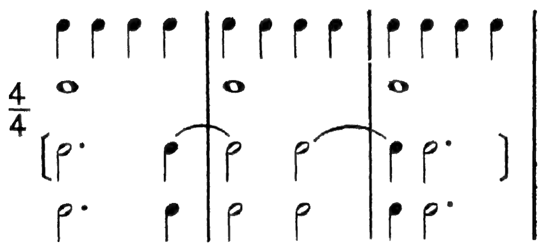


Figure 17.

superimposition of b.

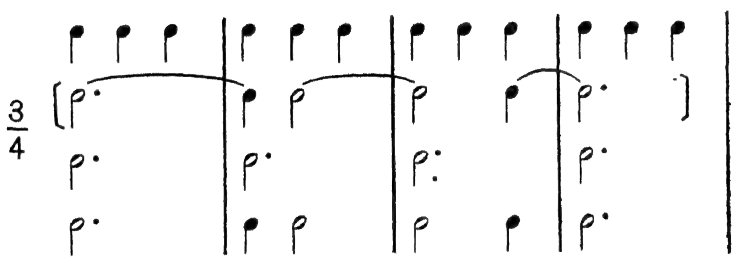


Figure 18.

All these diagrams represent the *natural nucleus* of a musical score,* in which c.d. units are arpeggio or obligato figures, *a* and *b* are chords, *r* rhythms of the theme, and c.p. sustained tones ("pedal point"). The resultants have the following characteristics:

*It may be useful to stress the fact that Schillinger means just what he says in this sentence (and other sentences); that is, he means not only that these patterns *could* be the prototype of a musical score, but—as he will show much later—they actually are the bases of scoring. (Ed.)

(1) recurrence
(2) balance (in $r_{4 \div 3}$: $2 + 2$)
(3) contrasts (in $r_{4 \div 3}$: $3 + 1$)
(4) inversion, through the axis of symmetry (center):

$$\overleftarrow{3 + 1 + 2} + \overrightarrow{2 + 1 + 3}$$

Thus, esthetic efficiency (harmony of form) is a product of physical efficiency.

All rhythmic patterns in music are either complete or incomplete resultants. Take, for example, a figure  ; it is $r_{3 \div 2}$. Take  ; it is two-thirds of the $r_{4 \div 3}$.

When all the resultants up to $a = 9$ have been found, one can obtain all the patterns of the past and present, and, to some extent, of the future.

In making your own diagrams, make them on graph paper, eliminating the c.d. units; they are the units of the cross sections.*

All the necessary generators for practical purposes are:**

$3 \div 2$				
$4 \div 3$				
$5 \div 2$	$5 \div 3$	$5 \div 4$		
$6 \div 5$				
$7 \div 2$	$7 \div 3$	$7 \div 4$	$7 \div 5$	$7 \div 6$
$8 \div 3$	$8 \div 5$	$8 \div 7$		
$9 \div 2$	$9 \div 4$	$9 \div 5$	$9 \div 7$	$9 \div 8$

Figure 19

When c.p. is greater than 15, use a and b superimposition only.

When the numbers get large, a musical eighth (♪) becomes the most practical musical denominator. All the reducible fractions are excluded from the above chart, for they give recurrences of the previous cases. For example, $6 \div 4$ would simply give $3 \div 2$ twice.

The a and b components present a clear idea of how "cross-rhythms" should be performed. Beating a and b with both hands, listen to the resultant, i.e., playing 3 against 2, play one-two, one, one, one-two ($2 + 1 + 1 + 2 =$ ♩ ♫ ♩), alternating hands.

*In the foregoing, Schillinger has given an extremely *rigorous* (as the logicians say) statement of the case so as to satisfy the most exacting demands of mathematicians and other scientists. It may be helpful to state the process in another, and less rigorous, way—for the benefit of those who are not directly interested in the scientific aspects of the matter. The process is this: (1) take a piece of graph paper, and regard each square from left to right as some unit in time, whether it be a sixteenth note, an eighth note, or what not; (2) mark off the larger of the two generators in a fashion similar to that seen in Figure 10 or Figure 47, that is, breaking the line—if the major generator be "4"—every four units; (3) then mark off the smaller generator until the two "come out even"; (4) then mark off the *resultant*, by making a line which breaks wherever either (or both) of the other two lines breaks. This can be done very rapidly, and the process is best learned by actually carrying it out. Schillinger generally used graph paper with twelve squares to the inch. (Ed.)

**When Schillinger presents a list of this kind, it is his intention that the student should work out all of these instances for himself. He planned the present manuscript so as to have it serve the double function of a workbook and theoretical study. (Ed.)

Let me add a few words on primitive rhythm: the true "primitive" rhythm (such as the rhythm used by some African cannibalistic tribes) is a *combination of various monomial periodicities in time-continuity.*

For example:

$2 + 2 + \ldots$

$3 + 3 + \ldots$

$4 + 4 + \ldots$ etc.

These, when combined in sequence, produce such rhythmic patterns as:

$$(2 + 2) + 1 + (1 + 1 + 1) + (1 + 1 + 1 + 1) + \ldots = \tfrac{4}{4}$$

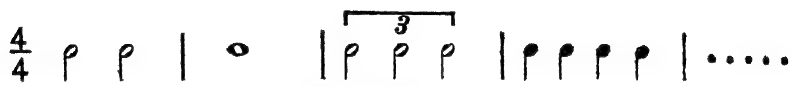


Figure 20.

CHAPTER 3

THE TECHNIQUES OF GROUPING

HAVING SEEN how two monomial periodicities produce a resultant, we have now to consider the manner in which these patterns may be grouped. There are three fundamental forms of grouping of $a \div b$.

(1) Grouping by the product (by ab);
(2) Grouping by the major generator (by a);
(3) Grouping by the minor generator (by b).

In order to group m elements by n, it is necessary to divide m by n. Thus grouping by ab is the *quotient* of $\frac{m}{ab}$.

As in the case of binary synchronization the duration of the entire score equals ab. The formula for grouping by ab is:

$$\frac{ab}{ab} = T \qquad (1)$$

i.e., grouping by ab produces one T with abt.

Example:

$3 \div 2 \qquad \frac{ab}{ab} = \frac{6}{6} = T$, one measure with 6t.

The 6t can be represented in musical notation as any measure with 6 single units. For instance, $\frac{3}{4}$ time, where t = ♪ , or $\frac{6}{4}$ time, where t = ♩ , or $\frac{6}{8}$ time, where t = ♪ .

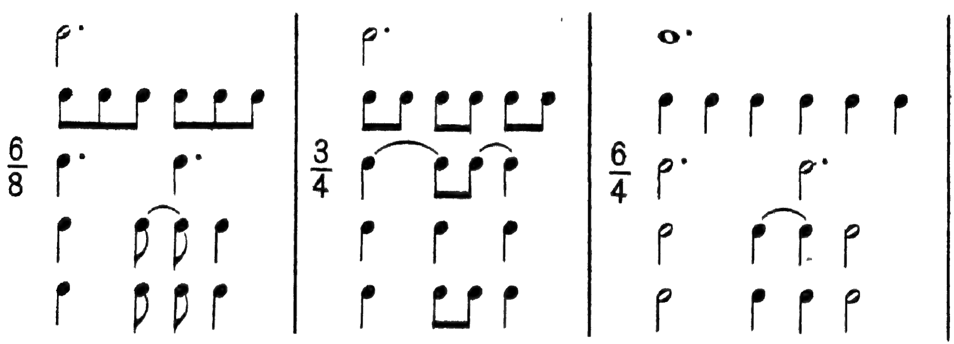


Figure 21.

$$\text{Grouping by a:} \qquad \frac{ab}{a} = bT \qquad (2)$$

In grouping by a, ab must be excluded from the score, as the presence of the latter neutralizes one of the accents, which as a result makes it sound like one T.

$3 \div 2 \qquad \frac{ab}{a} = \frac{6}{3} = 2T$, i.e., two measures with 3t.

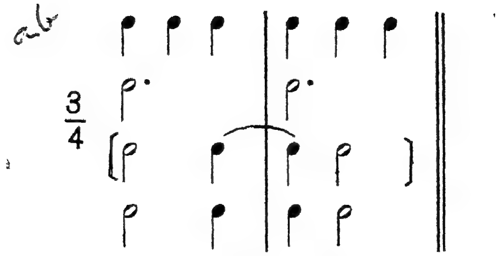


Figure 22.

When grouped by a, b becomes an optional component, causing an effect known as syncopation.

Grouping by b: $\frac{ab}{b} = aT$ (3)

Exclude ab from the score and assign a as an optional component.

$3 \div 2$ $\frac{ab}{b} = \frac{6}{2} = 3T$, i.e., three measures with 2t.

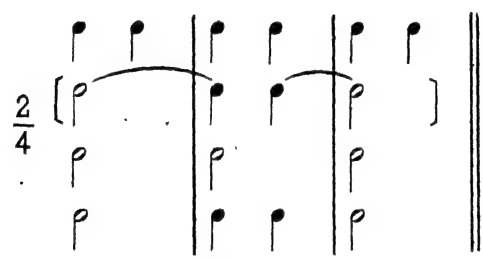


Figure 23.

It is practical to score all the 19 cases of binary synchronization by ab, by a, and by b, with the exception of cases in which ab is too great to be used as one T. The latter consideration is merely a concession to musical habits.

The following table includes all the necessary scores. The reason that some of the forms of T, like $\frac{10}{8}$ or $\frac{14}{8}$, are not in common use is merely due to the lack of adequate rhythmic patterns for their representation.

a ÷ b	Grouping by ab	Grouping by a	Grouping by b
3 ÷ 2	$\frac{6}{8}$; $\frac{3}{4}$ t = ♪	$\frac{3}{4}$	$\frac{2}{4}$
4 ÷ 3	$\frac{12}{8}$; $\frac{3}{4}$ t = 𝅘𝅥𝅯	$\frac{4}{4}$	$\frac{3}{4}$
5 ÷ 2	$\frac{10}{8}$	$\frac{5}{4}$	$\frac{2}{4}$
5 ÷ 3	$\frac{15}{8}$	$\frac{5}{4}$	$\frac{3}{4}$
5 ÷ 4	——	$\frac{5}{4}$	$\frac{4}{4}$
6 ÷ 5	——	$\frac{6}{8}$; $\frac{3}{4}$ t = ♪	$\frac{5}{4}$
7 ÷ 2	$\frac{14}{8}$	$\frac{7}{8}$	$\frac{2}{4}$
7 ÷ 3	——	$\frac{7}{8}$	$\frac{3}{4}$
7 ÷ 4	——	$\frac{7}{8}$	$\frac{4}{4}$
7 ÷ 5	——	$\frac{7}{8}$	$\frac{5}{4}$
7 ÷ 6	——	$\frac{7}{8}$	$\frac{6}{8}$; $\frac{3}{4}$ t = ♪
8 ÷ 3	——	$\frac{8}{8}$	$\frac{3}{4}$
8 ÷ 5	——	$\frac{8}{8}$	$\frac{5}{4}$
8 ÷ 7	——	$\frac{8}{8}$	$\frac{7}{8}$
9 ÷ 2	$\frac{18}{8}$	$\frac{9}{8}$	$\frac{2}{4}$
9 ÷ 4	——	$\frac{9}{8}$	$\frac{4}{4}$
9 ÷ 5	——	$\frac{9}{8}$	$\frac{5}{4}$
9 ÷ 7	——	$\frac{9}{8}$	$\frac{7}{8}$
9 ÷ 8	——	$\frac{9}{8}$	$\frac{8}{8}$

Figure 24.

CHAPTER 4

THE TECHNIQUES OF FRACTIONING

THE FIRST process by which rhythmic resultants are generated—the process just explained in the foregoing—is not entirely satisfactory for all musical purposes; it is too "rich" in its variety for all uses, and one may feel the need for a higher degree of uniformity which would complement this variety. Thus the second process by which rhythmic resultants may be generated is now offered with this purpose in mind.

Groups arrived at by means of this second process will be known as rhythmic resultants with fractioning around the axis of symmetry.

Symbols: $\underline{a \div b}$ (underlined) and $r_{\underline{a \div b}}$

The process of synchronization is:

(1) Take the product of a by a, i.e., a^2 (read: "a square"). a becomes its own complementary factor.

(2) Use a as a complementary factor of b, i.e., b appears a times.

(3) The minor generator completes itself before the major generator. Call the *first group* of the minor generator b_1 (the first b). Start the second b (b_2) at the beginning of the *second phase* of a. Start the third b (b_3) at the beginning of the *third phase* of a, when present. This procedure is continued until both generators complete at the same time. $b_1, b_2, b_3, \ldots$ always appear a times.

To find the total number of b groups this formula is used:

$\boxed{N_b = a - b + 1}$ i.e., the number of b groups equals a minus b plus 1.

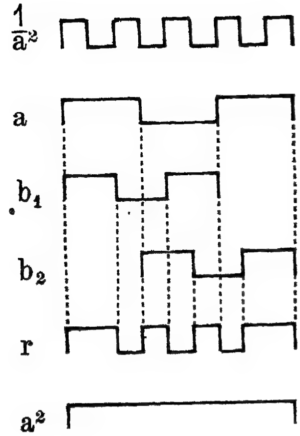
Figure 25.

Example:

$\underline{3 \div 2}$ find $r_{\underline{3 \div 2}}$

$$3 \times 3 = 3^2 = 9$$
$$3 \quad (3)$$
$$3 \quad (2)$$

$N_2 = 3 - 2 + 1 = 2$, i.e., b_1 and b_2



$$a^2\left(\frac{1}{a^2}\right) = 9\left(\frac{1}{9}\right)$$

$$= 3\times3 \quad a = \frac{3}{9} + \frac{3}{9} + \frac{3}{9}$$

$$= 3\times2 \quad b_1 = \frac{2}{9} + \frac{2}{9} + \frac{2}{9}$$

$$= 3\times2 \quad b_2 = \frac{2}{9} + \frac{2}{9} + \frac{2}{9}$$

$$r = \frac{2}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{2}{9}$$

$$a^2 = \frac{9}{9}$$

Figure 26.

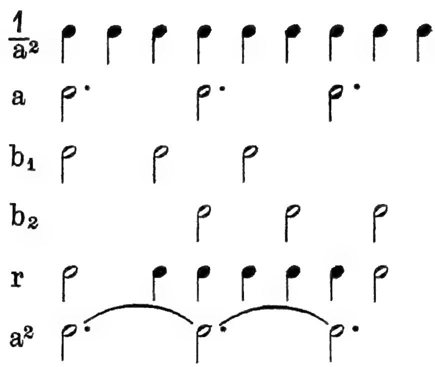


Figure 27.

Fundamental Grouping by a^2 or a only

Grouping by a^2

$$\boxed{\frac{a^2}{a^2} = T}$$

$$a^2=9 \qquad \frac{1}{a^2} = \frac{1}{9} = ♪; \quad \frac{9}{9} = T$$

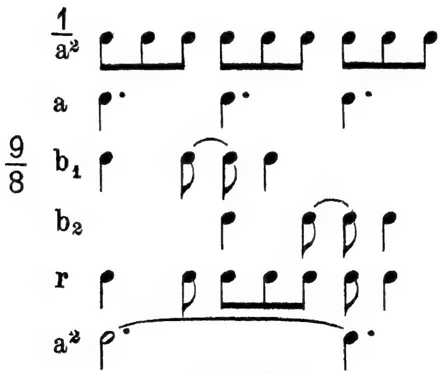


Figure 28.

Grouping by a

$$\boxed{\frac{a^2}{a} = aT}$$

Exclude a^2 from the score.

$a = \frac{3}{9}$ $\qquad$ $\frac{1}{a} = \frac{1}{9} =$ ♩; $\frac{9}{3} = 3T$

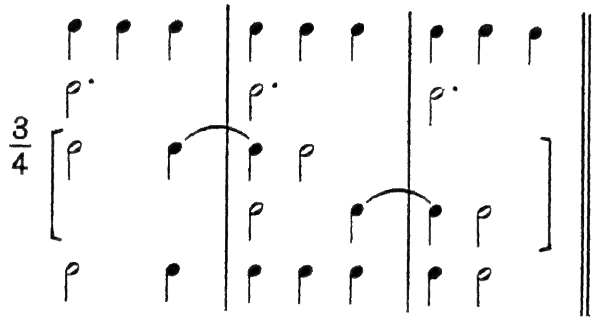


Figure 29.

Grouping by *b of the resultants with fractioning* serves the purpose of producing syncopated rhythms. In such cases the resultant and the bar do not close simultaneously in the first run of the resultant. Therefore, the resultant should be repeated from the point where it stops.

Just when the resultant and the bar come out even may be found in the following manner:

$$\boxed{\frac{a^2}{b} = Q}$$

Figure 30.

The "Q" stands for the quotient which indicates the number of bars. It always has a remainder. The denominator of the remainder indicates how many times the resultant will have to run. For the *b* grouping, the resultant is used alone.

$$\boxed{b \times Q = bQ}$$

Figure 31.

Example: (1)

$\frac{a^2}{b} = \frac{9}{2} = 4\frac{1}{2}$. $4\frac{1}{2}$ indicates the number of bars. 2 indicates the number of groups of r. $2\ (4\frac{1}{2}) = 9$.

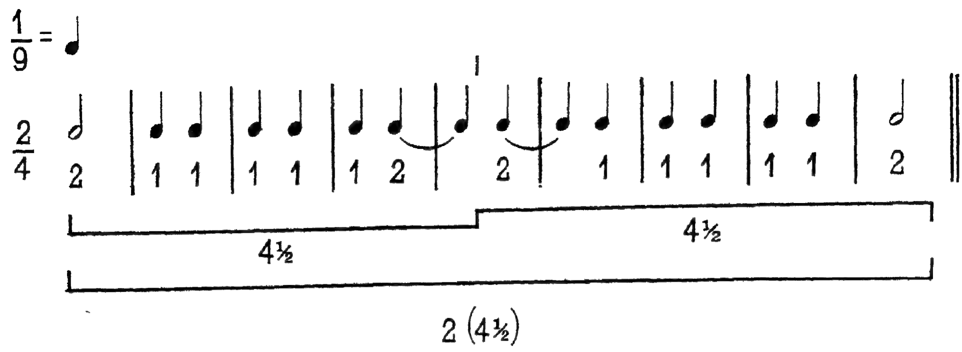


Figure 32.

Example: (2)

$$\underline{4 \div 3} \qquad \text{Find } r_{\underline{4 \div 3}}$$

$$4^2 = 16$$

$$4 \quad (4)$$

$$4 \quad (3)$$

$$N_3 = 4 - 3 + 1 = 2$$

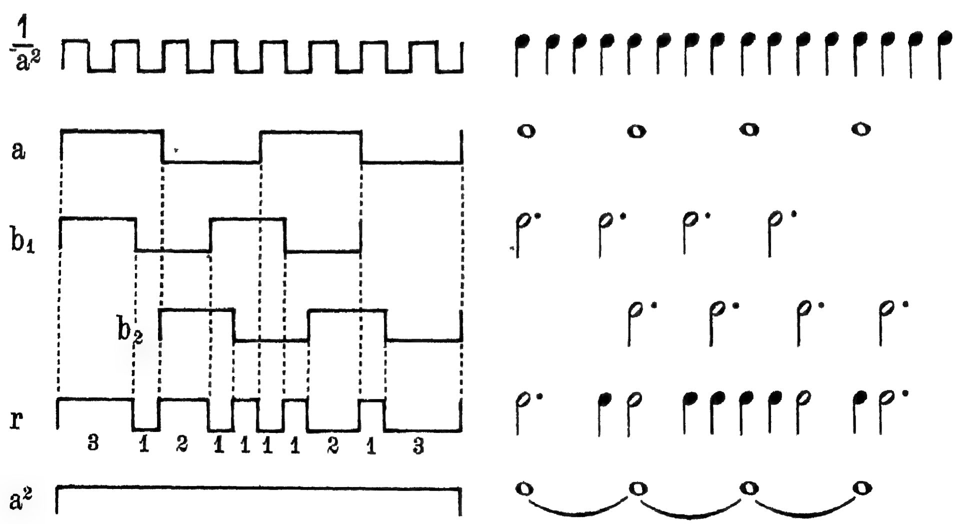


Figure 33.

Grouping by a^2

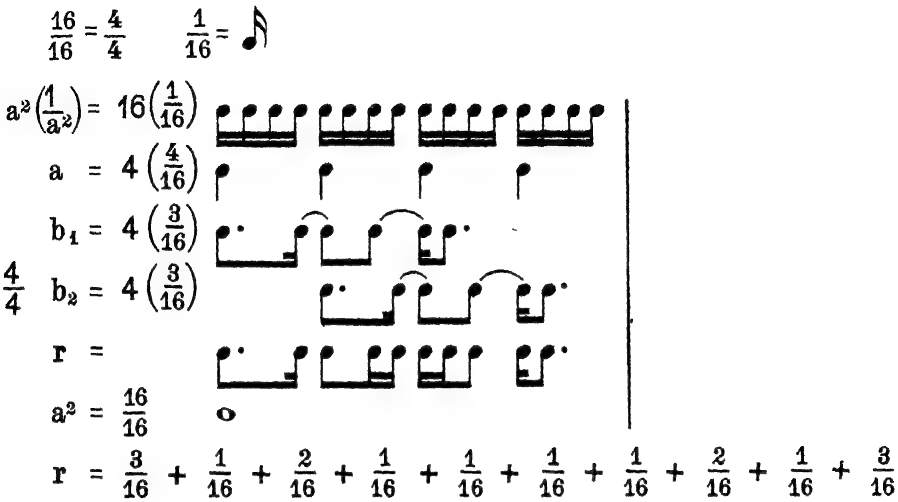


Figure 34.

Grouping by a

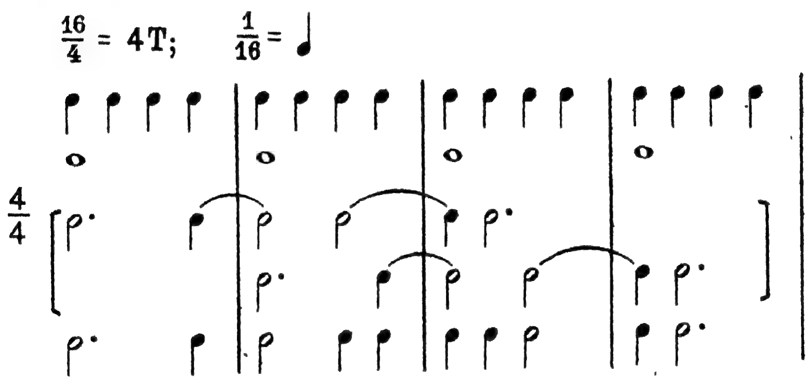


Figure 35.

Grouping by b

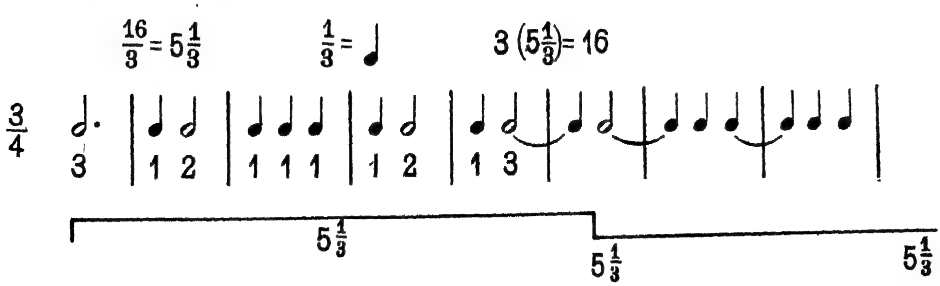


Figure 36 (continued).

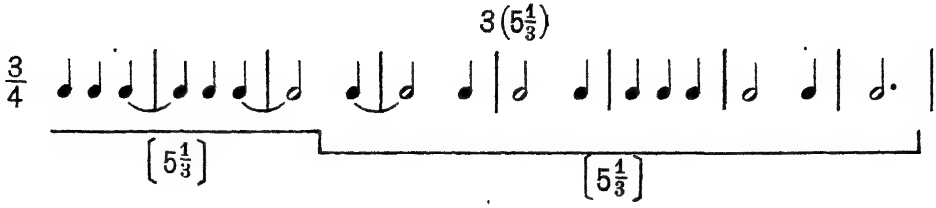


Figure 36 (concluded).

CHAPTER 5

COMPOSITION OF GROUPS BY PAIRS

THESE TECHNIQUES of obtaining resultants may be extended further so as to evolve processes by which we may compose rhythmic resultants in pairs.

In the ordinary exposition of a musical theme, it is customary to state the theme twice in such a way that for the first time the theme does not sound entirely completed, while for the second time it is brought to a completion. As composers of the past (as well as composers of the present) do not know how to do it, they usually resort to variations of the cadence *harmonically*. But it remains a pure problem of rhythm nevertheless.

Composers have also been confronted with the problems of expansion and contraction in the two adjacent groups. Moving from a long to a short group is what we mean by *contraction*, the opposite is *expansion*.

These procedures were performed crudely even by well-reputed composers. For instance, L. van Beethoven in his piano sonata, No. 1, in the first movement at the end of exposition, states a two-bar group three times. On the third statement, he makes an expansion by merely holding the chord through the whole bar (a whole note), thus adding one more bar. In his piano sonata, No. 7, (in the beginning of the finale) he has a four-bar group. There are many rests in this group, and the rests are injected *a priori* with the idea of taking them out afterwards. Thus he makes a three-bar group out of a four-bar group. Even this crude form of contraction was rarely attempted by Beethoven in his long career.

Here, we shall present a general method of *balancing*, *expanding*, and *contracting* a pair of adjacent groups, no matter what the rhythmic constitution of such groups may be.

As the resultants which have identical generators have a great deal in common, such performance gives the utmost esthetic satisfaction.

(A) **Balance** B = balance

$$\boxed{B = r_{\underline{a \div b}} + r_{a \div b} + a(a-b)}$$

Figure 37. (B) $a^2 = ab + a(a-b)$

The above means that in order to balance two resultants with identical generators, take the resultant of *a* to *b*, with fractioning, add the resultant of *a* to *b* and add *a* times *a* minus *b*. Grouping for such pairs is through *a* only.

Example:

$B = r_{\underline{3 \div 2}} + r_{3 \div 2} + 3(3-2) = [\,(2+1) + (1+1+1) + (1+2)\,] + [\,(2+1) + (1+2) + 3\,]$

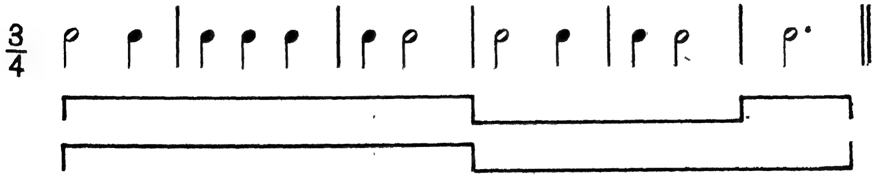


Figure 38.

$$B = r_{\underline{4\div3}} + r_{4\div3} + 4(4-3) = [\,(3+1)+(2+1+1)+(1+1+2)+(1+3)\,] + [\,(3+1)+(2+2)+(1+3)+4].$$

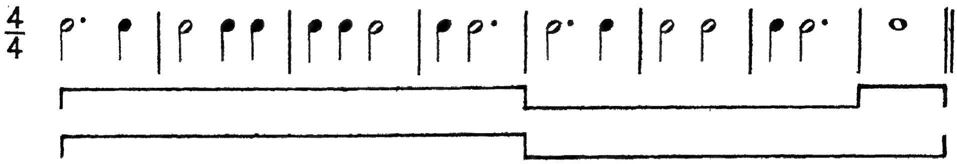


Figure 39.

Balance does not seem natural when a > 2b, a > 3b, i.e., when a is greater than 2b or greater than 3b. Yet it may be accomplished through a general procedure.

(1) Take $r_{\underline{a\div b}}$

(2) Take $r_{a\div b}$ as many times as it enters (as divisor) into a^2.

(3) Add one total duration which equals the difference between a^2 and 2ab, a^2 and 3ab, etc.

$$\boxed{B_{a>mb} = r_{\underline{a\div b}} + m r_{a\div b} + (a^2 - mab)}$$

Figure 40.

Example:

$5 \div 2$ $\qquad$ $5 > 2(2)$

(1) $r_{\underline{5\div2}} = (2+2+1)+(1+1+1+1+1)+(1+1+1+1+1)+(1+1+1+1+1)+(1+2+2) = 25$

(2) $\frac{25}{10} = 2\frac{1}{2}$

(3) $25 - 20 = 5$

$$B = r_{\underline{5\div2}} + r_{5\div2} + r_{5\div2} + 5 = [\,(2+2+1)+(1+1+1+1+1)+(1+1+1+1+1)+(1+1+1+1+1)+(1+2+2)\,] + [\,(2+2+1)+(1+2+2)\,] + [\,(2+2+1)+(1+2+2)\,] + 5$$

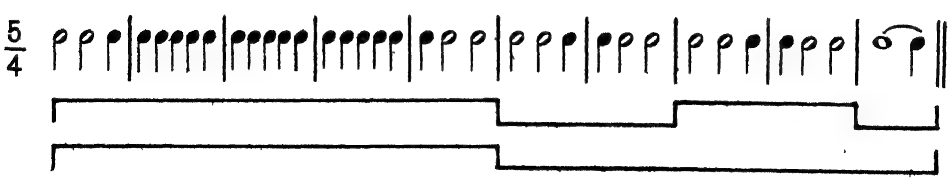


Figure 41.

(B) **Expansion** $\qquad$ E = expansion

$$\boxed{E = r_{a\div b} + r_{\underline{a\div b}}}$$

Grouping by *a* only.

Example:

$$E = r_{3\div 2} + r_{\underline{3\div 2}} = [(2+1) + (1+2)] + [(2+1) + (1+1+1) + (1+2)]$$

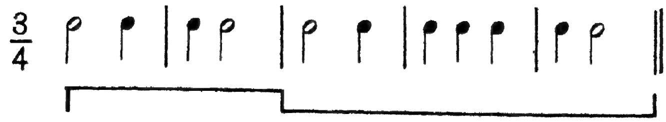


Figure 42.

$$E = r_{4\div 3} + r_{\underline{4\div 3}} = [(3+1) + (2+2) + (1+3)] + [(3+1) + (2+1+1) + (1+1+2) + (1+3)]$$

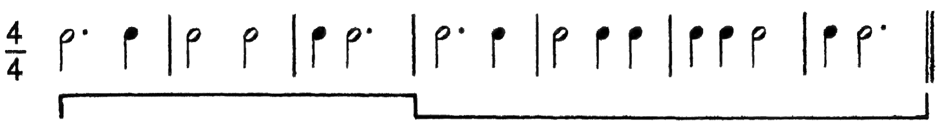


Figure 43.

(C) **Contraction** C = contraction

$$\boxed{C = r_{\underline{a\div b}} + r_{a\div b}}$$

Grouping by *a* only.

Figure 44.

Example:

$$C = r_{\underline{3\div 2}} + r_{3\div 2} = [(2+1) + (1+1+1) + (1+2)] + [(2+1) + (1+2)]$$

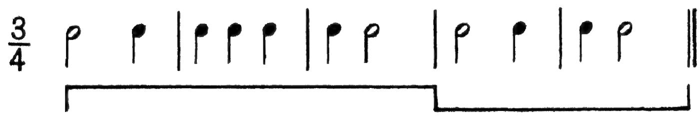


Figure 45.

$$C = r_{\underline{4\div 3}} + r_{4\div 3} = [(3+1) + (2+1+1) + (1+1+2) + (1+3)] + [(3+1) + (2+2) + (1+3)]$$

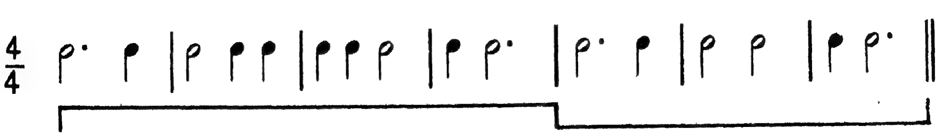


Figure 46.

CHAPTER 6

UTILIZATION OF THREE OR MORE GENERATORS

IT IS CLEAR that just as rhythmic groups may be developed by the use of two generators, so, too, may they be based on the use of three—or more than three—generators. In such a case, the selection of the third generator becomes important.

It happens that *all generators pertaining to one family of rhythm belong to the same series of number-values.** Such series are the series of growth; they control not only music and the arts in general, but also the proportions of the human body, as well as various forms of growth in nature. Horns, antlers, cockleshells, maple leaves, sunflower seeds and many other natural developments are controlled by the series of growth. Mathematically, one can produce an infinite number of *types* of the series of growth, and an infinite number of series of each type.

The series referring to the developments mentioned above constitute one specific type of *summation series*. In this type of summation series, every third number-value is the sum of the two preceding number-values. For instance, if in some series, numbers 2 and 3 occur, then the next number is 5, i.e., $2 + 3$. The best known of all series of this type is:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89 . . .

For example, the spiral tangent to a maple leaf grows through 90-degree arcs and each consecutive radius of each arc follows this very series. Formation of the seeds in a sunflower follows the same series. Professor Church of Oxford University devoted his life to this problem. He found that only slight deviations may be found and then in only two cases out of a thousand, the deviations being caused by exceptionally unfavorable climatic conditions.

An important portrait painter of New York City, Wilford S. Conrow, devoted many years of research in order to find out how this series works in relation to the human body. He found an overwhelming amount of material in the ancient Greek theories of proportions. Conrow's deductions are that it is this particular series that makes the human body beautiful to us.

I have found in the field of music that each style (or family) of rhythm evolves through the series of such types. Here are all the series that are useful for musical purposes:

I. 1, 2, 3, 5, 8, 13, ...
II. 1, 3, 4, 7, 11, 18, ...
III. 1, 4, 5, 9, 14, 23, ...

As previously mentioned, all rhythmic groups (or patterns) of one style are the resultants of the generators of the same series. For example, if a certain rhythmic group is identified with $r_{3 \div 2}$, then groups of the same style will be produced by $r_{5 \div 3}$ or $r_{5 \div 3 \div 2}$.

*For obvious reasons, Schillinger does not here present fully the entire case surrounding this statement, which is a statement of crucial significance in esthetics. His fuller statement is contained in his 'Mathematical Basis of the Arts,' which is to be published shortly. (Ed.)

The following are the important and practical combinations of generators to be worked out:

SERIES I. $2 \div 3 \div 5$ $\quad$ $3 \div 5 \div 8$
SERIES II. $3 \div 4 \div 7$
SERIES III. $4 \div 5 \div 9$

A. The Technique of Synchronization

In order to synchronize three or more generators, it is necessary first to find their common product and their complementary factors.

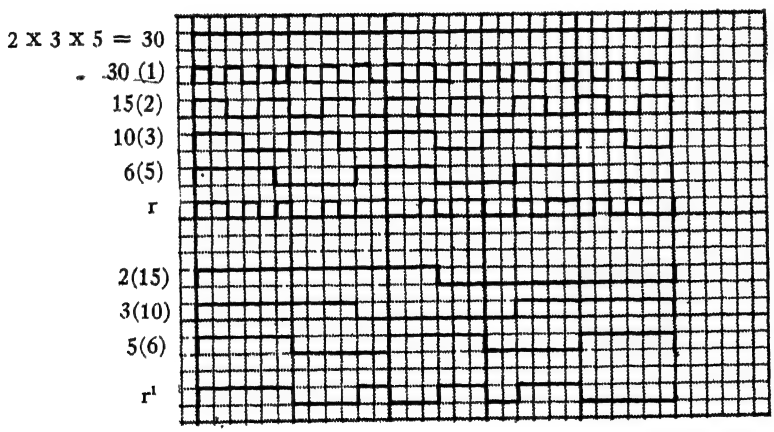
Let us take $2 \div 3 \div 5$
The product is $2 \times 3 \times 5 = 30$

The complementary factors are the quotients of the product by a generator. Thus, $\frac{30}{2} = 15$ means that 15 is a complementary factor of 2.

Therefore: 15 (2)
10 (3)
6 (5)

This method offers two resultants (r and r′) at a time, one serving as a theme, the other as a countertheme. Generators produce r, and the complementary factors produce r′.

$2 \div 3 \div 5$



$$r = 2 + 1 + 1 + 1 + 1 + 2 + 1 + 1 + 2 + 2 + 1 + 1 + 2 + 2 + 1 + 1 + 2 + 1 + 1 + 1 + 1 + 2$$
$$r' = 6 + 4 + 2 + 3 + 3 + 2 + 4 + 6$$

Figure 47.

The rule of grouping is: *group by any generator or any of the complementary factors*. In the case of $2 \div 3 \div 5$, grouping is available through 2, 3, 5, 6, 10, 15, i.e., $\frac{2}{4}, \frac{3}{4}, \frac{5}{4}, \frac{6}{8}, \frac{10}{8}, \frac{15}{8}$.

Grouped through $\frac{6}{8}$, r' appears as follows:

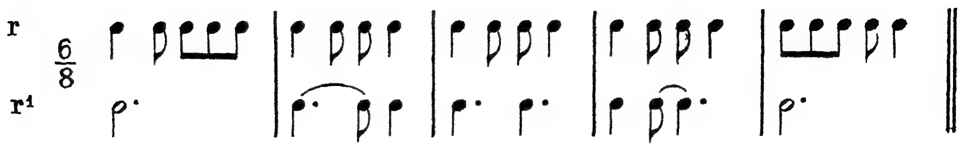


Figure 48.

It can be seen from this example that no more rhythmically suitable countertheme can be devised. The theme makes three recurrences while the countertheme makes continuous changes in much longer values. The listener has the opportunity to hear *both themes* together.

All resultants from three or more generators have these recurrences and variations as their chief characteristics.

CHAPTER 7

RESULTANTS APPLIED TO INSTRUMENTAL FORMS

WHEN WE speak of *time rhythm* we are referring to the periodicity of attacks, that is, the intervals of time at which the attacks occur.

A. Instrumental Rhythm

Instrumental rhythm is made up of the number of *places* of attack; for example, in beating two differently pitched kettle drums in sequence, we are dealing with two places of attack.

Synchronizations of these two types of rhythm—i.e., time rhythm and instrumental rhythm—are subject to the same laws of synchronization and interference as the time periodicity previously discussed.*

When the number of places in an instrumental group does not coincide with the number of terms in a time group, then a common denominator will define the number of time groups—and the number of instrumental groups—until their recurrence. For example, if we use two differently tuned kettle drums on $r_{3\div2}$, the entire figure will close after the first group is over because the number of *places* in the instrumental group is two (kettle drums) and the number of terms in the time group is four ($4 \div 2 = 2$). This means that while the instrumental group appears twice, the rhythmic resultant will appear once.

$r_{3\div2}$

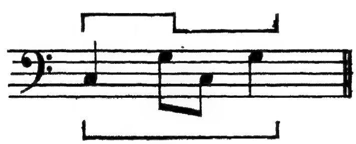


Figure 49.

Taking the same case of the two kettle drums for $r_{\underline{3\div2}}$, we get a totally different resultant. The number of attacks in the instrumental group remains the same (2). The number of terms in the rhythmic resultant is 7, $(2 + 1 + 1 + 1 + 1 + 1 + 2)$. $7 \times 2 = 14$. Seven has a complementary factor 2, and 2 has a complementary factor 7. The kettle drum 2 attack figure will appear 7 times, while the rhythmic resultant appears twice.

*Here we see the first of what will come to be a great many examples of the way in which Schillinger's theory of rhythm goes much further than the simple question of *time* rhythm. Note that Schillinger here states that instrumental rhythm is one thing (that is, the pattern according to which instruments enter or drop out of the ensemble) and time rhythm is another (that is, the pattern according to which the sounds are produced, regardless of which instruments are producing these sounds). Schillinger here discusses the application of his theory not only to the rhythm of the sounds produced, but also to the allocation of parts among various instruments. (Ed.)

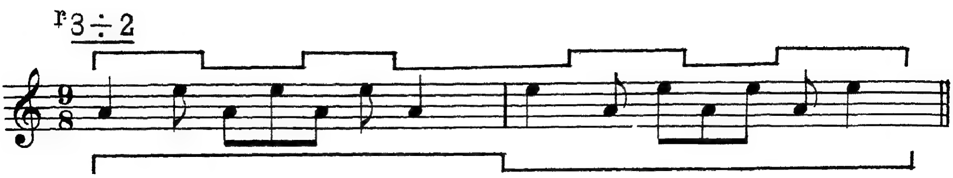


Figure 50.

This principle may be carried out to any desired degree of complexity, depending on the common denominator between the number of terms in a rhythmic group and the number of attacks in an instrumental group. The difference between two kettle drums and any melody or any instrumental form of harmony (accompaniment) with respect to this calculation is merely a quantitative difference.

Let us take a motif consisting of four different pitches, (for example: c, d, e, f); such sequence of pitches is merely one of the possible forms of melody. But superimposing $r_{3\div 2}$ we obtain one group without recurrence because the number of pitches (intonation attacks), and the number of terms in the rhythmic resultant (time attacks), are equal ($4 \div 4 = 1$). Taking the same four notes of the melody and superimposing $r_{\underline{3\div 2}}$, we get $7 \times 4 = 28$. The rhythmic group having 7 attacks acquires the complementary factor 4, i.e., it will run 4 times until its own recurrence, while the melody having 4 attacks will acquire the complementary factor 7, i.e., it will run 7 times until its own recurrence will coincide with the recurrence of the rhythmic resultant.

Figure 51.

This technique makes it possible to run a very simple motif practically to infinity, as the duration of continuous variability depends solely on a common denominator. A simple example of rhythmic continuity through instrumental interference may be found in many arrangements of fox-trots. The figure of 6 uniform attacks (two false triplets) placed in a common time measure ($=\frac{8}{8}$) produces an interference of $8 \div 6$. $8 \div 6$ reduces to $4 \div 3$. Six acquires the complementary factor 4, and 8 acquires the complementary factor 3, i.e., the instrumental figure with 6 attacks runs 4 times in the course of 3 $\frac{8}{8}$ measures.

Figure 52.

The principles of rhythmic (time) and instrumental interference have been known since time immemorial. They constitute one of the most striking characteristics in the composition of rhythmic continuity as it exists in the music of the Orient as well as through the entire African continent. This tendency is almost as fundamental as the superimposition of a major generator on any uniform group—as for example, the imaginary grouping of the attacks of a ticking clock by 2, 3, 4, or any other simple number we can think of.

B. Applying the Principles of Interference to Harmony

The principles of interference of rhythmic and instrumental groups, when applied to harmony, produce the most effective forms of accompaniment. They make it possible, as well, to correlate a number of accompaniments simultaneously.

At this point, the illustrations of harmony are restricted to three of the simplest instrumental forms. However, in the later part of the course, details of the instrumental forms of harmony will be discussed. Here we will cite:

(1) The two-attack instrumental figure (as in the polka). The first attack is the detached bass of harmony. The second attack includes all the remaining upper parts of a chord.

(2) The four-attack instrumental figure (as in the fox-trot). The first attack is the lower bass. The second attack, the upper part of the chord. The third attack is another detached bass. The fourth attack, the upper part of a chord.

(3) The six-attack instrumental figure (as in the rhumba). The first attack is the lower detached bass. The second attack is the upper part of the chord. The third attack is the middle detached bass. The fourth attack is the upper part of the chord. The fifth attack is the upper detached bass. The sixth attack is the upper part of the chord.

Figure 53.

It is easy to see that the waltz accompaniment figure is merely (1) above, with the upper chord having two attacks instead of one. The old tango and habanera figures are like (2), except that the last attack is made on the lower bass.

The following diagrams illustrate the continuous run of these instrumental forms of harmony with various simpler rhythmic resultants, all used on one chord:

Figure 54 (*continued*).

4÷3

Figure 54 (continued).

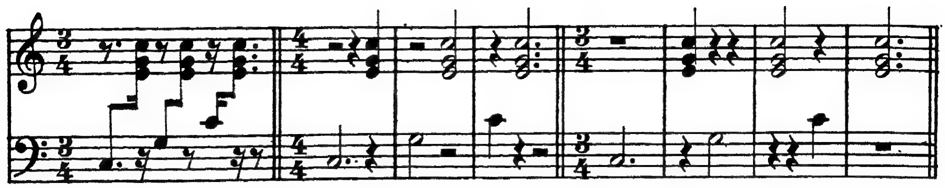


Figure 54 (concluded).

One may also compose other instrumental forms of harmony with as many as 16 attacks—such as an alternation of the four different notes in the bass, with the upper part of the chord doubled in two octaves:

Figure 55.

A still greater number of attacks in an instrumental figure may be produced by the common technique of arpeggio. Technically, any longer motif presents the same problem, except that its pitch commonly has a more limited range.

When one time-group is distributed through the different places of attack, different individual parts become *the resultants of interference between the time and the instrumental groups*. For example, if we have a figure of 4 places, as referred to in Item (2), page 29, and superimpose a time group, 2 + 1 + 1 (3 attacks), we obtain through the common denominator 12, 2 different instrumental resultants. One is the sequence of attacks on chords; the other, the sequence of attacks in the bass, when all the bass attacks are tied over. The upper part produces the resultant 2 (2 + 3 + 3) and the bass, 2 (3 + 3 + 2). This is a striking example of transformation of one type of rhythm into another —a result of the phenomenon of instrumental and time interference.

The 2 + 1 + 1 is a traditional classical figure, and, as expressed in the following musical example, consists of a quarter and two eighths. Yet the result

sounds like a rhumba. This is due to the *new* resultant which appears as a sequence of the attacks of the bass notes.

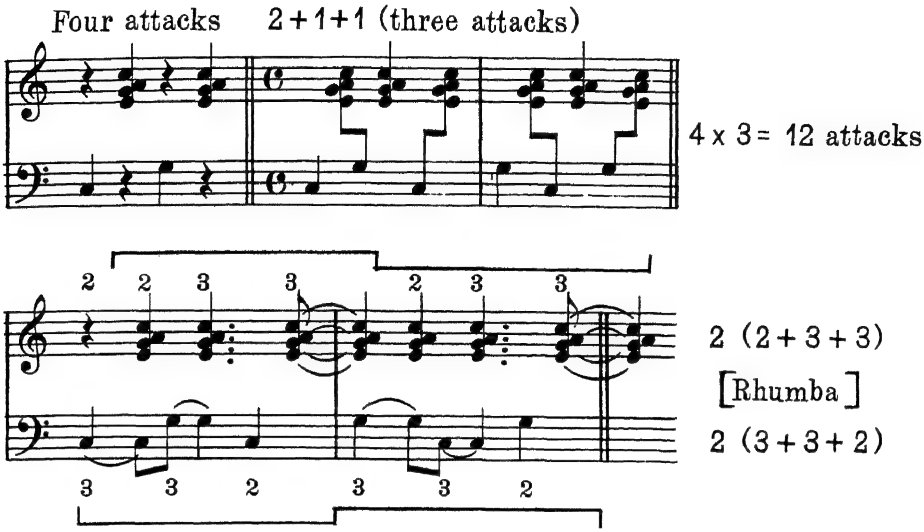


Figure 56.

The preceding technical items may also be treated in combination. The following example represents the application of two generators and their resultants, combined with the instrumental interference. The accompaniment represents the minor generator (2). The sustained chords represent the major generator (5). The melody represents the resultant (5 ÷ 2). In addition, the whole score is carried out through an *alien measure grouping*, $\frac{4}{4}$. While the entire rhythmic score would occupy 4 bars in $\frac{5}{4}$ time, it takes 5 bars in $\frac{4}{4}$ time. This example illustrates* the possibility of introducing various rhythmic resultants into music which is supposed to be written in common time.

Figure 57.

*The example in Figure 57 is of more than passing interest because it foreshadows the way in which full orchestral scores of unprecedented richness and complexity are developed logically and organically from the rhythmic raw materials now being discussed in this section. (Ed.)

CHAPTER 8

COORDINATION OF TIME STRUCTURES

MOTION—that is, changeability in time—is the most important intrinsic property of music. Different cultures of different geographical and historical localities have developed many types and forms of intonation. The latter varies greatly in tuning, in quantity of pitches employed, in quantity of simultaneous parts, and in the ways of treating them.

The types are as diversified as drum-beats, instrumental and vocal monody (one part music), organum, discantus, counterpoint, harmony, combinations of melody and harmony, combinations of counterpoint and harmony, different forms of coupled voices, simultaneous combinations of several harmonies, and many others. Any of these types—as well as any combinations of them—constitute the different musical cultures. In each case, musical culture crystallizes itself into a definite combination of types and forms of intonation. The latter crystallize into habits and traditions.

For example, people belonging to a *harmonic* musical culture want every melody harmonized. But people belonging to a monodic musical culture are disturbed by the very presence of harmony. Music of one culture may be *music* (meaningful sound) to the members of that culture; but the very same music may be *noise* (meaningless sound) to the members of another. The functionality of music is comparable to a great extent to that of a language.

Nevertheless, *all* forms of music have one fundamental property in common: *organized time*. The plasticity of the temporal structure of music, as expressed through its attacks and durations, defines the quality of music. Different types and forms of intonation—as well as different types of musical instruments—come and go like the fashions, while the everlasting strife for *temporal plasticity* remains a *symbol of the "eternal" in music.*

The temporal structure of music, usually known as *rhythm*, pertains to two directions: *simultaneity* and *continuity*. The rhythm of *simultaneity* is a form of coordination among the different components (parts). The rhythm of *continuity* is a form of coordination of the successive moments of one component (part).

People of our civilization have developed the power of reasoning at the price of losing many of the instincts of primitive man. Europeans have never possessed the "instinct of rhythm" with which the Africans are endowed. So-called European "classical music" has never attained the ideal it strived for, that ideal being: the utmost plasticity of the temporal organization. When J. S. Bach, for example, tried to develop a coordinated independence of simultaneous parts, he succeeded in producing only a resultant which is uniformity.* We find evidence of the same failures in Mozart and Beethoven. But a score in which the several coordinated parts produce, together, a resultant which

*That is to say, when the separate rhythms of the separate parts of a Bach score are "added up," the result tends to be simple uniformity. Schillinger suggests the desirability of scores, and develops a method of scoring, so that the separate parts, while satisfactory rhythmically by themselves, all "add up" to a new rhythm which is *not* uniformity. (Ed.)

has a distinct pattern—has been a "lost art" of the aboriginal African drummers. The age of this art can probably be counted in tens of thousands of years!

Today in the United States, due to the transplantation of Africans to this continent, there is a *renaissance* of rhythm. Habits form quickly—and the instinct of rhythm in the present American generation surpasses anything known throughout European history. Yet our professional "coordinators of rhythm," specifically in the field of dance music, are slaves to, rather than masters of, rhythm. There is plenty of evidence that the urge for coordination of the whole through individualized parts is growing. The so-called "pyramids" (sustained arpeggio produced by successive entrances of several instruments) is but an incompetent attempt to solve the same problem.

Fortunately, we do not have to feel discouraged or moan over this "lost art." The *power of reasoning* offers us a complete *scientific solution.*

This problem can be formulated as the *distribution of a duration-group through instrumental and attack-groups.*

The entire technique consists of five successive operations with respect to the following:

(1) The number of individual parts in a score;
(2) The quantity of attacks appearing with each individual part in succession;
(3) The rhythmic patterns for each individual part;
(4) The coordination of all parts (which become the resultants of instrumental interference) into a form which, in turn, results in a specified rhythmic pattern (the resultant of interference of all parts); and
(5) The application of such scores to any type of musical measures (bars).

Any part of such a score can be treated as melody, coupled melody, block-harmony, harmony, instrumental figuration—or as a purely percussive (drum) part. Aside from the temporal structure of the score, *the practical uses of this technique in intonation* depend on the composer's skill in the respective fields concerned, i.e., melody, harmony, counterpoint and orchestration.

A. Distribution of a Duration-Group (T) through Instrumental (i) and Attack (a) groups.

Notation

pli number of places in the instrumental group.
pla number of places in the attack-group.
a_a number of attacks in the attack-group.
a_T number of attacks in the duration-group.
PL the final number of places.
A the synchronized attack-group (the number of attacks synchronized with the number of places).
A′ the final attack group (number of attacks synchronized with the duration-group).
T the original duration-group.
T′ the synchronized duration-group.
T″ the final duration-group.
$N_{T''}$ the number of final duration-groups.

Procedures:

(1) Interference between the number of places in the instrumental group (pli) and the number of places in the attack-group (pla).

$$PL = \frac{pli}{pla}; \quad \frac{pla\ (pli)}{pli\ (pla)}$$

(2) The product of the number of attacks in the attack group (a_a) by the complementary factor to the number of places in the attack-group (pli after reduction).

$$A = a_a \cdot pli$$

(3) Interference between the synchronized attack-group (A) and the number of attacks in the original duration-group (a_T).

$$A' = \frac{A}{a_T} = \frac{a_a \cdot pli}{a_T}$$

(4) The product of the original duration-group (T) by the complementary factor to its number of attacks (A′).

$$T' = T \cdot A' = \frac{T \cdot a_a\ pli}{a_T}$$

(5) Interference between the synchronized duration-group (T′) and the final duration-group (T″).

$$N_{T''} = \frac{T'}{T''}$$

B. Synchronization of an Attack-Group (a) with a Duration-Group (T).

Distribution of attacks of an attack-group (a_a) through the number of attacks of a duration-group (a_T).

First Case: $\dfrac{a_a}{a_T} = 1$

$$A = a_T$$
$$T' = T$$

Example:

$a_a = 4a$; $T = r_{3 \div 2} = 6t$; $a_T = 4a$

$$A = 4a$$
$$T' = 6t$$

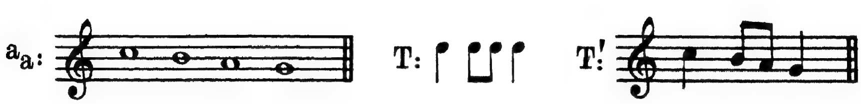


Figure 58.

Second Case: $\dfrac{a_a}{a_T} \neq 1$

$$A = a_T \cdot a_a$$
$$T' = T \cdot a_a$$

Example:

$a_a = 5a;\ T = r_{3 \div 2} = 6t;\ a_T = 4a$

$\frac{5}{4}$

$$A = 5a \cdot 4 = 20a$$
$$T' = 6t \cdot 5 = 30t$$

Figure 59.

Third Case: $\frac{a_a}{a_T} = \frac{a_{a'}}{a_{T'}}$ i.e., a reducible fraction

$$A = a_T \cdot a_{a'}$$
$$T' = T \cdot a_{a'}$$

Example:

$a_a = 6a;\ T = r_{3 \div 2} = 6t;\ a_T = 4a$

$$\frac{6}{4} = \frac{3}{2}$$
$$A = 4a \cdot 3 = 12a$$
$$T' = 6t \cdot 3 = 18t$$

Figure 60.

C. Distribution of a Synchronized Duration-Group (T′) through the Final Duration-Group (T″).

First Case: $\frac{T'}{T''} = 1$

$$T'' = T'$$

Example:

$T' = 6t;\ \ T'' = 6t$

Figure 61.

Second Case: $\frac{T'}{T''} \neq 1$

$$N_{T''} = T'$$

Example:

$T' = 6t;\ T'' = 5t$

$$N5t = 6$$

Figure 62.

Third Case: $\frac{T'}{T''} = \frac{T_{\prime}}{T_{\prime\prime}}$ i.e., a reducible fraction

$N_{T''} = T_{\prime}$

Example:

$T' = 6t;\ T'' = 4t$

$\frac{6}{4} = \frac{3}{2}$

$N_{4t} = 3$

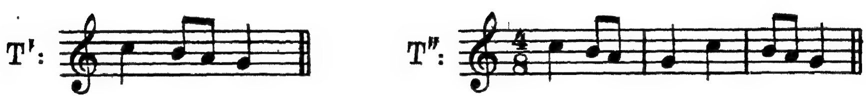


Figure 63.

Example:

$a_a = 5a$ $\qquad$ $T = r_{5\div2} = 10t;$ $\qquad$ $a_T = 6a$

Figure 64.

(1) $\frac{6}{5}$ $\quad$ $\frac{5\,(6)}{6\,(5)}$
(2) 10t is equivalent to 6 attacks; $10t \times 5 = 50t$
(3) When $T'' = \frac{8}{8}$, $\frac{50t}{8} = \frac{25\cdot4}{4} = 25T''$

Figure 65 (continued).

Figure 65 (concluded).

D. Synchronization of an Instrumental Group (pli) with an Attack-Group (pla).

Example:

pli = 4; pla = 3; $a_a = 3+2+3=8$; $T = r_{5\div2} = 10t$; 6a

Pl (1) $\frac{4}{3}$; $\frac{3}{4}\frac{(4)}{(3)}$ A' (3) $\frac{32}{6} = \frac{16}{3}$

A = (2) $8 \times 4 = 32$ (4) $\frac{16 \cdot 10}{3} = \frac{160}{3}$

(5) $T'' = 8t$; $\frac{160}{3 \cdot 8} = \frac{20}{3}$; $\frac{20 \cdot 3}{.3} = 20T''$

Figure 66.

Example:

pli = 3; pla = 3; $a_a = 3+2+2+3=10$; $T = r_{\underline{4\div3}} = 16t$; 10a

(1) $\frac{3}{3} = 1$

(2) $10 \cdot 1 = 10$

(3) $\frac{10}{10} = 1$

(4) $16 \cdot 1 = 16$

(5) $T'' = 8t$; $\frac{16}{8} = 2T''$

Figure 67.

Example:

pli = 6; pla = 8; $a_a = r_{5\div4} = 20$; $T = r_{\underline{4\div3}} = 16t$; 10a $T'' = 8t$

(1) $PL = \frac{6}{8} = \frac{3}{4}$; $\frac{4}{3}\frac{(6)}{(8)}$

(2) $A = 20 \cdot 3 = 60$

(3) $A' = \frac{60}{10} = 6$

(4) $T' = 16t \cdot 6 = 96t$

(5) $\frac{96}{8} = 12T''$

Example of composition of the resultant of instrumental interference.

pl (i) = pl (a) = 2

Form of distribution: 5+3

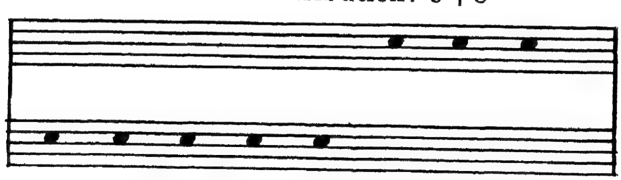


Figure 68.

(1) $\frac{2}{2} = 1$

(2) 2 is an equivalent of $5+3 = 8$

(3) Duration-group: $T = r_{5\div2} = 10t$ ≲ 6a

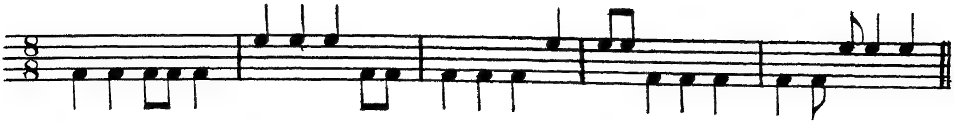
$a_T = 6$

$\frac{8}{6} = \frac{4}{3}$ $\frac{3}{4}\frac{(8)}{(6)}$

(4) $10t \times 4 = 40t$

(5) When $T'' = \frac{8}{8}$, $\frac{40t}{8} = 5T''$

Preliminary Scoring



Final Scoring

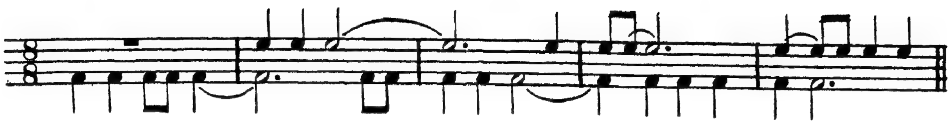


Figure 69.

Example of composition of the resultant of instrumental interference.

$pl(i) = 3 \qquad pl(a) = 3$

Form of distribution: 8+3+5+2

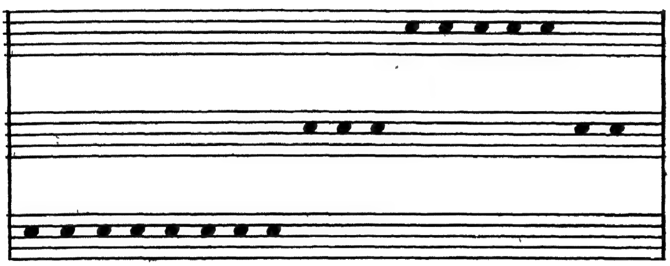


Figure 70.

(1) $\frac{3}{3} = 1$

(2) 4 is an equivalent of $8+3+5+2=18$

$$18 \times 1 = 18$$

(3) Duration group: $r_{5\div2} = 10t \qquad \frac{18}{6} = 3 \qquad 3(6)$

$$a_T = 6$$

(4) $10t \times 3 = 30t$

(5) When $T'' = \frac{8}{8}$, $\frac{30t}{8} = \frac{15}{4}$; $\frac{15 \cdot 4}{4} = 15T''$

Preliminary Scoring

Figure 71 (continued).

Figure 71 (concluded).

Final Scoring

Figure 72 (continued).

Figure 72 (concluded).

Example of composition of the resultant of instrumental interference.

$\mathrm{pl(i)} = 6; \quad \mathrm{pl(a)} = 8;$

Form of distribution: $r_{5 \div 4}$

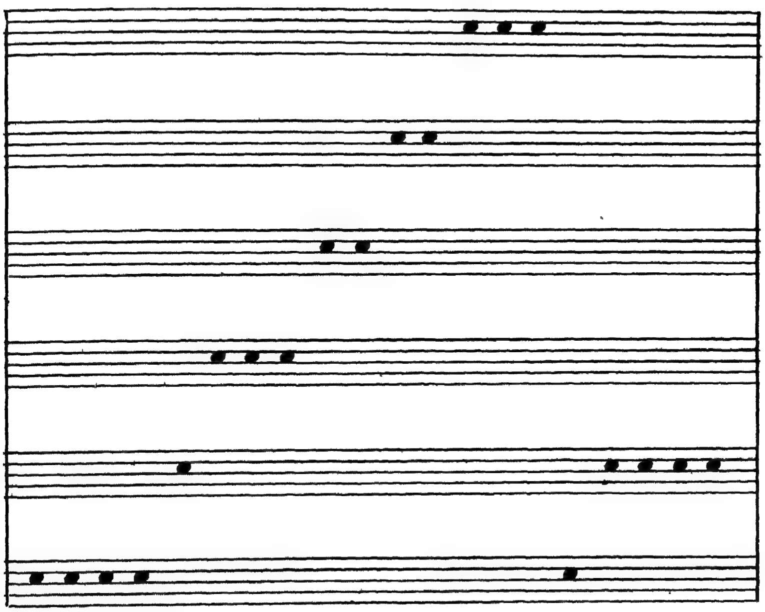


Figure 73.

(1) $\frac{6}{8} = \frac{3}{4} \quad \frac{4}{3}\frac{(6)}{(8)}$

(2) 8 is equivalent to 20 in $r_{5 \div 4}$ $\quad 20 \times 3 = 60$

(3) Duration-group $= \underline{r_{4 \div 3}}$ $\quad a_T = 10; \quad \frac{60}{10} = 6 \quad 6(10)$

$\underline{r_{4 \div 3}} = 16t$

(4) $16t \times 6 = 96t$; a given $T'' = \frac{8}{8}$

(5) $\frac{96t}{8t} = 12T''$

Preliminary Scoring

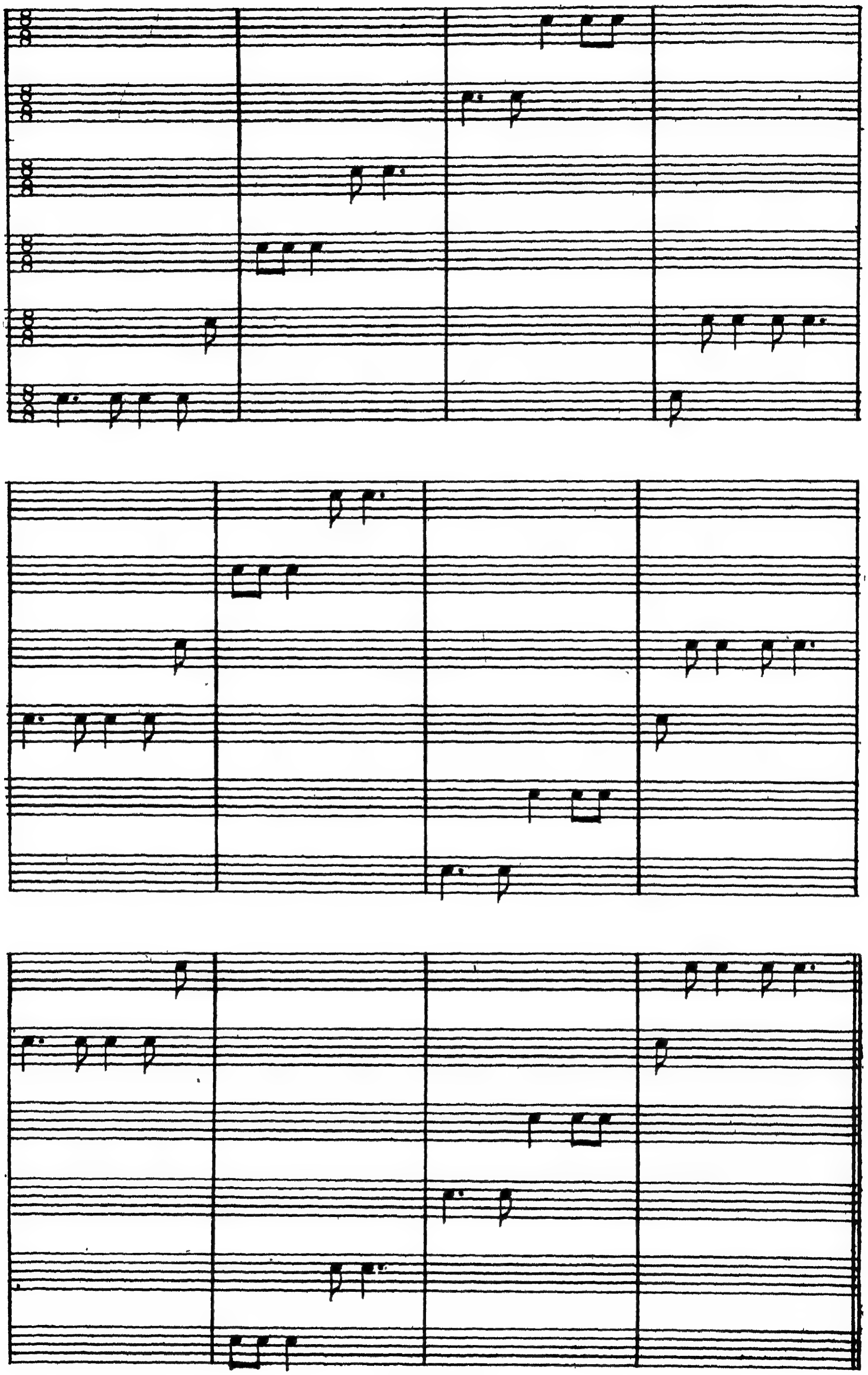


Figure 74.

Final Scoring

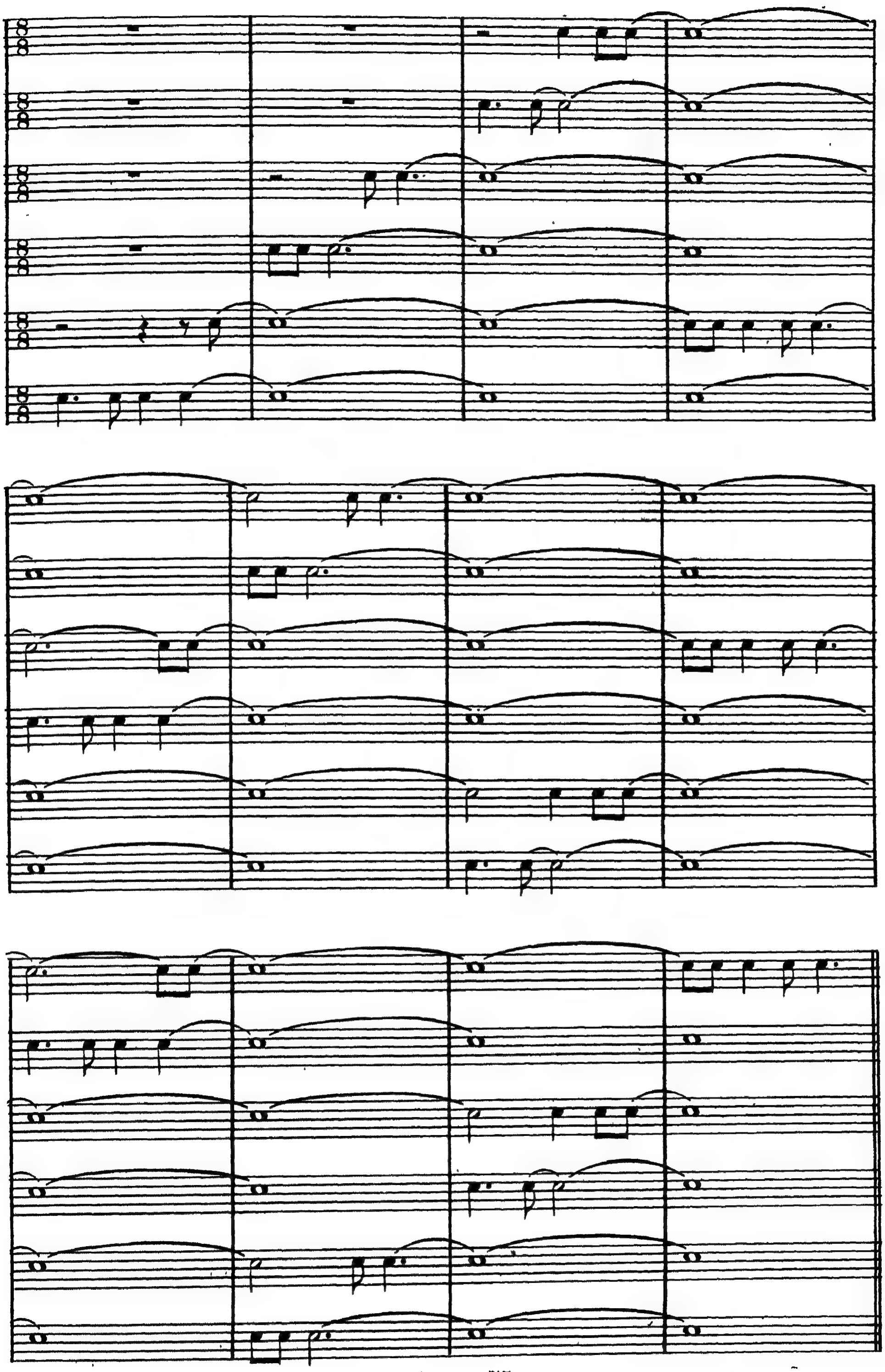


Figure 75.

CHAPTER 9

HOMOGENEOUS SIMULTANEITY AND CONTINUITY (VARIATIONS)

THE PRECEDING discussions show us that all rhythmic groups or rhythmic patterns are necessarily either the resultants of interferences or portions of such resultants.

A figure such as $2+1+1$ may be conceived as one of the elementary rhythmic patterns in $\frac{4}{4}$ time. Yet it is possible, with this method of analysis, to assign it directly to a definite place in a definite resultant—the second bar of $r_{\underline{4 \div 3}}$. The longer patterns, such as the resultants produced by higher number-values or by more than two generators, possess enough variation in themselves.

Musical memory does not emphasize a group of 20 or more bars as one indivisible pattern. Therefore, the recurrence of such pattern seems to be less monotonous than the recurrence of a short pattern. Short patterns obviously call for variations. There are many outstanding compositions in which direct recurrence of a short pattern is used throughout the entire composition—for example, the first movement of Beethoven's symphony No. 5; Chopin's waltz No. 7, the second theme. In such compositions, rhythmic monotony is usually compensated for by the variety of devices used on some other components—it may be the dynamic, the harmonic, or the melodic composition of a piece that makes this music sound interesting. The best method by which to detect the effect of the purely rhythmic patterns is to isolate them from all other components, i.e., to take a fragment of a composition, or the entire composition, and to perform the rhythm of it in a percussive manner.

The musical components of rhythm include durations, rests, accents, split-unit groups and groups in general. The inherent variability of any of these components of the time rhythm depends solely on their quantitative form, i.e., whether there are two or three, or more, elements involved in the pattern subjected to variations—for example, two elements, two durations, two forms of accent, as well as binary combinations of rests with durations, or durations with accents. The variability of groups follows the general principles of *permutations.*

A. General and Circular Permutations

There are two fundamental forms of permutations: first, general permutations; second, circular permutations (displacement). The quantity of general permutations is the product of all integers from unity up to the number expressing the quantity of the elements in a group. For example, the general number of permutations produced by 5 elements equals the product of $1 \times 2 \times 3 \times 4 \times 5$, i.e., 120. The number of circular permutations equals the number of elements in a group. Thus, five elements produce five circular permutations.

When an extremely large amount of material is used, general permutations become very practical. But in cases where limitations are imposed by a certain

type of esthetic necessity, circular permutations may solve the problem better than a vague selection from the entire number of general permutations.

In the following exposition, a bi-coordinate method will be applied to the composition of continuity. A linear sequence of the modified versions of one pattern produces the time coordinate (continuity). A correlation of the modified patterns produces the coordinate of simultaneity (or pitch). In other words, all modified forms of the original pattern may grow through the bi-coordinate system, i.e., they appear one after another in different parts, thus producing compensatory balance.

In terms of music the above simply means that a score may be evolved with a continuous variation of the original pattern following through the different parts.

Variations

2 Elements

Table of Permutations:

ab	ba

2 permutations

Examples of application:

(1) **Durations:** Binomial $2 + 1$ $a = 2; b = 1$

$$\frac{(2+1)+(1+2)}{(1+2)+(2+1)} =$$

Figure 76.

Binomial $5 + 3$ $a = 5; b = 3$

$$\frac{(5+3)+(3+5)}{(3+5)+(5+3)} =$$

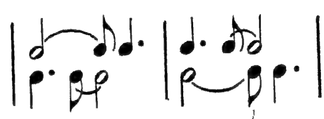


Figure 77.

(2) **Rests:** [indicated with a circle around the number]:

Binomial $1 + 1$ $a = \textcircled{1}; b = 1$

$$\frac{(\textcircled{1}+1)+(1+\textcircled{1})}{(1+\textcircled{1})+(\textcircled{1}+1)} =$$

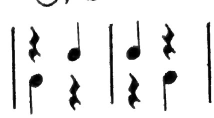


Figure 78.

Binomial $2 + 1$ $a = \textcircled{2}; b = 1$

$$\frac{(\textcircled{2}+1)+(2+\textcircled{1})}{(2+\textcircled{1})+(\textcircled{2}+1)} =$$

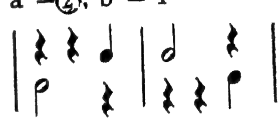


Figure 79.

Combined variations of durations and rests:

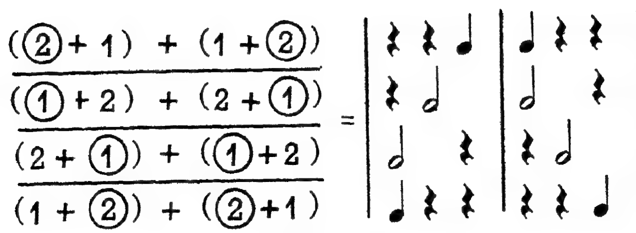
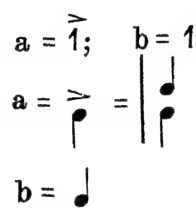


Figure 80.

(3) **Accents** [through superimposition of an additional component]:

Binomial 1 + 1



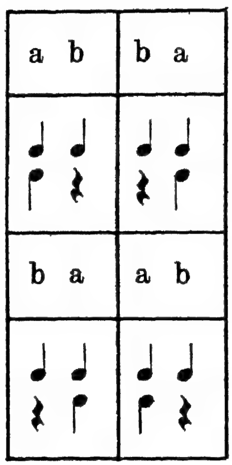


Figure 81.

Binomial 5 + 3

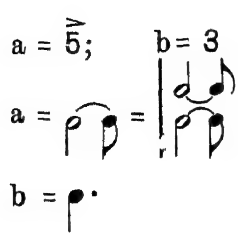


Figure 82 (continued).

$$\frac{(\overset{>}{5}+3)+(5+\overset{>}{3})}{(5+\overset{>}{3})+(\overset{>}{5}+3)} =$$

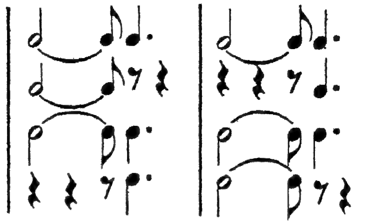
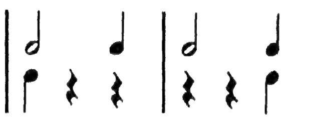


Figure 82 (concluded).

The additional component may emphasize the entire duration of the accented attack, as in the previous example, or be considerably shorter (just to single out the moment of attack).

Example:

Binomial $2 + 1$



or

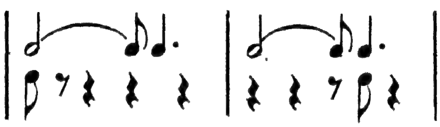


Figure 83.

Variations of rests may be combined with variations of the previous components.

(4) **Split-unit groups**

Binomial $2 + 2$ $\quad a = 1 + 1$
$b = 2$

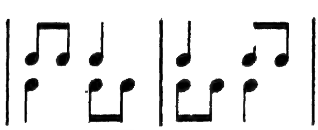


Figure 84.

When durations are non-uniform, either value may be split in a binomial.

$5 + 3 \qquad a = 3 + 2 \qquad b = 3$

$$\frac{[(3+2)+3]+[3+(3+2)]}{[3+(3+2)]+[(3+2)+3]} =$$

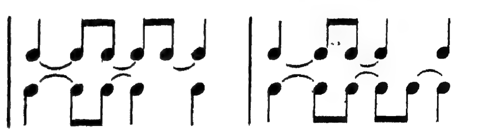


Figure 85.

$5+3 \qquad a=5 \qquad b=2+1$

$$\frac{[5+(2+1)]+[(2+1)+5]}{[(2+1)+5]+[5+(2+1)]} =$$

Figure 86.

(5) Groups in General

Any rhythmic group may become an element and be permuted with its converse.

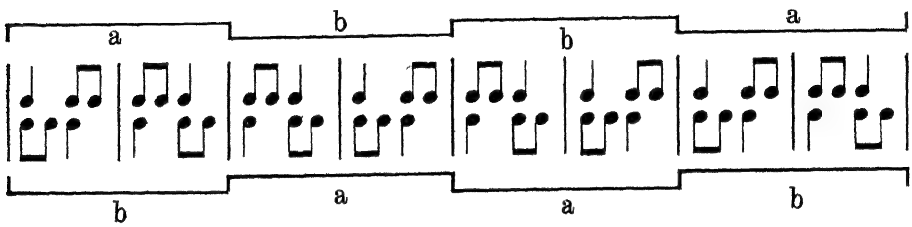


Figure 87.

* Song: "Pennies from Heaven"**

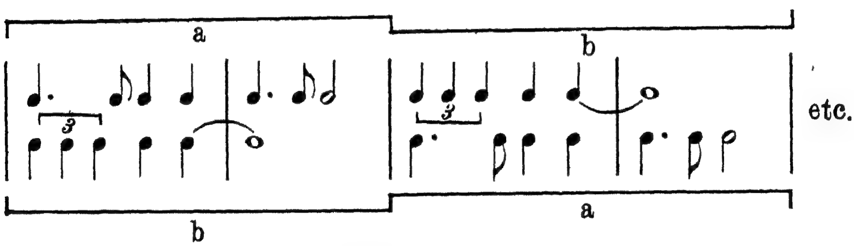


Figure 88.

$r_{\underline{4 \div 3}}$

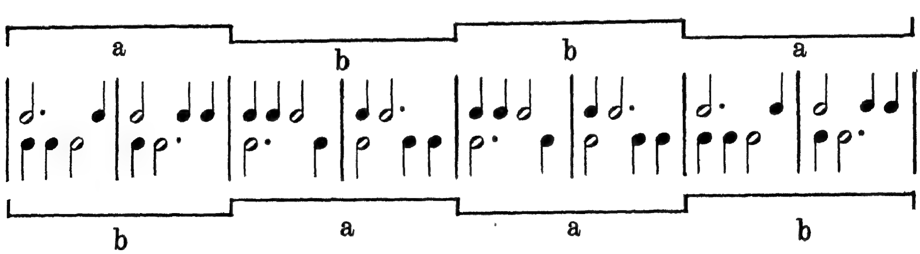


Figure 89.

*Schillinger's study of musical styles and the development of music took him from the earliest forms of recorded sound to contemporary popular American song. With an unusual catholicity of interest, Schillinger chooses illustrative materials frequently from popular songs. (Ed.)

As can be easily observed from these examples, the converse variation-group produces a rhythmic counterpart.

3 Elements

Table of General Permutations:

a	b	c	b	a	c
a	c	b	b	c	a
c	a	b	c	b	a

6 permutations

Figure 90.

Table of Circular Permutations:

(a) Clockwise circular permutations:

a	b	c	b	c	a	c	a	b

3 permutations

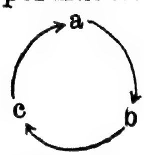


Figure 91.

(b) Counter-clockwise circular permutations:

a	c	b	c	b	a	b	a	c

3 permutations

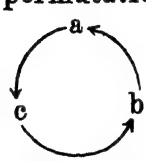


Figure 92.

When two elements in a group of three are identical, circular permutations *either* in clockwise or counter-clockwise direction are the only possible ones.

a	a	b	a	b	a	b	a	a	3 permutations
a	b	b	b	a	b	b	b	a	3 permutations

Figure 93.

Examples of Application:

(1) **Durations:**

Trinomial $2 + 1 + 1$; $a = 2$; $b = 1$

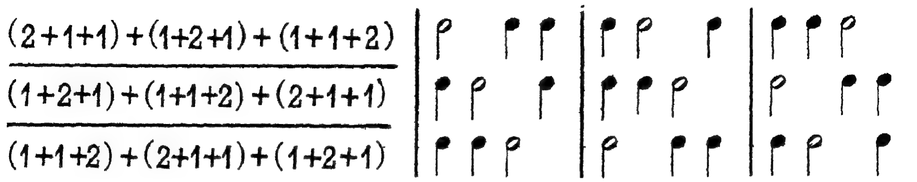


Figure 94.

Trinomial from $r_{8 \div 3}$ $3 + 3 + 2$
$a = 3$; $b = 2$

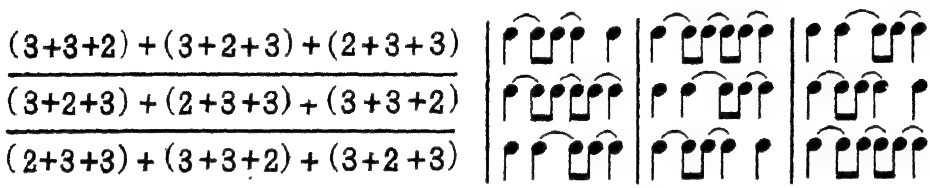


Figure 95.

Trinomial from $r_{4 \div 3}$ $3 + 1 + 2$
$a = 3$; $b = 1$; $c = 2$

$(a+b+c) + (a+c+b) + (c+a+b) + (b+a+c) + (b+c+a) + (c+b+a)$

Figure 96.

Using circular permutations of this continuity, we obtain the following simultaneity:

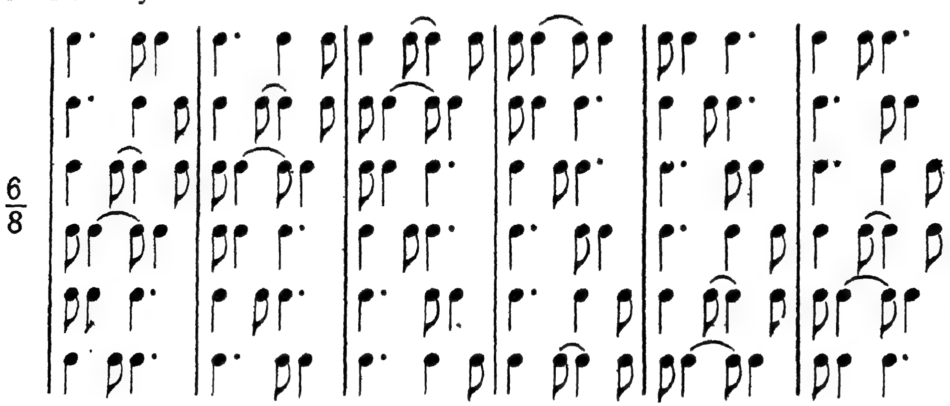


Figure 97.

(2) **Rests:**

Trinomial $1 + 1 + 1 \quad a = (1);\ b = 1$

(1+1+1) + (1+1+1) + (1+1+1)

(1+1+1) + (1+1+1) + (1+1+1)

(1+1+1) + (1+1+1) + (1+1+1)

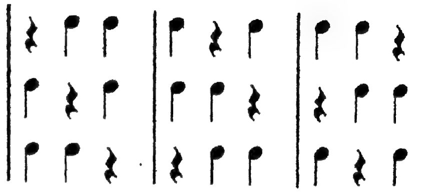


Figure 98.

Trinomial $2 + 1 + 1 \quad a = (2);\ b = 1$

(2+1+1) + (2+1+1) + (2+1+1)

(2+1+1) + (2+1+1) + (2+1+1)

(2+1+1) + (2+1+1) + (2+1+1)

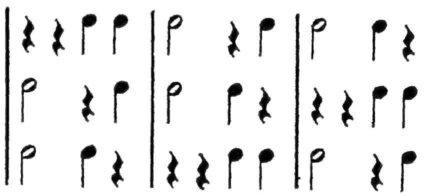


Figure 99.

(3) **Accents:**

Trinomial $1 + 1 + 1 \quad a = \overset{>}{1};\ b = 1$

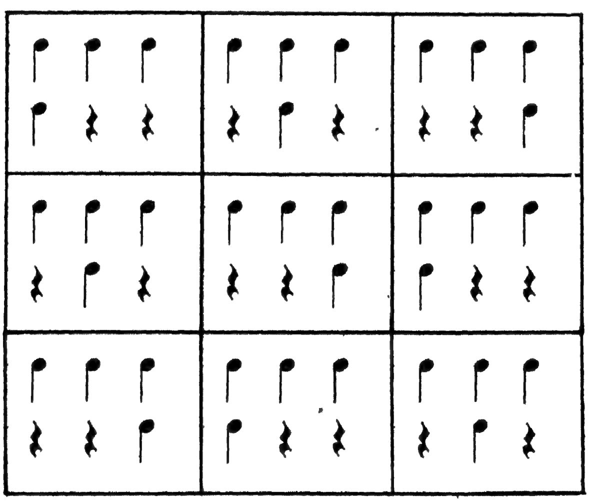


Figure 100.

Trinomial $2 + 1 + 1$ $\quad a = \overset{>}{2};\ b = 1$

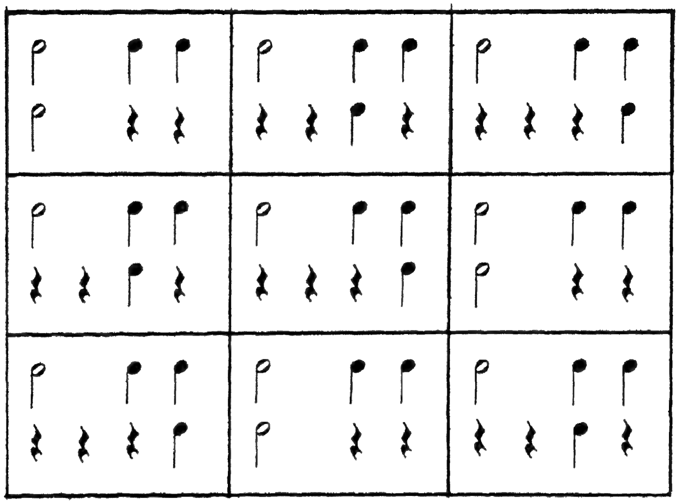


Figure 101.

Each group (with its additional component) of shifting accents may be used individually. Simultaneous application of all groups requires instruments of a different tone-quality for each group.

(4) **Split-Unit Groups:**

Trinomial $2 + 2 + 2$ $\quad a = 1 + 1;\ b = 2$

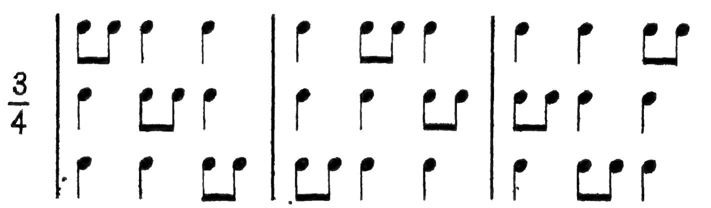


Figure 102.

Trinomial $4 + 1 + 3$ $\quad a = 4;\ b = 1;\ c = 2 + 1$

$[4+1+(2+1)]+[4+(2+1)+1]+[(2+1)+4+1]$

$[4+(2+1)+1]+[(2+1)+4+1]+[4+1+(2+1)]$

$[(2+1)+4+1]+[4+1+(2+1)]+[4+(2+1)+1]$

Figure 103.

5) **Groups in general:**

a = 2 + 1 + 1; b = 1 + 2 + 1;. c = 1 + 1 + 2

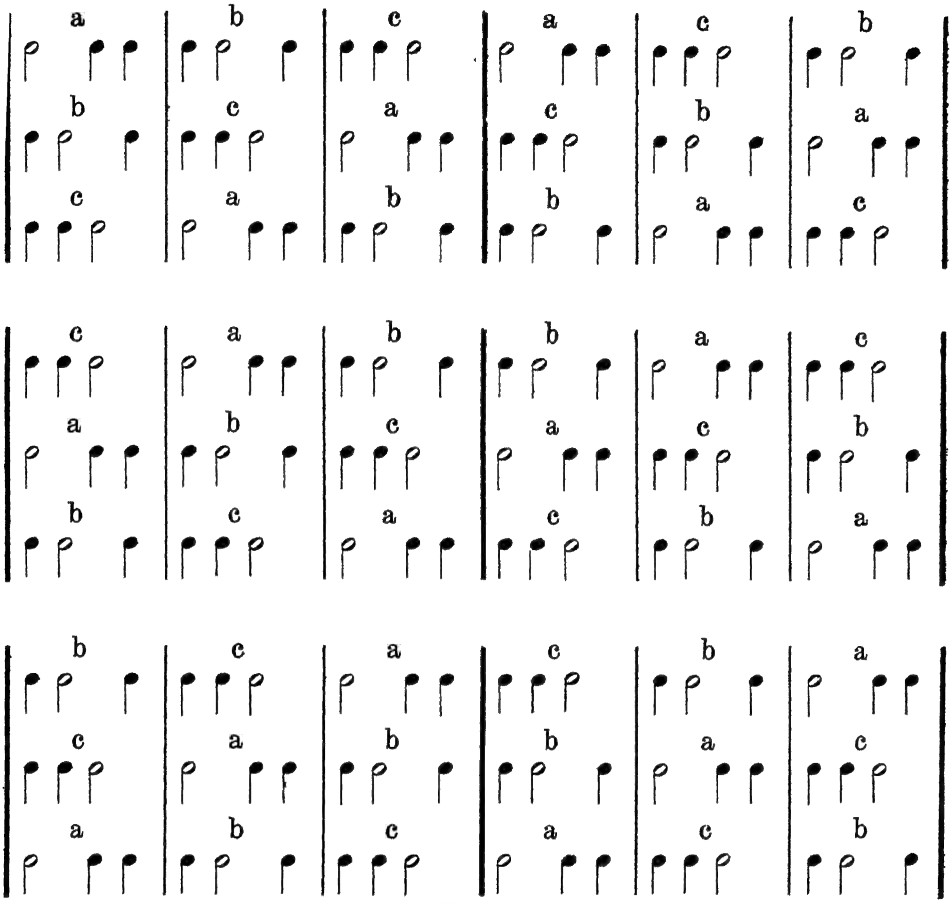


Figure 104.

Song: "Pennies from Heaven"*

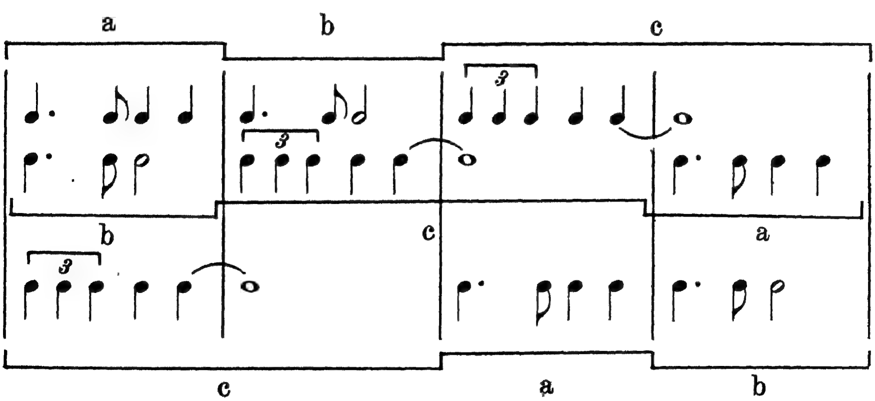


Figure 105.

*Copyright 1936 by Santly-Joy, Inc., New York, U.S.A. Reprinted by permission of the Publishers.

4 Elements

Table of General Permutations:

(1) All elements different:

abcd	acbd	cabd	bacd	bcad	cbad
abdc	acdb	cadb	badc	bcda	cbda
adbc	adcb	cdab	bdac	bdca	cdba
dabc	dacb	dcab	dbac	dbca	dcba

24 permutations

Figure 106.

(2) Two elements identical:

aabc	aacb	abac
abca	acba	baca
bcaa	cbaa	acab
caab	baac	caba

12 permutations

Figure 107.

(3) Two pairs identical:

aabb	abab
abba	baba
bbaa	
baab	

6 permutations

Figure 108.

(4) Three elements identical:

aaab	aaba	abaa	baaa

4 permutations

Figure 109.

Assuming that any of the permutations is an original group, each of the above groups may be limited to four circular permutations.

Example:

abcd	bcda	cdab	dabc

Figure 110.

Examples of application:

(1) **Durations:**

(a) All four elements different.

Quadrinomial from $r_{5\div4}$; $4 + 1 + 3 + 2$

$a = 4$; $b = 1$; $c = 3$; $d = 2$

$(4+1+3+2) + (4+1+2+3) + (4+2+1+3) + (2+4+1+3) +$
$+ (4+3+1+2) + (4+3+2+1) + (4+2+3+1) + (2+4+3+1) +$
$+ (3+4+1+2) + (3+4+2+1) + (3+2+4+1) + (2+3+4+1) +$
$+ (1+4+3+2) + (1+4+2+3) + (1+2+4+3) + (2+1+4+3) +$
$+ (1+3+4+2) + (1+3+2+4) + (1+2+3+4) + (2+1+3+4) +$
$+ (3+1+4+2) + (3+1+2+4) + (3+2+1+4) + (2+3+1+4)$

This 24-group continuity produces a 24-part simultaneity in 24 bars of $\frac{10}{8}$ time.

By limiting the original group $(4+1+3+2)$ to circular clockwise permutations, we obtain 4 parts in 4 bars of $\frac{10}{8}$ time.

(b) Two elements identical.

Quadrinomial from $r_{4\div3}$; $3 + 1 + 2 + 2$

$a = 2$; $b = 3$; $c = 1$

Form: $b + c + a + a$

Starting with the third permutation of the corresponding table, we obtain:

$(3+1+2+2) + (1+2+2+3) + (2+2+1+3) + (2+1+3+2) +$
$+ (1+3+2+2) + (3+2+2+1) + (2+3+2+1) + (3+2+1+2) +$
$+ (2+1+2+3) + (1+2+3+2) + (2+2+3+1) + (2+3+1+2)$

This 12-group continuity produces a 12-part simultaneity in 12 bars of $\frac{8}{8}$ time or in 24 bars of $\frac{4}{4}$ time.

(c) Two pairs identical.

Quadrinomial $r_{3\div2}$; $2 + 1 + 1 + 2$

$a = 2$; $b = 1$

Form: $a + b + b + a$

Starting with the second permutation of the corresponding table, we obtain:

$(2+1+1+2) + (1+1+2+2) + (1+2+2+1) + (2+1+2+1) +$
$+ (1+2+1+2) + (2+2+1+1)$

This 6-group continuity produces a 6-part simultaneity in 6 bars of $\frac{6}{8}$ time or in 12 bars of $\frac{3}{4}$ time (1 = ♩). Clockwise circular permutations give 4 parts in 4 bars of $\frac{6}{8}$ time or 4 parts in 8 bars of $\frac{3}{4}$ time (1 = ♩).

(d) Three elements identical.
Quadrinomial: $3 + 1 + 1 + 1$
$a = 1;\ b = 3$
Form: $b + a + a + a$

Starting with the fourth permutation of the corresponding table we obtain:

$$(3+1+1+1) + (1+1+1+3) + (1+1+3+1) + (1+3+1+1)$$

This 4-group continuity produces a 4-part simultaneity in 4 bars of $\frac{6}{8}$ time or in 8 bars of $\frac{3}{4}$ time.

Assigning different symbols to the same group we obtain the form $a+b+b+b$.

Then: $(a+b+b+b) + (b+a+b+b) + (b+b+a+b) + (b+b+b+a)$

This produces a continuity of perfect musical quality:

Figure 111.

Similar modification of the symbols assigned is possible with any group containing identical terms.

(2) **Rests:**
Quadrinomial: $1 + 1 + 1 + 1$
a = ①; b = 1

(①+1+1+1) + (1+①+1+1) + (1+1+①+1) + (1+1+1+①)
(1+①+1+1) + (1+1+①+1) + (1+1+1+①) + (①+1+1+1)
(1+1+①+1) + (1+1+1+①) + (①+1+1+1) + (1+①+1+1)
(1+1+1+①) + (①+1+1+1) + (1+①+1+1) + (1+1+①+1)

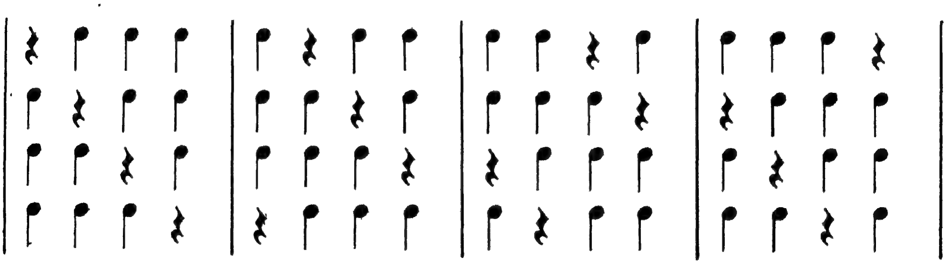


Figure 112.

Analogous permutations of rests may be devised in non-uniform groups.

(3) **Accents:**

Quadrinomial: $1 + \underset{>}{1} + 1 + 1$

$a = 1;\ b = 1$

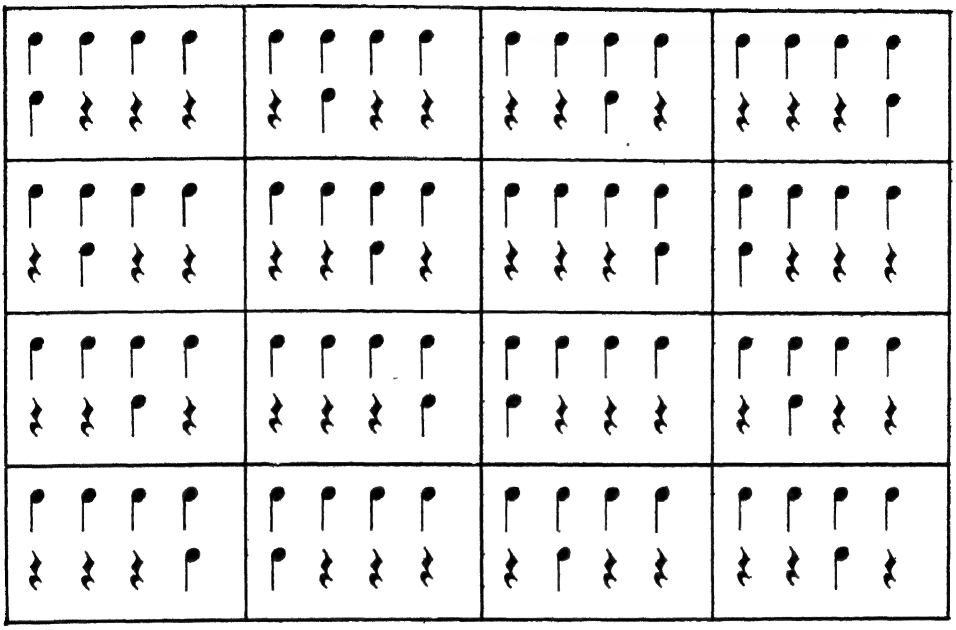


Figure 113.

Analogous permutations of accents may be devised in non-uniform groups.

(4) **Split-unit groups:**

Quadrinomial: $2 + 2 + 2 + 2$

$a = 1 + 1;\ b = 2$

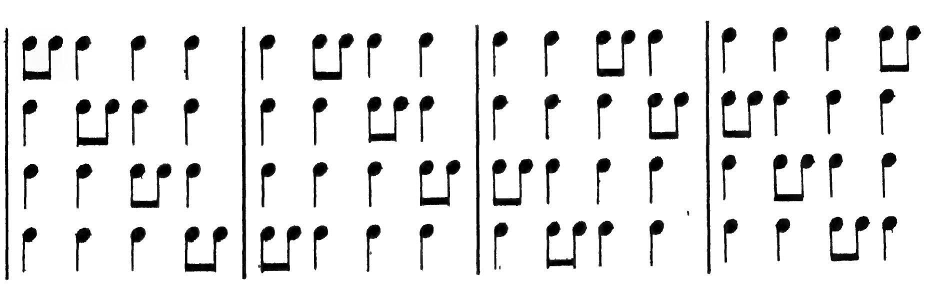


Figure 114.

Analogous permutations may be devised in non-uniform groups originally consisting of four places.

Example: $r_{\frac{5\div4}{2}} = 4 + 1 + 3 + 2$

Either of the numbers may be split into a group:

(a)	$4 = 2+2$	$(2+2) + 1+3+2$
	$4 = 2+1+1$	$(2+1+1) + 1+3+2$
	$4 = 1+2+1$	$(1+2+1) + 1+3+2$
	$4 = 1+1+2$	$(1+1+2) + 1+3+2$
	$4 = 1+1+1+1$	$(1+1+1+1) + 1+3+2$
(b)	$1 = \frac{1}{2} + \frac{1}{2}$	$4+(\frac{1}{2}+\frac{1}{2})+3+2$
(c)	$3 = 2+1$	$4+1+(2+1)+2$
	$3 = 1+2$	$4+1+(1+2)+2$
	$3 = 1+1+1$	$4+1+(1+1+1)+2$
(d)	$2 = 1+1$	$4+1+3+(1+1)$

Any of these versions may be used. Each version contains 4 circular and 24 general permutations.

(5) Groups in general:

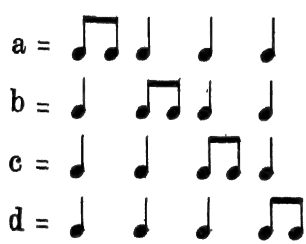


Figure 115.

This group produces the following simultaneity and continuity:

Simultaneity—4 parts.
Continuity through circular permutations—16 bars.
Continuity through general permutations—96 bars.

The original 4 bars take the appearance of the example in (4) [split-unit groups].

Any rhythmic resultant placed in 4 bars may constitute such a group. For example:

$r_{\underline{4 \div 3}}$ grouped by *a* in $\frac{4}{4}$ time:

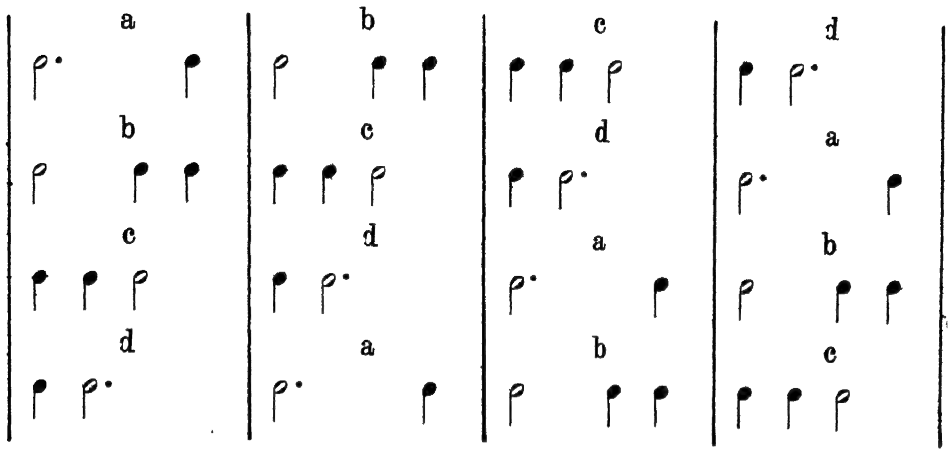


Figure 116.

The number of variations is the same as in the preceding group.

A group consisting of 4 elements may be produced from any rhythmic resultant, providing a non-uniform distribution is applied:

For example: $r_{5 \div 3}$ grouped by *b* in $\frac{3}{4}$ time:

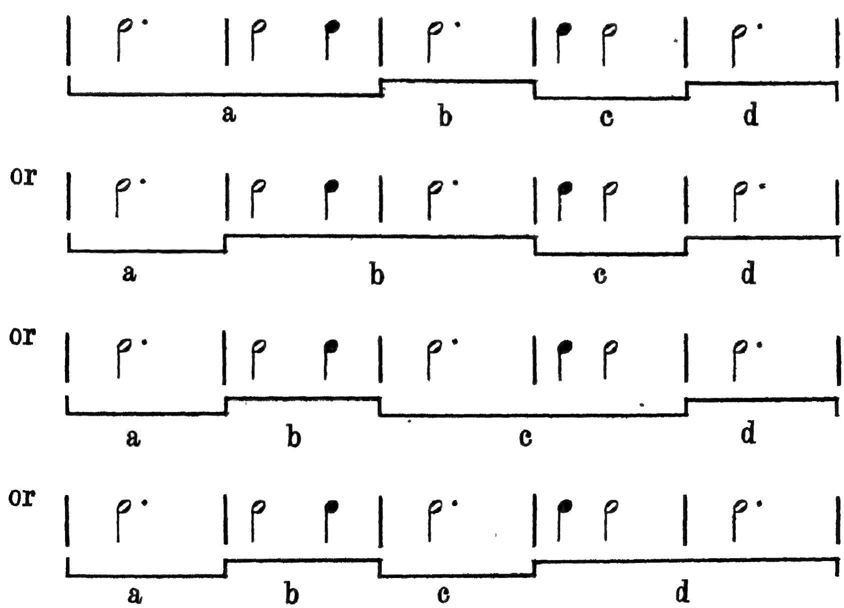


Figure 117.

Example from the popular song, *Pennies From Heaven.** When necessary a tie between the notes may be omitted, though it is not necessary if the same group repeats itself.

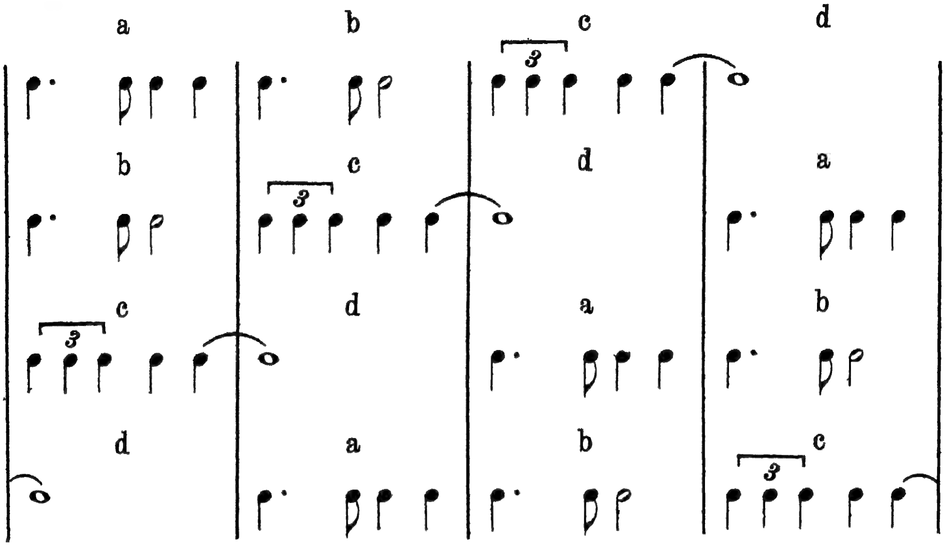


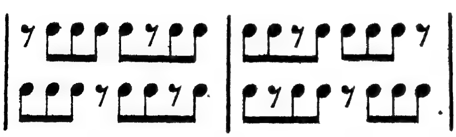
Figure 118.

You can see what extraordinary variety may be secured by a group as simple as this through this variation method.**

As the general *velocity* of musical time (tempo) is most essential in establishing one or another characteristic, many of the preceding examples, although similar in numbers, produce musical continuities as remote from each other in character as Händel is remote from the Cuban rhumba.

For example the group: (①+1+1+1) + (1+①+1+1) + (1+1+①+1) + (1+1+1+①) being written and performed as *largo* in $\frac{4}{4}$ time.

as compared with:



in the tempo of a fast rhumba.

Figure 119.

**The tables are worked out in detail on these and other pages not only for the sake of clarity; it is also a way of furnishing the practical composer with ready-made calculations so that each pattern need not be re-calculated whenever it is needed in actual composition. The time-saving way is to refer to the tables in this book, although a composer should also know how to calculate them afresh for himself, if necessary. (Ed.)

CHAPTER 10

GENERALIZATION OF VARIATION TECHNIQUES

A. Permutations of the Higher Order.

In order to increase the quantity of material evolving through the variation method from the original group, the method of *permutations of a higher order* may be used. The original element or group produces variations which in turn become the elements of the next order. The quantity of elements in the next successive order equals the square of the number of the elements of the preceding order. If the original number of elements in a group is 3, there will be 9 elements on the second order, 27 on the third, etc., through circular permutations. If the original number of elements in a group is 3, and general permutations are used, this will give 6 elements in the second order, 720 in the third order., etc.

Indicating the original elements as *a* of the first order (a_1), *b* of the first order (b_1) . . . and permuting them, the elements of the following order, which represent a group of the elements of the preceding order, are acquired. The technique of evolving the elements of the following order acquires this appearance:

$$\begin{array}{c} a_1 + b_1 = a_2 \\ \underline{b_1 + a_1 = b_2} \\ a_2 + b_2 = a_3 \\ \underline{b_2 + a_2 = b_3} \\ \cdots\cdots\cdots\cdots \\ \boxed{\begin{array}{c} a_{n-1} + b_{n-1} = a_n \\ b_{n-1} + a_{n-1} = b_n \end{array}} \end{array}$$

Figure 120.

This device is particularly important when one wishes to evolve a large quantity of material from the original group, or when the number of elements in the original group is exceedingly small. If the procedure of the permutations carried out through the sixth order concerned only 2 elements in a group, we would obtain ultimately only $2^6 = 64$ elements.

Music of animated motion often contains a much greater quantity of rhythmic elements (durations, rests, etc.). For example, take an average waltz. In ordinary printing we get at least 4 bars to a line, and 5 lines to a page. In music moving in eighth notes for 3 pages, we would get 360 durations

Example:

Application of the Permutations of the Higher Orders to the Original Group.

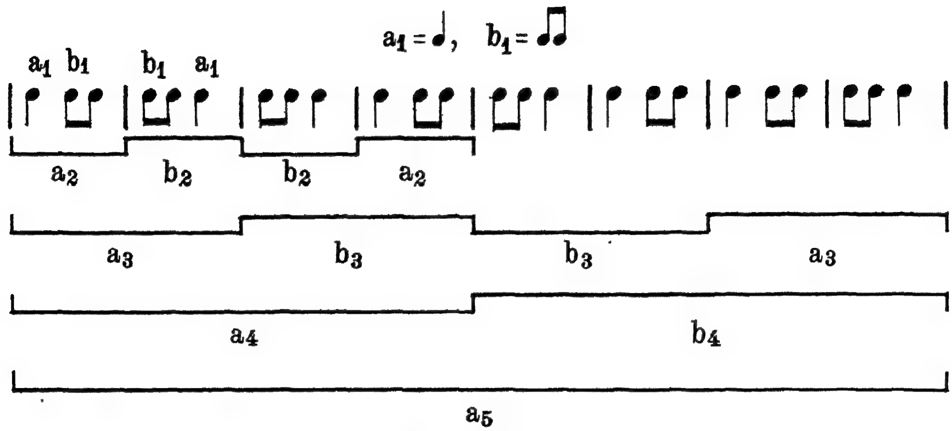


Figure 121.

Moving for 8 more bars, i.e., carrying out the permutations of the 4th order to their completion, we obtain 16 bars containing great variety as compared to the usual continuous recurrence. This device is particularly useful when the character of music must be retained for considerably longer time than the original rhythmic group permits. Instead of making continuous repetitions of the original group, or recurrences of the larger groups, it is possible with this device to go on continuously for an indefinite period of time.

In musical backgrounds for motion picture photoplays, when the scene develops in a definite locality—associated with definite rhythmic forms of expression, it may be desirable to extend this homogeneous rhythmic character to 10 or 15 minutes. In the case of a "Cuban" scene, rhumba rhythms are considered characteristic of the locality. The audience is distracted from action on the screen by the musical background when a definite dance composition is played repeatedly. This annoys the audience and never helps to bring out the dramatic plot. On the contrary, it produces conflicts with the plot. A neutral background, being homogeneous and yet continuously varied, will serve the purpose much better.

Permutations of the higher orders based on the original group with 3 elements (a_1, b_1, c_1) offer the following combinations by 2: $a_1 + b_1$, $a_1 + c_1$, $b_1 + c_1$. These are the three possible alternatives when 2 elements out of 3 are used. The 2 elements form a group of 3, following the regulations described in the preceding paragraph concerning the higher orders of the 2 elements.

The original group containing 3 elements has only one combination by 3: $a_1 + b_1 + c_1$. The second order permutations on the 3 elements appear as follows:

$$
\begin{aligned}
a_1 + b_1 + c_1 &= a_2 \\
a_1 + c_1 + b_1 &= b_2 \\
c_1 + a_1 + b_1 &= c_2 \\
b_1 + a_1 + c_1 &= d_2 \\
b_1 + c_1 + a_1 &= e_2 \\
c_1 + b_1 + a_1 &= f_2
\end{aligned}
$$

These 6 elements of the second order produce, in turn, combinations by 2, by 3, by 4, by 5 and by 6.

Combinations by 2:

$a_2 + b_2$	$b_2 + c_2$	$c_2 + d_2$	$d_2 + e_2$	$e_2 + f_2$
$a_2 + c_2$	$b_2 + d_2$	$c_2 + e_2$	$d_2 + f_2$	
$a_2 + d_2$	$b_2 + e_2$	$c_2 + f_2$		
$a_2 + e_2$	$b_2 + f_2$			
$a_2 + f_2$				

The total number of cases: $15 \times 2 = 30$

Combinations by 3:

$a_2 + b_2 + c_2$	$a_2 + c_2 + d_2$	$a_2 + d_2 + e_2$	$a_2 + e_2 + f_2$
$a_2 + b_2 + d_2$	$a_2 + c_2 + e_2$	$a_2 + d_2 + f_2$	
$a_2 + b_2 + e_2$	$a_2 + c_2 + f_2$		
$a_2 + b_2 + f_2$			
$b_2 + c_2 + d_2$	$b_2 + d_2 + e_2$	$b_2 + e_2 + f_2$	
$b_2 + c_2 + e_2$	$b_2 + d_2 + f_2$		
$b_2 + c_2 + f_2$			
$c_2 + d_2 + e_2$	$c_2 + e_2 + f_2$		
$c_2 + d_2 + f_2$			
$d_2 + e_2 + f_2$			

The total number of cases: $20 \times 6 = 120$

Combinations by 4:

$a_2 + b_2 + c_2 + d_2$	$a_2 + c_2 + d_2 + e_2$	$a_2 + d_2 + e_2 + f_2$
$a_2 + b_2 + c_2 + e_2$	$a_2 + c_2 + d_2 + f_2$	
$a_2 + b_2 + c_2 + f_2$		
$a_2 + b_2 + d_2 + e_2$	$a_2 + c_2 + e_2 + f_2$	
$a_2 + b_2 + d_2 + f_2$		
$a_2 + b_2 + e_2 + f_2$		
$b_2 + c_2 + d_2 + e_2$	$b_2 + d_2 + e_2 + f_2$	
$b_2 + c_2 + d_2 + f_2$		
$b_2 + c_2 + e_2 + f_2$		
$c_2 + d_2 + e_2 + f_2$		

Total number of cases: $15 \times 24 = 360$

Combinations by 5:

$a_2 + b_2 + c_2 + d_2 + e_2$ $\quad a_2 + b_2 + d_2 + e_2 + f_2$ $\quad a_2 + c_2 + d_2 + e_2 + f_2$

$a_2 + b_2 + c_2 + d_2 + f_2$

$a_2 + b_2 + c_2 + e_2 + f_2$

$b_2 + c_2 + d_2 + e_2 + f_2$

Total number of cases: $6 \times 120 = 720$

Combinations by 6:

$a_2 + b_2 + c_2 + d_2 + e_2 + b_2$

Total number of cases: $1 \times 720 = 720$

All the recurring elements are eliminated from these charts, which may be consulted for coefficients of recurrence. For example, a trinomial combination from 2 elements, a_1 and b_1, with a coefficient 2 for the first element becomes $2a_1 + b_1$. This is a trinomial with 2 identical elements, and is subjected to circular permutations only. Similar cases occur with 4 elements having 2 identical terms, 2 identical pairs or 3 identical terms. Similar cases occurring with 5 and 6 elements may contain 2, 3 and more identical elements. They will be treated as coefficients of recurrence.

Example:

Trinomial of the Third Order

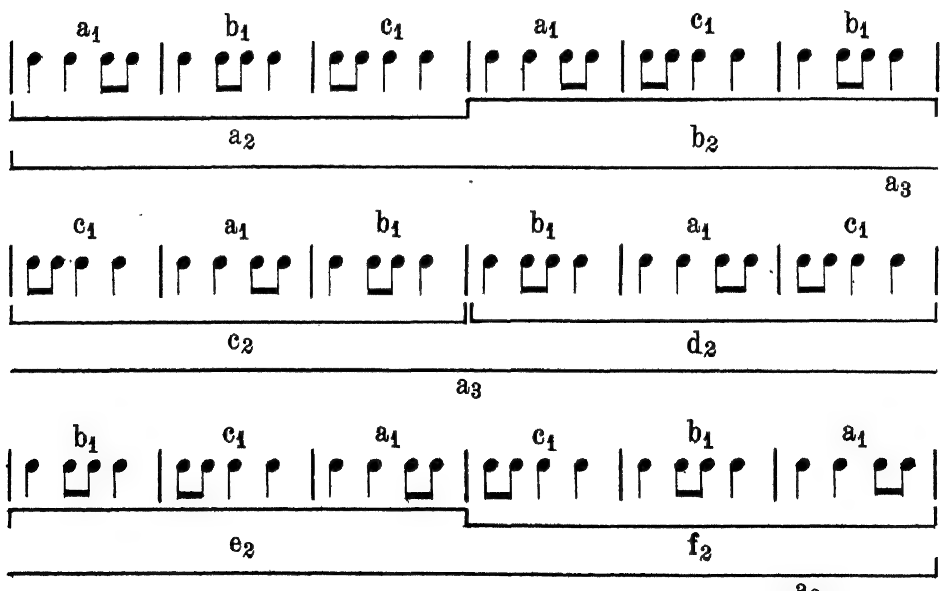


Figure 122.

When the quantities exceed the necessary amount, one can limit the number of variations by reducing them to circular permutations only. The illustrations above are applicable to rests, accents and other group formations.

CHAPTER 11

COMPOSITION OF HOMOGENEOUS RHYTHMIC CONTINUITY

ANY rhythmic group may be adapted to the processes of growth in simultaneity and continuity. There are three fundamental procedures, varying with regard to the quantity of material to be evolved. The first process gives the minimum quantity; the second, the intermediate; and the third, the maximum quantity. Select them in accordance with the requirements of each specific case.

(1) We may produce elements from a given rhythmic group by means of splitting the group through the *simplest divisor*. For example, the group $\underline{r_{4\div3}}$ (grouped by 4) represents a 4-bar continuity in $\frac{4}{4}$ time. 4 may be divided by 2 and thus we obtain two groups: a_1 comprising the first two bars, and b_1 comprising the second two bars. This gives us an 8-bar, 2-part continuity, i.e., the quantity of the original material is doubled both in simultaneity and continuity.

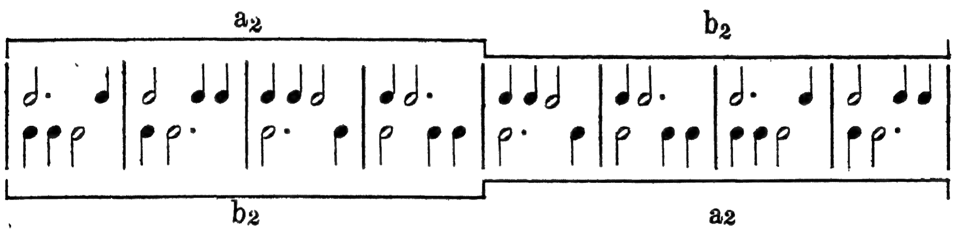


Figure 123.

When a group is not divisible by 2, like $r_{5\div3}$ (grouped by 5), it may become divisible by 3. In this case it produces 3 bars in $\frac{5}{4}$, $\frac{5}{8}$ or any other quintuple time—the first bar being a_1, the second b_1 and the third c_1.

(2) We may produce elements from a given rhythmic group by means of splitting it through individual bars. For example, in $\underline{r_{4\div3}}$ grouped in 4 bars, each individual bar becomes an element. The first is a_1, the second b_1, the third c_1, and the fourth d_1. This splitting process produces a 16-bar continuity in 4 parts—i.e., both simultaneity and continuity of the original group become quadrupled.

Figure 124.

This continuity is the result of circular permutations. Using general permutations for this group, and splitting it in this particular fashion, we obtain 4 bars in 4 parts with 24 different variations, i.e., 96 bars in 4 parts. In a case in which the simplest divisor corresponds to the splitting by individual bars, as in the above-mentioned case of $r_{5\div3}$, this becomes the only possible procedure.

Any bar splitting will ultimately give a score in which the number of parts equals the number of bars, and the number of bars equals the number of circular or general permutations available for such number. For example, taking $\underline{r_{8\div5}}$ and having it grouped in 8 bars, we obtain 8-part simultaneity in 64-bar continuity through circular permutations, and 276,480 bar continuity through general permutations as the total number of permutations of 8 elements equals 40,320.

(3) We may produce elements from a given rhythmic group by means of splitting the group through the *individual attacks* (terms). For example, if we take the group $\underline{r_{4\div3}}$, we obtain 10 individual terms. These 10 terms are subjected to growth in simultaneity and continuity. The original group arranged in 4 bars of the $\frac{4}{4}$ time produces a 10-part simultaneity. These 10 bars evolve into a 40-bar continuity (4 × 10). Thus the total original score has 40 bars in 10 parts.

$\underline{r_{4\div3}}$	3	1	2	1	1	1	1	2	1	3
	a_1	b_1	c_1	d_1	e_1	f_1	g_1	h_1	i_1	j_1

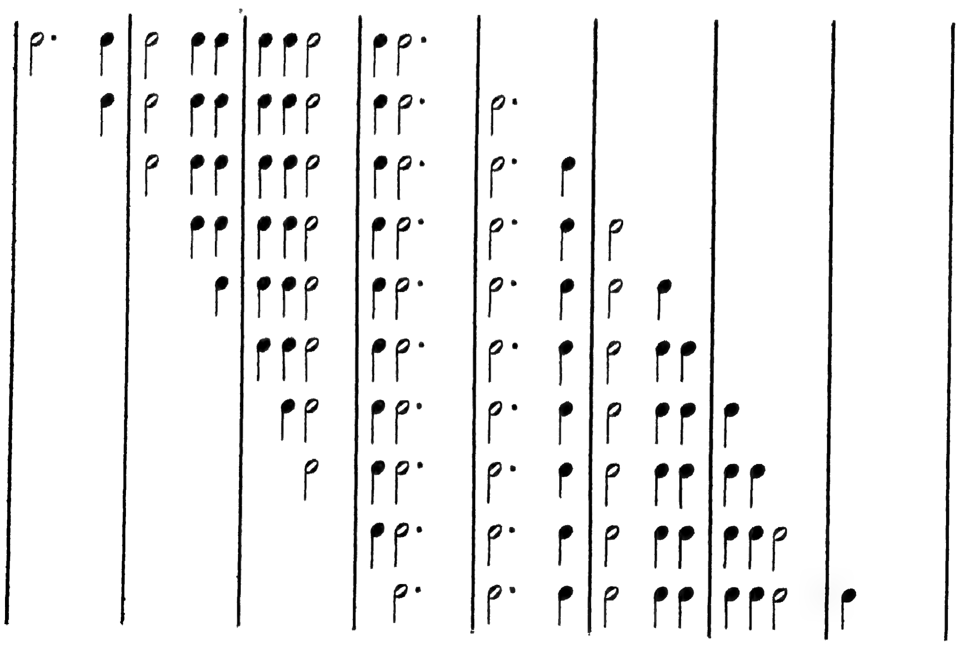


Figure 125 (continued).

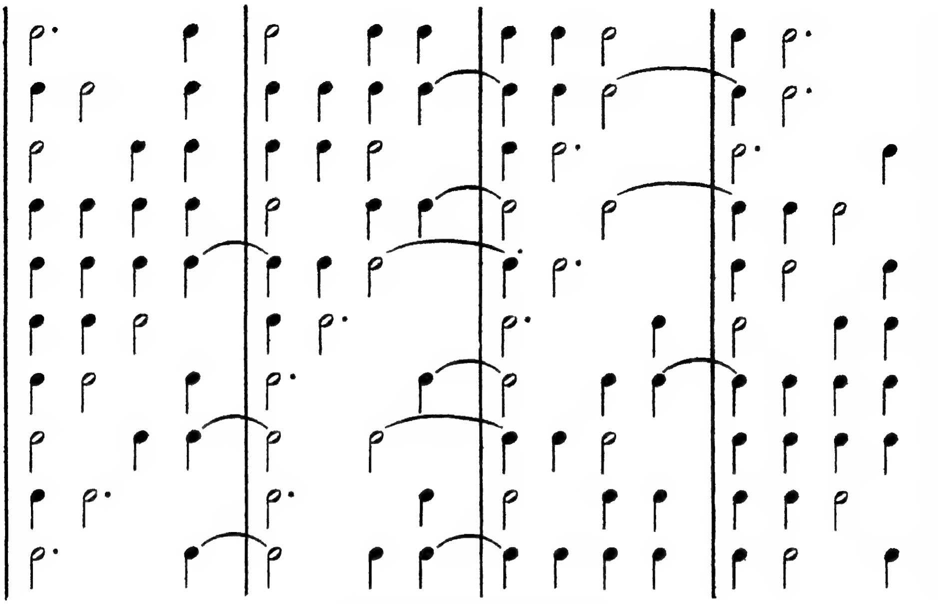


Figure 125 (concluded).

While in this case there is a coincidence of the figure 1 + 2 + 1, the number of parts moving simultaneously obscures it entirely to the human ear. This 40-bar, 10-part score produces 10 elements of 4 bars each. 10 elements give 3,628,800 permutations, which give a total of 145,152,000 bars in 10 parts.

CHAPTER 12

DISTRIBUTIVE POWERS

A. Continuity of Harmonic Contrasts

THE PROBLEM of producing contrasts in a rhythmic continuity concerns the two fundamental methods of evolving rhythm: *one*, the patterns generated through the common denominator and constituting rhythmic continuity within musical measures ($\frac{t}{t}$); and *two*, the patterns of the measures themselves growing into a complete form expressing the rhythm of measures and of groups of measures. The first form of continuity is called *fractional continuity;* the second, *factorial continuity.**

While rhythm evolves within musical measures, musical measures themselves also evolve their own rhythm. The correlation of the two in time sequence will be incorporated into the *series of factorial-fractional continuity.* Homogeneous series of factorial-fractional continuity are *power-series.* The original value ($\frac{t}{t}$) represents the *determinant of a series.* Powers express the evolution of a number through its own continuous factoring. Algebraic treatment of the power processes is quantitative and—being applied—does not bring the solution of esthetic problems. Esthetic problems are essentially the problems of *distribution* and *coordination*, and not problems of mere quantity. The process of evolving any initial ratio through its own factoring lies within the field of *distributive powers.*** The distributive powers organize not only the value but also the *quantity of the values* harmonically. Any binomial under distributive powers becomes a quadrinomial on the square ($2^2 = 4$). It becomes a group of 8 terms on the cube; a trinomial becomes a polynomial with 9 terms on the square ($3^2 = 9$). . . . It happens to be the fact that the art of music, with regard to its rhythm, has not yet exceeded the series with the $\frac{8}{8}$ determinant. In the later exposition of the evolution of rhythmic families, this subject will be treated in detail.

*This is an idea fundamental to Schillinger's system. He does not regard what is ordinarily called "musical form"—i.e., the organization of the entire composition by "phrases," etc.—as something separate from the rhythms of the measures themselves. Rather, he regards the two — fractional (rhythms *within* the measure) and factorial (rhythms *of* the measures)—as two aspects of the same central situation. (Ed.)

**For example, the algebraic square of a + b is: $a^2 + 2ab + b^2$. But the *distributive* square would be $a^2 + ab + ab + b^2$. In other words, the magnitudes are *distributed* rather than being grouped after coefficients. The use by Schillinger of *distributive powers* is one of the most extraordinary aspects of his system; these are used as well in other arts, especially in the spatial arts. (Ed.)

The following is a survey of the series which have been employed to date from the beginning of music by the inhabitants of this planet.

	$\frac{1}{t^n}\ldots$	$\frac{1}{t^3}\ldots$	$\frac{1}{t^2}\ldots$	$\frac{1}{t}\ldots$	$\frac{t}{t}\ldots$	$t\ldots$	$t^2\ldots$	$t^3\ldots$	t^n
$\ldots$	$\frac{1}{16}\ldots$	$\frac{1}{8}\ldots$	$\frac{1}{4}\ldots$	$\frac{1}{2}\ldots$	$\frac{2}{2}\ldots$	$2\ldots$	$4\ldots$	$8\ldots$	$16\ldots$
	$\ldots$	$\frac{1}{27}\ldots$	$\frac{1}{9}\ldots$	$\frac{1}{3}\ldots$	$\frac{3}{3}\ldots$	$3\ldots$	$9\ldots$	$27\ldots$	
	$\ldots$	$\frac{1}{64}\ldots$	$\frac{1}{16}\ldots$	$\frac{1}{4}\ldots$	$\frac{4}{4}\ldots$	$4\ldots$	$16\ldots$	$64\ldots$	
		$\ldots$	$\frac{1}{25}\ldots$	$\frac{1}{5}\ldots$	$\frac{5}{5}\ldots$	$5\ldots$	$25\ldots$		
		$\ldots$	$\frac{1}{36}\ldots$	$\frac{1}{6}\ldots$	$\frac{6}{6}\ldots$	$6\ldots$	$36\ldots$		
		$\ldots$	$\frac{1}{49}\ldots$	$\frac{1}{7}\ldots$	$\frac{7}{7}\ldots$	$7\ldots$	$49\ldots$		
		$\ldots$	$\frac{1}{64}\ldots$	$\frac{1}{8}\ldots$	$\frac{8}{8}\ldots$	$8\ldots$	$64\ldots$		
		$\ldots$	$\frac{1}{81}\ldots$	$\frac{1}{9}\ldots$	$\frac{9}{9}\ldots$	$9\ldots$	$81\ldots$		

. . . .

Each of the series whose determinants are indicated above represents centuries, sometimes millennia, of musical evolution. The most familiar of all is the series with the determinant 2. The $\frac{2}{2}$ series represents our own musical civilization, known to us as an important and glorious period of musical history, but it certainly does not appear very inventive from the viewpoint of objective analysis. The $\frac{2}{2}$ series has, in fact, caused more damage to the evolution of our musical culture than it has helped the development of our culture—with respect to rhythm. The number-values found on the right side of the determinant represent the constant growth of *factorial* groups (measures and their multiples). The left side represents the formation of rhythmic patterns within each consecutive measure.

The real reason for our musical civilization's being so elementary is the system of notation evolved in Europe during recent centuries. Just a few hundred years ago, the very idea of recording rhythm in relative durations seemed to be quite revolutionary (Mensural system). Even in our own day our schools teach that a whole note consists of two half-notes, and the two half-notes consist of four quarter notes, etc. But why mention notes, when this is an ordinary process of arithmetical division by two? The habit of thinking in two's and their multiples has retarded the development of our musical civilization to such an extent that the rhythms used on the African continent, perhaps thirty thousand years ago, seem to us to be quite exciting even today. The general field of classical music deals with division by two and multiplication by two. All classical rhythmic patterns are based on halfs, quarters, eights, sixteenths, etc. Measures accumulate through the same multiples.

Measure-groups (known as "phrases") appear in 2's, 4's, 8's, etc. The inefficiency of the accepted system of musical notation was sufficiently discussed at the beginning of this theory.* When a symbol called a quarter-note appears in musical writing, such a quarter-note does not necessarily represent a quarter of anything. It may be a half, a third, a fifth, or any other fraction.

*See Chapter 1 of *Theory of Rhythm*. See also Chapter 2 of Book IV, *Theory of Melody* which presents a history of musical notation and fully describes the inadequacy of the accepted system of notation that caused Schillinger to search for a new system. (Ed.)

Classical music developed very little efficiency in the $\frac{3}{3}$ series. The right side is entirely untouched, because when we find 3-bar phrases in such music, it is usually a 4-bar or 2-bar phrase—modified by means of expansion or contraction. The left side of the $\frac{3}{3}$ series is somewhat better developed. There are bars with three beats ($\frac{3}{4}$ time), there are bars with nine beats ($\frac{9}{8}$ time), and there are even a few rare cases when $\frac{27}{16}$ appears as a musical measure, as in some works of J. S. Bach. If music has been developed so consistently up to the seventh power on the determinant $\frac{2}{2}$, why should it not develop with the same consistency on any other determinant in use? Why has the $\frac{3}{3}$ series reached only its cube, and that only on very rare occasions? Why has it not developed beyond the first power on the factorial side? The answer is obvious: it is the system of musical notation attached to the $\frac{2}{2}$ determinant that has caused this conservatism.

Racial and national instincts in music, in contrast to acquired musical theories, work with much greater consistency although evolution by this means often requires centuries. Some of the American Indians, for example, exhibited such a degree of consistency with regard to the $\frac{3}{3}$ series. Their evolution did not reach high powers, yet these Indians are uniformly consistent as to both the factorial and fractional side. They use the first, the second, and the third powers of the above-mentioned series—see the musical example in Helen Roberts' book, *Form in Primitive Music*, page 39.*

The $\frac{4}{4}$ series, being a multiple of the $\frac{2}{2}$ determinant, does not exhibit strikingly new characteristics. We find such music frequently in many compositions. Groups like $4 + 1 + 1 + 1 + 1$ may be found in any music entitled "March" (♩ ♬♬). The accumulation of bars in groups of 4 and 16 is also quite common.

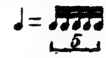
Classical music of the past evolving from the series with the $\frac{3}{3}$ determinant, deviating from the natural consistency of powers, resorts to simplification. The common case of music written in $\frac{3}{4}$ time is not a quarter-note split into a triplet of eighths but into two eighths. This means that $\frac{1}{3}$, instead of being multiplied by $\frac{1}{3}$ and becoming $\frac{1}{9}$, is multiplied by $\frac{1}{2}$ and becomes $\frac{1}{6}$. This is typical of a hybrid resulting from *unintentional* simplification. The eighth in $\frac{3}{4}$ time more frequently becomes ♫, and not ♫♪ (3) as it should.

This tendency toward simplification is philosophically puzzling. We must ask: is this number 2 an *unavoidable* condition in evolving any series, as in the multiplication of spermatozoa and other microbes and lower organisms? Or is the determinant 2 not as vital as it may seem at first, merely serving as an outlet for simplification?

If the former were true, one could not find any pure folk music with any other form of fractional development than that achieved through the determinant 2. Yet the music of Hindustan, very old and very traditional, uses a great many triplets representing a split-unit group of one beat in $\frac{5}{4}$ time (♩ = ♪♪♪ (3))

This shows that the $\frac{5}{5}$ determinant, which is characteristic of many old Asiatic civilizations, acquires its simplified fractioning through the $\frac{3}{3}$ series, i.e., $\frac{1}{5} \times \frac{1}{3} = 15$. The $\frac{5}{5}$ series, in addition to being characteristic of Hindustan, is also characteristic of Java, Bali and Siam, whence it moved westward influencing Afghanistan, Persia, Arabia and Russia.

*American Library of Musicology, 1932.

The present state of development of the $\frac{5}{5}$ series is still very elementary. Five-bar groups are as rare as the quintuplets in $\frac{5}{5}$ time (). It is difficult now to make a definite statement on the origin of the $\frac{5}{5}$ series. It may have been influenced by the forms of poetical rhythm known as pentameter; and it may have been influenced by the very same factors that influenced the formation of a pentacle in starfish.

The $\frac{6}{6}$ series, being a multiple of $\frac{2}{2}$ by $\frac{3}{3}$, is a typical European hybrid. It may be found throughout the southern coast of Europe, and especially in Portugal, Spain and Italy. Most of the barcarolles of the last-mentioned are written in $\frac{6}{8}$ time.

The $\frac{7}{7}$ series is also of eastern origin. In its trans-Asiatic travel it has crossed the Ural mountains and reached central Russia (Borodine, Rimsky-Korsakov).

The $\frac{8}{8}$ series is of African origin and is the most popular in dance music in the United States today. These patterns undoubtedly penetrated through the imported Negro slaves, as the patterns are common in South America, Puerto Rico, Cuba and the United States. In ancient times, these rhythms traveled northward and reached Arabia. During the late Middle Ages they got as far as North Russia and slowed down their pace, in the literal sense of the word. Folk music in the region of the White Sea and the Arctic Ocean on the north coast of European Russia has patterns identical with the Cuban rhumba of today; but the absolute *velocity* of the rhumba is doubled as compared with Russian music. This means that by taking a rhumba and slowing it down exactly twice, you will get the rhythms of North Russia, constructed in $\frac{8}{8}$ time and even with the same duration values ($\frac{8}{8} = \frac{3}{8} + \frac{2}{8} + \frac{3}{8}$). To make such music sound like a real rhumba, it would simply be necessary to transcribe it into a different pitch-scale.

The application of this method of series leads me to the conclusion that a consistent form of what is known as "jazz" is music which must be written in $\frac{8}{8}$ time, having $\frac{1}{8}$ as a common denominator and 8-bar phrases accumulating by eighths. The standard form of popular song usually includes a 32-bar chorus. The perfect structure is achieved when the chorus comprises a unit of factorial continuity and consists of 64 bars (8^2)—see *Cheek to Cheek*, and other 64-bar choruses.

The $\frac{9}{9}$ series is now in the making. There are some symptoms of it disclosing itself through different channels of musical time, one being the Viennese waltz and the other, the fox-trot. Today we have a hybrid of the old $\frac{8}{8}$ series and the coming $\frac{9}{9}$ series, which bears the trade-name of "swing."

A complete analysis of the phenomenon known as "swing," so prominent today, will be given at the end of the rhythm theory. It is a hybrid trying to crystallize itself through the intuitive efforts of musicians into the pure style of the $\frac{9}{9}$ series.

There are no difficulties in the way of producing any type of pure or hybrid series, because any of the series-determinants may become either major or minor generators of the rhythmic resultants, and may be incorporated in many ways. Any doubts as to the construction of a perfect 5-bar phrase may be dis-

solved by the utilization of the devices previously offered, such as $r_{5 \div 3 \div 2}$, grouped by 6; $r_{6 \div 5}$ grouped by 6, etc.

The above survey of the series of *factorial* and *fractional* continuity shows that these series belong to the category of power series. Since each number in any of these series represents a monomial, further evolution of the monomial into a polynomial will express the more developed patterns of factorial and fractional continuity. The latter, like the original, are subjected to powers.

The method of *distributive powers* offers a solution for producing harmonic contrasts developed from the original polynomial ratio. This solves the problem of composing counterthemes to any theme, whether the contrast appears in simultaneity (counterpart) or continuity (sequence). The law of distributive powers is a common esthetic law of proportionate distribution of harmonic contrasts.

B. Composition of Rhythmic Counterthemes by Means of Distributive Powers

I. *Square of a Binomial*

Formula: (a) Factorial: $(a+b)^2 = a^2 + ab + ab + b^2$

(b) Fractional: $\left(\frac{a}{a+b} + \frac{b}{a+b}\right)^2 =$

$$\frac{a^2}{(a+b)^2} + \frac{ab}{(a+b)^2} + \frac{ab}{(a+b)^2} + \frac{b^2}{(a+b)^2}$$

To obtain the distributive second power of a binomial, it is necessary to multiply the first term of a binomial by itself, then by the second term; then the second term by the first, then the second by the second.

Formula for Synchronization:

(a) Factorial: $S = a(a+b) + b(a+b)$

(b) Fractional: $S = \frac{a}{a+b} \cdot \left(\frac{a+b}{a+b}\right) + \frac{b}{a+b} \cdot \left(\frac{a+b}{a+b}\right)$

To synchronize the initial binomial with its distributive square, it is necessary to multiply the first term by the sum of the binomial, then the second term by the sum of the binomial.

Example:

Series $\frac{2}{3}$ Factorial binomials: $2 + 1$ and $1 + 2$

Fractional binomials: $\frac{2}{3} + \frac{1}{3}$ and $\frac{1}{3} + \frac{2}{3}$

$\left(\frac{2}{3} + \frac{1}{3}\right)^2 = \frac{4}{9} + \frac{2}{9} + \frac{2}{9} + \frac{1}{9}$ (squaring)

$\frac{3}{3}\left(\frac{2}{3} + \frac{1}{3}\right) = \frac{6}{9} + \frac{3}{9}$ (synchronization)

$\left(\frac{1}{3} + \frac{2}{3}\right)^2 = \frac{1}{9} + \frac{2}{9} + \frac{2}{9} + \frac{4}{9}$ (squaring)

$\frac{3}{3}\left(\frac{1}{3} + \frac{2}{3}\right) = \frac{3}{9} + \frac{6}{9}$ (synchronization)

The initial binomials synchronized with their distributive squares represent the themes. The distributive squares represent the counterthemes.

The proportion $\frac{a^2}{ab} = \frac{ab}{b^2}$ produces harmonic contrast, and gives esthetic satisfaction as to both simultaneity and continuity.

Here is a graph and the musical notation of the entire score:

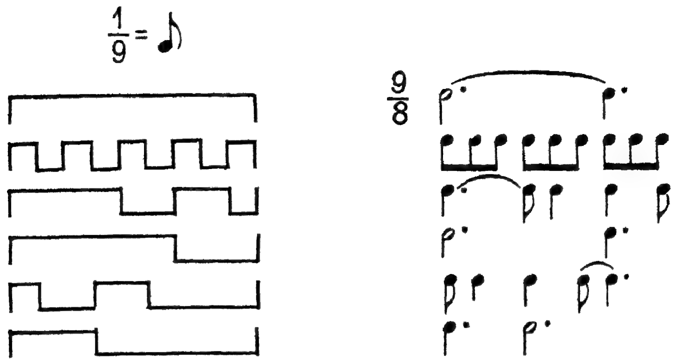


Figure 126.

The same score may be expressed in $\frac{3}{4}$ time.

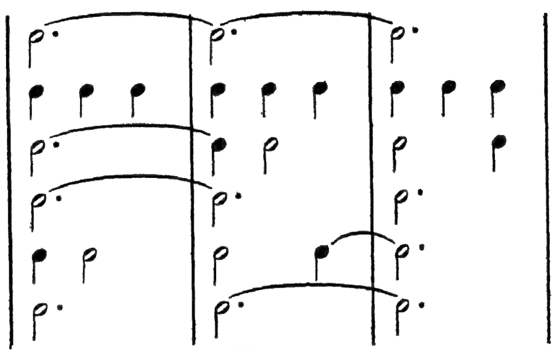


Figure 127.

$\frac{6}{9} + \frac{3}{9}$ and $\frac{4}{9} + \frac{2}{9} + \frac{2}{9} + \frac{1}{9}$ in continuity:

Figure 128.

Musical intuition in some cases approximates these harmonic groups. Here is a musical pattern which is the nearest approximation to the case above:

Figure 129.

i.e., $7 + 1 + 3 + 1$

instead of $4 + 2 + 2 + 1$.

As the numbers grow, it becomes practical to find the resultants of interference between the initial binomials (synchronized with their squares) and the resultants of the distributive squares. Having these resultants available, such power groups may later be utilized in scoring (when more than one orchestral

part is desirable), and the resultants of such groups may be used when one part must express the same rhythm.

In addition to this, it is important to supplement the score by $r_{\underline{a \div b}}$ where *a* is the determinant of a series. For example, in the foregoing case the determinant is 3; therefore $r_{\underline{3 \div 2}}$ may be added to the score.

Here is a complete graph and musical notation of the power groups, their resultants and $r_{\underline{a \div b}}$.

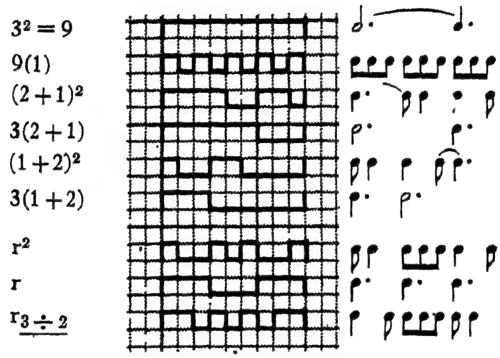


Figure 130.

Chart of the binomials for squaring and synchronization:

$\frac{3}{3}$ 2+1 1+2	$\frac{4}{4}$ 3+1 1+3	$\frac{5}{5}$ 3+2 4+1 2+3 1+4	$\frac{6}{6}$ 5+1 1+5
$\frac{7}{7}$ 4+3 5+2 6+1 3+4 2+5 1+6		$\frac{8}{8}$ 5+3 7+1 3+5 1+7	$\frac{9}{9}$ 5+4 7+2 8+1 4+5 2+7 1+8

Factorial groups of rhythm build the entire continuity in terms of bars, while fractional groups build the bars in terms of duration-units (attacks).

II. *Square of a Trinomial*

Formula: $(a + b + c)^2 = (a^2 + ab + ac) +$
$+ (ab + b^2 + bc) + (ac + bc + c^2)$.

The distributive square of a trinomial is the sum of the products of *a* by itself, of *a* by *b*, of *a* by *c*, of *b* by *a*, of *b* by itself, of *b* by *c*, of *c* by *a*, of *c* by *b* and of *c* by itself.

The number of terms in a distributive square of any polynomial equals the square of this number. Thus, a binomial gives 4 terms ($2^2 = 4$), a trinomial gives 9 terms ($3^2 = 9$), etc. The denominator of all terms in the distributive power-groups equals the quantitative square of the sum. In a trinomial it equals $(a + b + c)^2$, like $(3 + 2 + 1)^2 = 36$.

In order to synchronize any initial polynomial with its distributive square, it is necessary to find the products of each term by the sum of the polynomial. For example, to synchronize a trinomial with its distributive square:

$$\frac{a}{a+b+c} \cdot \frac{(a+b+c)}{(a+b+c)} + \frac{b}{a+b+c} \cdot \frac{(a+b+c)}{(a+b+c)} + \frac{c}{a+b+c} \cdot \frac{(a+b+c)}{(a+b+c)}$$

Series: $\frac{4}{4}$

$\frac{2}{4}+\frac{1}{4}+\frac{1}{4} \quad \frac{1}{4}+\frac{2}{4}+\frac{1}{4} \quad \frac{1}{4}+\frac{1}{4}+\frac{2}{4}$

$$(\tfrac{2}{4}+\tfrac{1}{4}+\tfrac{1}{4})^2 = (\tfrac{4}{16}+\tfrac{2}{16}+\tfrac{2}{16}) + (\tfrac{2}{16}+\tfrac{1}{16}+\tfrac{1}{16}) + (\tfrac{2}{16}+\tfrac{1}{16}+\tfrac{1}{16})$$
$$\tfrac{4}{4}(\tfrac{2}{4}+\tfrac{1}{4}+\tfrac{1}{4}) = \tfrac{8}{16}+\tfrac{4}{16}+\tfrac{4}{16}$$
$$(\tfrac{1}{4}+\tfrac{2}{4}+\tfrac{1}{4})^2 = (\tfrac{1}{16}+\tfrac{2}{16}+\tfrac{1}{16}) + (\tfrac{2}{16}+\tfrac{4}{16}+\tfrac{2}{16}) + (\tfrac{1}{16}+\tfrac{2}{16}+\tfrac{1}{16})$$
$$\tfrac{4}{4}(\tfrac{1}{4}+\tfrac{2}{4}+\tfrac{1}{4}) = \tfrac{4}{16}+\tfrac{8}{16}+\tfrac{4}{16}$$
$$(\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{2}{4})^2 = (\tfrac{1}{16}+\tfrac{1}{16}+\tfrac{2}{16}) + (\tfrac{1}{16}+\tfrac{1}{16}+\tfrac{2}{16}) + (\tfrac{2}{16}+\tfrac{2}{16}+\tfrac{4}{16})$$
$$\tfrac{4}{4}(\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{2}{4}) = \tfrac{4}{16}+\tfrac{4}{16}+\tfrac{8}{16}$$

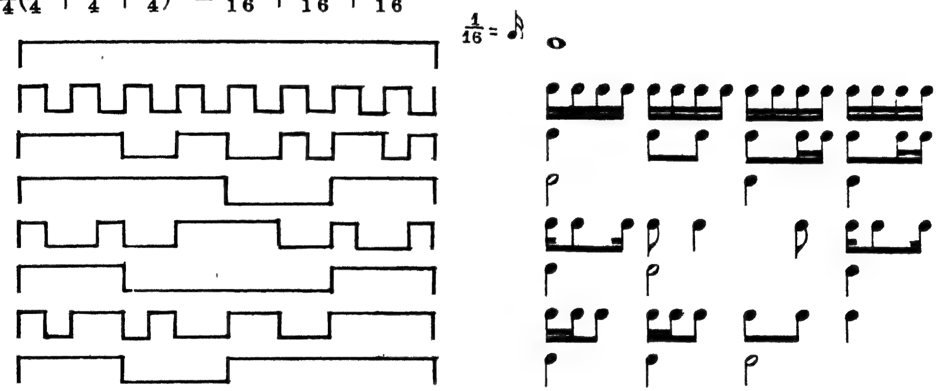


Figure 131.

The same score may be expressed in four bars in $\frac{4}{4}$, assuming $\frac{1}{16}$ = ♩. The above computation can be made in integers, i.e., using the numerators only.

As in the case of binomials, it is desirable to supplement this score by the first and second power resultants and the $r_{\underline{a \div b}}$.

Here is the entire score:

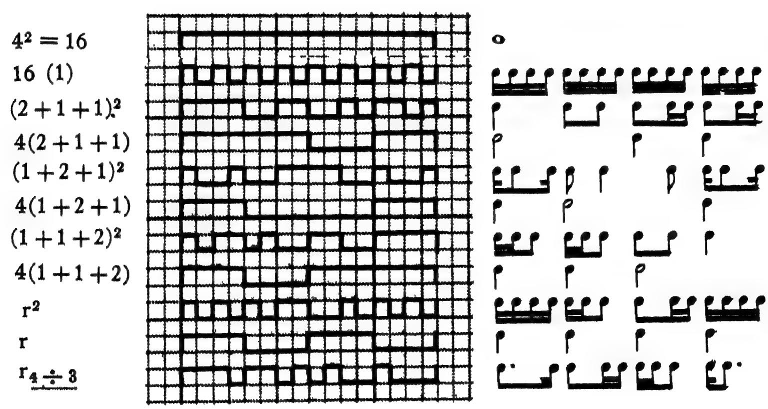


Figure 132.

It is interesting to note that in this particular case, i.e., $2 + 1 + 1$ and $1 + 1 + 2$, classical composers found *intuitively* the exact distributive squares. As you can see from this score, they could not find the square of $1 + 2 + 1$. This figure, i.e., $(1 + 2 + 1) + (2 + 4 + 2) + (1 + 2 + 1)$, or assuming $\frac{1}{16} =$ ♪), $\frac{4}{4}$  is very practical for the tango.

Chart of Trinomials

	2+1+1		2+2+1	3+1+1		4+1+1
$\frac{4}{4}$	1+2+1	$\frac{5}{5}$	2+1+2	1+3+1	$\frac{6}{6}$	1+4+1
	1+1+2		1+2+2	1+1+3		1+1+4

	3+3+1	2+2+3	5+1+1		3+3+2	6+1+1
$\frac{7}{7}$	3+1+3	2+3+2	1+5+1	$\frac{8}{8}$	3+2+3	1+6+1
	1+3+3	3+2+2	1+1+5		2+3+3	1+1+6

	4+4+1	2+2+5	7+1+1
$\frac{9}{9}$	4+1+4	2+5+2	1+7+1
	1+4+4	5+2+2	1+1+7

The reason for selecting these particular trinomials will be given later when we discuss the evolution of style in rhythm.

III. *Generalization of the Square*
(Any Polynomial)

Formula:

$$(a + b + c + \dots + m)^2 = (a^2 + ab + ac + \dots + am) + \\ + (ab + b^2 + bc + \dots + bm) + (ac + bc + c^2 + \dots + cm) + \\ + \dots +) \, am + bm + cm + \dots + m^2)$$

The following graphs and scores on quintinomials of the $\frac{8}{8}$ series should be worked out.

$2 + 1 + 2 + 1 + 2$
$2 + 1 + 2 + 2 + 1$
$2 + 2 + 1 + 2 + 1$
$1 + 2 + 1 + 2 + 2$
$1 + 2 + 2 + 1 + 2$

The following is an illustration of the first one:

$$(2+1+2+1+2)^2 = (4+2+4+2+4) + (2+1+2+1+2) + (4+2+4+2+4) + \\ + (2+1+2+1+2) + (4+2+4+2+4)$$

Synchronization:

$$8(2+1+2+1+2) = 16 + 8 + 16 + 8 + 16$$

Assuming $\frac{1}{64} =$ ♪

Figure 133.

This is the square of the real "hot" rhythms and it has the utmost plasticity. Nobody realizes, listening to this, that the eight bars are over.

Any bar of $\frac{4}{4}$ treated as $\frac{8}{8}$ will give a perfect countertheme for 8 bars. Take, for example, the song used earlier, *Pennies from Heaven.** The first bar (it may be any bar) is ♩. ♪♩ ♩ , i.e., $3 + 1 + 2 + 2$. It is now squared in order to obtain a countertheme for the first eight bars.

$$(3 + 1 + 2 + 2)^2 = (9 + 3 + 6 + 6) + (3 + 1 + 2 + 2) + \\ + (6 + 2 + 4 + 4) + (6 + 2 + 4 + 4)$$

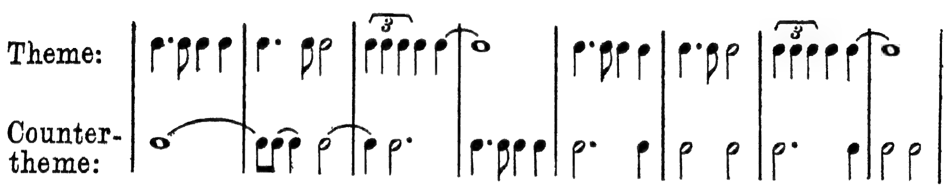


Figure 134.

IV. *Cube of a Binomial*

Cubes produce a new degree of harmonic contrasts. Distributive cubes serve as a new countertheme to the groups of the first and the second power with which they will be synchronized. Cubes are related to squares as the squares are related to the first powers. The number of terms in a distributive binomial of the third power equals 2^3. The recurrence of the central binomial is an invariant of a distributive binomial of the third power.

To obtain the distributive third power of binomials, multiply the distributive second power binomial by the first term of the first power binomial, then by the second term of the first power binomial, and add the products in the same sequence.

Formula:

$$(a+b)^3 = a(a^2 + ab + ab + b^2) + b(a^2 + ab + ab + b^2) = \\ = a^3 + a^2b + \underline{a^2b + ab^2} + \underline{a^2b + ab^2} + ab^2 + b^3$$

The denominator is the quantitative cube of the sum.

To synchronize the distributive square with the distributive cube, it is necessary to multiply each term of the square by the sum of the first power binomial.

To synchronize the first power binomial with its distributive cube, it is necessary to multiply each term of the first power binomial by the square of the sum of the binomial.

$$(2 + 1)^3 = 2(4 + 2 + 2 + 1) + (4 + 2 + 2 + 1) = \\ = (8 + 4 + 4 + 2) + (4 + 2 + 2 + 1) = 27$$

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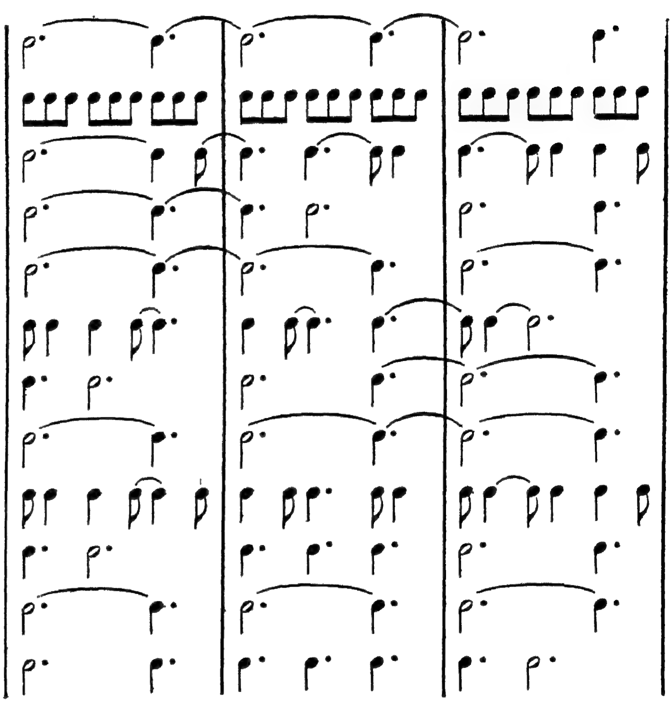
Synchronization of the square with the cube:

$$3(4 + 2 + 2 + 1) = 12 + 6 + 6 + 3 = 27$$

Synchronization of the first power with the cube:

$$9(2 + 1) = 18 + 9 = 27$$

1 + 2 gives the converse of these groups. Assuming $\frac{1}{27}$ = ♪ , we obtain 3 bars in $\frac{9}{8}$ time.



r of the cube

r of the square

r of the original

r $\underline{3 \div 2}$ (synchronized)

Figure 135.

This produces three harmonically contrasting pairs. Using the first, the second and the third power groups in sequence, we obtain a harmonically growing animation.

As cubes become relatively great number-values, it is practical to limit them for musical purposes by the value 3. Thus, the only practical binomials are:

in $\frac{3}{3}$	in $\frac{4}{4}$	in $\frac{5}{5}$
2 + 1	3 + 1	3 + 2
1 + 2	1 + 3	2 + 3

The previous second power resultants can be easily synchronized, being multiplied by the corresponding determinants.

V. *Cube of a Trinomial*

The procedure remains the same, i.e., each term of second power groups must be multiplied consecutively by each term of the first power groups, and the products added in sequence.

Formula:

$$(a + b + c)^3 = a[(a^2 + ab + ac) + (ab + b^2 + bc) + (ac + bc + c^2)] +$$
$$+ b[(a^2 + ab + ac) + (ab + b^2 + bc) + (ac + bc + c^2)] +$$
$$+ c[(a^2 + ab + ac) + (ab + b^2 + bc) + (ac + bc + c^2)] =$$
$$= (a^3 + a^2b + a^2c + a^2b + ab^2 + abc + a^2c + abc + ac^2) +$$
$$+ (a^2b + ab^2 + abc + ab^2 + b^3 + b^2c + abc + b^2c + bc^2) +$$
$$+ (a^2c + abc + ac^2 + abc + b^2c + bc^2 + ac^2 + bc^2 + c^3).$$

The denominator equals the quantitative sum of the trinomial cubed.

Synchronization of the first and the second power trinomials with the distributive third power trinomial must be performed by consecutive multiplication of each term of the first power trinomial by the square of the sum of its terms —and for synchronization of the square—by the sum of its terms.

Example:

$\frac{4}{4}$ $2 + 1 + 1$

$$(2 + 1 + 1)^3 = 2[(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)] +$$
$$+ [(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)] +$$
$$+ [(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)] =$$
$$= [(8 + 4 + 4) + (4 + 2 + 2) + (4 + 2 + 2)] +$$
$$+ [(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)] +$$
$$+ [(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)].$$

Synchronization of the square:

$$4(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1) = (16 + 8 + 8) +$$
$$+ (8 + 4 + 4) + (8 + 4 + 4).$$

Synchronization of the first power:

$$16(2 + 1 + 1) = 32 + 16 + 16$$

Assuming $\frac{1}{64}$ = 𝅘𝅥𝅯

Figure 136.

Trinomials to be cubed and synchronized with their second and first power groups:

$\frac{4}{4}$	$\frac{5}{5}$	
$2+1+1$	$2+2+1$	$3+1+1$
$1+2+1$	$2+1+2$	$1+3+1$
$1+1+2$	$1+2+2$	$1+1+3$
$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$
$3+2+1$	$2+2+3$	$3+3+2$
$3+1+2$	$2+3+2$	$3+2+3$
$1+3+2$	$3+2+2$	$2+3+3$
$2+3+1$		
$2+1+3$		
$1+2+3$		

VI. *Generalization of the Cube*

(Any Polynomial)

To obtain the distributive cube of any group (polynomial) it is necessary to obtain the distributive square first, and multiply all its terms by the terms of the first power polynomial consecutively; then add the products in sequence.

Formula:

$$
\begin{aligned}
(a+b+c+\ldots+m)^3 = {} & a[\,(a^2+ab+ac+\ldots+am)+ \\
& +(ab+b^2+bc+\ldots+bm)+ \\
& +(ac+bc+c^2+\ldots+cm)+\ldots \\
\ldots & +b[\,(a^2+ab+ac+\ldots+am)+ \\
& +(ab+b^2+bc+\ldots+bm)+ \\
& +(ac+bc+c^2+\ldots+cm)\,]+\ldots \\
\ldots & +c[\,(a^2+ab+ac+\ldots+am)+ \\
& +(ab+b^2+bc+\ldots+bm)+ \\
& +(ac+bc+c^2+\ldots+cm)\,]+\ldots \\
\ldots & +m[\,(a^2+ab+ac+\ldots+am)+ \\
& +(ab+b^2+bc+\ldots+bm)+ \\
& +(ac+bc+c^2+\ldots+cm)\,]\ldots
\end{aligned}
$$

Synchronization must be obtained in the manner previously described, i.e., through consecutive multiplication by the square of the sum, or by the sum respectively.

One bar in $\frac{8}{8}$ will give an entire countertheme of 64 bars. *Charts and scores should be made on the following quintinomials:*

$2+1+2+1+2$
$2+1+2+2+1$
$2+2+1+2+1$
$1+2+1+2+2$
$1+2+2+1+2$

VII. *Generalization of All Powers*
(Any polynomial to any power)

When further development of contrasting parts is desirable, powers higher than the cube may be used. In practical application they will concern mostly small groups and small number-values.

The procedure remains the same. To obtain the distributive n^{th} power of any group, it is necessary to obtain the distributive $n - 1$ power of the same group, multiply each term of such group by the terms of the first power group consecutively, and then add the products in sequence.

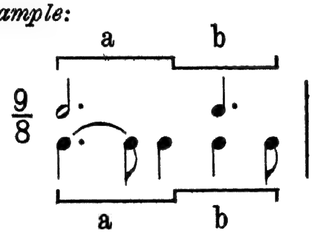
If G stands for a group, this may be expressed through the formula:

$$\boxed{G^n = G(G^{n-1}) \text{ with distribution.}}$$

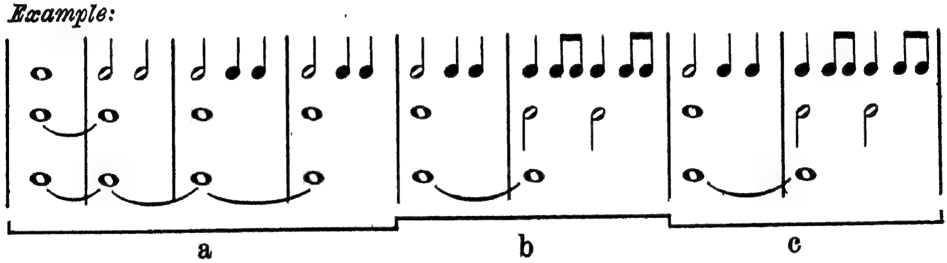
To synchronize the first power group with the n^{th} power group, it is necessary to multiply each term of the first power group by the quantitative $n - 1$ power of the same group. To synchronize the second power group with the n^{th} power group, it is necessary to multiply each term of the second power group by the quantitative $n - 2$ power of the same group, etc.

All permutations in the power groups must be performed through the terms of the preceding power.

Example:



Example:



Example:

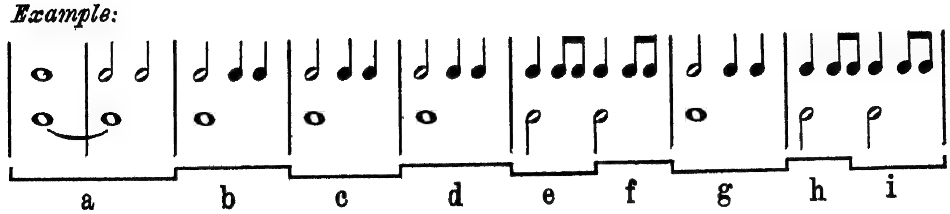


Figure 137

CHAPTER 13

EVOLUTION OF RHYTHM STYLES (FAMILIES)

WE MAY note that uniform groups, as well as non-uniform groups, generate various resultants. Whereas synchronized monomial periodicities generate symmetric polynomial resultants, distribution within any T (the determinant of a series) produces binomials or trinomials characteristic of all resultants where such T is a major generator.

Taking all the possible *binomials* of a certain T and synchronizing them with their converses, trinomial resultants may be obtained. Through synchronizing all permutations of such trinomial resultants (of one series), quintinomials are obtained. The resultants of quintinomials and their permutation-groups produce groups with *nine terms*.

This is a normal serial development as observed in various phenomena (for instance, in crystal formation).

Formula:

$$\boxed{i_n = 2nt_{n-1} - 1}$$

The number of terms in the n^{th} interference-group equals the product of the number of terms in the $n-1^{st}$ interference-group by 2, minus 1.

Example:

The first interference-group		$i_1 = 2$
The second	" "	$i_2 = (2\times2) - 1 = 3$
The third	" "	$i_3 = (2\times3) - 1 = 5$
The fourth	" "	$i_4 = (2\times5) - 1 = 9$
The fifth	" "	$i_5 = (2\times9) - 1 = 17$

With the limit 9 as a determinant of a series, the maximum non-uniform resultants are quintinomials. Uniform resultants follow the maximum non-uniform resultants. The greater the number-value of a determinant, the more interference-groups it produces. While the determinant 3 produces only one non-uniform interference-group, 9 produces three non-uniform interference-groups.

All the consecutive interference-groups generated by one determinant constitute the evolution of all rhythmic patterns in the corresponding family (style).

This makes it possible to predict all future rhythmic patterns of one family as well as to trace the origin of more involved rhythms.

As previously mentioned, the original (binomial) interference-groups may be obtained directly from a determinant. For example, the distribution of a determinant 5 gives 3 + 2 and 4 + 1, and their permutations. These binomials are the first and the last binomials of the resultants obtained from two uniform monomial generators in which the determinant of a series is a *major* generator (a).

Therefore, 3 + 2 are the first two terms of a resultant where a = 5; 4 + 1 are the last two terms of a resultant where a = 5.

In order to trace the origin of a binomial with respect to two uniform generators, it is necessary to take the greater number-value of the binomial and to assign it as a minor generator (b). The sum of the binomial is the major generator.

Example:

 is a given binomial.

Find the determinant of the series.
$5 + 3 = 8$ The determinant is $\frac{8}{8}$
Find the *a* and *b* generators.
$b = 5$ $a = 5 + 3 = 8$

The binomial represents the first two terms of $r_{8 \div 5}$.

Existing music often works on more than one determinant, thus producing various hybrids. It is very easy to trace the origin of any rhythmic hybrid, as such groups which are alien to the family are indicated in musical notation by the numbers. For instance, the triplets in $\frac{4}{4}$ time; the duplets in $\frac{3}{4}$ time, etc.

Leaving theories aside for the moment, I believe that the actual cause of any new interference-binomial appearing in the world is the urge toward *unbalancing*, that is, the *centrifugal* tendency.

In the light of such a hypothesis, the origin of the "Charleston" $5 + 3$ binomial may be explained as a tendency to disturb the balance of $\frac{2}{4} + \frac{2}{4}$ in $\frac{4}{4}$ or $\frac{4}{8} + \frac{4}{8}$ in $\frac{8}{8}$.

Chronologically, the more unbalanced binomials (such as $\frac{7}{8} + \frac{1}{8}$) appear later than the balanced ones (such as $\frac{5}{8} + \frac{3}{8}$), regardless of their structural complexity. While $5 + 3$ has been known in the American dance-music for some time, $1 + 7$ appeared as a prominent pattern only with the song, "Organ Grinder's Swing."

The prediction of ***new rhythmic families*** to come is based on the principle of the growth-through-power series.

So far we have had, during the entire range of recorded history, the evolution of $\frac{2}{2}$ into its second power $\frac{4}{4}$, and into its third power $\frac{8}{8}$. Most probably $\frac{9}{9}$ will take its place in the near future as the second power of $\frac{3}{3}$. The series, $\frac{6}{6}$ is an exhausted European hybrid, being the product of 2×3. The $\frac{5}{5}$ and $\frac{7}{7}$ series are Oriental series of old origin. They may become fashionable for a while in the Western musical world.

Thus, the series of factorial-fractional continuity express the evolutionary forms in the two-coordinate system.

A. "Swing" Music

The following is an analysis of the phenomenon known as "swing music" —it is an analysis of "swing" as it is *performed*, not as it is written out on paper.

In view of the fact that triplets of eighths in common time are very prominent in this type of playing, particular attention must be given to the value 3, its multiples and its powers. Knowing from the previous analysis that $\frac{9}{9}$ is the most probable candidate for the new style, I have studied all the "waltz-like" phenomena which have appeared during the last few decades. The utmost plasticity

of the Viennese type of waltz (such as the waltz from *Rosenkavalier* by Richard Strauss) is due to this figure:

Figure 138.

The above is $3 + 1 + 5 + 1 + 5 + 1 + 2$ etc., where the characteristic grouping is one appearing between the two fives.

The trinomial $4 + 1 + 4$ can be found in the second interference group in the $\frac{8}{8}$ series. Here again (as in the case of powers) is the ***intuitive approximation*** of the correct patterns. The *waltz* pattern is "trying to evolve" into its second-power. The idea of unity between the two greater number-values is right, but the number-values are only approximately correct.

There are other hybrids which are characteristic of Viennese waltzes. For example, a hybrid between $\frac{3}{8}$ and $\frac{4}{4}$ series: $3 + 1$ coming from $\frac{4}{4}$ series and placed into $\frac{6}{8}$ series ($\frac{3}{4}$ time).

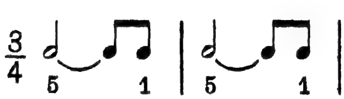
Figure 139.

Most jazz ("Charleston Rhythm") is a hybrid between $\frac{8}{8}$ and $\frac{6}{8}$ series. Examine the following:

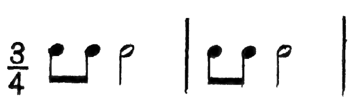
Figure 140.

And so, too, with all patterns of $\frac{6}{8}$ series put into $\frac{8}{8}$ time.

The original binomial of this style is most characteristic of Viennese waltzes:



Also the trinomial of $\frac{6}{8}$ series: $1 + 1 + 4$



See "Rosenkavalier"

Figure 141.

This, being placed into $\frac{8}{8}$ time, produces:

Figure 142.

An approach to the $\frac{9}{9}$ family from another angle is "swing." The foundation of the latter is the fox-trot in triplets. Rhythms of $\frac{4}{4}$ and $\frac{8}{8}$ are modified on a basis of $\frac{12}{12}$ or $\frac{12}{8}$ and $\frac{4}{4}$ (in triplets) musically.

The common denominator units are the eighths.

Figure 143.

Through syncopation tendencies, plus the $\frac{3}{3}$ series binomial, we obtain all the possible patterns of "swing."

The original patterns:

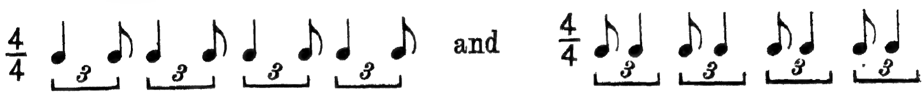


Figure 144.

The syncopated patterns:

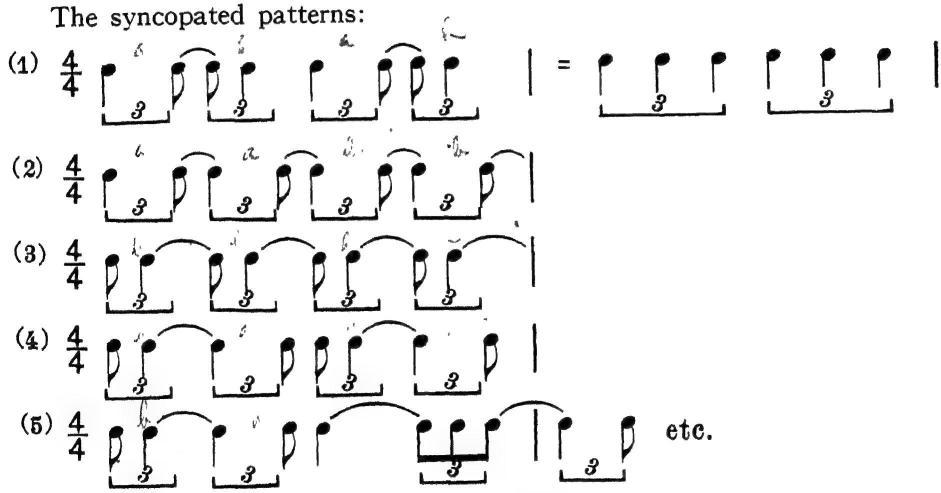


Figure 145.

The characteristic values are:

2, i.e., ♪♪ or ♩ (an eighth tied to an eighth, or a quarter).

3, i.e., ♩♪ or ♪♩ (a quarter tied to an eighth, or an eighth tied to a quarter).

4, i.e., ♩♩ (a quarter tied to a quarter).

Often some of these number-values appear as rests.

It is interesting to note that, even in bands such as Benny Goodman's, all orchestral parts are *written* either in the $\frac{4}{4}$ series patterns or $\frac{8}{8}$ series patterns, but are then translated into "swing" while being played.

Figure 145, line 4, is the first true pattern of a $\frac{9}{9}$ series trinomial $(4 + 1 + 4)$. This pattern, with greater consistency, would appear as in line 4a. The number-values are correct but the group unit is wrong. It is applied in the wrong type of measure, $\frac{12}{12}$ instead of $\frac{9}{9}$.

Thus, we can see that both the Viennese waltz and the fox-trot are engaged in a struggle for crystallization of the $\frac{9}{9}$ family.

All rhythmic interference groups have as the only alternatives in their evolution: *either to evolve the higher powers of the same patterns, or to evolve into the higher powers of the same determinant.*

The entire process of the evolution of rhythmic families may be expressed as follows:

$$
\begin{array}{ccccccc}
r_1 & \ldots & Pr_1 & \ldots & SPr_1 & \ldots & i_1 \\
r_2 & \ldots & Pr_2 & \ldots & SPr_2 & \ldots & i_2 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
r_1^2 & \ldots & Pr_1^2 & \ldots & SPr_1^2 & \ldots & i_1 \\
r_2^2 & \ldots & Pr_2^2 & \ldots & SPr_2^2 & \ldots & i_2 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
r_1^n & \ldots & Pr_1^n & \ldots & SPr_1^n & \ldots & i_1^n \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
r_n^n & \ldots & Pr_n^n & \ldots & SPr_n^n & \ldots & i_n^n \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}
$$

r—is the resultant
P—Permutations
S—Synchronization
i—Interference

Continuous dotted line represents uniformity.

The first resultant (r_1) produces its permutations (Pr_1) which form the first interference-group; these being synchronized (SPr_1) produce the first interference. The resultant of this interference is the second resultant (r_2), etc.

The following graphs should be converted into musical notation:

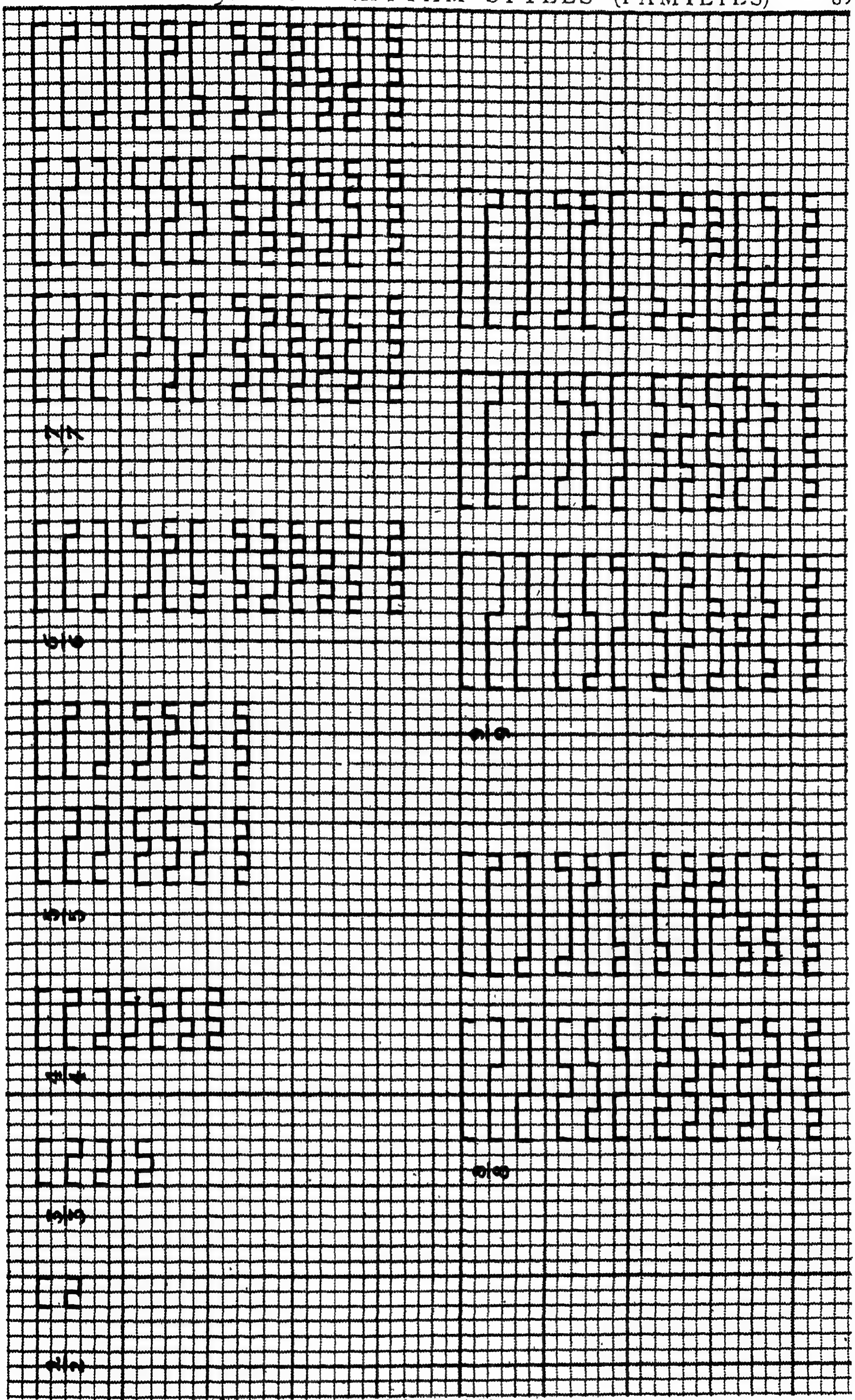


Figure 146.

CHAPTER 14

RHYTHMS OF VARIABLE VELOCITIES

THE only constant velocity known in the physical world is that of light By introducing *constant* velocities into the art forms we try to simplif the scheme of surrounding phenomena. But different forms of progressive *variable* speed are also well known to us—through biological growth, througl gravity, through different series of acceleration and different ratios of acceleration

The ratios of acceleration through gravity grow rapidly. Series like th natural harmonic series reveal a much greater graduality in their rates of ac celeration. The urge for freedom in a musical performance often reveals itsel by the speeding up or slowing down of a certain passage written out in musica notation as a uniform group. The increase and the decrease of speed may be however, merely two reciprocals of the same series.

Rhythms based on constant velocities are either continuous repetitions o the terms of a monomial periodicity, or of several monomial or binomia. per iodicities synchronized. But rhythms based on *variable velocities* or progression are single terms belonging to different periodicities.

In the following list of different mathematical series, the series of gravit has been eliminated as it is too "intense" for esthetic purposes. The simple the ratios of acceleration, the more obvious the forms of acceleration will appea to the listener of music. Such is the case of doubling or quadrupling the origina speed. In all three forms of representation—number, graph and music—ther is a constant speed which reveals various forms of acceleration through th actual number-values. Music expressed in such a way may be performed o conducted in a constant tempo—counting out all the durations exactly as the are represented by the number-values.

Here is a list of various series of acceleration:

(1) *Natural Harmonic Series.*
1, 2, 3, 4, 5, 6, 7, 8, 9 . . . .

(2) *Arithmetical Progressions.*
+ n constant
+ 2 — 1, 3, 5, 7, 9 . . . .
+ 3 — 1, 4, 7, 10, 13 . . .

(3) *Geometrical Progressions.*
× n
× 2 — 1, 2, 4, 8, 16 . . . .
× 3 — 1, 3, 9, 27 . . . .
× 2 — 3, 6, 12, 24, 48 . . . .
× 3 — 2, 6, 18, 54. . . .

(4) *Power Series.*
n^{th} power
2, 4, 8, 16, 32 . . . .
3, 9, 27, 81 . . . .
5, 25, 125. . . .

(5) *Summation Series.*
1, 2, 3, 5, 8, 13, 21 . . . .
1, 3, 4, 7, 11, 18 . . . .
1, 4, 5, 9, 14, 23 . . . .

(6) *Arithmetical Progressions with Variable Differences.*
$1^{+1}, 2^{+2}, 4^{+3}, 7^{+4}, 11^{+5}, 16^{+6}, 22^{+7}, \ldots.$

(7) *Prime Number Series.*
1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 . . . .

It is important to note that the rate of acceleration varies within each given series. For instance, in the first series, the ratio between the second and the first value is $2 \div 1$, while the ratio between the sixth and the seventh number-values is $7 \div 6$. Therefore, when a greater speed of acceleration is desired, it is advisable to take a number-value considerably higher than unity. This concerns mostly the type of series that grow with greater speed, such as arithmetical progressions.

Although any series may be used for carrying out different types of accelerando and rallentando, the most suitable are those belonging to a given rhythmic family. For example, to obtain a proper type of "accel. rall." in the $\frac{8}{8}$ series ("Charleston" family), we should use the most suitable series, which is the first summation series—for all the number-values of this series represent generators of the $\frac{8}{8}$ series. Likewise, for any music written in the $\frac{4}{4}$ series, such as marches, polonaises, mazurkas, etc., we shall have the most appropriate "accel. rall." expressed through the second summation series. Similarly, music in the $\frac{9}{8}$ series requires the third summation series for its "accel. rall."

In many cases the freedom of the performer leads to the use of numbers *alien* to the series in which a given piece of music is composed; this causes an obvious dissatisfaction and many listeners—with a natural sense of rhythm—feel that something is wrong but cannot explain the cause of such rhythmic irregularity. Speeding up and slowing down is a natural tendency in much folk music. Some of the most striking examples may be found in Hungarian music (see Liszt's *Second Hungarian Rhapsody*) and music of gypsies in various countries.

The technique of variable speeds becomes extremely important in dealing with stage productions, compositions for the dance, and especially film music.

In film music, the animation technique in particular requires absolute precision of timing. In illustrating a "chase," one has to time the corresponding music in such a fashion that the whole period of acceleration will be limited to a definite portion of time, with the precision of $\frac{1}{24}$ of a second (the duration of a single shot in a film synchronized with sound). In some instances of descriptive music, especially those dealing with the speeding up and slowing down of mechanisms, similar precision adds a great deal to the esthetic effects. This method permits the use of the "accel. and rall." of the same rate and series as counter-rhythms, as well as the resultants of their interference (we shall call them the *resultants of acceleration*).

Thus, we acquire four fundamental forms to be used as material for acceleration groups:

(1) Increasing velocity (accelerando).
(2) Decreasing velocity (rallentando).
(3) The two [(1) and (2) velocities] combined.
(4) The resultant of acceleration.

Forms (1) and (2) may be used for the introductions and conclusions (codas); forms (3) and (4)—for the climaxes.

In either case it is more practical to find the decreasing velocity (increasing number-values) first, as it is more practical to have a definite *initial* number-value.

For example, if we use the natural harmonic series and start with unity, we find a practical stopping point at 8, because $1 + 2 + 3 + 4 + 5 + 6 + 7 + + 8 = 36$. The sum must be in a simple relation to the multiples of the group-unit (measure). The value 36 offers a number of possibilities. Four-and-a-half of $\frac{4}{4}$ time (in eighths); nine bars in $\frac{4}{4}$ time (in quarters); four bars of $\frac{9}{8}$ time (in eighths); six bars in $\frac{6}{8}$ time (in eighths); twelve bars in $\frac{3}{4}$ time (in quarters).

Using the first summation series for accelerando in the $\frac{8}{8}$ family, we obtain a practical value of 32 by summing up $1 + 2 + 3 + 5 + 8 + 13$. This offers exactly four bars of $\frac{8}{8}$ time, and in music of eight-bar groups, the ratio becomes very simple ($\frac{8}{4} = \frac{2}{1}$).

A. Acceleration in Uniform Groups.

Examples:

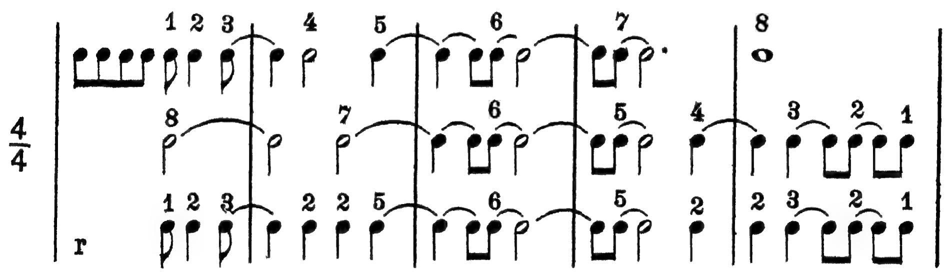


Figure 147.

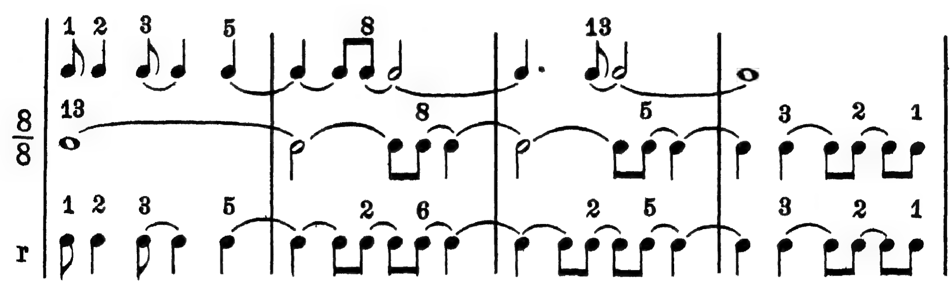


Figure 148.

B. Acceleration in Non-Uniform Groups.

The technique of progressive addition to the original number-value now becomes an addition of respective values to the terms of the original group.

Here is an example of progressive addition through natural harmonic series:

The original group: $3 + 1 + 2$

$(3 + 1 + 2) + (6 + 2 + 4) + (9 + 3 + 6) + \dots$

Figure 149.

Thus, the $3 + 1 + 2$ group appears with the coefficients which are the terms of a certain series (natural series in this case).

$$\begin{aligned} (3 + 1 + 2) &= 6 \\ 2\,(3 + 1 + 2) &= 12 \\ 3\,(3 + 1 + 2) &= 18 \end{aligned}$$

C. Rubato.

"Rubato" is the process of unbalancing a balanced binomial, or the process of balancing an unbalanced binomial. In terms of quantities, the first process *increases the complexity* of an original ratio; the second—*decreases the complexity* of an original ratio.

The process of unbalancing a balanced binomial must be carried out by means of a *unit of deviation.* This unit of deviation, supposedly an infinitesimal (in the calculus, dx) becomes a rational fraction in the field of musical rhythm. The most satisfactory results are produced by means of a *standard unit of deviation,* which is defined in this theory as $\frac{1}{t}$, i.e., the unit of a series of fractional continuity. We shall call it τ (the Greek letter, tau).

Formula for a standard unit of deviation:

Example I: $\tau = \frac{1}{t}$

Take the second theme of Chopin's *Valse in c♯ minor.* All bars of this theme have the following construction: $\frac{3}{4}$. Many performers play the first bar of this theme like this: . Let us see what causes the transformation of , i.e., $2 + 2$ into , i.e., $3 + 1$. The way this binomial is written makes it $\frac{2}{4} + \frac{2}{4}$. In this case we have $\frac{4}{4}$ series, where $\tau = \frac{1}{4}$. Therefore, the process of unbalancing the original binomial $(\frac{2}{4} + \frac{2}{4})$ may be expressed as follows:

$(\frac{2}{4} + \tau) + (\frac{2}{4} - \tau) = (\frac{2}{4} + \frac{1}{4}) + (\frac{2}{4} - \frac{1}{4}) = \frac{3}{4} + \frac{1}{4}$, which means .

Example II.

Take any fox-trot where you find $\frac{4}{4}$ 𝅗𝅥 𝅗𝅥 in the printed copies. In a performance you hear it as 𝅗𝅥 ♪♩. or ♩. ♪𝅗𝅥 . Let us follow the previous procedure. The two half-notes in a fox-trot time belong in reality to the $\frac{8}{8}$ series. They must be expressed as $\frac{8}{8} = \frac{4}{8} + \frac{4}{8}$. In this case $\tau = \frac{1}{8}$. By adding $\frac{1}{8}$ to the first term and subtracting from the second we obtain $(\frac{4}{8} + \frac{1}{8}) + (\frac{4}{8} - \frac{1}{8}) = \frac{5}{8} + \frac{3}{8}$.

In both examples the process of unbalancing may be reversed, i.e., $\frac{1}{4} + \frac{3}{4}$ and $\frac{3}{8} + \frac{5}{8}$.

The process of balancing an unbalanced binomial is a typical case of the ratio simplification as we find it in "swing" performance. Write ♬ in $\frac{4}{4}$ time, and the "swingsters" will play it ♩ ♪ (3) in the same $\frac{4}{4}$ time. The same thing happens in "boogie-woogie," where the written accompaniment of broken octaves $\frac{8}{8}$ ♫♫♫♫ is played $\frac{4}{4}$ ♩♪ ♩♪ ♩♪ ♩♪ (3, 3, 3, 3), and the upper parts are handled as "Charleston." The ratio 3 + 1 becomes 2 + 1, which is closer to balance.

All the other forms of "rubato" playing are either small groups of accelerando and rallentando, or are the same as:

D. Fermata (Hold).

There are two types of fermata; the two may seem to have an entirely different character, but in reality this difference is purely quantitative.

The first type produces the effect of a full stop. It is commonly used at the very beginning, at the very end, at the moment of a climax, or before a new theme enters.

In writing out such a fermata, it is best to make it a simple multiple of the preceding or the following values, or the sum of the preceding group of uniform values.

An example of transcription of a fermata of the first type:

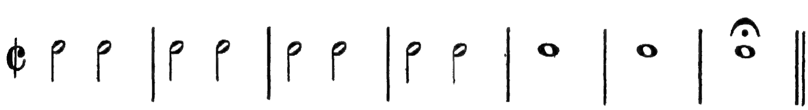


Figure 150.

The first four bars move in halves, the following two—in wholes. By assigning a double value to the last note, we satisfy two requirements. *One:* we produce a simple ratio of 1 + 1 + 2 in the last three bars, i.e., our last note is the simplest multiple for each of the two preceding notes; *two:* such a multiple compensates the first four bars, thus creating a balance 4 + 4.

Transcription:

Figure 151.

Obviously, this gives the utmost satisfaction.

The second type of fermata is a *temporary delay.* The method of creating simple ratios is most effective in transcribing this type of fermata. Here is a transcription of the following example:

Figure 152.

By isolating the group preceding the fermata we obtain $\frac{3}{4}$; by isolating the group with a fermata we obtain $\frac{2}{8}$

Now the entire group appears as follows:

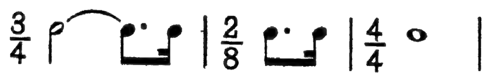


Figure 153.

This being transcribed into number-values gives:

$$\left(\tfrac{11}{16}+\tfrac{1}{16}\right)+\left(\tfrac{3}{16}+\tfrac{1}{16}\right)+\tfrac{16}{16}$$

The first bar is related to the second bar as $3 \div 1$ because $\frac{3}{4}=\frac{6}{8}$; $\frac{6}{8}\div\frac{2}{8}=\frac{3}{8}\div\frac{1}{8}$.

Simplifying the ratio $3 \div 1$ into $2 \div 1$, we obtain the following musical measures: $\frac{3}{4}+\frac{3}{8}$. Making the first duration in the second bar (the fermata) longer, we obtain a binomial $\frac{2}{8}+\frac{1}{8}$, or in musical notation:

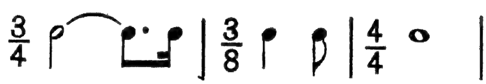


Figure 154.

This procedure makes the absolute value of the fermata note increase in a very subtle way, $\frac{1}{16}$ longer than the original duration. Here are the numbers from the musical transcription:

$$\left(\tfrac{11}{16}+\tfrac{1}{16}\right)+\left(\tfrac{4}{16}+\tfrac{2}{16}\right)+\tfrac{16}{16}$$

By comparing the original and the transcription, it is easy to see that the fermata note (which originally was $\frac{3}{16}$) became $\frac{4}{16}$, thus gaining $\frac{1}{16}$.

The rhythm of variable velocities presents a fascinating field for study and exploration. The very thought that various rhythmic groups may speed up and slow down at various rates, appearing and disappearing, is overwhelming.

This idea stimulates one's imagination towards the complex harmony of the universe, where different celestial bodies (comets, stars, planets, satellites) coexist in harmony of variable velocities.*

*So ends the exposition of Schillinger's theory of rhythm, to be followed next by the theory of pitch-scales. Although the casual reader may not be entirely aware of it, the specific techniques have now been set forth whereby all possible rhythms, of any nature whatsoever, may be derived. And these "all possible" rhythms have been grouped into related families, sub-families and "styles," so that what is an infinity of rhythms may be rapidly and practically utilized in the actual composition of music. (Ed.)

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK II

THEORY OF PITCH-SCALES

BOOK TWO

THEORY OF PITCH-SCALES

PRELIMINARY REMARKS ON THE THEORY OF PITCH-SCALES

Just as the first book developed the theory and practice of rhythms, which concern durations in *time*, so does the present portion of the Schillinger system develop the theory and practice of that other basic factor in music, *pitch*. The theory of pitch-scales concerns pitch considered in continuity, i.e., one tone sounding after another. When pitch is considered in simultaneity, i.e., tones sounding at the same time, questions of harmony and counterpoint are involved. These are discussed in later sections of the entire work. Schillinger approaches the pitch question as, first, a problem in primary selective systems—or tuning; then, as a problem of abstracting from all the tones made available by the tuning system those particular tones which are to be used in any composition. These sets of tones, called pitch-scales or—more commonly—just "scales," furnish raw material for both melody and harmony. (Ed.)

CHAPTER 1

PITCH SCALES AND EQUAL TEMPERAMENT

THE INTONATION units and the intervals between them constitute the elements of the pitch-scales. The intonation units are named *pitch-units* (p) in the following exposition, and the intervals between the pitch-units are called *pitch-intervals* (i).

A pitch-scale is a sequence of pitch-units following in consecutive order (increasing or decreasing frequency). The number of pitch-units in scales, constructed within the equal temperament of twelve, ranges from 1 to 144. Families of pitch-scales, as well as families of time-scales (rhythm), serve as esthetic material for racial, national and local expressions.

The subject of this portion of my theory consists of the following items:

(1) Construction of melodic forms from pitch-scales;

(2) Modification of melodic forms;

(3) Composition of melodic continuity;

(4) Deduction of harmonic forms from pitch-scales.

All pitch-scales may be classified into the following four groups:

Group One: One root-tone. Range limit = 11.

Group Two: One root-tone. Range over 12.

Group Three: More than one root-tone. Range = 12.

Group Four: More than one root-tone. Ranges: 24, 36, 60, 132.

The number values here express the number of semitone units which will serve as standard units for measuring pitch within the equal temperament of 12.*

*A few additional words of explanation may be useful here. Pitch is, of course, a question of the frequencies of the sound waves, i.e., the number of vibrations per second. In order to produce music, it is first necessary to determine *which particular* frequencies will be used as points of reference. We take this for granted now, but working it out was a subject of much theoretical struggle over the centuries. From the set of *all possible* frequencies (in this case, all audible frequencies), it was thus necessary to *select* a smaller set which becomes a *primary selective system.*

In equal temperament tuning, the 12 "tones" comprising the system are c, c sharp, d, d sharp, e, f, f sharp, g, g sharp, a, a sharp, b—followed by another c, the latter c being one octave higher than the former (one octave higher means that the frequency is exactly *doubled*). The flatted tones are considered in equal temperament to be identical to (that is, enharmonics of) the sharped tones.

How are these twelve basic tones tuned, that is, what are their frequency ratios? They are related in the following manner: if we construct a series of the twelfth root of 2, $\sqrt[12]{2}$, in such a fashion that the root remains 12 while the power of 2 increases from zero to 12, we will have a series which corresponds to the actual frequencies of equal temperament tuning. Note that the first term is 1, for the zero power of 2 is 1 and the 12th root of 1 is also 1. Here

The mathematical expression for this system of tuning (developed by Andreas Werckmeister in Germany, in 1691) is $\sqrt[12]{2}$. Two expresses the octave ratio of frequencies, i.e., $2 \div 1$; the exponent 12 expresses the number of the uniform ratios within one octave. The semitones are integers when they express the logarithms to the base $\sqrt[12]{2}$.

we will express the root powers as fractions, in which the denominators represent roots and the numerators represent powers. The series is:

$2^{\frac{0}{12}}$	$2^{\frac{1}{12}}$	$2^{\frac{2}{12}}$	$2^{\frac{3}{12}}$	$2^{\frac{4}{12}}$	$2^{\frac{5}{12}}$	$2^{\frac{6}{12}}$
C	C♯	D	D♯	E	F	F♯
$2^{\frac{7}{12}}$	$2^{\frac{8}{12}}$	$2^{\frac{9}{12}}$	$2^{\frac{10}{12}}$	$2^{\frac{11}{12}}$	$2^{\frac{12}{12}}$	
G	G♯	A	A♯	B	C	

The frequencies are logarithmically related as the terms of the above series are related. In reality, as Schillinger points out elsewhere, there are at least two additional matters to keep in mind: (1) music as actually performed exhibits a much higher variety of intonation than the equal temperament system would suggest, the tuning system supplying simply the points of reference; (2) music tends toward a greater fluidity of intonation more nearly approximating actual curves, as in the instance of what are popularly called "scoops" in pitch in violin performances, or in "hot" trumpet intonations. (Ed.)

CHAPTER 2

FIRST GROUP OF PITCH-SCALES

"DIATONIC" AND RELATED SCALES

IN THE FOLLOWING discussion, all the scales are constructed from *c*; they are classified according to the number of pitch-units and the number-values for the intervals.

In each case the full number of technical possibilities is described.

A. ONE-UNIT SCALES. ZERO INTERVALS.

[The number of scales: one.]

Scales with one constant pitch-unit constitute the so-called "monotone" music and may actually be found among the primitives.

The natives of southern Patagonia (Tierra del Fuego) have one pitch-unit scale and are not familiar with any other form of musical intonation. This music has been recorded on dictaphone cylinders by Erich von Hornbostel, Berlin University, and the records are—or were—located in the phonogram archive of the Psychological Institute of Berlin University. One copy exists in the archives of the New School in New York City. This music compensates for its lack of variety in intonation by the variety of its rhythm.

Music of our civilization quite frequently deals with one pitch-unit scales. Instances are to be found in sustained tones (pedal points) and many rhythmic passages executed by individual instruments, such as rhythmic trumpet passages.

The only technical procedure possible with such scales is: *superimposition of the time-rhythm.*

Scale Time-Rhythm = $\underline{r_{4\div3}}$

Figure 1.

B. TWO-UNIT SCALES. ONE INTERVAL.

[The number of scales: eleven.]

Table of Intervals

Scale	1	2	3	4	5	6	7	8	9	10	11
Interval	c–d♭	c–d	c–e♭	c–e	c–f	c–f♯	c–g	c–a♭	c–a	c–b♭	c–b

Technical procedures:

(1) Definition of the number of melodic forms.
(2) Combinations of melodic forms.
(3) Continuity of melodic forms through permutations.
(4) Coefficients of recurrence of the melodic forms.
(5) Superimposition of the time-rhythm.

(1) There are two melodic forms; one combination of the latter is possible with the two-unit scales.

(2) The melodic forms are: $a_1 + b_1$ and $b_1 + a_1$, where a_1 and b_1 are the pitch-units. Thus, the two forms become a_2 and b_2 respectively.

(3) A neutral continuity of melodic forms may be obtained by means of permutations of the higher order. In order to give individual expression to the continuity of melodic forms, it is necessary to introduce a specified recurrence into such continuity.

(4) Two elements (a_2 and b_2) permit the application of binomial and polynomial coefficients (with an even number of terms). For example:

$3a_2 + b_2$ or $5a_2 + 3b_2$ etc.;
or $3a_2 + b_2 + 2a_2 + 2b_2 + a_2 + 3b_2$
or $4a_2 + b_2 + 3a_2 + 2b_2$ etc.

By placing such a continuity of melodic forms into various types of bars and using uniform durations for each pitch-unit, one may achieve an extraordinary diversity of effects.

Scale: 5; Melodic Form: $2a_2 + b_2$

Figure 2.

(5) The final procedure is superimposition of time-rhythm on the preselected form of melodic continuity.

As it follows from the Theory of Rhythm [see Book I], such a continuity is subjected to synchronization and interference. The components of synchronization are the number of attacks in melodic continuity and the number of attacks in the rhythmic group.

Rhythm: $r_{4\div3}$; Measure: $\frac{3}{4}$

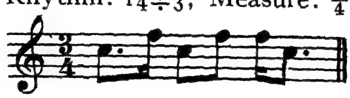


Figure 3.

Rhythm: $r_{\underline{3 \div 2}}$; Measure: $\frac{9}{8}$

Figure 4.

C. Three-unit Scales. Two Intervals.

[The number of scales: 55.]

Table of Intervals

1+1	2+1	3+1	4+1	5+1	6+1	7+1	8+1	9+1	10+1
1+2	2+2	3+2	4+2	5+2	6+2	7+2	8+2	9+2	
1+3	2+3	3+3	4+3	5+3	6+3	7+3	8+3		
1+4	2+4	3+4	4+4	5+4	6+4	7+4			
1+5	2+5	3+5	4+5	5+5	6+5				
1+6	2+6	3+6	4+6	5+6					
1+7	2+7	3+7	4+7						
1+8	2+8	3+8							
1+9	2+9								
1+10									

Material:

(1) The number of melodic forms = 6.

$a_1 + b_1 + c_1 = a_2$
$a_1 + c_1 + b_1 = b_2$
$c_1 + a_1 + b_1 = c_2$
$b_1 + a_1 + c_1 = d_2$
$b_1 + c_1 + a_1 = e_2$
$c_1 + b_1 + a_1 = f$

(2) Combinations of melodic forms.

(a) Combinations by two:

$a_2 + b_2$	$b_2 + c_2$	$c_2 + d_2$	$d_2 + e_2$	$e_2 + f_2$
$a_2 + c_2$	$b_2 + d_2$	$c_2 + e_2$	$d_2 + f_2$	
$a_2 + d_2$	$b_2 + e_2$	$c_2 + f_2$		
$a_2 + e_2$	$b_2 + f_2$			
$a_2 + f_2$				

Each combination has two permutations. The number of cases: $15 \times 2 = 30$.

(b) Combinations by three:

(α) two places identical:

$a_2 + a_2 + b_2$
- - - - - - - Total number: $15 + 15 = 30$
$a_2 + b_2 + b_2$ Three permutations each.
- - - - - - -

The number of cases: $30 \times 3 = 90$.

(β) all three places different:

$a_2 + b_2 + c_2$	$a_2 + c_2 + d_2$	$a_2 + d_2 + e_2$	$a_2 + e_2 + f_2$
$a_2 + b_2 + d_2$	$a_2 + c_2 + e_2$	$a_2 + d_2 + f_2$	
$a_2 + b_2 + e_2$	$a_2 + c_2 + f_2$		
$a_2 + b_2 + f_2$			
$b_2 + c_2 + d_2$	$b_2 + d_2 + e_2$	$b_2 + e_2 + f_2$	
$b_2 + c_2 + e_2$	$b_2 + d_2 + f_2$		
$b_2 + c_2 + f_2$			
$c_2 + d_2 + e_2$	$c_2 + e_2 + f_2$		
$c_2 + d_2 + f_2$			
$d_2 + e_2 + f_2$			

Twenty combinations, six permutations each. The number of cases: $20 \times 6 = 120$.

(c) Combinations by four:

(α) Three identical places:

$a_2 + a_2 + a_2 + b_2$
- - - - - - - - - - Total number: $15 + 15 = 30$.
$a_2 + b_2 + b_2 + b_2$ Four permutations each.

The number of cases: $30 \times 4 = 120$.

(β) Two identical pairs:

$a_2 + a_2 + b_2 + b_2$ Total number $= 15$.
Six permutations each.

The number of cases: $15 \times 6 = 90$.

(γ) Two identical places:

$a_2 + a_2 + b_2 + c_2$
- - - - - - - - - - Total number: $20 \times 3 = 60$
$a_2 + b_2 + b_2 + c_2$
- - - - - - - - - - Twelve permutations each.
$a_2 + b_2 + c_2 + c_2$

The number of cases: $60 \times 12 = 720$.

(δ) All four places different.

$a_2 + b_2 + c_2 + d_2$ $a_2 + b_2 + d_2 + e_2$ $a_2 + b_2 + e_2 + f_2$
$a_2 + b_2 + c_2 + e_2$ $a_2 + b_2 + d_2 + f_2$
$a_2 + b_2 + c_2 + f_2$

$a_2 + c_2 + d_2 + e_2$ $a_2 + c_2 + e_2 + f_2$
$a_2 + c_2 + d_2 + f_2$

$a_2 + d_2 + e_2 + f_2$

$b_2 + c_2 + d_2 + e_2$ $b_2 + c_2 + e_2 + f_2$
$b_2 + c_2 + d_2 + f_2$

$b_2 + d_2 + e_2 + f_2$

$c_2 + d_2 + e_2 + f_2$

Fifteen combinations, 24 permutations each. The number of cases: $15 \times 24 = 360$.

(d) Combinations by five may contain four, three or two identical places, or two identical pairs. As the material begins to grow to enormous quantities, this exposition will be limited by referring to the combinations with five different places.

$a_2 + b_2 + c_2 + d_2 + e_2$ $a_2 + b_2 + c_2 + e_2 + f_2$
$a_2 + b_2 + c_2 + d_2 + f_2$

$a_2 + b_2 + d_2 + e_2 + f_2$

$a_2 + c_2 + d_2 + e_2 + f_2$

$b_2 + c_2 + d_2 + e_2 + f_2$

Six combinations, 120 permutations each. The number of cases: $6 \times 120 = 720$.

(e) One combination by six:

$a_2 + b_2 + c_2 + d_2 + e_2 + f_2$

720 permutations.

The number of cases: $1 \times 720 = 720$.

(3) Continuity of melodic forms through permutations.

Circular permutations can be used as well. They give the best combinations by the number of elements.

$a_1 + b_1 + c_1 = a_2$
$b_1 + c_1 + a_1 = b_2$ Three melodic forms
$c_1 + a_1 + b_1 = c_2$

Combinations by 2:

$a_2 + b_2$ $b_2 + c_2$
$a_2 + c_2$

Three combinations, 2 permutations each. Total: $3 \times 2 = 6$.

Combinations by 3:

$a_2 + b_2 + c_2$

One combination, 6 permutations. Total: $1 \times 6 = 6$.

(4) **Coefficients of recurrence of the melodic forms.**

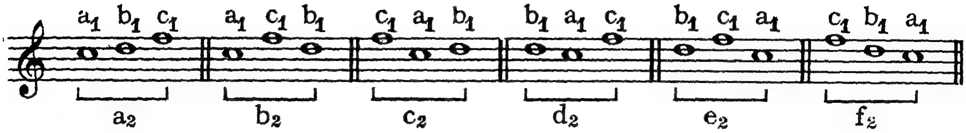


Figure 5.

(5) **Superimposition of time-rhythm.**

Melodic Form: $3a_2 + b_2 + 2c_2$; Rhythm: $r_{4 \div 3}$

Measure: $\frac{3}{4}$

Rhythm: $r_{5 \div 3}$ Measure: $\frac{3}{4}$

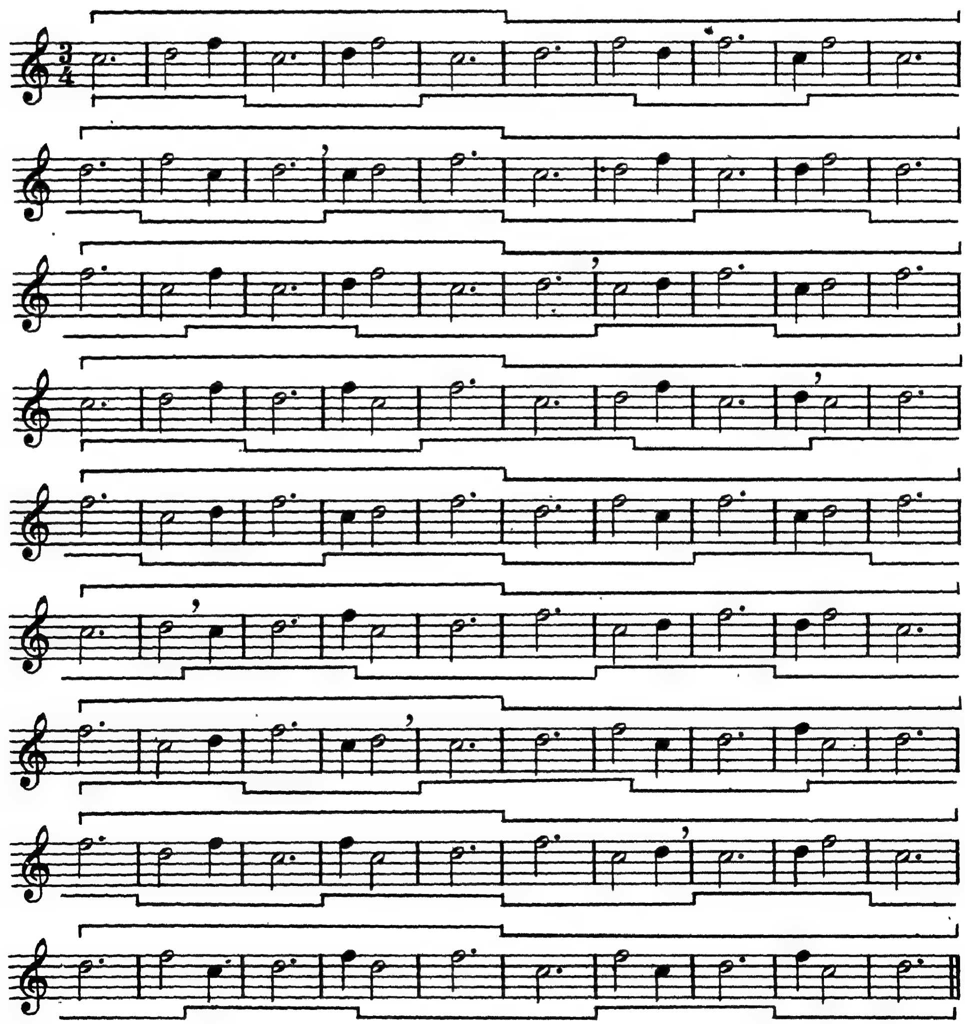


Figure 6

D. Four-unit Scales. Three Intervals.
[The number of scales: 165.]

Table of Intervals

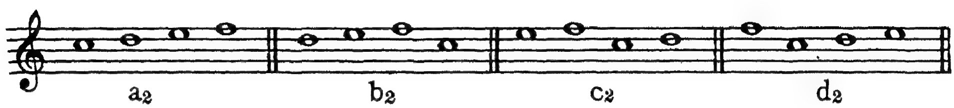
1+1+1	2+2+1	3+3+1	4+4+1	5+5+1
1+1+2	2+1+2	3+1+3	4+1+4	5+1+5
1+2+1	1+2+2	1+3+3	1+4+4	1+5+5
2+1+1	2+2+2	3+3+2	4+4+2	
1+1+3	2+2+3	3+2+3	4+2+4	
1+3+1	2+3+2	2+3+3	2+4+4	
3+1+1	3+2+2	3+3+3	4+4+3	
1+1+4	2+2+4	3+3+4	4+3+4	
1+4+1	2+4+2	3+4+3	3+4+4	
4+1+1	4+2+2	4+3+3		
1+1+5	2+2+5	3+3+5		
1+5+1	2+5+2	3+5+3		
5+1+1	5+2+2	5+3+3		
1+1+6	2+2+6			
1+6+1	2+6+2			
6+1+1	6+2+2			
1+1+7	2+2+7			
1+7+1	2+7+2			
7+1+1	7+2+2			
1+1+8				
1+8+1				
8+1+1				
1+1+9				
1+9+1				
9+1+1				

1+2+3	1+2+4	1+2+5	1+2+6	1+2+7	1+2+8
1+3+2	1+4+2	1+5+2	1+6+2	1+7+2	1+8+2
3+1+2	4+1+2	5+1+2	6+1+2	7+1+2	8+1+2
2+1+3	2+1+4	2+1+5	2+1+6	2+1+7	2+1+8
2+3+1	2+4+1	2+5+1	2+6+1	2+7+1	2+8+1
3+2+1	4+2+1	5+2+1	6+2+1	7+2+1	8+2+1

1+3+4	1+3+5	1+3+6	1+3+7
1+4+3	1+5+3	1+6+3	1+7+3
4+1+3	5+1+3	6+1+3	7+1+3
3+1+4	3+1+5	3+1+6	3+1+7
3+4+1	3+5+1	3+6+1	3+7+1
4+3+1	5+3+1	6+3+1	7+3+1

1+4+5	1+4+6
1+5+4	1+6+4
5+1+4	6+1+4
4+1+5	4+1+6
4+5+1	4+6+1
5+4+1	6+4+1

2+3+4	2+3+5	2+3+6	2+4+5
2+4+3	2+5+3	2+6+3	2+5+4
4+2+3	5+2+3	6+2+3	5+2+4
3+2+4	3+2+5	3+2+6	4+2+5
3+4+2	3+5+2	3+6+2	4+5+2
4+3+2	5+3+2	6+3+2	5+4+2



Melodic Form: a_2; Rhythm: $r_{5 \div 2}$; Measure: $\frac{4}{4}$

Melodic Form: $a_2 + b_2 + c_2 + d_2$

Figure 7.

Material:

(1) The number of melodic forms = 24 by all permutations, and 4 by circular permutations.

(2) Combinations by 2, 3, 4, . . . 24 through the first group, and by 2, 3, and 4 through the second group.

When it is desirable to *limit* the material, use circular permutations. Limited material is practical for thematic development of motifs. In writing studies for an instrument (in animated motion), often hundreds of attacks are necessary. Then a very large amount of material becomes desirable and all the permutations may be used.

(3) Continuity of melodic forms through permutations.

(4) Coefficients of recurrence of the melodic forms.

(5) Superimposition of time-rhythm.

The technique described above may be applied to variations, thematic development and composition of melodic continuity.

In arranging a song, the technique of varying melodic forms can be utilized for modification of the theme and for the counter-motifs, (thus becoming modified thematic motifs), to fill in the time intervals during the long durations and the rests in a theme.

The following is an illustration of this procedure. [George Gershwin's *The Man I Love*, first four bars of the refrain.]*

Figure 8.

All four motifs have the form a_2 ($= a_1 + b_1 + c_1$), i.e., the sequence of appearance of the pitch-units is a_2 despite the recurrences. The scale is obviously a three-unit scale, and in the fourth bar the scale shifts its root-tone, following the harmony.

The next step is the modification of the second, the third and the fourth bars, using b_2, c_2 and d_2 respectively, and preserving the original form of recurrence.

The melody then acquires the following appearance:

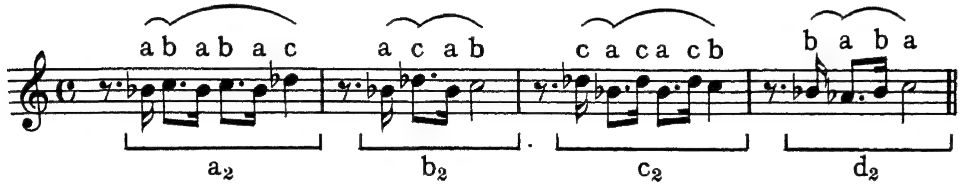


Figure 9.

Such modified motifs, being placed in any of the parts of harmony, produce a thematic "fill-in." It may be compared with the original neutral scalewise "fill-in" in Gershwin's own version.

E. Scales of Seven Units

Technical procedures similar to the foregoing are possible with the scales having more than four pitch-units. Any desired number of scales can be built with pitch-units exceeding 4.

There is no need to have complete charts of all 2,048 scales of this group, as all the necessary procedures will be generalized in the succeeding pages.

Seven-unit scales constitute the musical language of our civilization and serve as the material of harmony. There are 462 seven-unit scales, but only a few will be offered in the following description.

Major and minor scales are constructed from four-unit scales known as "tetrachords."

By uniting two tetrachords separated by the interval 2, one can produce all major and minor scales with the repeated upper tonic. This form of presentation of the scale material helps to emphasize the different structures of so-called "major" and "minor" scales.

The three fundamental tetrachords are:

Major (M) = 2 + 2 + 1
Minor I (m_1) = 2 + 1 + 2
Minor II (m_2) = 1 + 2 + 2

In addition to these tetrachords, European music of the last few centuries also uses the tetrachord coming from the Mohammedan East (Arabia, Persia). This is a tetrachord which penetrated into Europe partly through the Crusaders and partly through the immediate influence of the Turks upon the Balkans. It still prevails in the southern part of Europe (Jugoslavia, Hungary, Rumania). It can be found in the music of Franz Liszt, Ludwig van Beethoven and many other composers. We shall call it the *harmonic* tetrachord (h). Its structure is: 1 + 3 + 1.

All major and minor scales are classified according to musical tradition into:

(1) Natural
(2) Harmonic
(3) Melodic

Though melody may be based on one unaltered scale, *hybrids* appear quite frequently. There is no law or reason for playing the melodic minor upward—and the natural minor downward, the way many instrumentalists do. As long as one intends to use hybrids, any hybrids may be used.

Major

Upward	*Downward*
natural	harmonic
natural	melodic
harmonic	natural
harmonic	melodic
melodic	natural
melodic	harmonic

Analogous hybrids exist in the minor group.

Table of Scales

Major		**Minor**
	Natural	
M + 2 + M		$m_1 + 2 + m_2$
	Harmonic	
M + 2 + h		$m_1 + 2 + h$
	Melodic	
M + 2 + m_2		$m_1 + 2 + M$

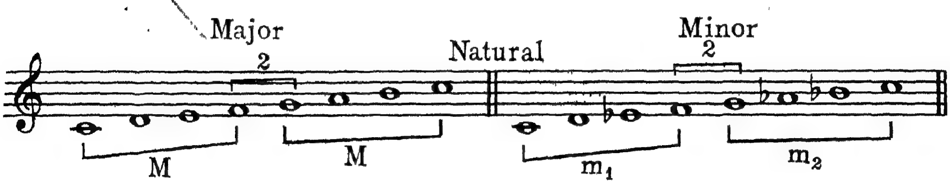


Figure 10 (continued).

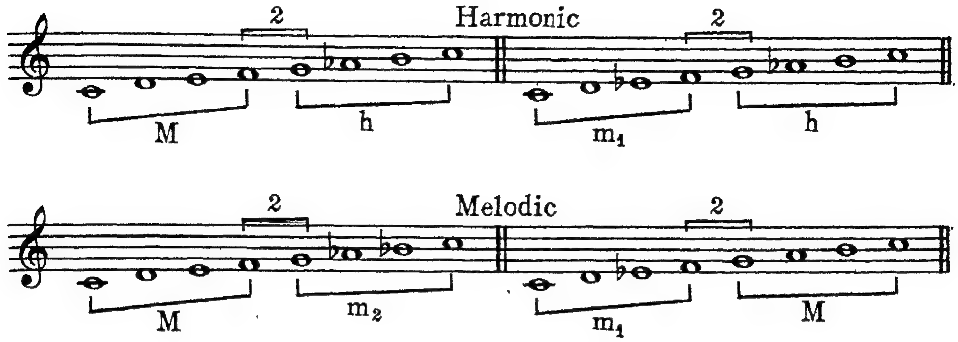


Figure 10 (concluded).

Comparing the two groups one finds that all lower parts of the major groups are M; all lower parts of the minor groups are m_1; all connections in all groups are 2; the natural scales in both groups have individual upper tetrachords; the upper tetrachords are in common for all harmonic scales; the melodic scales in both groups have individual upper tetrachords; the upper tetrachords in the natural and melodic scales exchange their structures, being, in the natural scales $2+2+1$ for the major, and $1+2+2$ for the minor; in the melodic scales, $1+2+2$ for the major, and $2+2+1$ for the minor.

Here are a few more scales in common use.

Neapolitan Minor: $m_2 + 2 + h$

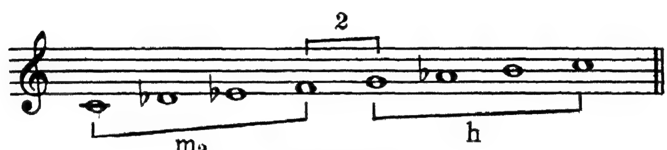


Figure 11.

Hungarian Minor: $2 + (1 + 3 + 1) + (1 + 3 + 1) = 2 + h + h$

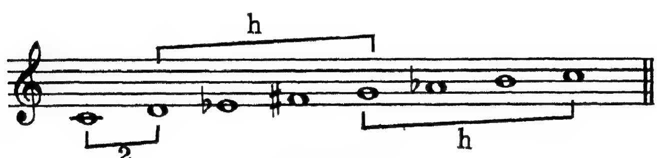


Figure 12.

Hungarian Major or "Blue": $3 + 1 + 2 + 1 + 2 + 1 + 2$

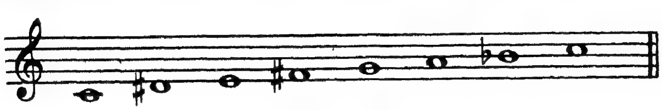


Figure 13.

Persian or Double Harmonic Scale: h + 2 + h

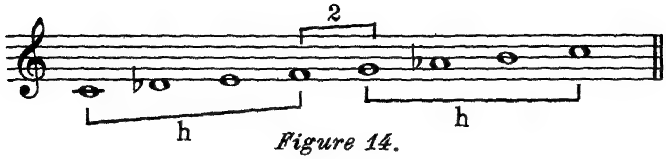


Figure 14.

The so-called "ecclesiastic modes" may be regarded as derived from natural major.

The entire technique of scale derivation, as well as the evolution of scales within the families, will be explained in the succeeding pages.

CHAPTER 3

EVOLUTION OF PITCH-SCALE STYLES

PITCH-SCALES, like time-scales (rhythms), are subject to serial development. The number-values express pitch intervals. Each scale with two pitch-units and one interval becomes a *generator* of its family. Splitting the number-value expressing the interval into a binomial, we acquire a three-unit scale with two intervals. The modified forms of the binomial interval fall into synchronization and produce a resultant scale with four units and three intervals. The modified forms of the trinomial interval fall into synchronization and produce a resultant scale with six units and five intervals. The modified forms of the quintinomial interval fall into synchronization and produce a resultant scale with ten units and nine intervals.

A. Relating Pitch-Scales through the Identity of Intervals

All scales identified by the original interval, or the consequent resultants, belong tc one family. This is the process of relating pitch-scales through the *identity of intervals.*

Example:

Two-unit scale. Interval = 5 = c − f

(a) 5 = 3 + 2 = c − e♭ − f
5 = 2 + 3 = c − d − f

This interference group produces the resultant trinomial =
= 2 + 1 + 2 = c − d − e♭ − f
2 + 2 + 1 = c − d − e − f
1 + 2 + 2 = c − d♭ − e♭ − f

The following quintinomial (the resultant of the second interference-group) is uniformity, i.e., = 1 + 1 + 1 + 1 + 1 = c − d♭ − d♮ − e♭ − e♮ − f
Uniformity, being neutral, belongs to all families (as the last interference) and does not possess any distinctive characteristics.

(b) 5 = 4 + 1 = c − e − f
5 = 1 + 4 = c − d♭ − f

The resultant of this interference-group =
= 1 + 3 + 1 = c − d♭ − e − f
1 + 1 + 3 = c − d♭ − d♮ − f
3 + 1 + 1 = c − d♯ − e − f

The following quintinomial is neutral.

Example:

Trinomial:
4 + 4 + 3 = c − e − g♯ − b
4 + 3 + 4 = c − e − g − b
3 + 4 + 4 = c − e♭ − g − b

The resultant quintinomial of $4 + 4 + 3$ with permutations, equals:

$3 + 1 + 3 + 1 + 3 = c - e\flat - e\natural - g - g\sharp - b$
$1 + 3 + 1 + 3 + 3 = c - d\flat - e - f - a\flat - b$
$3 - 1 + 3 + 3 + 1 = c - d\sharp - e - g - a\sharp - b$
$1 + 3 + 3 + 1 + 3 = c - d\flat - e - g - a\flat - b$
$3 + 3 + 1 + 3 + 1 = c - e\flat - f\sharp - g - a\sharp - b$

The resultant nine-term polynomial equals:

$1+2+1+1+1+1+1+2+1 = c-d\flat-e\flat-f\flat-f\natural-g\flat-g\natural-a\flat-b\flat-b\natural$
$2+1+1+1+1+1+2+1+1 = c-d-d\sharp-e-f-f\sharp-g-a-a\sharp-b$
$1+1+1+1+1+2+1+1+2 = c-c\sharp-d-d\sharp-e-f-g-g\sharp-a-b$
$1+1+1+1+2+1+1+2+1 = c-c\sharp-d-d\sharp-e-f\sharp-g-g\sharp-a\sharp-b$.
$1+1+1+2+1+1+2+1+1 = c-d\flat-d\natural-e\flat-f-g\flat-g\natural-a-b\flat-b\natural$
$1+1+2+1+1+2+1+1+1 = c-c\sharp-d-e-f-f\sharp-g\sharp-a-a\sharp-b$
$1+2+1+1+2+1+1+1+1 = c-d\flat-e\flat-e\natural-f-g-a\flat-a\natural-b\flat-b\natural$
$2+1+1+2+1+1+1+1+1 = c-d-d\sharp-e-f\sharp-g-g\sharp-a-a\sharp-b$
$1+1+2+1+1+1+1+1+2 = c-c\sharp-d-e-f-f\sharp-g-g\sharp-a-b$

Thus, one may start with a monomial, trinomial or quintinomial, and evolve scales of corresponding complexity which *belong to one family*, i.e., which provide a homogeneous melodic continuity.

Taking a scale from folklore, one may compose music of different degrees of complexity—yet secure an authentic style.

This method also provides material from which one may evolve themes of different complexity, to be used in one musical continuity (as primary and secondary subjects or counter-subjects). For example, if we use the original scales of the Stony Indians (Alberta, Canada) [$3 + 2 = c - e\flat - f$ and $2 + 3 = c - d - f$] for one subject, it is desirable to adopt the resultant scale [$2 + 1 + 2 = c - d - e\flat - f$] for another subject.

B. Relating Pitch-Scales Through the Identity of Pitch-units

Another device through which scales of one family may be evolved is *the process of circular permutations* of the pitch-units of the original scale. This is the process of relating pitch-scales through the *identity of pitch-units.*

Take the scale $c - d - e - g - a$. Let the original scale be indicated as d_0 (zero displacement.) The derivative scales resulting from circular permutations of these pitch-units will be indicated as d_1 (the first displacement scale), d_2 (the second displacement scale), etc. The number of displacement scales resulting from the original scales equals the number of units in the original, minus one.

$$\boxed{N_d = N_u - 1}$$

There are 5 units in the $c - d - e - g - a$ scale.

$N_d = 5 - 1 = 4$

Here is a chart of derivative scales:

d_0	c – d – e – g – a
d_1	d – e – g – a – c
d_2	e – g – a – c – d
d_3	g – a – c – d – e
d_4	a – c – d – e – g

As can be seen from the above chart, each of the derivative scales has a different group of intervals.

The following chart represents the transposition of these scales to the tonic *c*.

d_0	c – d – e – g – a
d_1	c – d – f – g – b♭
d_2	c – e♭ – f – a♭ – b♭
d_3	c – d – f – g – a
d_4	c – e♭ – f – g – b♭

Scales derived from permutations of the *intervals* are different from scales derived from permutations of the *pitch-units*, though there are *some* coincidences. Original scale:

c – d – e – g – a = 2 + 2 + 3 + 2

Permutation of the Intervals:

c	d	e	g	a
2	2	3	2	
c	d	e	f♯	a
2	2	2	3	
c	e♭	f	g	a
3	2	2	2	
c	d	f	g	a
2	3	2	2	

Only the last scale in this group coincides with one in the preceding group (d_3).

Using either form of producing derivative scales of the same family, one can evolve a melodic continuity. Other devices previously presented, such as permutations of pitch-units in the scales following as one group, combinations of such melodic forms, coefficients of recurrence and superimposition of time-rhythm, can be used in the composition of continuity.

The following examples offer illustrative comparisons of the application of these different devices.

The original scale: Melodic forms derived from circular permutations:

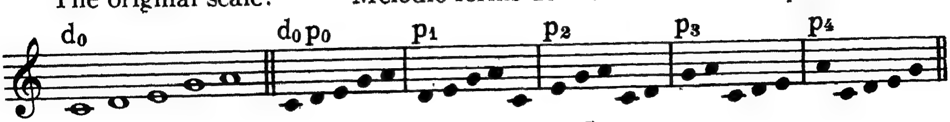


Figure 15. (continued).

The original scale: Derivative scale through pitch permutations:

Melodic forms derived from pitch permutations:

Transposition of the pitch permutation scales:

Melodic forms derived from the above scales:

The original scale: Derivative scales through interval permutations:

Melodic forms derived from the above scales:

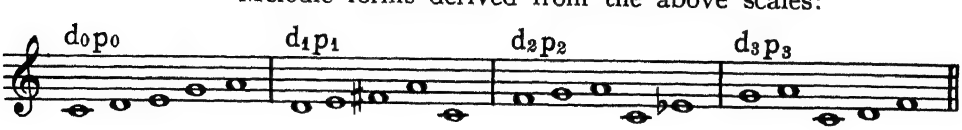


Figure 15 (concluded).

C. Evolving Pitch-Scales through the Method of Summation

There are two methods of evolving scales each with a different number of units but belonging to the same family.

The first method was described as the method of interference, as applied to the number-values expressing intervals. Through this method we can evolve scales with a greater number of units than in the original one. When a scale with many units is the original scale, the simpler derivative scales may be evolved through reversal of the first procedure, i.e., through *summing up* the number-values expressing intervals. For example, if the original scale is: $2 + 2 + 1 + + 2 + 2 + 1$, i.e., c – d – e – f – g – a – b♭, simpler scales may be evolved in the following ways:

(2+2)+1+2+2+1 = 4+1+2+2+1 = c – e – f – g – a – b♭
2+(2+1)+2+2+1 = 2+3+2+2+1 = c – d – f – g – a – b♭
2+2+(1+2)+2+1 = 2+2+3+2+1 = c – d – e – g – a – b♭
2+2+1+(2+2)+1 = 2+2+1+4+1 = c – d – e – f – a – b♭
2+2+1+2+(2+1) = 2+2+1+2+3 = c – d – e – f – g – b♭

(2+2+1)+2+2+1 = 5+2+2+1 = c – f – g – a – b♭
2+(2+1+2)+2+1 = 2+5+2+1 = c – d – g – a – b♭
2+2+(1+2+2)+1 = 2+2+5+1 = c – d – e – a – b♭
2+2+1+(2+2+1) = 2+2+1+5 = c – d – e – f – b♭

(2+2+1+2)+2+1 = 7+2+1 = c – g – a – b♭
2+(2+1+2+2)+1 = 2+7+1 = c – d – a – b♭
2+2+(1+2+2+1) = 2+2+6 = c – d – e – b♭

(2+2+1+2+2)+1 = 9+1 = c – a – b♭
2+(2+1+2+2+1) = 2+8 = c – d – b♭

(2+2+1)+(2+2+1) = 5+5 = c – f – b♭
etc.

D. Evolving Pitch-Scales through the Selection of Intervals

The second method consists of taking a smaller group of intervals or units from the original scale in the sequence of their appearance.

(a) We may evolve partial scales through selecting pitch-units from the original scale.

The Original scale:
c – d – e – f – g – a – b♭
Partial six-unit scales:
c – d – e – f – g – a
d – e – f – g – a – b♭
Partial five-unit scales:
c – d – e – f – g
d – e – f – g – a
e – f – g – a – b♭

Partial four-unit scales:

c − d − e − f
d − e − f − g
e − f − g − a
f − g − a − b♭

Partial three-unit scales:

c − d − e
d − e − f
e − f − g
f − g − a
g − a − b♭

Partial two-unit scales:

c − d
d − e
e − f
f − g
g − a
a − b♭

(b) We may evolve partial scales through selecting intervals from the original scale, and in the sequence of their appearance.

The original scale:

$2 + 2 + 1 + 2 + 2 + 1$

Partial Scales:

$2 + 2 + 1 + 2 + 2$	= c − d − e − f − g − a
$2 + 1 + 2 + 2 + 1$	= c − d − e♭ − f − g − a♭
$2 + 2 + 1 + 2$	= c − d − e − f − g
$2 + 1 + 2 + 2$	= c − d − e♭ − f − g
$1 + 2 + 2 + 1$	= c − d♭ − e♭ − f − g♭
$2 + 2 + 1$	= c − d − e − f
$2 + 1 + 2$	= c − d − e♭ − f
$1 + 2 + 2$	= c − d♭ − e♭ − f
$(2 + 2 + 1)$	
$2 + 2$	= c − d − e
$2 + 1$	= c − d − e♭
$1 + 2$	= c − d♭ − e♭
$(2 + 2)$	
$(2 + 1)$	

Scales with identical structures are omitted (numbers in parentheses).

E. Historical Development of Scales

Analysis of historic material in the field of melody reveals that the laws of identity described above (pitch, interval) develop intuitively with different races and civilizations.

Primitive American Indian music, such as that of the Canadian Stony Indians in Alberta already cited, has two 3-unit scales, both belonging to the same family through identity of intervals (3 + 2 and 2 + 3). The ancient Greeks had their fundamental tetrachord (4-unit scale) 2 + 2 + 1. They called it a "Lydian" tetrachord. Through their own procedures, which were quite different from the procedures described in this theory, they found two other fundamental tetrachords: the "Phrygian" (2 + 1 + 2), and the "Dorian" (1 + 2 + 2). This is another case of evolving scales through interval identity. Ancient China used a scale which has still survived and which is used throughout Asia among the Mongols. It is usually known as a "pentatonic" scale. Naturally, this is only one of the large number of the "pentatonic"—i.e., 5-unit—scales. The construction of this scale is 2 + 2 + 3 + 2. Another scale used by the Chinese has the construction, 2 + 3 + 2 + 2.

It is interesting to note, also, that the last-mentioned two scales have frequently been employed in many American popular songs in the course of the last two decades.

What is still more important is that the Americans have developed intuitively—and perhaps even through the channels of harmony—two other scales used together with the two Chinese scales and incorporated into the same musical continuity. These scales have been described in the preceding text, and possess the following structures:

$$2 + 2 + 2 + 3 \text{ and } 3 + 2 + 2 + 2$$
$$c - d - e - f\sharp - a \qquad c - e\flat - f - g - a$$

A similar analysis of the more developed scales, such as the 7-unit scales of our major and minor groups, and the Greek and the Ecclesiastic modes, reveals that musical intuition, with the investment of centuries of experience, has led to the evolution of scale families through a proper channel.

Through our method of analysis, we find that the so-called "Ecclesiastic modes," i.e., scales used during the Middle Ages in Europe, are *displacement* scales of the natural major. Natural major was known as the Ionian mode; d_1 was known as the Dorian mode; d_2 was known as the Phrygian mode; d_3 was known as the Lydian mode; d_4 was known as the Mixolydian mode; d_5 was known as the Aeolian mode; and d_6 was the Locrian or Hypo-Phrygian mode. These scales all conform to one family through the identity of their pitch-units.

There are two different systems of terminology which conflict with each other in relation to the above-mentioned scales (modes). The one offered here is the *medieval* terminology used by musicians. The other is the ancient Greek

terminology used by historians only with reference to the ancient Greek music. When such discrepancies occur as the Ecclesiastic Dorian mode being called the Greek Phrygian, the explanation is quite apparent—that when Greek manuscripts were studied during the Middle Ages many things were misinterpreted, and this change of the names is merely due to misunderstanding of the Greek terms.

Taking advantage of the fact that the whole European culture of music is an outcome of circular pitch displacement in the natural major or Ionian mode, this evolution can be continued from any other forms of major and minor, thus yielding 21 more displacement-scales: 7 from harmonic major; 7 from harmonic minor; and 7 from melodic major. Upon comparison of the major and the minor natural scales, it may be observed that the natural minor is the d_5 of the natural major, and the melodic minor is the d_3 of the melodic major.

As it follows from the previous text, all the pitch-displacement scales may be transposed to the same pitch-axis (key note). When we apply this method to natural major scale and its derivative modes, this entire group appears in different normal key signatures. Starting the natural major (Ionian) scale on c, the key signature is zero. Starting the Dorian (d_1) on c places this music in the key of B♭ major, to which the two flats (b♭ and e♭) belong. The Phrygian mode (d_2) starting on c acquires the four normal flats pertaining to A♭ major. The Lydian mode (d_3) acquires one sharp pertaining to G major. The Mixolydian mode (d_4) acquires one flat pertaining to F major. The Aeolian mode (d_5) acquires three flats pertaining to E♭ major. The Locrian mode (d_6) acquires five flats pertaining to D♭ major.

All the displacement scales derivative from the natural major (through which my system of key signatures, not commonly in use, has been evolved) may be automatically transposed to one axis, in which the different displacement scales will have the same name for their pitch-units, but differ in their key signatures. If the great composers of the past had known anything about this procedure (i.e., that the same music can acquire different characteristics without loss of any of its ingredients and without distortion of any of its components), they would have overcome difficulties in finding the proper type of chords, their progressions and the forms of voice leading—all of which was one of the most difficult tasks they faced in their intuitive attempts at modal writing. Their difficulty was not only in finding the proper chord relations, but also in finding *all* the chords belonging to any one of the displacement scales.

Rimsky-Korsakov, who is considered one of the best composers in modal writing, is helpless enough when he tries to find the proper chord progressions for such modes as Dorian, or Mixolydian, but he becomes entirely helpless when he attempts to *modulate* through various modes. The first problem is merely a problem of automatic key signature adjustment; the second will be explained in the next chapter.

TABLE OF MODAL TRANSPOSITIONS

Original Key	Derivative Scale (Mode)	Derivative Key	Relative Signature
c	Dorian d_1	B♭	2♭
c	Phrygian d_2	A♭	4♭
c	Lydian d_3	G	1♯
c	Mixolydian d_4	F	1♭
c	Aeolian d_5	E♭	3♭
c	Locrian d_6	D♭	5♭

Figure 16.

The above signature variations are relative to their original keys. All the additional sharps mean the addition of sharps to the naturals, and the addition of naturals to the flats. All the additional flats mean the addition of flats to the naturals, and the addition of the naturals to the sharps.

For example, if one desires to play music written in the key of A major directly in Phrygian mode, and A major contains three sharps in its key signature (f♯, c♯, g♯), translation into the Phrygian mode will require the addition of four flats, i.e., the cancellation of the three sharps into naturals and the addition of one flat (b♭). Music originally written in the key of natural C minor (Aeolian), to be played in Mixolydian scale, requires cancellation of e♭ and a♭. C minor is d_5 in the key of E♭ major. E♭ major has three flats in its key signature (b♭, e♭, a♭). The Mixolydian mode starting on c belongs to F major, which has one flat in its key signature (b♭). The difference between the Aeolian of E♭ major, and the Mixolydian of F major excludes the two above-mentioned flats from the key signature. This explains how through a more complicated procedure one can perform modal transpositions automatically.

There is room in this description to present one illustration of the inadequate modal manipulations of the composers of the past—manipulations considered to be acceptable only by reason of the present level of musical competence.

For a classical example, take a record or the music of the *Song of the Viking* from Rimsky-Korsakov's opera, *Sadko*. Play it first as it is written by the composer; then cancel all the accidentals. The two versions should be compared, and the component scales analyzed. It will be sufficient to take the first refrain where modulation returns it to the original Dorian d (C major).

As musical key signatures in their customary form refer only to the natural scales, all other alterations of pitch appear as accidentals. Therefore, automatic modal transposition refers only to the Ionian scale and its derivatives. But if the musical world faced the fact squarely, it would agree that most key signatures are a pure myth; that there is scarcely a piece of music which really evolves in a natural scale throughout; that scales change and are modified, and so does the key-axis. Then all could agree that the application of *real* key signatures would solve the problem of universal automatic transposition which is possible now only for the natural scales. For example, if one would like to play music

written in natural major, in the scale which is d_3 of G melodic minor, it would be necessary only to add both b♭ and f♯ to the key signature.

Existing musical theories offer such vague notions on this matter that they even explain such scales as being in the key of F major and confuse the f♯ alteration with g♭. This is true of one of the attempts to explain music written by Scriabine.

CHAPTER 4

MELODIC MODULATION AND VARIABLE PITCH AXES

OUR SENSORY orientation—with respect to static and kinetic forms—is based on our *general associative orientation.* The prerequisite of the latter is *memory.* Real or imaginary guiding lines help us to apprehend, to analyze, to study and to construct different forms. In geometry, we use the coordinates, the bisector, the directrix, the radius-vector; in astronomy, we use the geodetics (equator, eliptic); in painting, design and sculpture, we apply geometrical lines and centers (the coordinates, the medians, the area boundaries, the center of gravity, the harmonic [rhythmic] center).

When it comes to music, we must all confess that previously accepted musical "theories" do not provide us with such luxuries. Musical notation does not suggest any quantitative or directional data. Fortunately for the art of music and for musicians, our general associative orientation is not obscured by our own acquired musical education.

A. Primary Axis

When we listen to a melody we hear and identify (owing to our memory) that pitch-unit which is more predominant. Our auditory centers register the quantity of attacks and durations on various sound-wave frequencies which constitute a certain melody. Then our memory sums them up, thus producing an imaginary line (which can be registered graphically)—*the primary axis of a melody.*

A *primary axis* (P.A.) may be defined as the *pitch-time* maximum of an entire melody or of any portion of it. This means that, when we hear only the first two measures of a certain melody, the axis may be one pitch-unit, but when we hear the first eight measures of the same melody, it may be *another.* We re-orientate ourselves as time flows. It is very noticeable that while we move away from certain nearby objects, the center of scenery modulates—as, for example, on the ferry-boat trip from Manhattan to Staten Island.

The P.A. of a melody is the root-tone (the tonic) of a *real* scale. If a melody is written in the standard signature of three flats (b♭, e♭, a♭), it may be in any of the displacement scales of the natural E♭ Major. If the P.A. of such melody is *g*, then it is a case of Phrygian *g* scale. Only through associations with harmony may we think of e♭ being a root-tone under such circumstances. But any of the derivative scales may be harmonized by the chords of any other derivative scale from the same d_0. Thus, *the number of axis-relations between a melody and its harmony equals the square of the number of derivative scales* (from one d_0 and including d_0). Therefore, any of the five-unit scales offers 25 axis-relations between melody and harmony. Any seven-unit scale offers 49. A melody in d_0 may be accompanied by harmony in d_0, d_1, d_2 . . . A melody in d_1 may be accompanied by harmony in d_0, d_1, d_2 . . . etc.

There are four forms of these axis-relations which will be considered in this discussion in time-continuity (as modulations from one axis to another). Later they will be considered in the theory of simultaneous correlation of melodies (counterpoint)* and correlation of melodies with harmonies (melodization of harmony and harmonization of melody).**

B. The Key-Axis

When harmony is absent, the P.A. of a melody is the real *key-axis.* The term "tonal" in the following classification will pertain to *intervals;* the term "modal" pertains to *pitch-units.* "Unimodal" means "in identical mode," i.e., the scale remains the same. "Polymodal" means "in different modes," i.e., the scale varies. "Unitonal" means "in identical tonality," i.e., the key remains the same. "Polytonal" means "in different tonalities," i.e., the key varies.

Modulations may be performed from key to key without a change in the scale-structure, and modulations from a scale of one structure into a scale with a different structure may be accomplished without a change in key.

C. Four Forms of Axis-Relations:

(1) Unitonal — Unimodal U — U
(2) Unitonal — Polymodal U — P
(3) Polytonal — Unimodal P — U
(4) Polytonal — Polymodal P — P

The key does not change in the first two forms. U — U (1) represents the affirmation of P.A. and zero modulation. The process of establishing a key and a scale includes: 1) introduction of the *pitch-units* of a selected scale in any desirable sequence; 2) movement of the *leading tones* (pitch-units adjacent to the tonic—P.A.) into P.A.; and 3) the quantitative predominance of P.A.
Time: $(r_{3\div2})$ Pitch: $2 + 2 + 3 + 2$

1. *Melodic continuity* (*circular permutations of the scale*):

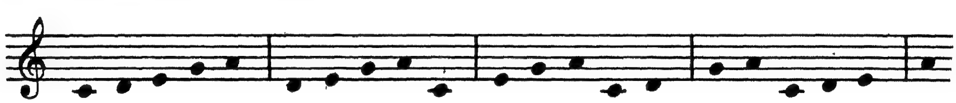


Figure 17.

2. *Melodic continuity with time-rhythm superimposed:*

Figure 18.

*See Book VII.
**See Book VI.

Unit a, whose durations sum up to 7, forms the P.A. of this melody. This shows that any pitch-unit of a scale may become a P.A. U−P (2) represents modulations on scales derived from one common d_0. Such modulations may be achieved through one procedure: transposition of the melody into any derivative scale.

The key-axis of the scale in figure 18 is c, while the P.A. of the melody is a. The key-axis of d_1 scale is d, while the P.A. of the melody becomes c. The key-axis of d_2 scale is e while the P.A. of the melody becomes d, etc.

The following is the original melody together with its transposed versions to all the other axes:

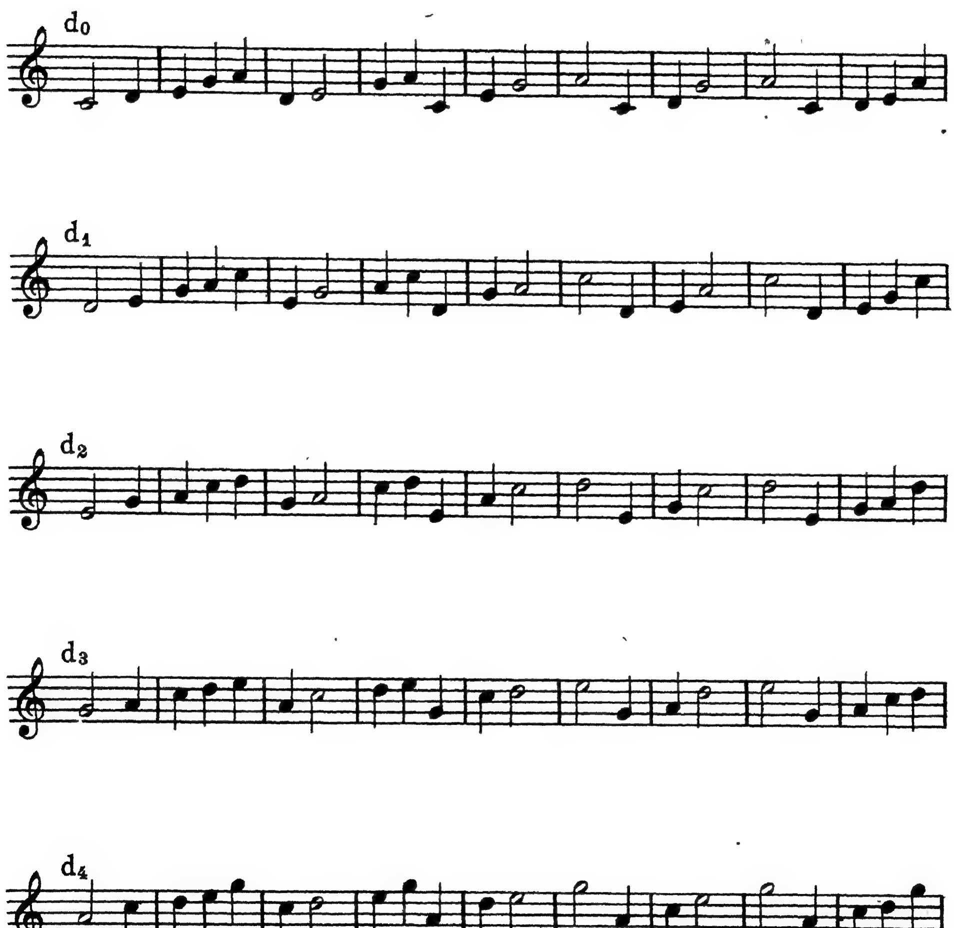


Figure 19.

These five different axes become elements of continuity. Five elements produce 120 permutations. Any of these 120 forms may be used for esthetic purposes.

Here is a composition employing the following arrangement: $d_3 - d_2 - d_1 - d_4 - d_0$.

Figure 20.

The entire continuity of all 120 forms would extend to 5,400 measures (45×120).

By varying the key signatures (which remain constant every time for the entire continuity), we can multiply the number of possible compositions by 330, the number of all five-unit scales.

The relationship, P — U (3), represents a more general form of variation of the key-axis. In this case all or some of the pitch-units are not in common in the two adjacent key-axes (the preceding and the following keys). The structure of the scale remains the same.

P — P (4) represents a case in which both the ***key-axis and the scale vary***, and ***the pitch-units are not entirely in common.***

Cases (3) and (4) emphasize modulations as they are usually known, i.e., from one key to another, with or without modification of the scale structure.

When these axis-relations concern seven-unit scales, some of the pitch-units are in common because all the combinations by seven, taken from twelve elements and with more or less uniform distribution, have some elements in common. For example, natural C Major and natural E♭ Major have four units in common: c, d, f, g. Pitch-units which are written differently, but sound the same (enharmonics), must be considered identical.

Thus in modulations from natural C Major to harmonic f♯ minor, four units are in common: a, b, d, f (= e♯). Scales with fewer pitch-units, being constructed from different key-axes, may not have any tones in common. Such is the case in $2 + 3 + 2$ = c — d — f — g and an identical scale from an *e* key-axis: *e* — f♯ — a — b.

In the older civilizations, where the number of units in a scale is restricted to a very few, modulations exist in the (1) and (2) type of the axis relations only. Types (3) and (4) are unknown. *To make such scales practical for type (3) and (4) modulations, the themes must already be modulating through type (2).* This increases the number of pitch-units in a theme and produces potential common tones.

The technique of transition from one key-axis to another for types (3) and (4) consists of three different devices, each having a different esthetic value:

(a) common units
(b) chromatic alterations
(c) identical motifs

D. Modulating through Common Units

In order to modulate through common units, it is necessary:

(1) to detect the pitch-units which are in common between the preceding and the following key.

(2) to produce motifs on common units long enough to eliminate the potential discrepancy between units of the preceding and the following key that are not in common (i.e., long enough to let the memory forget the possible discrepancy). The motifs are melodic forms with time rhythm superimposed. It is best to take rhythm material from the theme.

The theory of planning variable key-axes will be fully explained in the *Special Theory of Harmony.** For the present, it is best to modulate into *any key-axis which is identical with one of the pitch-units of the original scale.*

If c – d – e – g – a is the original scale, the best modulations are to the keys of d, e, g and a. The corresponding scales assume the following appearances:

Key of c = c – d – e – g – a
Key of d = d – e – f# – a – b
Key of e = e – f# – g# – b – c#
Key of g = g – a – b – d – e
Key of a = a – b – c# – e – f#

The sequence of different keys in one melodic continuity composes the possible permutations. For contrast, use as adjacent keys those which have fewer units in common; for similarity, do the contrary. In the case above, with the following planning—Key c – Key g – Key e – Key a—similarity is obtained by modulating between the first two keys, extreme contrast between the second and the third keys, and much less contrast between the last two keys.

The following is an example of modulatory continuity obtained through the application of *common units*. It is desirable *not* to show the axis of the following key in the course of modulation. The reasons for this will appear later in the *Theory of Melody.***

* See Book V. **See Book IV.

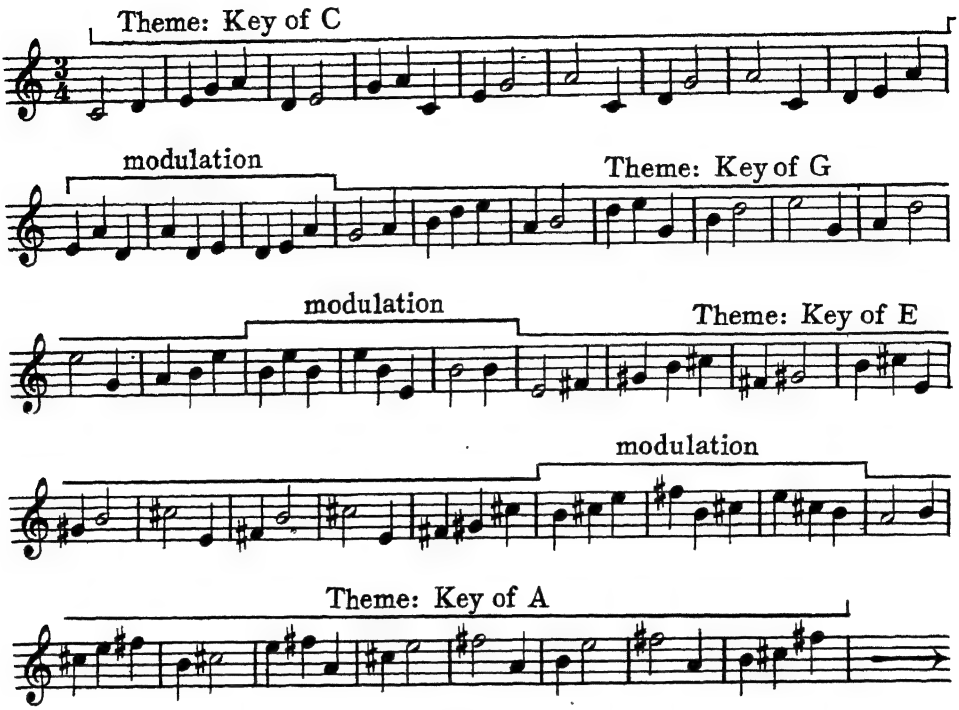


Figure 21.

Modulation through common units is the most subtle of all modulatory devices. A question may arise as to *how long* a modulation should be. The only answer to this question is: long enough to let the listener forget the potential pitch-discrepancy of the adjacent keys. The example given above illustrates a reasonable average. The length of modulation depends on the absolute velocity (tempo) of music, as well as on the audience for which the music is written. It must be longer for the conservative listener and shorter for the advanced one. At the present time there are many listeners who object to any modulatory transition from key to key; instead, they prefer to go there directly. Some modern composers object to the very phenomenon of the axis.

E. Modulating through Chromatic Alteration

If the common-unit method of transition is regarded as the process of *dodging* the conflicts (as in diplomacy) then the chromatic alteration method of transition is entirely bold; it takes advantage of the possible conflict and goes about it directly (as in war).

In order to modulate through chromatic alterations, it is necessary:

(1) to find the units that are not in common but have identical musical names (like c — c♯).

(2) to perform one or more chromatic operations with such units. A single chromatic operation consists of *demonstrating the preceding and the following units in reasonably long durations* with their following into the

next unit bearing a different musical name (like c — c♯ — d or c — c♭ — b♭). In the case of *more than one* chromatic operation, it is necessary to *proceed immediately* with the other intended chromatic operations and to use the *third term* of a chromatic group *in the last group only.*

Example:

From natural C Major to natural E♭ Major. Units not in common: b — b♭; e — e♭; a — a♭.

One operation: b — b♭ — a♭; e — e♭ — d; a — a♭ — g.

More than one operation: b — b♭ — e — e♭ — a — a♭ — g.

Modulatory Continuity Obtained Through the Application of Chromatic Alterations:

(Theme: from the preceding example of modulation through common units; key-sequence: C — E).

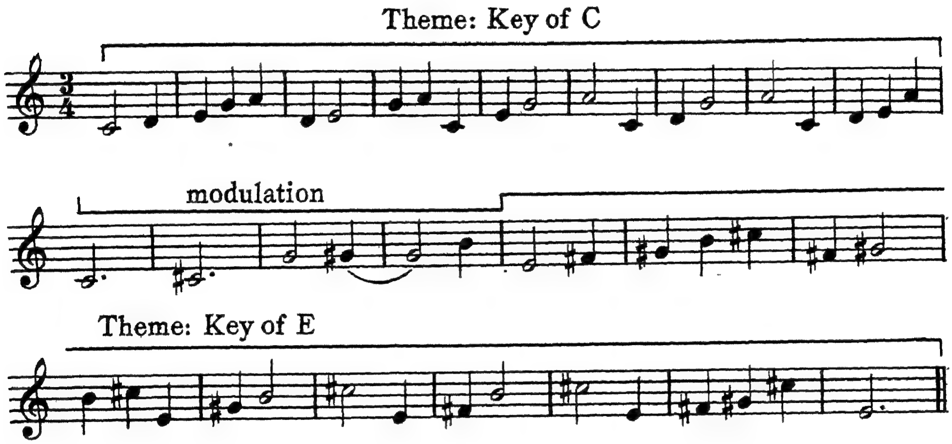


Figure 22.

F. Modulating through Identical Motifs

The *identical-motif* method of transition is the process of imitating appearances and is like adapting oneself to a surrounding medium which constantly varies (as in mimicry; compare with the behavior of a chameleon). It is the most obvious and the most commonly used of all three methods of transition.

In order to modulate through identical motifs, it is necessary:

(1) to select a motif from the theme which immediately precedes the modulation.

(2) to construct another motif identical or similar in appearance and to adapt it to the signature of the succeeding key. The second motif may consist of the pitch-unit bearing the same musical names as the first motif, or it may be located in the adjacent lower or higher position.

This method of transition is very typical of popular songs, or of anything that must have an obvious character. It is also found in most of the well-known classical symphonies.

The following illustration is one of modulatory continuity obtained through the application of *identical motifs*. The theme is from the preceding example: Key sequence: C – E – G – A.

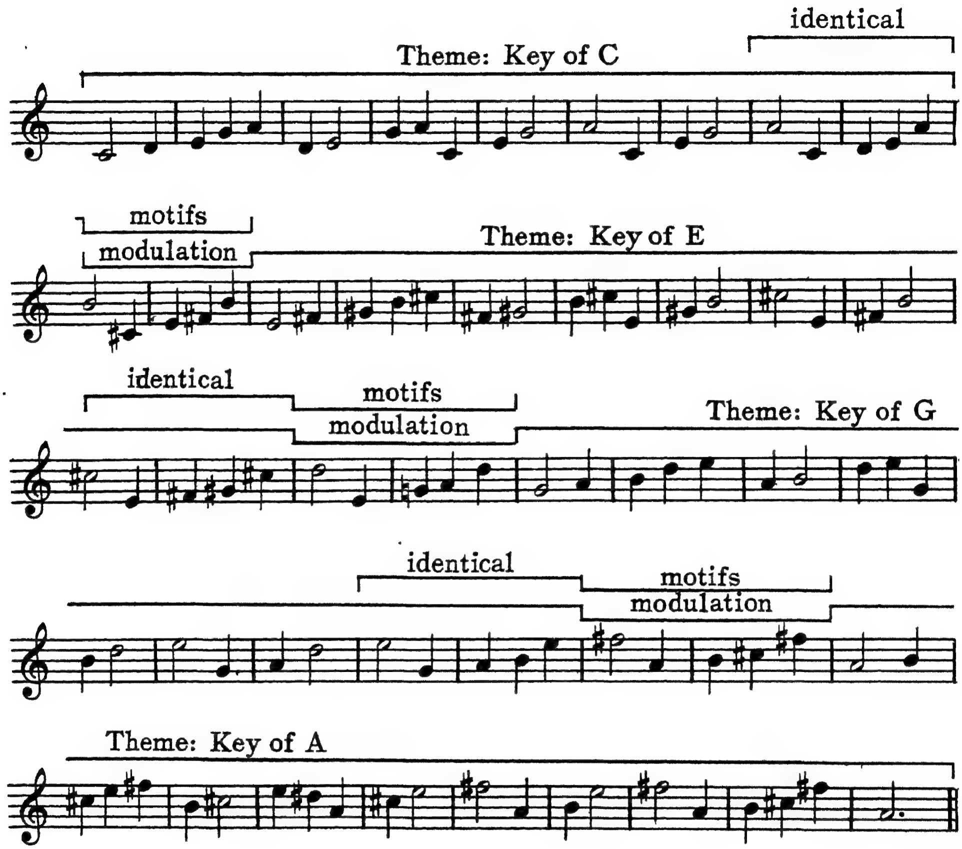


Figure 23.

CHAPTER 5

PITCH-SCALES: THE SECOND GROUP

Scales in Expansion

THE second group of pitch-scales emphasizes scales produced from one constant pitch-unit, and *exceeding* the range of one octave. These scales do not necessarily conform to two- or three-octave range. The range may be more than one and less than two octaves; more than two and less than three, etc.

A. Methods of Tonal Expansion

Scales constituting this group may be obtained by means of *tonal expansion* (expansion of invariant pitch-units through rearrangement of their mutual positions) of the scales of the first group.

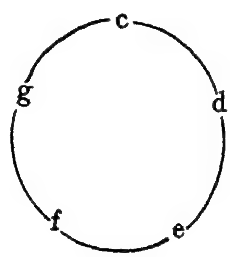
The first expansion (E_1) of a scale may be obtained through circular permutation *over* one pitch-unit of the original scale.

There are two cases: first, when the number of units in a scale is an odd number.

Example:

Scale: c – d – e – f – g

Circular arrangement:



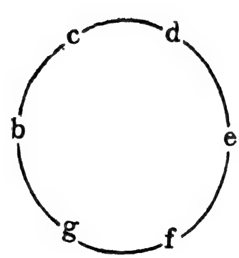
The first expansion: c – e – g – d – f

Second, when the number of units is even. Then, through the same form of permutation over one unit, ***the recurring unit is omitted*** in addition to the normal omission of the respective number of units.

Example:

Scale: c – d – e – f – g – b

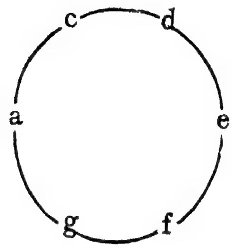
Circular arrangement:



The first expansion: c – e – g – d – f – b

Scale: c – d – e – f – g – a

Circular arrangement:



The first expansion: c – e – g – d – f – a

With the increase of the number of units omitted between the selected units, further expansions may be obtained. The total number of tonal expansions of one scale equals the number of units therein minus one.

$$\boxed{N_E = N_P - 1}$$

This includes the original scale.

A scale that cannot be contracted in a given system of tuning will be considered as being in the *zero* expansion (E_0).

All further expansions will be E_1, E_2, ... E_n, where the subnumeral represents the number of units omitted between the number of units selected in circular permutation.

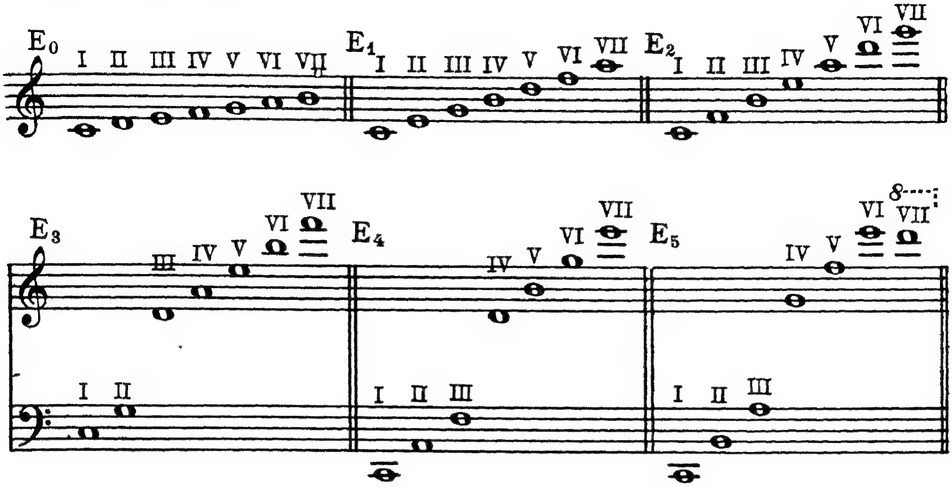
The process of tonal expansion is applicable to any melodic form—a scale being merely a special case of melodic form. Different expansions of a melody provide means for *variation* as well as for composition of *melodic continuity*.

The technique of *transcribing a melody from one expansion into another* consists in finding the scale in both expansions, in enumerating all the units in consecutive order from the root-tone (scale axis) in both scales, and in translating units of one melody into the units of another through the identical numbers.

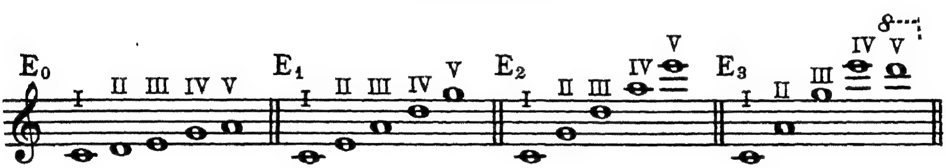
The octave-adjustment (range) for compounding continuity out of different expansions must be performed by placing the root-tone (the axis from which expansions have been obtained) of all the expansions on the same pitch level. With the adjustment, fragmentary melodies in different expansions become elements of one intonation-group, and as such, are permutable in time continuity.

Examples of Tonal Expansions

Natural Major Scale



Chinese Scale



Melodic Minor (g) d_3

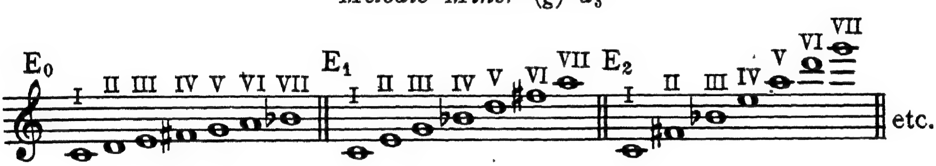


Figure 24.

The last case (melodic minor) is particularly interesting, as it illustrates how music written in the 17th or 18th century can be transformed directly into the style of Debussy or Ravel by means of E_1; how music written by Handel or Bach may be converted into the style of Scriabine's *Poem of Ecstasy* by means of E_2.

This device of tonal expansion is the device for modernization of the music of the past. If the music of the present were written consistently, following its own tendency in any of the expansions, it could be contracted back into E_0. Thus, two styles three centuries apart could be compared under the same coefficients of expansion. This device gives the critics of music something to think about. One cannot really draw any comparisons between music of the present and music written two or three centuries ago because they exist in different states of expansion.

B. Translation of Melody Into Various Expansions

Melody is a special type of scale. When a melody contains, among others, the adjacent musical names (musical seconds), it may be considered part of a complete 7-unit scale (containing all musical names). The following melody may be considered part of a natural major scale with the root-tone on c. In such a case the expansion must be performed from c as the axis of expansion. This melody is formed on both sides of the axis and the same pattern will remain in all expansions.

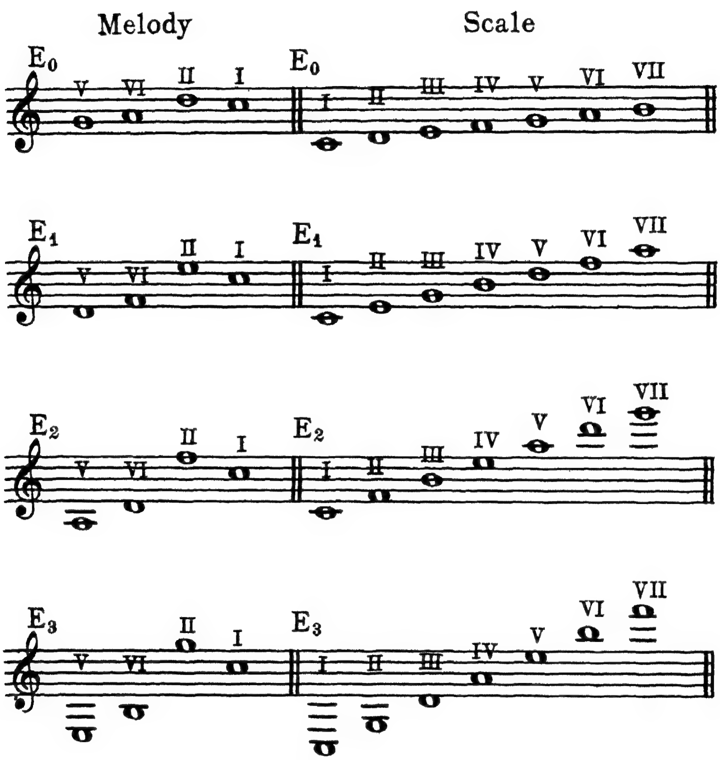


Figure 25.

Different expansions of the same melody produce melodic continuity in similar forms, evolving in different ranges. They are permutable in time continuity.

The following is an example of melodic continuity produced by different expansions.

The original setting:

Figure 26.

Continuity produced by circular permutations:

Figure 27.

As presented in the foregoing example, this device may be employed to produce studies for a solo instrument and is particularly suitable for instruments with wide ranges, such as the violin, clarinet and French horn. In order to obtain more expressive melodies, time-rhythm must be superimposed on the melodic continuity. The interference between the number of units in a melodic layout and a rhythmic group often results in a complete solo composition of considerable length. By playing this type of melody in different modal transpositions, one may obtain a number of compositions, each distinctly different in character, and each esthetically equivalent to the original.

Examples of the tendency toward tonal expansion resulting from purely intuitive processes may be found extensively in the works of modern composers. For example, Prokofiev in his *Song, Opus 27, No. 2* for voice and piano, has a melody evolving mostly in the E_2, while the accompaniment is a hybrid of E_2, E_3, and E_0. The last two bars on the first page reveal E_0 in the melody, E_1 in the right hand of the piano accompaniment. These forms are hybrid and naturally produce various deviations from the pure style. In No. 3 of the same *Opus*, the vocal part is a hybrid between E_0 and E_1 while the right hand of the piano accompaniment is consistently carried out in E_1, and the left hand in E_3.

C. Variable Pitch Axes (Modulation)

All techniques with regard to changes of scale structure or key signature are applicable to the second group of scales as well. Modal transpositions as well as modulations can be carried out in any form of tonal expansion, providing that the expansion remains constant in the two portions of melodic continuity connected by any form of modulatory transition.

It is unsatisfactory to vary expansion in the two portions of melodic continuity belonging to two different axes with modulatory transitions between them. Therefore, all the variations of E must be performed from one axis. The entire scheme of modulatory continuity, including expansions, may appear as follows:

Key I E_0 + Key I E_1 + Mod. +
+ Key II E_1 + Key II E_2 + Mod. +
+ Key III E_2 + Key III E_1 +
+ Key III E_0 + ...

In a melodic continuity evolving from one thematic melody, this device becomes invaluable as it introduces variety into unity. It eliminates the necessity of having several melodies as different themes in one composition. One or two subjects are enough to evolve a diversified melodic continuity when tonal expansions and modulations are used. This form of composition may be applied on a limited scale for the purpose of arranging music where the "fill-in" groups are to appear as imitations of a preceding motif in one or another tonal expansion.

There are many popular melodies which are intuitively written in the first expansion. For example, *Without a Song*, *You Hit the Spot*, and others. Note also Debussy's *La fille aux cheveux de lin.* Such themes present the possibility of reversing the whole procedure, i.e., tonal *contraction* of the original theme. Vincent Youmans' *Without a Song** starts on *c* in the key of F (F is the axis and being in the E_1 is the third degree of E_1). The same melody, being rewritten into E_0 and translated into the corresponding degrees, acquired a new musical appearance that can be utilized wherever thematic motifs are desired. It may serve as an introduction or provide the interludes between the portions of thematic continuity. The processes of expanding and contracting music often lead to startling discoveries. For example, in the case of *Without a Song*, this melody when translated into E_0 has a great deal in common with the theme by Rimsky-Korsakov from his opera *Coq d'Or* commonly known as *Hymn to the Sun.*

Contraction: E_0

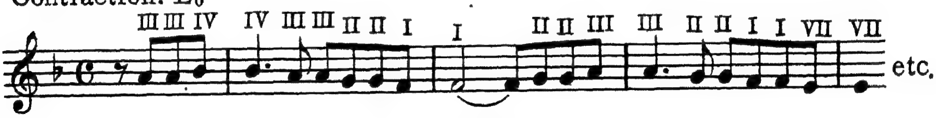


Figure 28.

D. Technique of Modulation in Scales of the Second Group

As transition from key to key—based on chromatic alteration—does not offer any definite procedure for tonal expansion and may lead to pitch-units alien to both the preceding and the succeeding key, it has to be eliminated. Thus, the two available devices are:

1. The common tones.
2. The identical motifs.

*The process of tonal contraction, as described by Schillinger, is most easily executed in the following manner. The first expansion (E_1) of the scale of F has the following notes: f-a-c-e-g-b♭-d-f. These are numbered from 1 to 8. The zero expansion (E_0) of the scale of F—f-g-a-b♭-c-d-e-f—is likewise numbered from 1 to 8. Now, we take the notes of Vincent Youman's song *Without A Song* and number them according to their position in the expanded scale. To discover the tonal contraction of this melody, we simply substitute the notes of the contracted scale corresponding to these numbers. When this has been done to *Without A Song*, the notes of the opening bars change as follows: c-c-e-e-c-c-a-a-f become a-a-b♭-b♭-a-a-g-g-f, etc. The latter will, of course, be readily identified with the theme of *Hymn to the Sun.* (Ed.)

As previously stated, both the preceding and the succeeding key have the same coefficient of expansion (whatever it is). The common tones can be easily found. For any given pair of keys, these common tones are invariant in any given scale, since tonal expansion does not alter the original pitch-units but merely arranges them in a new fashion. It is important, however, to realize that any usage of such common tones for a transition from one key axis to another must be carried out within the type of intervals inherent in the selected tonal expansion. For example, in the major or minor diatonic scales, the first expansion intervals are 3rds, 5ths, 7ths, 9ths, etc. There are no 2nds or 4ths in the same octave. One should refrain from using 2nds when they are really 9ths. This concerns all the intervals.

Here is an example of melodic continuity modulating through common tones.

Figure 29 (continued).

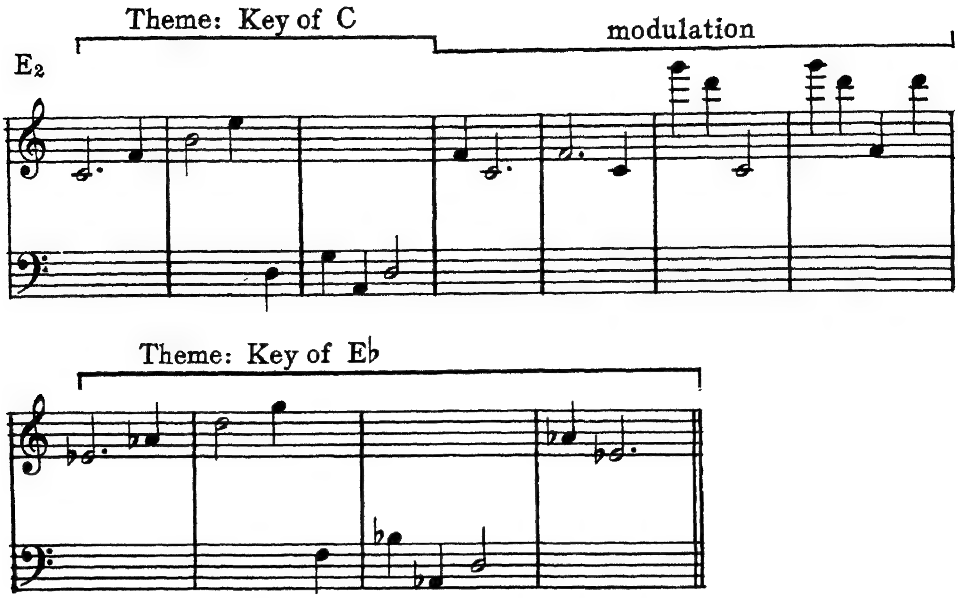


Figure 29 (concluded).

The use made of common tones in the above example was different in each case, i.e., E_1 and E_2 were not direct translations of the melodic pattern of the modulation in E_0 into the corresponding expansions. Modulation through identical motifs in any given expansion can be accomplished in a similar way, providing there are enough common tones to manipulate, and depending on the complexity of the structure of the motif itself.

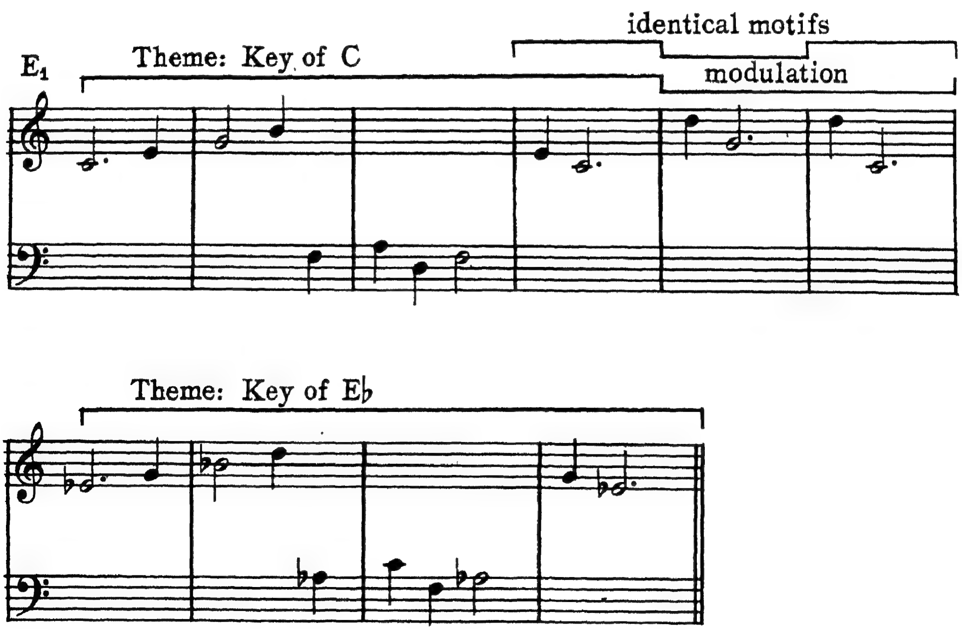


Figure 30.

The two forms of modulation may be combined in the same melodic continuity.

In order to translate melodic continuity which already contains modulations based on common tones or identical motifs, it is necessary to introduce the principle of *common degrees*, as the corresponding degrees of one expansion do not correspond to the respective degrees of another. For example, e in the key of c is the second degree of E_1, and the same e is the third degree of E_0, and the same e is the fourth degree of E_2. Naturally when a certain tone does not correspond to itself in one key, by reason of the different arrangements produced by different expansions, it will not correspond in the same relation to any other key. Take f, which is the fourth degree of the key of C in E_0, and the fifth degree of the key of B♭ in the same expansion; the note f in the key of C is the sixth degree of the E_1 and the same note is the third degree in the key of B♭ in the same expansion; the note f in the key of C on E_2 is the second degree while the same note in the key of B♭ in the corresponding expansion is the seventh degree.

Modulatory continuity must be translated into any other expansion by means of *common degrees*. A new pitch-unit representing the identical degrees of the original expansion must be used directly in place of the corresponding pitch-unit of the same expansion. For example, if the modulation in E_0 was carried out from the key of C to B♭, through the common tone f—f being the fourth degree of the first and the fifth degree of the second key—it would change its pitch-units in such a way that the identity of degrees, i.e., IV = V, would be preserved.

The fourth degree of the key of C in the first expansion is b while the fifth degree of the key of B♭ in E_1 is c. Therefore, the transition must take place through these two pitch-units placed in immediate sequence. The corresponding modulation in E_2 will take the following form: the fourth degree of the key of C in E_2 is e while the fifth degree in the key of B♭ in E_2 is g. The immediate sequence from e to g constitutes the transition. In this case, g following e must be placed one-tenth above e as this is the proper placement of a third in E_2.

Key of C	IV = V Key of B♭
E_0	f – f
E_1	b – c
E_2	e – g

Figured 31 (continued)

Figure 31 (*concluded*)

The identity of motifs in the process of modulating through the different forms of expansions, has dual significance. Firstly, it permits modulation through common tones, yet preserves the identity of the melodic material. This effect was illustrated above with reference to identical motif modulation. Secondly, through direct changes of key signatures in the adjacent identical motifs, one may achieve arpeggio-like modulatory progressions. To the listener's ear, the latter will appear as the customary modulations moving through arpeggio chords

Figure 32.

In the above example, a group of three identical motifs gradually becomes modified through variation of the key signatures from zero signature (key of C)

through one-flat signature (key of F) to a two-flat signature calling for e♭ and d♭ (which would permit the motif's being interpreted as fitting into the key of harmonic b♭ minor, which in its full form has four flats).

The whole field of tonal expansion technique is suggestive of harmony, and therefore presents more elaborate forms of arpeggio-making than the usual harmonic arpeggio.

This device may be successfully utilized when the effect of forming, of growing or of decreasing has to be expressed through a thematically homogeneous melodic form. These effects—when combined with corresponding dynamic treatment—suggest any mechanical form associated with spiral development, i.e., forming, increasing in size, or becoming louder on the one hand, and moving away, decreasing in size, or fading out on the other. Motion picture backgrounds offer a very fertile field for the application of such devices.

CHAPTER 6

SYMMETRIC DISTRIBUTION OF PITCH-UNITS

THE problem of the symmetric distribution of sequences within a given acoustical range of a simple ratio is not new. Musical cultures of the Orient—such as the Javanese, Siamese, Balinese, and Arabian—attempted to produce such symmetries in their systems of tuning. They were not mathematically equipped to solve this problem in its general form, i.e., by means of logarithms, but they intentionally sought to distribute the pitch relations of an octave into five and seven uniform intervals, or to produce more complex forms of periodicity of pitch, such as in the Arabian scale introduced in the 7th century A.D. The latter differs from the Javanese and Siamese scales. The first two are symmetrical systems of tuning (primary selective systems), while the Arabian is a scale constructed within a given tuning system (secondary selective system).

Ancient civilizations were fascinated by the properties of prime numbers. This perhaps explains why they used a symmetric breaking-up of an octave into such numbers as 5 and 7. The actual motivation behind the use of these particular numbers may be an inclination which results from the primeval pentadic and heptadic forms of symmetry. The creation by nature of lower forms of animal life in forms of pentagonal symmetry and snow-flakes in hexagonal symmetry, is merely an outcome of electro-chemical processes which may also take place in our brain-functioning as well as in the general evolution of species.

There is no acoustical reason, or "natural inclination" in the human ear, for differentiating the octave into heptadic or pentadic symmetric relations. Intervals thus produced do not conform to a simple acoustical ratio. Habit and heredity are more important factors in the development of artistic taste than is the perfection of the structural constitution of the raw material. Listening to Javanese, Siamese or Balinese music—authentically recorded from the original sources, one can easily get accustomed to it in a very short time.

While the apparent reasons for the tuning symmetry in Oriental musical cultures were religious and symbolic considerations, the apparent reasons behind the system now in use in the civilized world are acoustical considerations. But these apparent reasons are misleading. They are not true in the light of unbiased scientific analysis. The real reason for the evolution from the system of the symmetry of 12 to an octave is the versatility of the number 12 as compared to 5 and 7. While 5 and 7 are prime numbers, i.e., they may be divided by themselves or by unity only, the number 12 has *four additional* divisors, (2, 3, 4, 6). The next number which would have one more divisor is 60, i.e., no other number between 12 and 59 exhibits greater versatility with respect to combinations of the sub-systems than does the number 12 itself. Being a limited value, it becomes very practicable for the solution of many problems of musical composition. The lack of versatility in the prime numbers, with respect to tuning, becomes apparent after a continuous experience of listening to Javanese or Balinese music. Music of our culture also becomes monotonous, regardless of its esthetic quality, and for the same reason.

When a composer like Debussy begins to use the symmetry of 6 (whole-tone scale) *consistently*, his music becomes monotonous—despite the abundant use of various devices. Using 6 instead of 12 makes the system lose one divisor; the loss of this one divisor makes such music considerably more monotonous to our ear.

All the above-mentioned systems of symmetry are evolved within the range of a ratio of $2 \div 1$, which, being the simplest ratio, produces the effect of greatest likeness to our ear. Musical experience considers this likeness so great that all the tones of such ratio bear identical musical names. With the further evolution of pitch discrimination, this likeness may become assigned to ratios of somewhat greater complexity, such as $3 \div 2$, $5 \div 4$, etc. Then it will be possible to evolve the primary selective systems on the basis of symmetry within such ratios.

Generalizing this idea (symmetric splitting into uniform ratios) we can express it in a formula:* $S = \sqrt[m]{\frac{a}{b}}$

Thus, the system of Javanese music is a special case of symmetry in which $S = \sqrt[5]{2}$; Siamese, in which $S = \sqrt[7]{2}$; and the European so-called "equal temperament," $S = \sqrt[12]{2}$. The latter was developed by Andreas Werckmeister in 1691.

The need for such a system in Europe in the 17th century was created by the desire to produce greater versatility of the pitch axes. The limited key relations satisfactory to the community at that time were compensated for by the acoustical perfection of the system then in use. This system, known as "mean temperament," was a bi-coordinate acoustical system of tuning. The two ratios were $3 \div 2$ and $5 \div 4$, one giving a so-called "perfect 5th," and the other, a so-called "major 3rd"—and between the two coordinate systems developed from these two ratios, compromises were reached.

While in full agreement with the requirements of the Church—as well as with the simpler natural phenomena, this system gave the utmost satisfaction with regard to the consonant quality of harmony. The ideal of early homophonic music was consonant quality of a few chords rather than versatility of harmony at the price of an acoustical compromise. The technical expediency of the new system won, and the entire cultural inheritance of the preceding century's vocal music was automatically transplanted to the new system to which the instruments were tuned, even at the time of J. S. Bach.

J. S. Bach was the first composer to take advantage of the key versatility offered by the new system. Variation of key relations was used by him with the boldness of a catalogue rather than in the form of harmonious continuity. Each prelude and fugue is in a different key in place of a greater variety of key modulations within a single composition.

Musical culture, the stronghold of which was consonance, had eventually to give up its way in favor of the harmonic versatility offered by the new system. Simple harmonic forms used in music of the period preceding equal temperament lost their acoustical perfection in the new system to such an extent that at later times treatises were written trying to explain the reason why certain simple chord structures and chord progressions were "false" in the equal temperament of 12.

*See footnotes on pages 101-2.

It took two hundred years to realize, at least intuitively, the nature and the possibilities of $\sqrt[12]{2}$ system. Even today we are dealing with hybrids produced by music—the sources of which go centuries back—and by forms derived from equal temperament. The intuitive start on the new track was due not to any discrimination in favor of symmetry, but rather to the consequences of a habit formed in the early 16th century. For the interval of an augmented 4th, which occurs between steps II and V of the "Neapolitan" minor scale, was the actual stimulus which historically influenced music to take this particular direction. This interval which exhibits the *symmetry of 2* within one octave ($\sqrt{2}$), is at the same time the simplest form of symmetry within the $\sqrt[12]{2}$ system.*

The first phases of this evolution of harmonic forms produced the $\sqrt[4]{2}$ (Wagner), and the $\sqrt[3]{2}$ (Liszt). While Wagner operated on the $\sqrt[4]{2}$ with $4 + 3$ structures (major triad), he attempted and failed in the application of the $\sqrt[3]{2}$, using the structure $3 + 4$ (minor triad). Liszt used the $\sqrt[3]{2}$ on $4 + 3$ structures exclusively; it took a few decades until the $3 + 4$ structures on the same roots came into existence with Rimsky-Korsakov.

An early application of the $\sqrt[6]{2}$ to melodic forms, as well as to harmonic forms of the $\sqrt[3]{2}$ ($4 + 4$ structure: equals augmented triad), was in the opera, *Stone Guest* written by Dargomishsky in the middle of the 19th century. Further application of this system appeared at the end of the 19th century in the music written by Debussy and Ravel. The $\sqrt[12]{2}$ in $4 + 3$ structures is characteristic of Wagner and of post-Wagnerian opera written by Russian composers.

All the group forms of symmetry within the $\sqrt[12]{2}$ are the derivatives (sub-systems) of this system. Various combinations of the various sub-roots of the $\sqrt[12]{2}$ produce various forms of group symmetry—such as binomials, trinomials and more complicated polynomials. At the beginning of the 20th century the binomial form of symmetry becomes quite apparent (as in Rimsky-Korsakov's *Coq D'Or*). The influence of symmetry on chord structures as well as on chord progressions begins to flourish with Debussy and Ravel.

Chord structures comprising five and more functions (such as 9th chords and 11th chords) take the place of the more archaic triads and 7th chords. Where Beethoven would move his melodies in the inversions of triads, Debussy prefers the 7th and 9th chords.

The most recent forms of symmetry of pitch belong to the type of writing known as "polytonality." Polytonality is a symmetric superimposition of chord structures related through the roots of the octave. The sub-roots of the $\sqrt[12]{2}$ become autonomous tonalities. Such simultaneous superimposition of sym-

*Referring once more to the footnote on pages 101-2, the symmetry of 2 is mathematically analogous to—and much simpler than—the symmetry of 12 characteristic of equal temperament. The ratio series for symmetry of 2 is (expressed in fractional powers): $2^{\frac{0}{2}}$, $2^{\frac{1}{2}}$, $2^{\frac{2}{2}}$. In the first term, the zero power of 2 is 1, and the square root of 1 is 1; in the third and final term, the square root of two squared is, of course, simply 2. The middle term is the point of symmetry. It fits the symmetry of 12 because the square root of 2 to the first power is the same as $2^{\frac{6}{12}}$, which is the middle term of the longer series described in the footnote cited earlier. The same procedure may be generalized to cover the symmetry of 3, of 4, of 6. (Ed.)

metrically related keys may be observed in homophonic as well as polyphonic writing.

The first intentional superimposition of chords belonging to two symmetrical roots of the octave occurred in Stravinsky's *Petrouchka.* The maximum saturation caused by symmetric superimposition derives from the simultaneous composition of all the roots of the octave. Equal temperament of 12 is the complete expression of the symmetry of 12 in one octave. The sub-systems of this general symmetry are: the sub-systems derived from $\sqrt[6]{2}$ as a ratio unit; the $\sqrt[4]{2}$; the $\sqrt[3]{2}$; and the $\sqrt{2}$, the latter being the simplest form of octave symmetry.

CHAPTER 7

PITCH-SCALES: THE THIRD GROUP

Symmetrical Scales

THE THIRD group of pitch-scales is made up of those scales derived from various roots of the number 2—the square (second) root, the cube (third) root, the fourth root, the sixth, and the twelfth roots.

The first table below shows the manner in which the interval between one tone (say, c) and its octave may be divided into *twelve symmetric* parts, thus producing a 12-tonic system; the second table shows the same octave symmetrically split into six tonics; the third, division into four symmetric tonics; the fourth, division into three tonics; the fifth, into two tonics.*

A. Table of Symmetric Systems Within $\sqrt[12]{2}$.

(1)

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_1
1	$\sqrt[12]{2}$	$\sqrt[6]{2}$	$\sqrt[4]{2}$	$\sqrt[3]{2}$	$\sqrt[12]{2^5}$	$\sqrt{2}$	$\sqrt[12]{2^7}$	$\sqrt[3]{2^2}$	$\sqrt[4]{2^3}$	$\sqrt[6]{2^5}$	$\sqrt[12]{2^{11}}$	2
C	C#	D	E♭	E	F	F#	G	A♭	A	B♭	B	C^1

(2)

T_1	T_2	T_3	T_4	T_5	T_6	T_1
1	$\sqrt[6]{2}$	$\sqrt[3]{2}$	$\sqrt{2}$	$\sqrt[3]{2^2}$	$\sqrt[6]{2^5}$	2
C	D	E	F#	A♭	B♭	C^1

(3)

T_1	T_2	T_3	T_4	T_1
1	$\sqrt[4]{2}$	$\sqrt{2}$	$\sqrt[4]{2^3}$	2
C	E♭	F#	A	C^1

(4)

T_1	T_2	T_3	T_1
1	$\sqrt[3]{2}$	$\sqrt[3]{2^2}$	2
C	E	A♭	C^1

(5)

T_1	T_2	T_1
1	$\sqrt{2}$	2
C	F#	C^1

The capital "T's" in the preceding table represent the corresponding tonics (axis-points of the corresponding symmetric systems). These tonics serve as root tones of the structures evolving in simultaneity and continuity.

The first such evolution (in simultaneity) produces *chord* structures. The second (in continuity) produces the individual *pitch-scales* of one compound symmetric scale and also the progression of roots for the chord sequence.

*These symmetric scales and the symmetric harmony derived therefrom are of the utmost importance in modern and future music; they constitute one of the most brilliant theoretical and practical discoveries of the Schillinger System. (Ed.)

All *sectional scales* of the third group, starting from their symmetrical points, have *identical construction.* The number of scales is limited by the intervals between the two adjacent symmetrical roots.

B. Table of Arithmetical Values Expressing Intervals in Semitones.

The Third Group

Number of roots.	Complete range.	Interval between roots.	Limit-range of sectional scales.
2	12	6	5
3	12	4	3
4	12	3	2
6	12	2	1
12	12	1	0

Scales of the Third Group

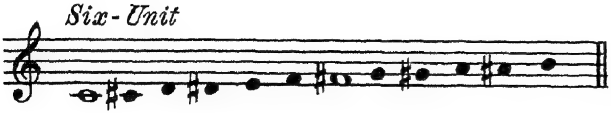
2 Tonics

Figure 33 (continued).

Four-Unit

Five-Unit

Six-Unit



3 Tonics

One-Unit

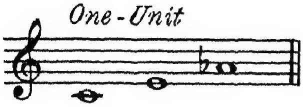
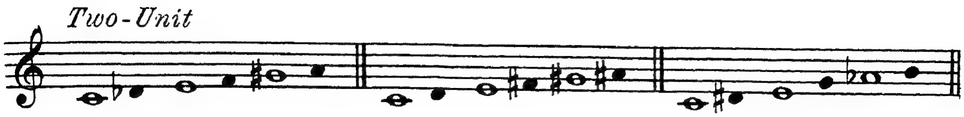
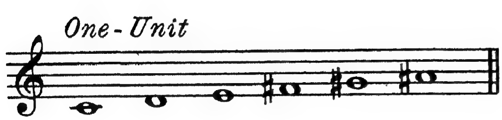


Figure 33 (continued)



4 Tonics

6 Tonics



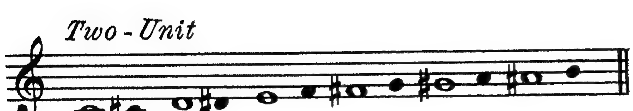


Figure 33 (continued).

12 Tonics

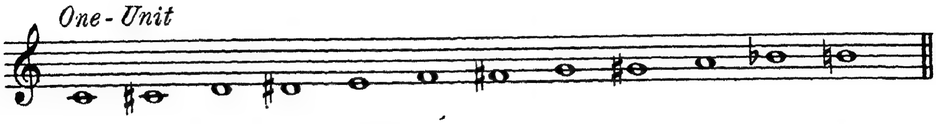


Figure 33 (concluded).

The scale on $\sqrt{2}$ with four-units in each sectional scale of the structure $2+1+2$ was known to the Arabs in the 7th century A.D. Their conception of the structural scheme was: a large step and a small step. Thus, they had obtained a binomial periodicity which, in its nearest approximation to our tuning system, produces $4(2+1)$, and the derivative of it, $4(1+2)$.

This scale came into existence in our music through the realization of $\sqrt{2}$ and $\sqrt[4]{2}$ as influenced by harmonic structures. It results automatically from a continuous chain of the simple chord-structures following the above-mentioned roots. We find portions of it as far back as the music of Bellini.

No composer until Rimsky-Korsakov was aware of this by-product of harmony. It is evidenced in his operas, *Kaschey* and *Mlada*.

Wagner used the scale on $\sqrt[3]{2}$ with three-unit sectional scales $(2+1)$ in his prelude to *Parsifal*. Naturally, neither of these composers was conscious of the symmetric systems as such.

Arabians called their $4(2+1)$ scale a "string of pearls" (Zer ef Kend), drawing an analogy between the alternation of large and small beads in a string of pearls and the large and small steps between the pitch-units of the scale.

Further study of this and other symmetric scales as by-products of chord progressions will be found in the *Special Theory of Harmony*.*

C. Composition of Melodic Continuity in the Third Group

The Third Group of scales offers the following possibilities for composition of melodic continuity:

Scales with Two Tonics:
Total number equals 32.

1-Unit sectional scales on two tonics produce 1^2 equals 1 melodic form. Total number of scales 1.

2-Unit sectional scales on two tonics produce 2^2 equals 4 melodic forms. Total number of scales 5.

3-Unit sectional scales on two tonics produce 6^2 equals 36 melodic forms. Total number of scales 10.

4-Unit sectional scales on two tonics produce 24^2 equals 576 melodic forms. Total number of scales 10.

5-Unit sectional scales on two tonics produce 120^2 equals 14,400 melodic forms. Total number of scales 5.

6-Unit sectional scales on two tonics produce 720^2 equals 518,400 melodic forms. Total number of scales 1.

*See Book V.

Scales with Three Tonics:
Total number equals 8.

1-Unit sectional scales on three tonics produce 1^3 equals 1 melodic form. Total number of scales 1.
2-Unit sectional scales on three tonics produce 2^3 equals 8 melodic forms. Total number of scales 3.
3-Unit sectional scales on three tonics produce 6^3 equals 216 melodic forms. Total number of scales 3.
4-Unit sectional scales on three tonics produce 24^3 equals 13,824 melodic forms. Total number of scales 1.

Scales with Four Tonics:
Total number equals 4.

1-Unit sectional scales on four tonics produce 1^4 equals 1 melodic form. Total number of scales 1.
2-Unit sectional scales on four tonics produce 2^4 equals 16 melodic forms. Total number of scales 2.
3-Unit sectional scales on four tonics produce 6^4 equals 1,296 melodic forms. Total number of scales 1.

Scales with Six Tonics:
Total number equals 2.

1-Unit sectional scales on six tonics produce 1^6 equals 1 melodic form. Total number of scales 1.
2-Unit sectional scales on six tonics produce 2^6 equals 64 melodic forms. Total number of scales 1.

Scales with Twelve Tonics:
Total number equals 1.

1-Unit sectional scales on twelve tonics produce 1^{12} equals 1 melodic form. Total number of scales 1.

Scales of the roots, the exponents of which are multiples of the original roots, give coincidences in the corresponding symmetric points. Thus the scales built through the $\sqrt[4]{2}$ coincide with some of the scales built on the $\sqrt{2}$ where the sectional scales move through the points coinciding with the points of the $\sqrt[4]{2}$. If the two tonics are c and f♯, then all sectional scales which include e♭ and a coincide with the four tonics having identical c — e♭ — f♯ — a as their roots.

The technique of evolving a continuity in symmetric scales must be carried out through sectional scales used either in their complete form or in parts. The complete utilization of the sectional scales follows the methods of circular or general permutations of melodic forms, application of the coefficients of recurrence of melodic forms, superimposition of time rhythm on melodic form, etc. When some of the sectional scales, or all of the sectional scales, are used in parts, a definite rhythmic procedure must be established. The method of elimination of pitch-units must follow with a system of circular permutations or any other pre-arranged method of distribution.

Example of composition of melodic continuity from
a scale of the third group:

c – d – f – f♯ – g♯ – b

Through circular permutations we obtain:

c–d–f – g♯–b–f♯ – f–c–d – f♯–g♯–b – d–f–c – b–f♯–g♯

Using 1-unit at a time on the second tonic and all three units on the first tonic, and applying the method of circular permutations, we obtain:

c–d–f – f♯ – d–f–c – g♯ – f–c–d – b

Example of Melodic Continuity:

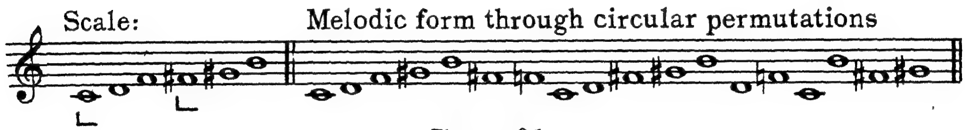


Figure 34.

Rhythm of durations: $3(2+1)+(2+1)^2$

Melody: 18 attacks
Durations: 6 attacks
Interference: $\frac{18}{6} = 3$

Continuity:

Figure 35.

CHAPTER 8

PITCH SCALES: THE FOURTH GROUP

Symmetrical Scales of More Than One Octave in Range

THE FOURTH group of pitch-scales is based on the following roots: $\sqrt[3]{4}$, $\sqrt[4]{8}$, $\sqrt[6]{32}$, $\sqrt[12]{2048}$. The ranges of these systems are, respectively: 2 octaves (3 tonics); 3 octaves (4 tonics); 5 octaves (6 tonics); 11 octaves (12 tonics).

(1)

T_1	T_2	T_3	T_1
1	$\sqrt[3]{4}$	$\sqrt[3]{4^2}$	4
C	A♭	E	C^1

Read the tones upward; the C^1 is *two* octaves above the C.

(2)

T_1	T_2	T_3	T_4	T_1
1	$\sqrt[4]{8}$	$\sqrt{8}$	$\sqrt[4]{8^3}$	8
C	A	F♯	E♭	C^1

The C^1 is three octaves above C.

(3)

T_1	T_2	T_3	T_4	T_5	T_6	T_1
1	$\sqrt[6]{32}$	$\sqrt[3]{32}$	$\sqrt{32}$	$\sqrt[3]{32^2}$	$\sqrt[6]{32^5}$	32
C	B♭	A♭	F♯	E	D	C^1

The C^1 is five octaves above C.

(4)

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9
1	$\sqrt[12]{2048}$	$\sqrt[6]{2048}$	$\sqrt[4]{2048}$	$\sqrt[3]{2048}$	$\sqrt[12]{2048^5}$	$\sqrt{2048}$	$\sqrt[12]{2048^7}$	$\sqrt[3]{2048^2}$
C	B	B♭	A	A♭	G	F♯	F	E

T_{10}	T_{11}	T_{12}	T_1
$\sqrt[4]{2048^3}$	$\sqrt[6]{2048^5}$	$\sqrt[12]{2048^{11}}$	2048
E♭	D	D♭	C^1

The C^1 is eleven octaves above C.

All sectional scales of the fourth group starting from their symmetrical points have *identical construction*. The number of scales is limited by the interval between the two adjacent symmetrical roots.

The Fourth Group

The number of roots	The complete range	The interval between the roots	The limit-range of sectional scales
3	24	8	7
4	36	9	8
6	60	10	9
12	132	11	10

The tonality of all scales of the fourth group may be discovered by utilizing all combinations by 2, 3 . . . for each sectional scale until it fills out the sectional limit-range.

The total number of scales of the fourth group amounts to two thousand. Here are a few illustrations:

The Fourth Group of Pitch Scales

Excerpts from Complete Table

THREE TONICS

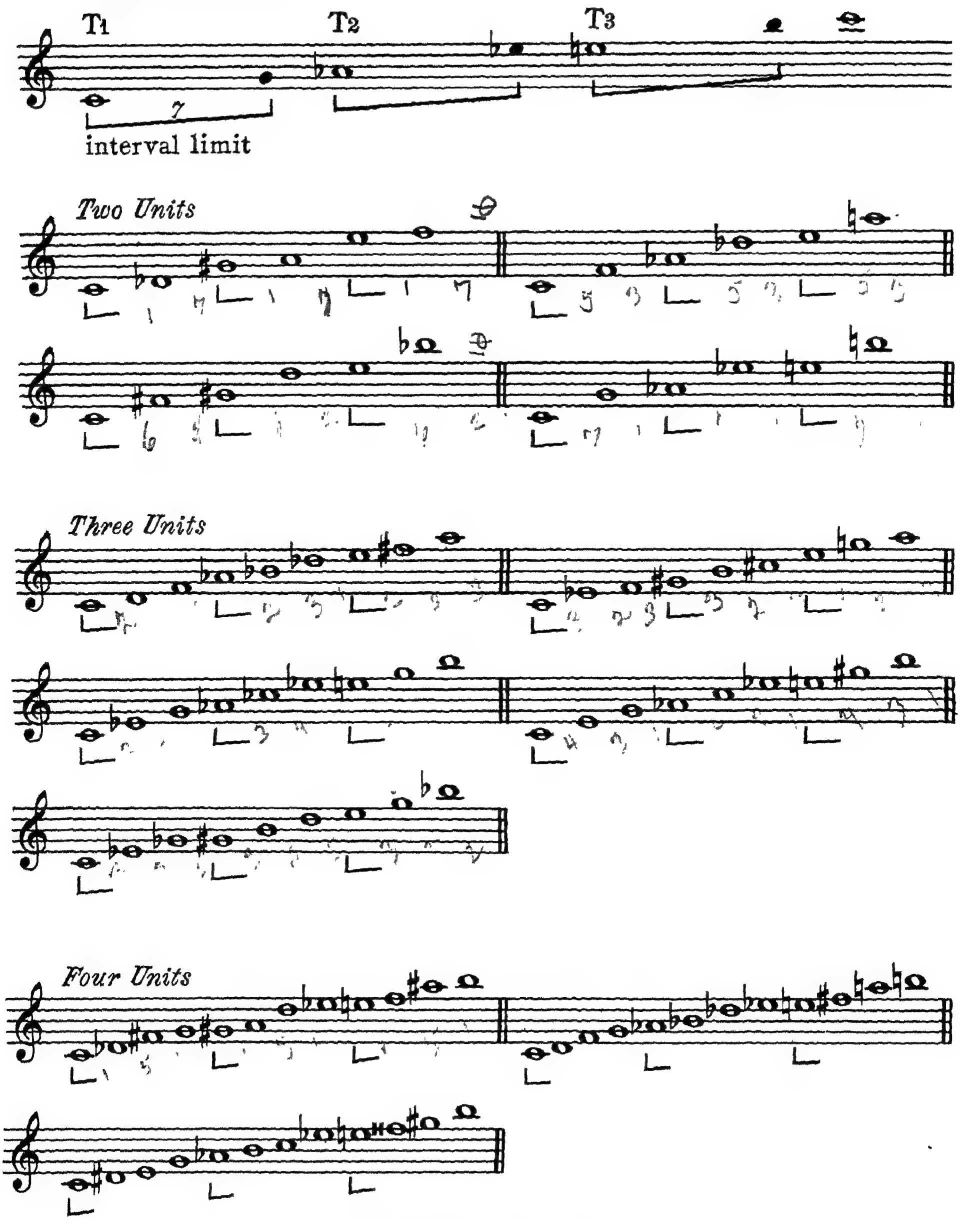
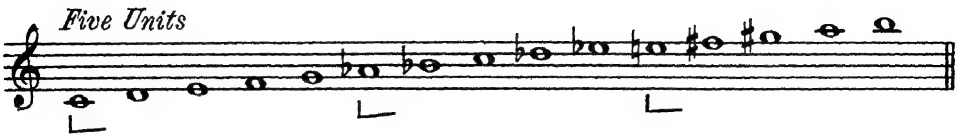


Figure 36 (continued).



FOUR TONICS

Figure 36 (continued).

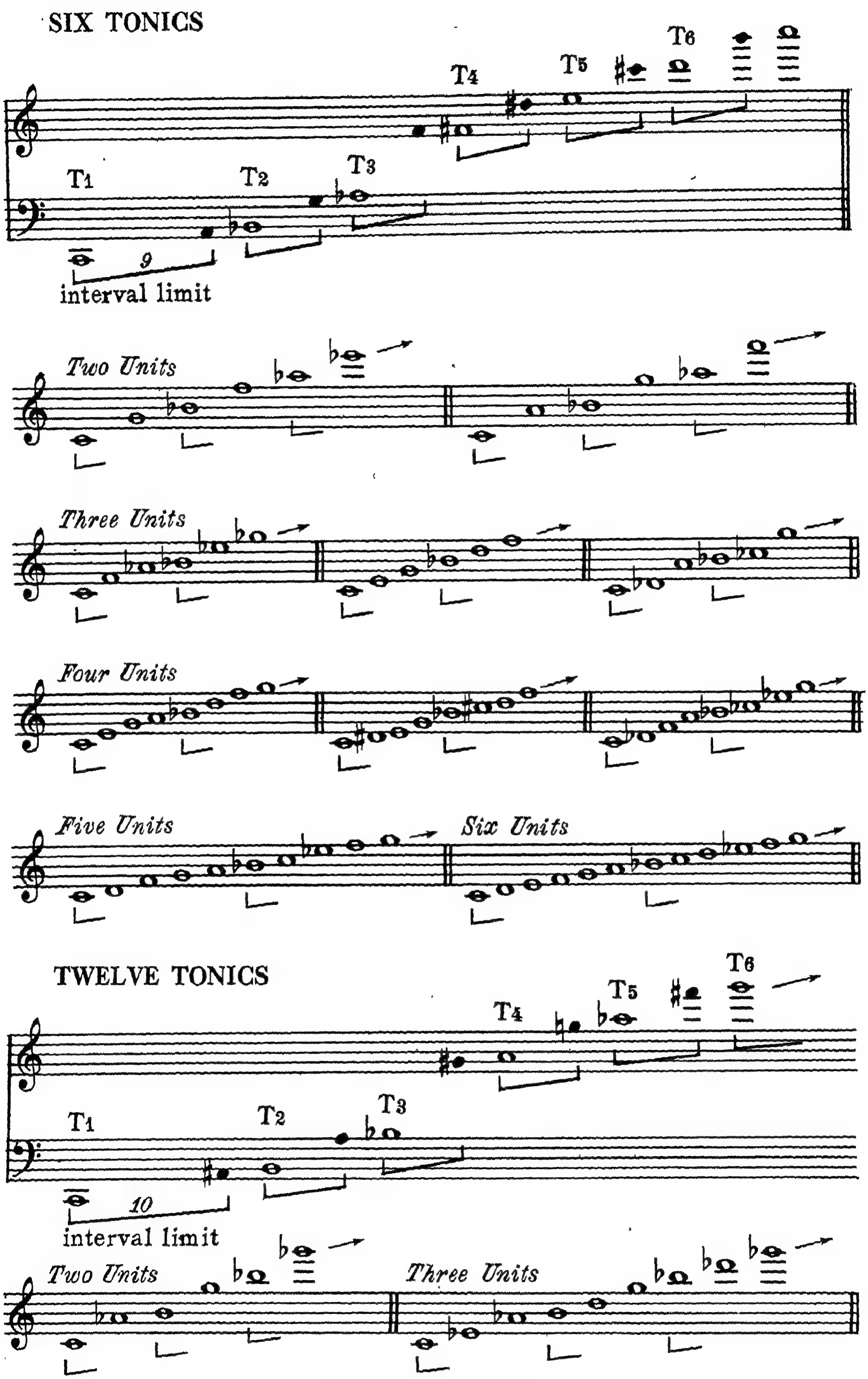


Figure 36 (continued).

etc.

Figure 36 (concluded)

The definition of the quantity of melodic forms in the fourth group is based on the same method of computation as in the third group. The general number of melodic forms available from any symmetric scale equals the number of permutations of the units of the sectional scale to the power expressing the number of roots (tonics).

$$N_m = (u!)^{NT}$$

A numeral with an exclamation mark on its right represents the product of all integers from 1 to such numeral value, i.e., 5! equals $1 \times 2 \times 3 \times 4 \times 5 = 120$. For example, the number of melodic forms in a 5-unit sectional scale on four tonics equals 207,360,000 melodic forms [$(5!)^4 = 120^4 = 207{,}360{,}000$].

A. Melodic Continuity in Scales of the Fourth Group

Composition of melodic continuity from the scales of the fourth group may originate from the three forms of settings:

(1) The original scale
(2) The first contraction
(3) The final contraction

The procedure from the setting (1) is the usual procedure as described for the scales of the third group.

The second setting can be obtained in the following way; first, construct the sectional scale on the first tonic; then the sectional scale above it on the tonic nearest in pitch to the first tonic; then the sectional scale below it on the tonic nearest in pitch to the first tonic. When the number of tonics is even, the further addition of the remaining sectional scales may be either above or below the first tonic. This always offers two forms of distribution for the second setting. For example, take a 4-unit sectional scale on four tonics like c – d – f – g – a – – b – d – e – f# – g# – b – c# – d# – e# – g# – a#. Centering the first tonic and surrounding it by the nearest tonics, we obtain the following setting of type (2):

– d# – e# – g# – a#

– c – d – f – g –

a – b – d – e

Figure 37

Adding to this the remaining tonic (f♯), we may place it either above or below:

(a)

— f♯ — g♯ — b — c♯
— d♯ — e♯ — g♯ — a♯ —
— c — d — f — g —
a — b — d — e —

(b)

— d♯ — e♯ — g♯ — a♯
— c — d — f — g —
— a — b — d — e —
f♯ — g♯ — b — c♯ —

Figure 38

The second setting is always an overlapping one. There are definite contractions corresponding to each system of tonics. Scales on three tonics in their first contraction emphasize the range of 15 semitones. Scales on four tonics in their first contraction emphasize the range of 17 semitones. Scales on 6 tonics in their first contraction emphasize the range of 21 semitones. Scales on 12 tonics in their first contraction emphasize the range of 22 semitones.

The final contraction (3) generally produces a complete chromatic scale with the same range as the first contraction when the distribution of tonics must be preserved. The pitch-units of the sectional scales become rest tones and all the intermediate tones become auxiliary. Each sectional scale consists of *directional units.* Rest tones may move into the auxiliary tones, but their return to the rest tones is required. Otherwise, the auxiliary tones become passing notes between the rest tones.

In the following example of the final contraction on the scale previously illustrated, the black notes indicate the tones to be used as passing or auxiliary, and the white notes indicate the rest tones, i.e., the original pitch-units of the sectional scale in their original sequence.

Figure 39.

The final contraction of symmetrical scales, though it produces a chromatic scale, is different from a chromatic scale used on a basis of atonality, where all tonal centers are completely eliminated. Composition of melodic continuity from melodic forms obtained through either of the melodic settings of the scales of the fourth group follows the same principle as the composition of melodic continuity in the third group of scales.

Hybrid forms of melodic continuity in the fourth group of scales may be derived by the application of mixed settings, i.e., different states of contraction the rest following the usual methods of variation of melodic forms.

Examples of Composition of Melodic Continuity from Scales of the Fourth Group

Scale: Rhythm: $r_{5 \div 3}$

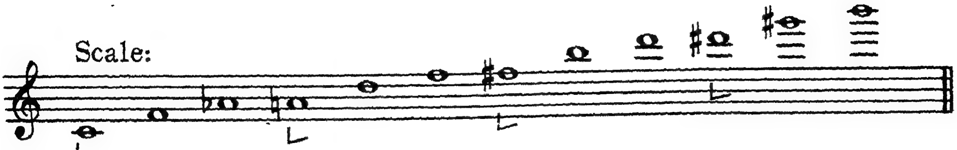
Continuity from the Original Setting (Circular Permutations)

The First Contraction

Continuity from the First Contraction (Circular Permutations)

Figure 40 (continued).

Scale:



Rhythm: $(2+1+1)^3$; $\frac{1}{64}$ = ♪

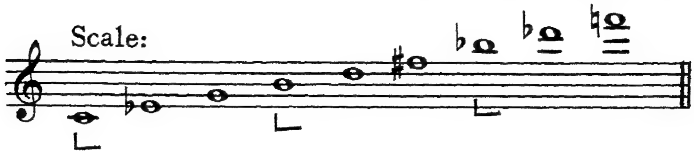
Continuity from the Original Setting (without Permutations)

The First Contraction

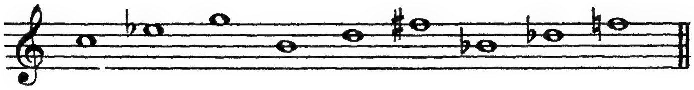
Melodic Form (Circular Permutations)

Continuity from the First Contraction

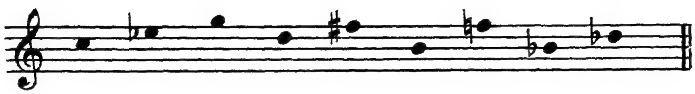
Figure 40 (continued).



The First Contraction



Melodic Form



Rhythm: $3(2+1) + (2+1)^2$

Continuity from the First Contraction (*Circular Permutations*)

Figure 40 (concluded).

The procedure of composing melodic continuity from sectional scales of the symmetric system may also be reversed for the purpose of evolving symmetric continuity from a given motif. A root-tone of such motif corresponds to a root-tone of the symmetric sectional scale. The selection of the system of symmetry is left to the composer's discretion. However the choice of such system may well be suggested by the *harmonic* structure of the original composition.

Almost any music contains some symmetric chord-progressions, at least in a fragmentary form. For example, in the repertory of popular songs, you find as a characteristic progression (often the beginning of the chorus) the $\sqrt[3]{2}$. See such songs as *Night and Day*, *Everything I Have Is Yours*, and *High on a Windy Hill* which uses $\sqrt{2}$. In such cases, the above form of symmetry would be the one most organically related to the whole harmonic constitution of the chorus.

By varying the rhythm of such motifs, it is possible to build different kinds of "cadenzas" which may be used in the introductory section or in transitions.

The following illustration exemplifies an application derived from a well-known motif:

GEORGE GERSHWIN'S *I Got Rhythm**

Theme:

Four Tonic Setting (without Permutations)

Four Tonic Setting (with Circular Permutations)

Figure 41.

B. Directional Units

A scale which contains directional units may appear in all of its three settings. The directional units consist of the original tones of a sectional scale and of the tones of the allied sectional scales. The tones of the original sectional scale may be surrounded on both sides by the tones of the allied sectional scales. In such a case, a system of upper and lower auxiliary tones to the same tones of the original sectional scale is possible. In some cases, the units from the allied sectional scales appear once between the tones of the original sectional scale—and in some cases twice. In other cases, the lower one serves as the upper auxiliary tone to the preceding tone of the original sectional scale, and the upper one as the lower auxiliary tone to the following tone of the original scale.

A *directional unit* consists of a tone of the original sectional scale together with any of the auxiliary tones leading into it. When permutations are used, such a directional unit does not change its own form, i.e., if the original directional unit consists of a lower auxiliary tone leading into the tone of the original sectional scale, this particular form of sequence of the directional unit must be preserved. Permutations refer to different directional units only.

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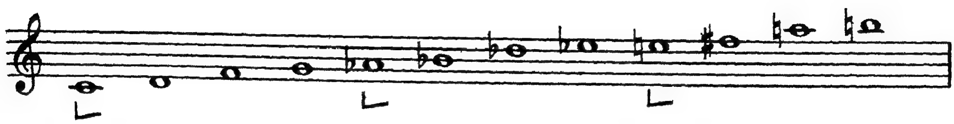
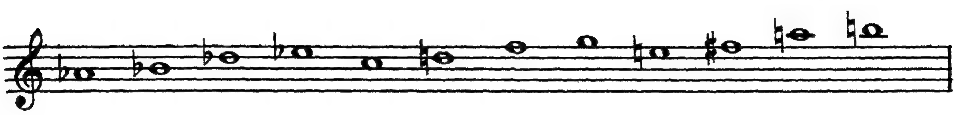
Figure 42, which follows, contains examples of this technique. The arrows indicate the direction taken by the auxiliary tone as it moves into the original tone. The letters indicate directional units.

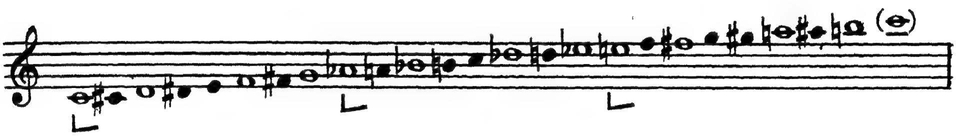
The Original Scale



The First Contraction

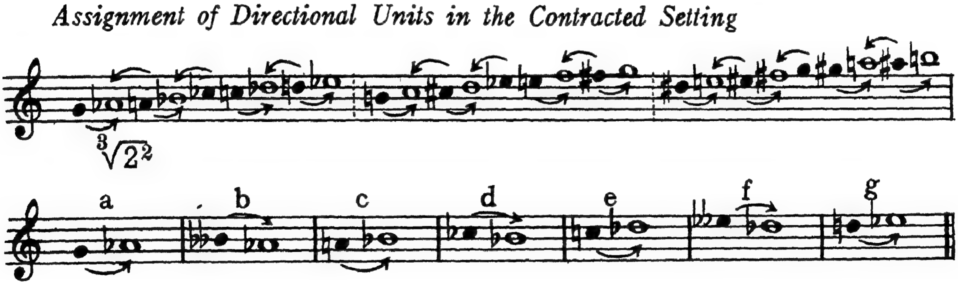


The Original Scale with Auxiliary Tones



The First Contraction with Auxiliary Tones

Assignment of Directional Units in the Contracted Setting



The Number of Melodic Forms: $2^7 = 128$

Figure 42.

By selecting coefficients of recurrence for certain directional groups (as shown in the *Theory of Rhythm*, Book I) and superimposing the rhythm of durations, we may obtain a different type of melodic continuity from that presented in previous examples evolved from the scales of the Fourth Group.

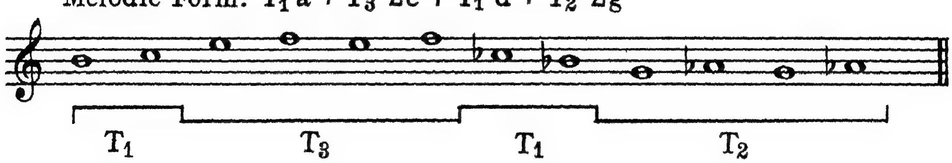
When such melodies are harmonized, the original tones of the sectional scale become rest tones of a chord (chordal functions), and the auxiliary tones become the elements of melodic *figuration.*

Melodic Form: a + 2c + d + 2g

Time rhythm: $r_{\underline{3 \div 2}}$—with circular permutations

Continuity:

Melodic Form: T_1 a + T_3 2c + T_1 d + T_2 2g



Time rhythm: $r_{4 \div 3}$

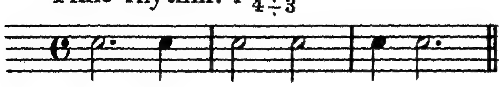


Figure 43 (continued).

Continuity:

$(T_1 + T_3 + T_1 + T_2) + (T_3 + T_1 + T_2 + T_1) + (T_1 + T_2 + T_1 + T_3) +$
$+ (T_2 + T_1 + T_3 + T_1)$

Figure 43 (concluded).

This process is *reversible* when applied to a given melody. In such a case it is necessary before harmonizing to determine which are the auxiliary and which are the rest tones, and then to assign the rest tones as definite chordal functions.

CHAPTER 9

MELODY-HARMONY RELATIONSHIPS IN SYMMETRIC SYSTEMS

CHORDS, or "harmony", and pitch-scales are interrelated. There is sufficient evidence that *simultaneous pitch aggregations* (groups which usually are called "chords") are *tonal expansions* of the corresponding pitch scales.* Such expansions produce an acoustically more suitable form of distribution of pitch-units. Wider intervals are characteristic of the lower groups of harmonics and our ear accepts narrow intervals more easily in the higher frequencies.

Certain groups produce an unsatisfactory effect merely because of their low pitch placement. Our hearing is not capable of discriminating simultaneous groups of pitch when their position is so low in frequency that the corresponding fundamental tone of the series cannot be heard in reality, i.e., when it would be below 30 cycles per second. It is easy to verify this phenomenon by simply placing such intervals as thirds, which are supposed to be "consonant," in an unusually low register. Yet when even the most "dissonant" intervals are located according to the series of harmonics, our hearing accepts them as consonant intervals. For example, the c of the large octave sounding simultaneously with the c♯ of the second octave on the scale of harmonics are the fundamental and the 17th harmonics. In this case the correspondence to the actual intonation of harmonics is so close that the effect is definitely consonant to our ear. It may be easily verified through placing any melody in such parallel couplings.

The process of building harmonic groups (chords) is a process of redistributing pitch-units so that the latter are spread through a greater tonal range. As many scales in symmetric systems of pitch appear in already wide interval-distribution, many of these scales sound like chords when played simultaneously. Any simultaneous combinations of pitch-units of such scales or of their sectional scales produce chords of varying complexity and therefore of varying *tension* (the degree of dissonant quality).

The following figure is an illustration of a scale belonging to the fourth group and based on the symmetry of 3 tonics ($\sqrt[3]{2}$, i = 4).

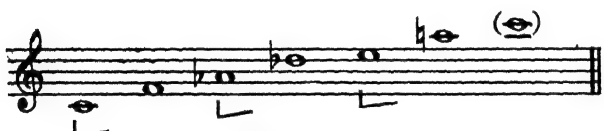


Figure 44.

*This chapter introduces matters having to do with harmony, and it should therefore be noted that the complete development of the theory of harmony occurs at a later point in the text. When all the tones of any scale are sounded simultaneously, the resulting sound provides the raw material for harmonization of melodic forms in that scale. But such a sounding of all tones at once is acoustically dissonant, except in the case of scales of comparatively few tones. Therefore, the scale is subjected to the first expansion, E_1, in order to locate the tones in a manner that will yield better acoustic results. The result of the expansion is the master-structure denoted by the Greek letter, sigma. From this total master-structure, sections are taken to provide the raw material for specific groups ("chords"). But this approach, although fundamental to all harmony, does not constitute the entire Schillinger theory of harmony which is very much more extensive than this chapter would indicate. (Ed.)

Σ (sigma) indicates the compound chord which emphasizes *all* the pitch units of the scale, whether such chord appears as a scale in its original settin or in any of the tonal expansions.

The original setting of the above scale, appears in the form of the following Σ:

Figure 45.

From this compound harmonic group, smaller groups may be devised of different degrees of complexity and classified on the basis of the number of component units. Starting each time with the succeeding pitch-unit of the original scale-setting, harmonic groups for all degrees of the scale are obtained. Being classified on the basis of the number of pitch-units, they become: diads, triads, tetrads, pentads, or hexads—for this particular scale consists of 6 units (3 tonics, 2-unit scales).

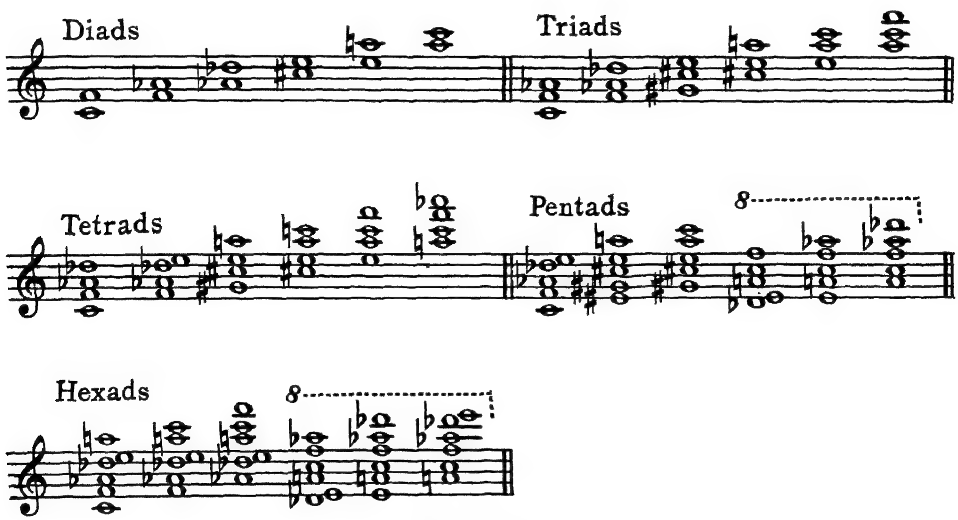


Figure 46.

Without studying harmony as such at all, one can produce perfect forme of harmonic continuity through the application of consecutive pitch-units in the form of harmonic groups of the different degrees of tension. In order to perform what is usually known as "voice-leading", it is necessary to find the *nearest* pitch-units of the succeeding group as they appear in relation to the preceding group. Groups of the different degrees of tension, as treated later in the *Special Theory of Harmony**, have many forms of transformations constituting their positions and voice-leading. In this case the principle of nearest tones is merely *one* of the special cases of such transformations.

*See Book V and IX.

Harmonic Continuity of Diads:

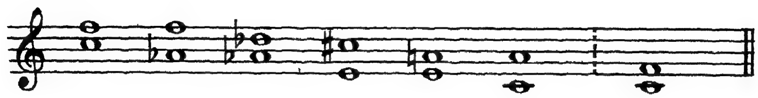


Figure 47.

By knowing which is the fundamental tone for each of such diads and through the addition of this same constant fundamental tone in the bass, a *hybrid form* of harmony is obtained (2p var. + 1 const.)

Hybrid Three-Part Harmony

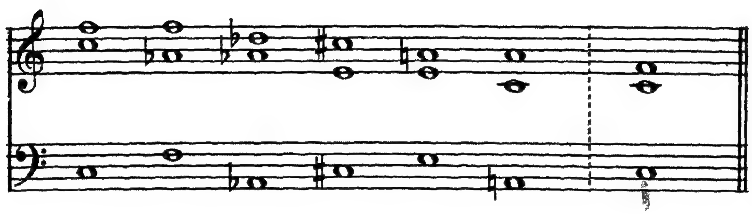


Figure 48.

Likewise, a continuity of triads may be devised.

Harmonic Continuity of Triads:

Figure 49.

By adding a constant fundamental to 3-part harmony, a hybrid 4-part harmony is obtained. (3p var. + 1 const.)

Hybrid Four-Part Harmony:

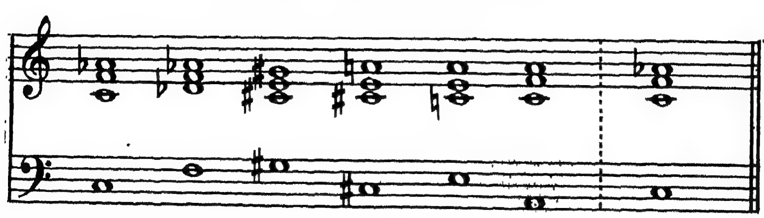


Figure 50.

The following example represents a harmony of tetrads evolved through the same principle:*

Harmonic Continuity of Tetrads:

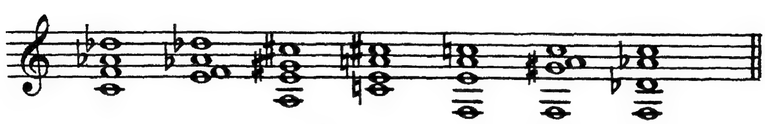


Figure 51.

Through analogous addition of a constant fundamental, a hybrid 5-part harmony thereof is obtained (4p var. + 1 const.)

Hybrid Five-Part Harmony:

Figure 52.

The following examples represent the harmony of pentads and hexads respectively. Through the addition of the constant fundamental an additional part is obtained, thus producing a hybrid 6-part harmony (5p var. + 1 const.) and 7-part harmony (6p var. + 1 const.)

Harmonic Continuity of Pentads:

Figure 53.

*With the tetradic examples given here, structures of considerable tension are introduced—and their dissonant characteristics may be too "extreme" for some, but hardly for all, tastes. Schillinger here is developing, however, *a general method*, which in this case happens to be applied to a master-structure (denoted by sigma) of high tension. Exactly the same methods may be applied to structures of much lower tension, if one desires to produce progressions, etc., which seem "less extreme." For example, the same processes set forth here may be applied to a master structure consisting of the first expansion, E_1, of G melodic minor—i.e., the minor scale starting on G with a one-flat signature and a sharped F. When expanded this produces (reading upward in thirds) c — e — g — b flat — d — f sharp — a. This master-structure happens to be a very "popular" one; examples which use this as raw material will sound "less extreme." (Ed.)

Harmonic Continuity of Hexads:

Figure 54.

In addition to harmonic possibilities as the by-products of symmetric systems of pitch, there is a method of harmonization of melody and melodization of harmony, based on the different relations of tension with respect to tonics used in melody and harmony. The word "tension" refers *both* to the harmonic group, with respect to its dissonant quality, and to the relationship which exists between melody and harmony.

In the following text, M will signify a sectional scale or a melodic form derived therefrom. H will express a harmonic group (chord structure) built on the pitch-units of one or more sectional scales (treated as total groups).

Different forms of relations between M and H produce different degrees of tension. The *minimum* tension occurs when M and H have *identical* groups (like $\frac{M}{H} = T_1$). The increase of tension depends upon the remoteness of the T's expressing M and H. The system of tension relations is symmetrical, i.e., it follows the arrangement of the tonics: T_1 is followed by T_2, T_2 is followed by T_3 . . . . T_n is followed by T_1.

The relationship between $\frac{M}{H}$ may be constant (with specified degrees of tension) or variable (with specified range of tension). The variable range of tension is subject to distributive processes assuming centrifugal or centripetal form. Forms are centrifugal when moving from the center to periphery, and centripetal when moving from periphery to the center. With regard to a scale of tension, a centrifugal form means movement from medium tension to low tension, and then to high tension, and then to low tension. Centripetal form means from high tension to low tension, and then to medium tension, or from low tension to high tension, and then to medium tension.

SYMMETRIC SYSTEMS OF PITCH $\frac{M}{H}$ RELATIONS

(1) $\frac{M}{H}$

$$\frac{M}{H}=T_1;\ \frac{M}{H}=T_2;\ \frac{M}{H}=T_3\ \ldots\ \frac{M}{H}=T_n$$

(2) $\frac{2M}{H}$

$$\frac{2M}{H}=\frac{T_1+T_2}{T_1};\ \frac{2M}{H}=\frac{T_2+T_3}{T_1};\ \ldots\ \frac{2M}{H}=\frac{T_{n-1}+T_n}{T_1};\ \frac{2M}{H}=\frac{T_n+T_1}{T_1}$$

(3) $\frac{M}{2H}$

$$\frac{M}{2H}=\frac{T_1}{T_1+T_2};\ \frac{M}{2H}=\frac{T_1}{T_2+T_3};\ \ldots\ \frac{M}{2H}=\frac{T}{T_{n-1}+T_n};\ \frac{M}{2H}=\frac{T_1}{T_n+T_1}$$

(4) $\frac{3M}{H}$

$$\frac{3M}{H}=\frac{T_1+T_2+T_3}{T_1};\ \ldots\ \frac{3M}{H}=\frac{T_{n-2}+T_{n-1}+T_n}{T_1};\ \frac{3M}{H}=\frac{T_{n-1}+T_n+T_1}{T_1};\ \frac{3M}{H}=\frac{T_n+T_1+T_2}{T_1}$$

(5) $\frac{M}{3H}$

$$\frac{M}{3H}=\frac{T_1}{T_1+T_2+T_3};\ \ldots\ \frac{M}{3H}=\frac{T_1}{T_{n-2}+T_{n-1}+T_n};\ \frac{M}{3H}=\frac{T_1}{T_{n-1}+T_n+T_1};\ \frac{M}{3H}=\frac{T_1}{T_n+T_1+T_2}$$

(6) $\frac{nM}{H}$

$$\frac{nM}{H}=\frac{T_1+T_2+T_3+\cdots+T_n}{T_1};\ \frac{nM}{H}=\frac{T_2+T_3+\cdots+T_n+T_1}{T_1};\ \frac{nM}{H}=\frac{T_3+\cdots+T_n+T_1+T_2}{T_1};$$
$$\frac{nM}{H}=\frac{T_n+T_1+T_2+T_3+\cdots}{T_1}$$

(7) $\frac{M}{nH}$

$$\frac{M}{nH}=\frac{T_1}{T_1+T_2+T_3+\cdots+T_n};\ \frac{M}{nH}=\frac{T_1}{T_2+T_3+\cdots+T_n+T_1};\ \frac{M}{nH}=\frac{T_1}{T_3+\cdots+T_n+T_1+T_2};$$
$$\frac{M}{nH}=\frac{T_1}{T_n+T_1+T_2+T_3+\cdots}$$

(8) $\frac{aM}{bH}$

$$\frac{aM}{bH}=\frac{T_1+T_2+\cdots+T_a}{T_1+T_2+\cdots+T_b}$$

Figure 55

The lower forms of tension pertain to music which corresponds chronologically to the earlier forms of $\frac{M}{H}$ relations. Such music is typical of Scarlatti, Haydn, Mozart, etc. The higher forms of tension lead to modernity of effect. The actual musical effect depends on the original structure of the sectional scales and their compound sonority from all symmetrical points simultaneously. Many of the scales in their $\frac{M}{H}=\frac{T_2}{T_1}$ relation produce the effect of moderately modern music in the way it sounds to the listener today. It belongs to the type of music in which the tension is analogous to that of Chausson, Debussy, Ravel, early Stravinsky, etc. Further forms of tension are characteristic of the later Stravinsky, of Casella, Malipiero, Auric, Poulenc, Milhaud, etc.

By using the multiple-tonic system, such as six or twelve tonics, still higher tensions than those mentioned above can be obtained.

As it follows from the table, the emphasis of more than one group of M against one group of H, and vice-versa, may include different degrees of tension as a constant characteristic of style. In the esthetic sense such a method offers a moderation of the extremities. Thus, any symmetrical scale offers a multiplicity

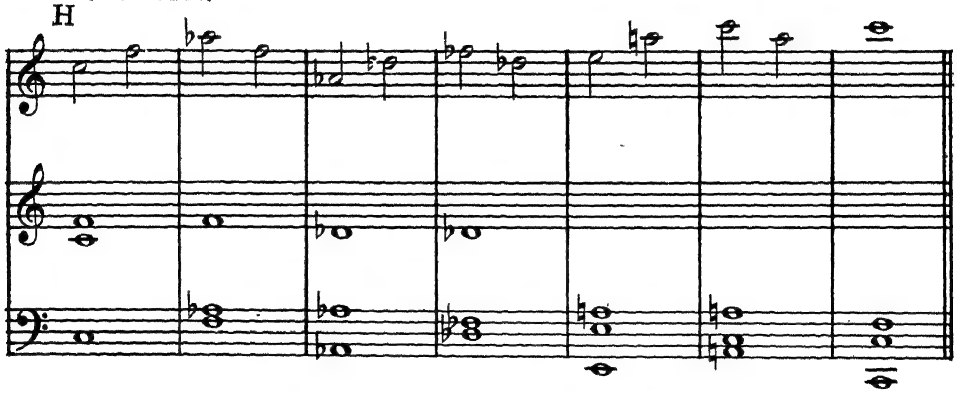
of styles where each individual style is an outcome of the forms of setting of the original scale as well as the specifications of $\frac{M}{H}$ relations.

In the following examples, melodies from the 2-unit sectional scales are produced from two melodic forms $(a_2 + b_2)$. Constant T appears in various degrees of tension, occurring between melody and harmony.

In the first example, the melody emphasizes the pitch-units of the corresponding harmonic group only.

In the second example the 2-unit melody group is displaced one phase upward, i.e., it emphasizes the second unit of the first sectional scale and the first unit of the second sectional scale.

$\frac{M}{H}$ = T const.



$\frac{M}{H}$ = T const.

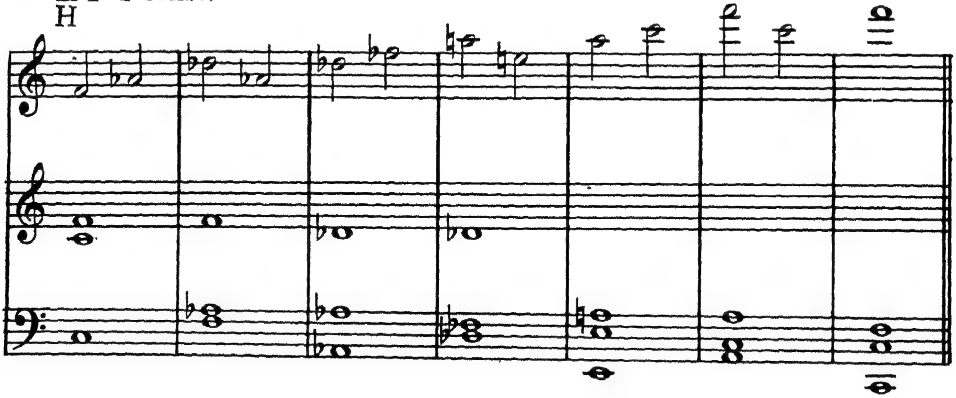


Figure 56.

The constant degrees of tension acting within the restricted limits of sectional scales represent different degrees of tension between melody and harmony, according to the table of $\frac{M}{H}$ relations set forth on page 173.

Melodic form is realized through the same structure as in the preceding example.

$$\frac{M}{H} = \frac{T_2}{T_1} + \frac{T_3}{T_2} + \frac{T_1}{T_3}$$

$$\frac{M}{H} = \frac{T_3}{T_1} + \frac{T_1}{T_2} + \frac{T_2}{T_3}$$

$$\frac{M}{H} = \frac{T_1+T_2+T_3}{T_1} + \frac{T_1+T_2+T_3}{T_2} + \frac{T_1+T_2+T_3}{T_3} + \dots$$

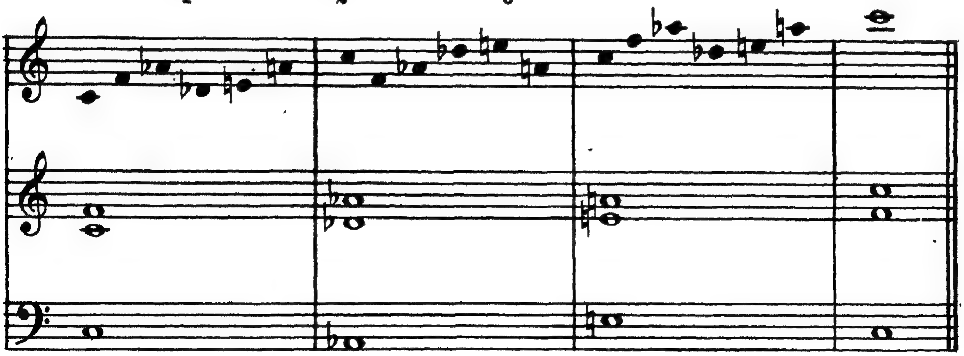


Figure 57.

As previously stated, when the original setting of a symmetrical scale is *not acoustically acceptable,* its sectional scales must undergo tonal expansions in order to acquire the acoustical appearance of harmonic groups. For example, take a scale of the fourth group with 4-unit sectional scales on 4 tonics ($\sqrt[4]{2}$, $i = 2 + 3 + 2$).

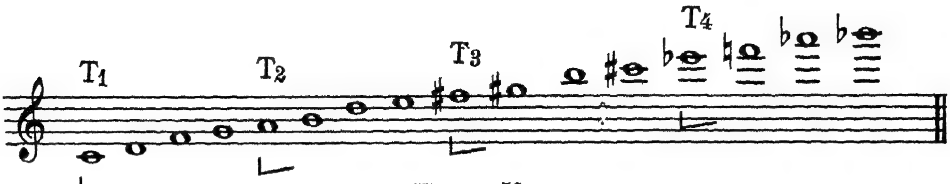


Figure 58.

Through E_1, harmonic groups may be obtained on all 4 tonics. Here are the chord structures (S) obtained through tonal expansion of the sectional scales.

Figure 59.

These tetrads produce the following form of harmonic continuity when voice-leading is obtained through the same principle (moving to the nearest tone).

Figure 60.

By establishing various relations of tension between melody and harmony, different forms of accompanied melody may be devised.

$$\frac{M}{H} = T_1 + T_2 + T_3 + T_4$$

$$\frac{M}{H} = \frac{T_2}{T_4} + \frac{T_3}{T_2} + \frac{T_4}{T_3} + \frac{T_1}{T_4} + \cdots$$

$$\frac{M}{H} = \frac{T_3}{T_1} + \frac{T_4}{T_2} + \frac{T_1}{T_3} + \frac{T_2}{T_4} + \cdots$$

Figure 61.
(continued)

$$\frac{M}{M}=\frac{T_4}{T_1}+\frac{T_1}{T_2}+\frac{T_2}{T_3}+\frac{T_3}{T_4}+\cdots$$

$$\frac{M}{H}=\frac{T_1+T_2}{T_1}+\frac{T_2+T_3}{T_2}+\frac{T_3+T_4}{T_3}+\frac{T_4+T_1}{T_4}+\cdots$$

Figure 61 (concluded).

In this procedure, the removal of harmony from melody produces the same effect of increasing tension as does the removal of melody from harmony. The entire scale may thus be harmonized in two fundamental ways: when a chord is constant and the sectional scale varies, and when the sectional scale is constant and the chord varies.

The following example illustrates the combination of both procedures, i.e., the first sectional scale is accompanied by all chords; then the second sectional scale is accompanied by all chords, etc.

$$\frac{M}{H}=\frac{T_1}{T_1+T_2+T_3+T_4}+\frac{T_2}{T_1+T_2+T_3+T_4}+\frac{T_3}{T_1+T_2+T_3+T_4}+\frac{T_4}{T_1+T_2+T_3+T_4}+\cdots$$

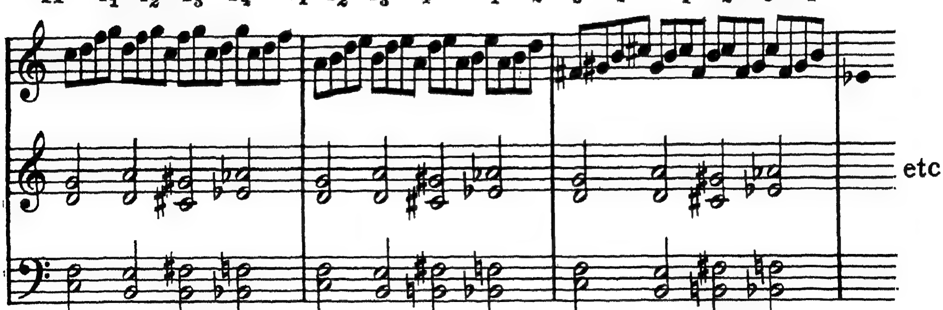


Figure 62.

The reversal of this procedure is applicable to various phases of arranging and composing music. An illustration might be taken from George Gershwin's song, *I Got Rhythm*. As a possible form of introduction, the first two bars of melody represent the original two-bar motif; the following three two-bar motifs represent circular permutations of the original motif on the $\sqrt[4]{2}$ setting. The original harmony is left for accompaniment which naturally undergoes, under such conditions, one of the procedures described in $\frac{M}{H}$ relations. In this particular example, the degree of tension between melody and harmony is constant.

This method is applicable in many ways and potentially includes an inconceivable amount of music, as the number of scales consists of two thousand, and practically every scale gives an infinite number of melodies and a great number of $\frac{M}{H}$ relations.

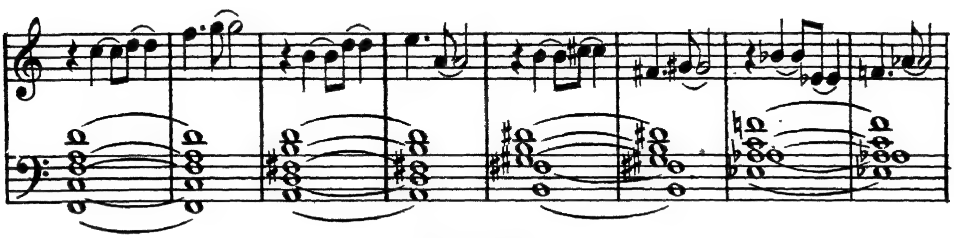


Figure 63.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK III

VARIATIONS OF MUSIC
BY MEANS OF GEOMETRICAL PROJECTION

BOOK THREE

VARIATIONS OF MUSIC BY MEANS OF GEOMETRICAL PROJECTION

CHAPTER 1

GEOMETRICAL INVERSIONS

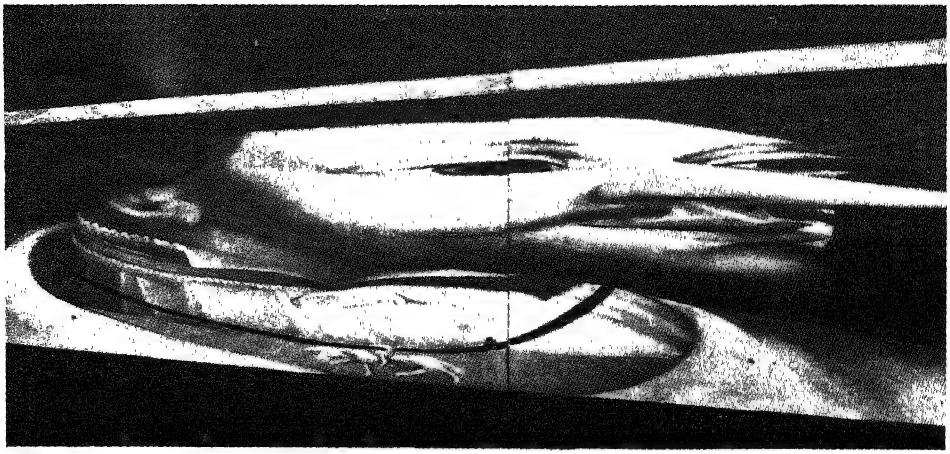
MUSIC in any equal temperament, when it is recorded graphically in rectangular projection, expresses the equivalent of musical *notation* in equal temperament. Such a geometrical projection of music is expressed on a plane, and as such is subject to quadrant rotation of the plane through three dimensional space. Rotation may be either *clockwise* or *counterclockwise.**

The conception of time, which is based on the common denominator and not on the logarithmic series, implies two possible positions: (1) the original, under zero degrees to the field of vision (parallel to the eyes); (2) the 180° position derived from the first one through rotation around the ordinate axis. Such an ordinate axis is either the starting or the ending limit of the vertical cross-section of the graph (duration limits). If the original (zero degree position) is conceived as a forward motion of music in time continuity, then the respective variation of it (180° position) is the backward motion of the original, when the ordinate is the ending limit in time.**

The *logarithmic* contraction of time corresponds to the logarithmic contraction of space on the graph—and if our music were not bound to a common denominator system of measurement, it would be possible to apply such projection practically. This same form of variation has been known in *visual* art since about 1533 A.D., in skillful paintings made by German and Italian artists. They are based on the principle of angle-perspective and have to be looked at (that is, held at an angle) from right to left, instead of under the zero angle to the field of vision.

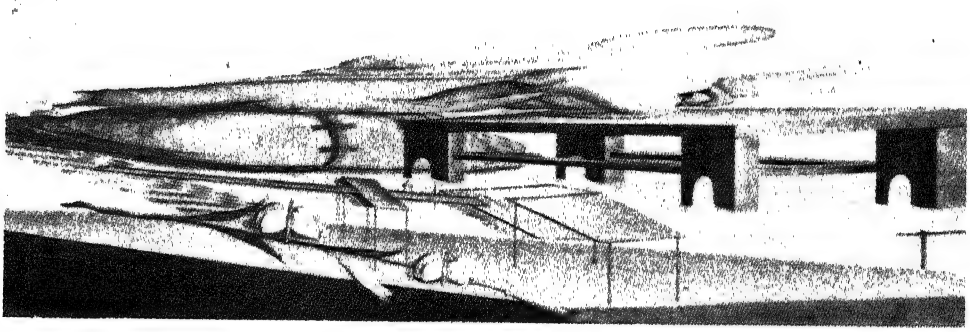
*It may be helpful to add at this point the following: geometric inversion of music consists of "a" of the original form of the music, to start with; then, as the "b" inversion, the same thing *backwards*; as the "c" inversion, the original but backwards and *upside down*; as the "d" version, forwards and upside down. (Ed.)

**When you turn a left-hand page of this book as you normally do, you are revolving it through 180° around—what may be conceived as—its "ordinate" axis. Now if the page were transparent and there was reading matter on only one side, you would find—after you turned the page—that the material at the right side would then be at the left side: that is, you would be reading it backwards. This is position ⓑ of geometrical inversion. See part A of figure 4. (Ed.)



*German School, 16th Century; Charles V, 1533**

Figure 1.



*Unknown Master, 16th Century: St. Anthony of Padua **

Figure 2.

*Courtesy of Museum of Modern Art, exhibited in *Fantastic Art, Dada and Surrealism*, 1937.

By revolving the second position of a musical graph through the abscissa (which becomes the axis of rotation) 180° in a clockwise direction, we obtain the third position of the original. The axis of rotation must represent a *pt* (pitch-time) maximum and the direction of the third position is backwards upside-down of the original, and forward upside-down of the second position.* Further 180° clockwise rotation of the third position about its ordinate produces the fourth position, which is the backwards of the third position, the backwards upside-down of the second position and the forward upside-down of the original.** The respective four positions will be expressed in the following exposition through ⓐ, ⓑ, ⓒ and ⓓ.

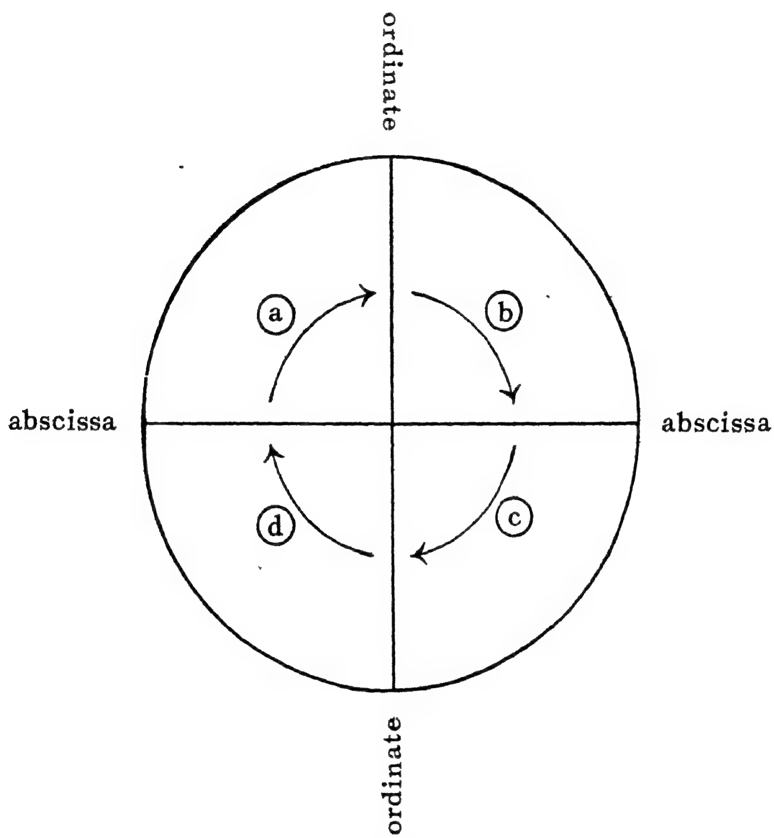


Figure 3.

*To continue from the point at which we stopped in the footnote on page 185, imagine that you could turn the transparent page downward. You would then be revolving it 180° around—what may be conceived as—its "abscissa" axis. The reading matter would appear upside down and backwards with reference to its original position on the left-hand page. This is the third position ⓒ of geometrical inversions. See part A of figure 4. (Ed.)

**Continuing with the illustration of the previous footnote, imagine that you could now turn our transparent page back toward the front cover. The reading matter would then appear upside down and forwards with reference to the original left-hand page, position ⓐ. It would appear backwards with regard to position ⓒ—and upside down and backwards with regard to position ⓑ. See part A of figure 4. (Ed.)

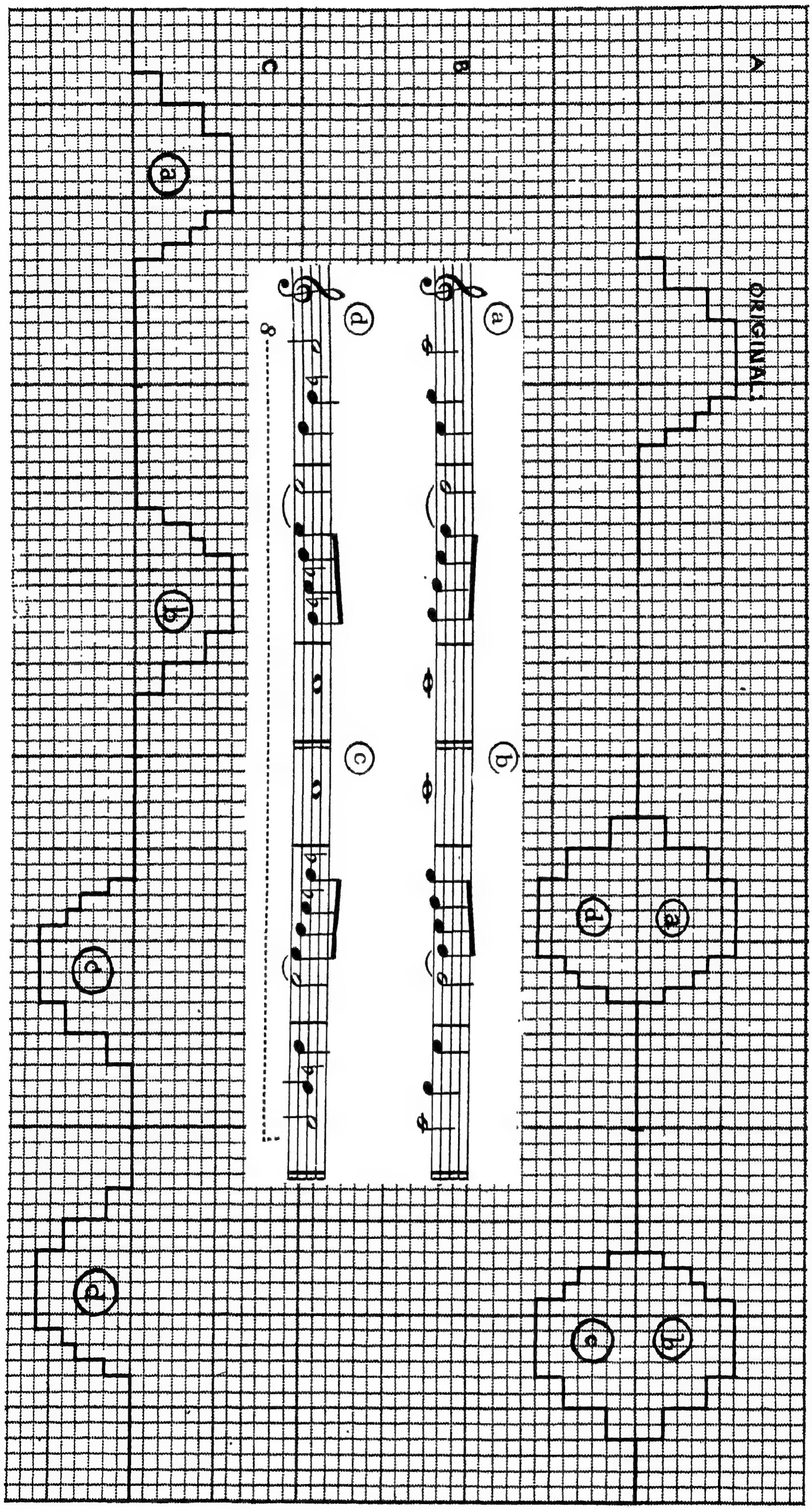


Figure 4. Evolving the Four Geometrical Inversions of a Given Melody.

*Graphs on this page are slightly reduced in scale.

These four geometrical inversions may be used individually as variations of a given melody. They may also be developed into a continuity in which the different positions are given different coefficients. Under such conditions the recurrence of the different positions is subject to rhythm.

Melodic continuity derived from geometrical inversions is exemplified in the following illustrations:

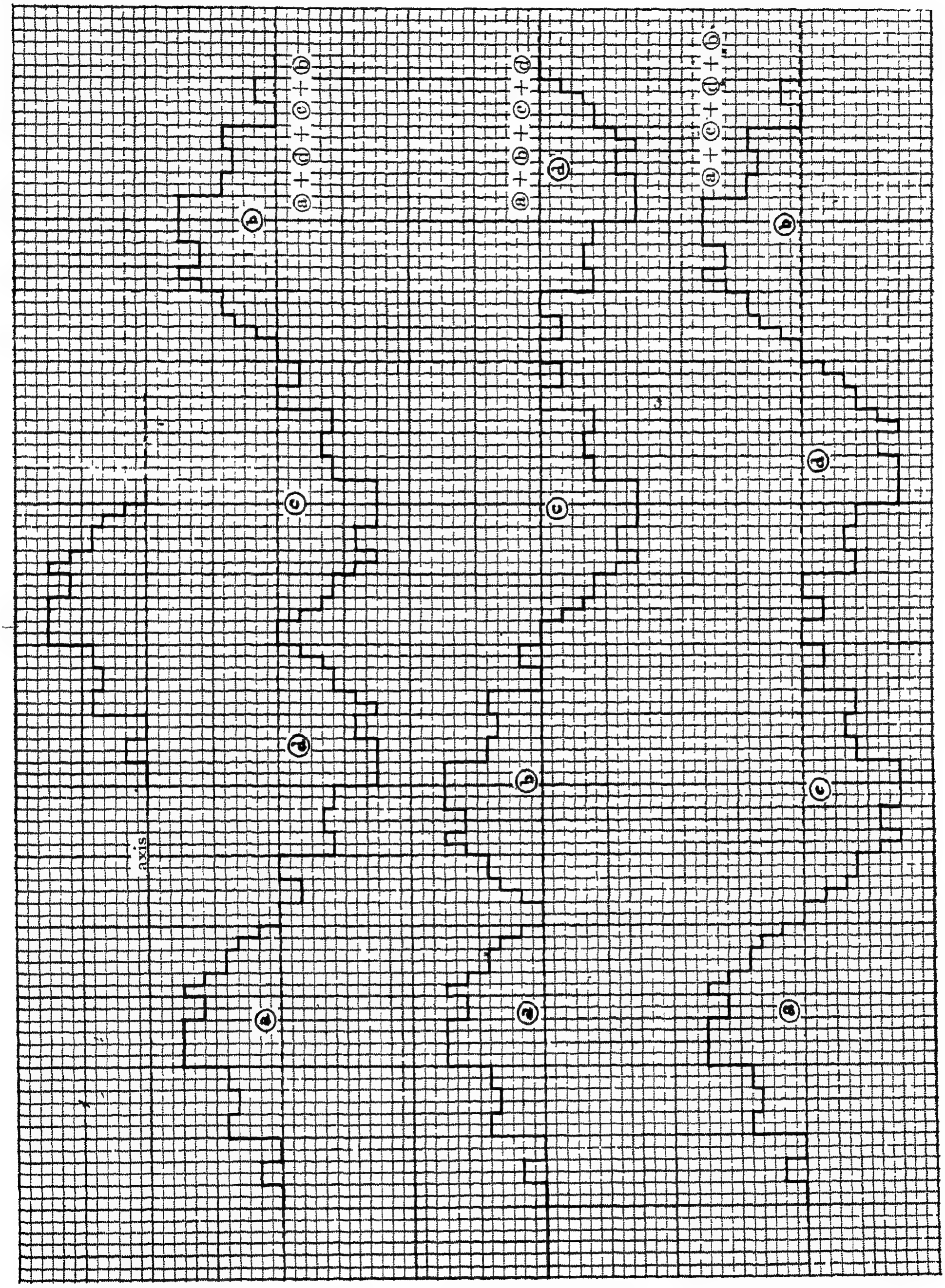


Figure 5. Some permutations of geometrical inversions. Graph representation.

Figure 6. Musical representation of graphs in Figure 5.

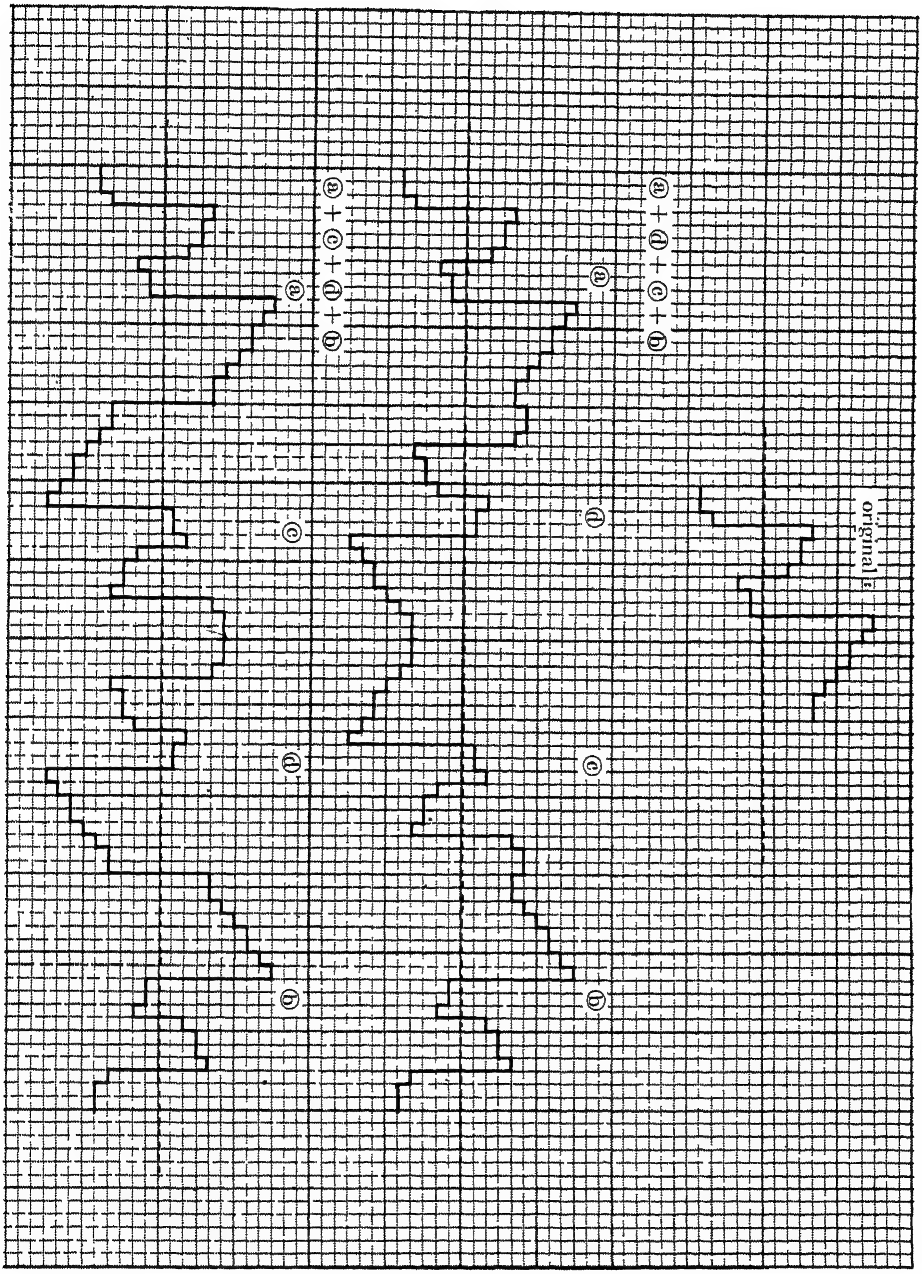


Figure 7. Geometrical inversions represented graphically.

Theme:

a+d+c+b

a+c+d+b

Figure 8. Musical representation of Figure 7.

This method of geometrical inversion, when applied to the composition of melodic continuity, offers much greater versatility—yet preserves the unity more—than any composer in the past was able to achieve. For example, by comparing the music of J. S. Bach with the following illustrations, the full range of what he could have done by using the method of geometrical inversions becomes clear.

In Invention No. 8, from his *Two-Part Inventions*, during the first 8 bars of the leading voice (upper part after the theme ends), the first 2 bars fall into the triple repetition of an insignificant melodic pattern lasting one and one-half times longer than the entire theme.

Figure 9. J. S. Bach, Two-Part Inventions, No. 8.

Using the method of geometrical inversion (even with a compromise of the recurrence of the original position), we obtain the following version of thematic continuity.

Figure 10. Inversion of J. S. Bach, Two-Part Inventions, No. 8 (continued).

Figure 10. Expansions of J. S. Bach, Two-Part Inventions, No. 8 (concluded).

In another case, that of Fugue No. 8 from Bach's *Well-Tempered Clavichord*, Volume I, if we compare the first 12 bars of the original with the version evolved from this same theme by means of geometrical inversion, we cannot fail to see the esthetic advantage of this method of composition over the more casual one derived partly from dogmatic and partly from intuitive channels.

Figure 11. J.S.Bach, Vol. 1, Fugue VIII.

Figure 12. J. S. Bach, Well-Tempered Clavichord, Vol. 1, Fugue VIII.

In some cases geometrical inversions of music give new and often more interesting character to the original. When a composer feels dissatisfied with his theme, he may try out some of the inversions—and he may possibly find them more suitable for his purpose, discarding the original. Such was the case when George Gershwin* wrote a theme for his opera *Porgy and Bess*, where position c was used instead of the original which was not as expressive and lacked the character of the latter version.

*In the *Musical Courier* of Nov. 1, 1940 Leonard Liebling, editor, wrote: "After George Gershwin had written over 700 songs, he felt at the end of his inventive resources and went to Schillinger for advice and study. He must have valued both, for he remained a pupil of the theorist for four and a half years." *Porgy and Bess*, which took Gershwin more than two years of work under his teacher's supervision, was composed according to the Schillinger System. (Ed.)

An analysis of well-known works of the composers of the past often throws new light upon them, revealing hidden characteristics that become more apparent in the geometrical inversions. For example, the harmonic minor scale combined with certain rhythmic forms produces an effect of Hungarian dance music. In L. van Beethoven's Piano Sonata No. 8, the first theme of the finale in its position ⓑ reveals a decidedly Hungarian character which is not as noticeable in its original form. This analysis also discloses that position ⓓ of the same theme has a more archaic character than the original, linking Beethoven's music with that of Joseph Haydn.

Figure 13. Geometric inversions of L. van Beethoven, Piano Sonata No.8, Finale.

In composing continuity through geometrical inversions, it is important to attend to the rhythmic structure of time elements in the original theme. According to the principles of this theory, whenever rhythmic groups assume natural forms, i.e., have an axis of symmetry, the quality of the melody will not be debased in the ⓑ and ⓒ positions of the original, and the rhythmic resultants as well as the permutation-groups are reversible.

While the principle of inversion does not interfere in any way with the intonation, it may produce undesirable, lasting durations which become exaggerated when the forward and backward moving positions are adjacent. If the original has a long duration at the end and position ⓑ follows immediately, this duration will be doubled. In such a case a rhythmic readjustment is desirable and the elimination of one of the long durations becomes necessary. Complete elimination of the final points having excessive durations may produce, in some cases, even a more satisfactory melodic continuity, as in the example below. (The melody is taken from Figure 4).

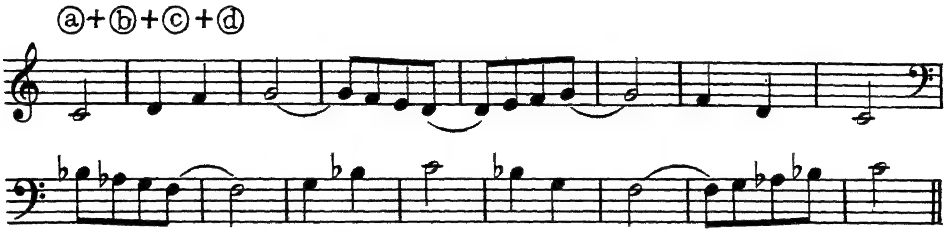


Figure 14. Adjusting rhythms in geometric inversion.

Thus, it is possible to plan in advance the composition of melodic continuity through combining geometrical inversions of the original material with a rhythmic group pre-selected for the coefficients of recurrence of the different positions.

Rhythm of Coefficients: $r_{4 \div 3}$
Geometrical Positions: ⓐ, ⓓ, ⓑ
Continuity: 3 ⓐ + ⓓ + 2 ⓑ + 2 ⓐ + ⓓ + 3 ⓑ

The actual technique of transcribing music from one position to another may be worked out in three different ways. The student may take his choice.

1. Direct transcription of the inverted positions from the *graph* into musical notation.
2. Direct transcription from a complete manifold of chromatic tables representing ⓐ and ⓓ positions for all the 12 axes.

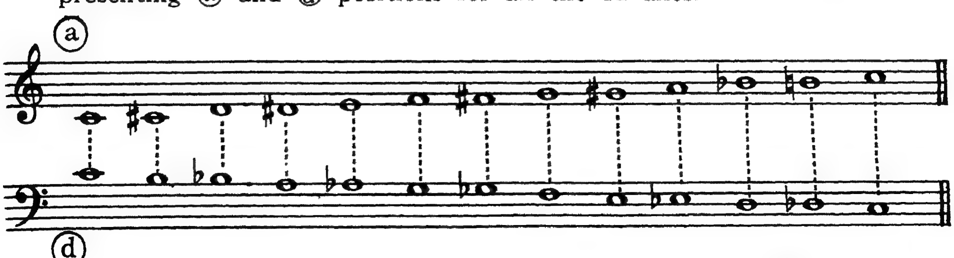


Figure 15. Manifold of chromatic tables for ⓐ and ⓓ.

3. Step by step (melodic) transcription from the original.

The unconscious urge toward geometrical inversions was actually realized in music of the past through those backward and contrary motions of the original pattern which may be found in abundance in the works of the contrapuntalists of the 16th, 17th and 18th centuries. As they did not do it geometrically but tonally, they often *misinterpreted* the tonal structure of a theme appearing in an upside-down position. They tried to preserve the tonal unity instead of preserving the original pattern. Besides these thematic inversions of melodies, evidence of the tendency toward unconscious geometrical inversions may be observed in the juxtaposition of major and minor as the *psychological* poles. In reality, the commonly used harmonic minor is simply an erroneous geometrical inversion of the natural major scale. The correct position (d) of the natural major scale is the Phrygian scale and not the harmonic minor. The difference appears in the 2d and 7th degrees of that scale.

In the following examples, d$_{(b)}$ indicates the upward reading of the (d) scale.

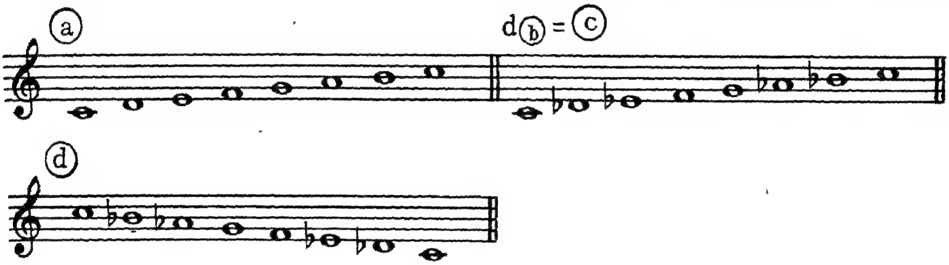


Figure 16. Inversion of natural major.

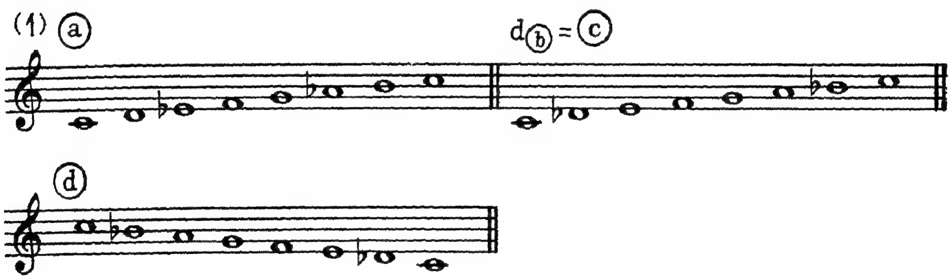


Figure 17. Inversion of harmonic minor.

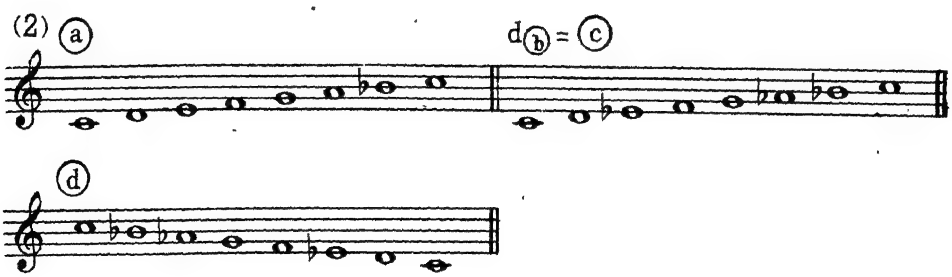


Figure 18. Inversion of Mixolydian.

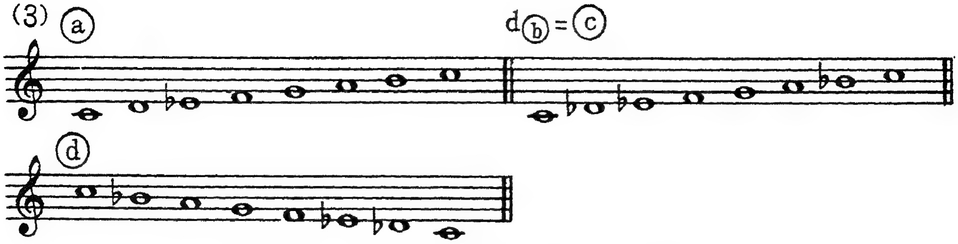


Figure 19. Inversion of melodic minor.

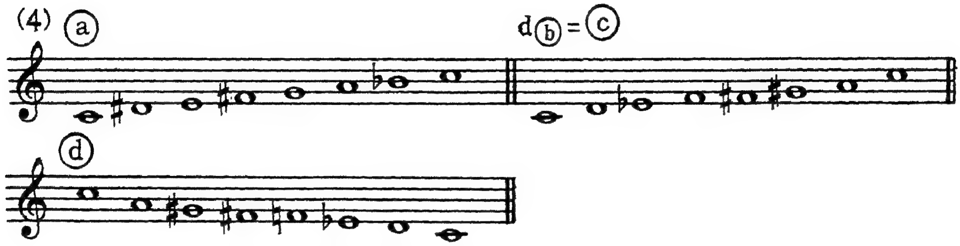


Figure 20. Inversion of Hungarian major.

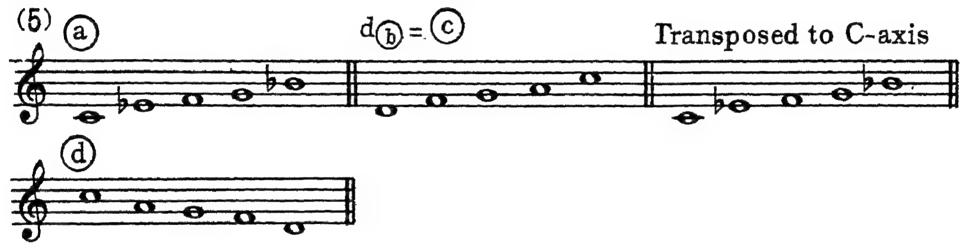


Figure 21. Inversion produces an axis of symmetry.

Thus, we see that the "psychological major" of the harmonic minor is an entirely new scale, figure 17; that the "psychological minor" of the Mixolydian scale is the Aeolian scale, figure 18; that the "psychological major" of the melodic minor scale is d_4 of the melodic major scale, figure 19; that the psychological minor of the Hungarian major scale is not the Hungarian minor scale but a new scale, figure 20; that some of the scales being inverted through their axis of inversion produce an axis of symmetry, i.e., their compensating scales are identical in structure with the original scale, i.e., ⓐ = ⓓ, figure 21.

The transcription of polyphonic continuity into different geometrical inversions must be performed from the pitch-axis of inversion in such a way that not only individual counterparts but also their mutual pitch relations are inverted accordingly. For example, if the pitch-axis of inversion is g and the theme enters on d, the same melody will start on c in position ⓓ—seven semitones in the opposite direction from the axis of inversion as compared with the seven semitones of the original direction. In the following excerpt from a fugue, the theme starts on d and the reply on g; g being the axis of inversion sets the theme on the starting point c and the reply on the starting point g (the invariant of inversion).

Figure 22. J. S. Bach, Well-Tempered Clavichord, Vol. 1, Fugue XVI.

The effect of psychological contrasts, to which I have referred with regard to scales, takes place with chord structures and their progressions as well. The most obvious illustration is a major triad (c — e — g; 4 + 3) with its reciprocal structure minor triad (c — e♭ — g; 3 + 4). When such a chord is to be inverted from c as an axis, all pitch-units take corresponding places in the opposite direction, i.e., c remains constant (the invariant of inversion), e becomes a♭, and g becomes f. Here is a comparative chart of positions ⓐ and ⓓ of the chords commonly known as triads [S(5)] and 7th chords [S(7)].

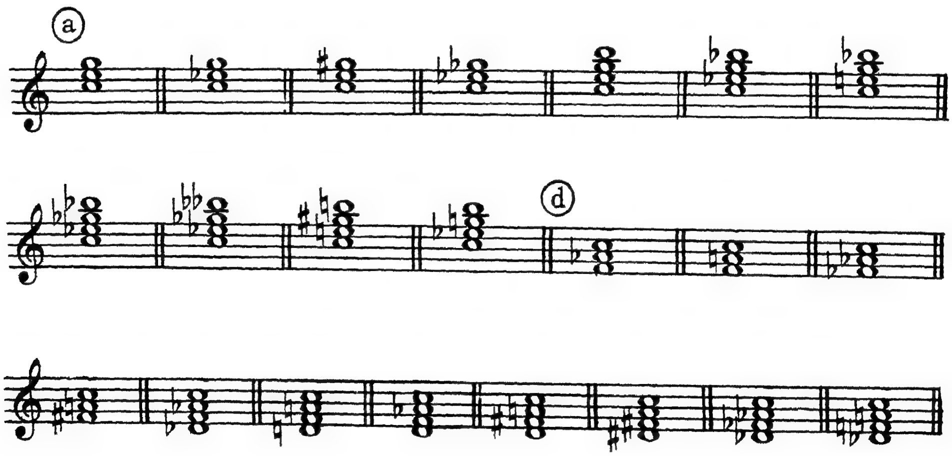


Figure 23. ⓐ and ⓓ positions of the triads.

This method of inverting chords as well as scales in order to find the psychological reciprocal is particularly useful in cases *where there is doubt* as to what the reciprocal chord structure or progression may be. It also provides an exact way of finding the reciprocal structures and progressions in those cases in which the latter are entirely unknown—and the trial and error method does not bring

any satisfactory result. For example, the reciprocal of the structure in the following example may sound quite surprising—yet the above chart shows that position ⓓ does not distort the original structure but merely changes its position.

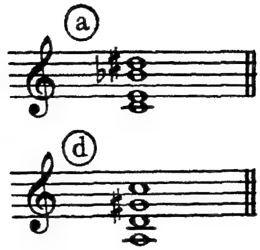


Figure 24.

From all this, it is easy to see that not only an individual melody or a group of melodies (counterpoint), but also a melody with harmonic accompaniment, may be transcribed via various geometrical inversions. The melody of the earlier example is offered here with an accompaniment of harmony and its inversion into position ⓓ.

Figure 25.

Geometrically, a melody appearing above harmony in the original appears below harmony in position ⓓ . It may also be rewritten, without any damage to the music, by being placed *above* the harmony.

The technique of transcribing any harmonic continuity into different geometrical positions can be greatly simplified by using the method of *enumeration of each voice* of the harmony. Each voice becomes a melody and it is only necessary to know the entire chord, (i.e., the starting-points of such melodies) for the starting-point, after which all voices may be transcribed horizontally (as melodies).

Figure 26. ⓐ *and* ⓓ *of melody with chords.*

If position ⓓ makes the upper voice appear in the bass, the opposite must be true: i.e., the placement of the bass of the original above all other voices.

Figure 27. ⓐ *and* ⓓ *of melody with chords.*

The above-mentioned operations make it clear that any of the variations in the original distribution of voices of a chord may serve as a starting point for any harmonic continuity. Thus, a 4-part harmony offers 24 versions in each of the four geometrical positions. This device is superior to the ingenuity of any composer using an intuitive method in order to achieve variety of instrumental forms of the same harmonic continuity.

The following chart represents 24 original forms of distribution of the starting chord (according to the 24 permutations of 4 elements), for the harmonic continuity offered in the preceding figures 26 and 27. When the starting chord has the same structure but different distribution, the resulting sonority of each version also becomes different.

d c c c	d b b b	d b b b	d c c c	d a a a	d a a a
c d b b	b d c c	b d a a	c d a a	a d c c	a d b b
b b d a	c c d a	a a d c	a a d b	c c d b	b b d c
a a a d	a a a d	c c c d	b b b d	b b b d	c c c d
4 3 3 3	4 2 2 2	4 2 2 2	4 3 3 3	4 1 1 1	4 1 1 1
3 4 2 2	2 4 3 3	2 4 1 1	3 4 1 1	1 4 3 3	1 4 2 2
2 2 4 1	3 3 4 1	1 1 4 3	1 1 4 2	3 3 4 2	2 2 4 3
1 1 1 4	1 1 1 4	3 3 3 4	2 2 2 4	2 2 2 4	3 3 3 4

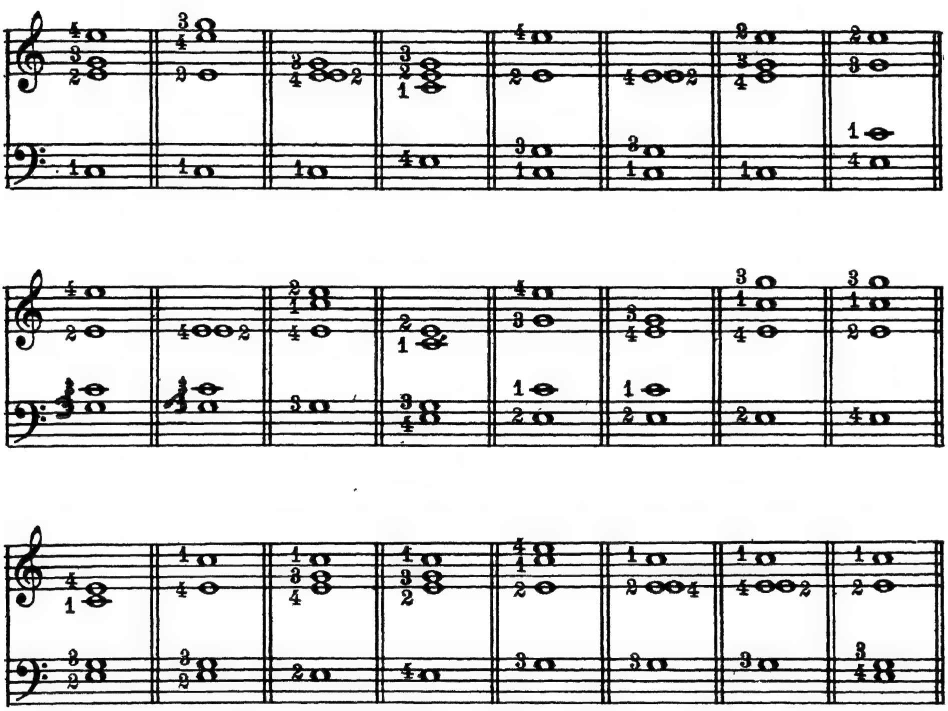


Figure 28. Twenty-four original forms of distribution of starting chord.

When portions of harmonic continuity which are short enough to be retained in the memory are used in different geometrical positions, the result is natural contrasts between adjacent sections of such continuity. In the following example, the preceding harmonic content was used in groups of three chords to one geometrical position. Each group of three chords is followed by its own position d of the same group. The continuity appears as follows:

[1 + 2 + 3] a + [1 + 2 + 3] d +
+ [4 + 5 + 6] a + [4 + 5 + 6] d . . .

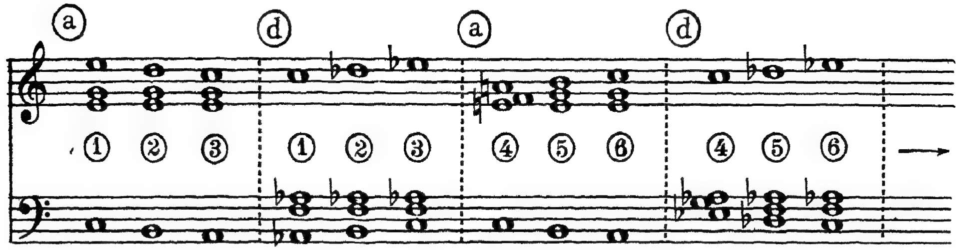


Figure 29. Contrasting harmonic continuity.

Another form of contrasting harmonic continuity derived from the same material may be evolved through consecutive progressions from chord to chord, emphasizing every three chords for one geometrical position.

[1 + 2 + 3] a + [4 + 5 + 6] d +
+ [7 + 8 + 9] a + [10 + 11 + 12] d + . . .

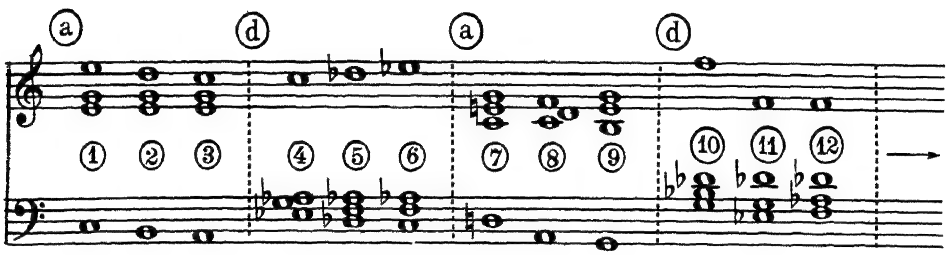


Figure 30. Contrasting harmonic continuity.

The two preceding forms of harmonic continuity are satisfactory only when different orchestration or different registration is applied to each individual group of inversion. When it is desirable to get a mixture of different geometrical positions forming one harmonic continuity and containing contrasts, then every movement of transition to a new geometrical position must be readjusted with regard to voice-leading. It may be accomplished by those students who are not as yet familiar with the theory of harmony *by connecting the two adjacent chords belonging to two different geometrical positions* through their nearest tones. Thus, Figure 29 takes the following appearance:

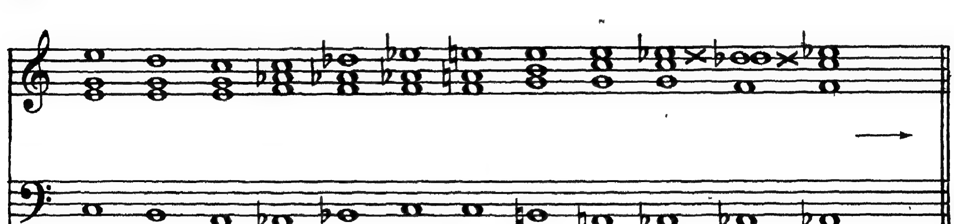


Figure 31. Connecting adjacent chords through nearest tones.

Figure 30 takes the following form when connecting tones are added:

Figure 32. Adding nearest tones to connect adjacent chords of different geometrical positions.

It is possible to create compositions of harmonic continuity from any original chord-progression, where different geometrical positions may appear in any desirable order and with any desirable coefficients of recurrence. In order to obtain a clear presentation of the scheme of progressions, it is necessary to take the entire progression in position ⓐ and to enumerate all chords in the order of their appearance. In order to enumerate the same progression in position ⓑ, it is necessary to start the numbers reading backward. The next step is to enumerate the entire position ⓓ of the chord progression starting at the beginning, and position ⓒ starting at the end, proceeding backwards.

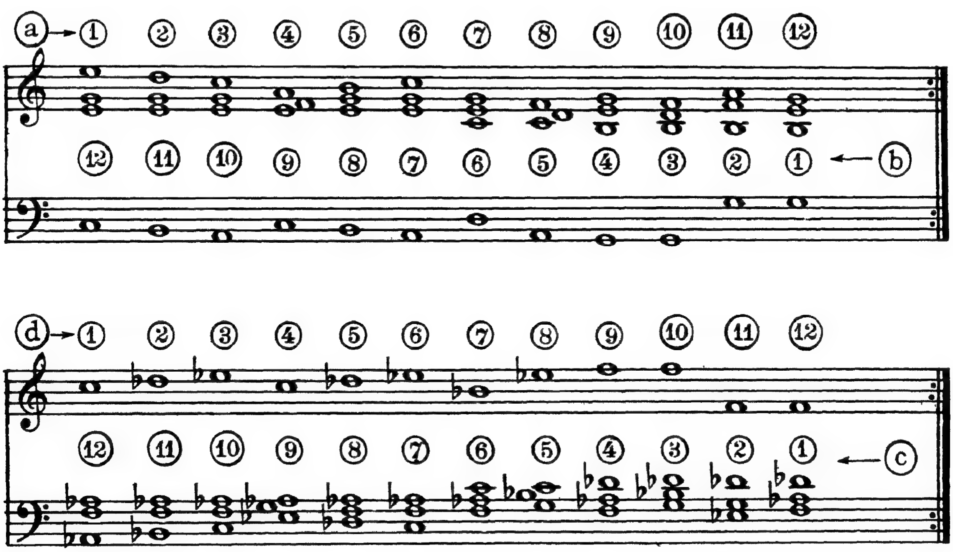


Figure 33. ⓐ ⓑ ⓒ *and* ⓓ *enumerated.*

The following is an example of the composition of a continuity containing all geometrical inversions and based on the rhythm of coefficients of $r_{5\div4}$:

Rhythm: $r_{5\div4}$

4 Ⓑ + Ⓒ + 3 Ⓓ + 2 Ⓐ + 2 Ⓑ + 3 Ⓒ + Ⓓ + 4 Ⓐ

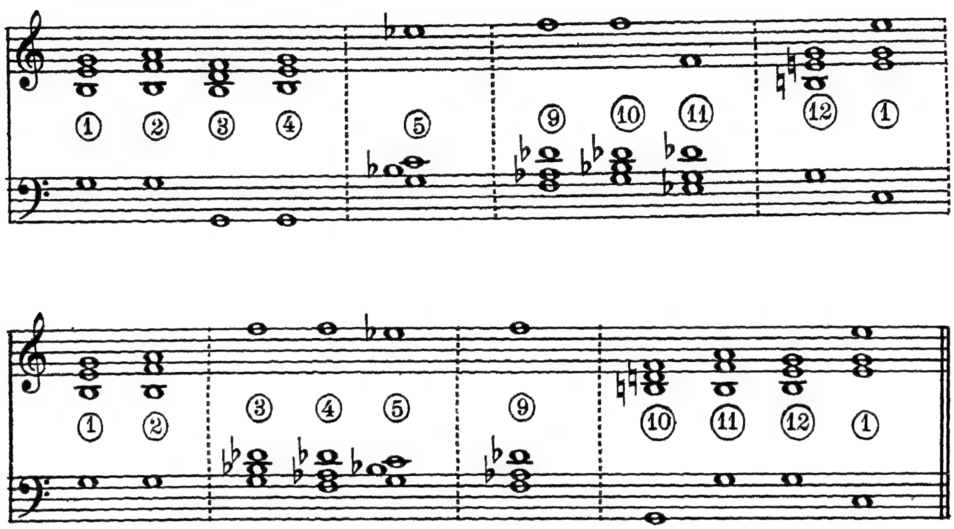


Figure 34. Composition of a continuity containing all geometric inversions.

In the above example, the direction of position changes with each coefficient The entire scheme starting in position Ⓑ includes the 1, 2, 3 and 4 moving from the end towards the beginning in the original harmonic setting. The next chord moves in the same backward direction but it is a pitch inversion of the preceding progression. Thus, this chord becomes 5. The succeeding three chords are in position Ⓓ which means that the time direction changes to forward. As the last chord was the 5 in backward motion, it corresponds to the 8 in forward motion. Therefore, the first chord to be obtained in position Ⓓ is 9. As this inversion consists of three chords, they include 9, 10 and 11. Proceeding in a similar fashion, one can evolve any number of harmonic progressions from a limited group of chords.

The last case—harmonic continuity, Figure 34, being adjusted through voice-leading to the nearest positions for adjacent chords belonging to different geometrical inversions—takes the following appearance.

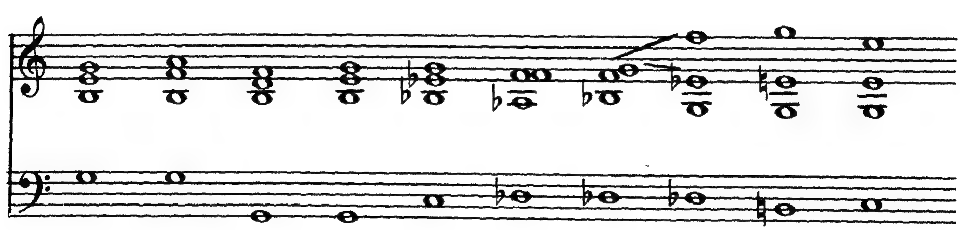


Figure 35 (*continued*)

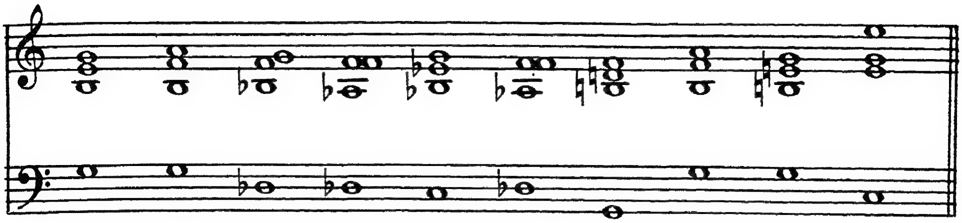


Figure 35. Adjusting harmonic continuity of Figure 34 through voice leading (concluded).

In its final form as above, such a continuity may be assigned to a homogeneous orchestration or registration.

CHAPTER 2

GEOMETRICAL EXPANSIONS

HAVING DISCUSSED the technique of geometrical *inversions*, we may now consider an additional set of techniques, those leading to geometrical *expansions*. Tonal expansions, as distinct from geometrical expansions, were discussed in the *Theory of Pitch-Scales*, Book II.*

On an ordinary graph, the unit of measurement is equivalent to $\frac{1}{12}$ of an inch, and it represents, in this system of notation, the standard pitch-unit, i.e., $\sqrt[12]{2}$ (a semitone). Such units are expressible in arithmetical integers as logarithms to the base of $\sqrt[12]{2}$. Thus, a semitone consists of one unit, a whole tone of two units, etc., along the ordinate.

A melodic graph may be translated into different absolute pitch values by substituting different coefficients for the original p.

To translate a musical graph into $\sqrt[6]{2}$ we would simply use double units on the ordinate for the original single units, while preserving all the other relations within a given melodic continuity. In this case, p = 2p. By using greater coefficients such as 3, 4, 5, 6 or 7 ($\sqrt[4]{2}$, $\sqrt[3]{2}$, $\sqrt[12]{2^5}$, $\sqrt{2}$, $\sqrt[12]{2^7}$), we obtain the respective units for the pitch intervals.

This form of projection is known as an *optical projection through extension of the ordinate.* It is one of the natural tendencies in visual arts. When artists attempt to produce a distortion (variation) of the original proportions, they are unconsciously attempting to achieve one or another form of geometrical projection.

These variations, when executed geometrically and in accordance with optics, give a greater amount of esthetic satisfaction because they are more natural.

On the next page you will find an example of the translation of one system of proportions into another, as applied to linear design.

*Schillinger describes various methods of tonal expansion in Chapter 5 of Book II, pp. 133-7. In tonal expansion, as contrasted with geometrical expansion, the original pitch-units are not altered; they are merely rearranged. In the first tonal expansion (E_1) of c—d—e—f—g, for example, these pitch-units reappear in the following order: c—e—g—d—f. The student will recall that this new form is obtained by circular permutation in which alternate units are skipped. In geometrical expansion, however, the original pitch-units are not retained. The process, as the student will learn, is one of extending the semitone to a full tone, or more. Thus, c—d—e—f—g would become c—e—g♯—a♯—d. The unit of measurement may be extended so that p, instead of equalling 2p, equals 3p, 4p, 5p, etc. (Ed.)

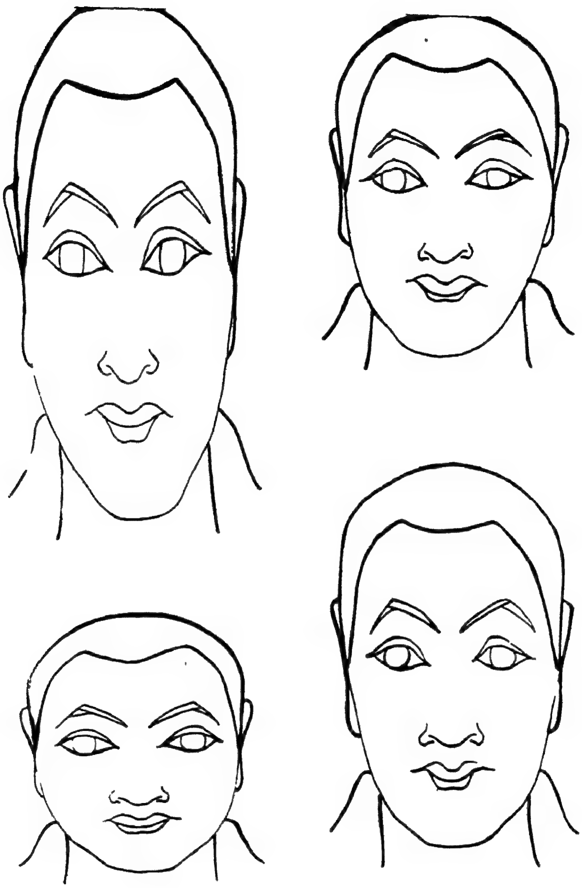


Figure 36. Translating one system of proportions into another.

In the illustration above, the same configuration is presented under different coordinate ratios. The technique of such translation consists of producing a network on the original drawing (with as many units as is desirable with regard to precision) and then transcribing this network into a differently proportioned area, preserving the same number of lines on both coordinates of the network. Then all points of the drawing acquire their respective positions in the corresponding places of the network.

Compare these geometrical projections with the distortions in these and other paintings by El Greco and Modigliani.

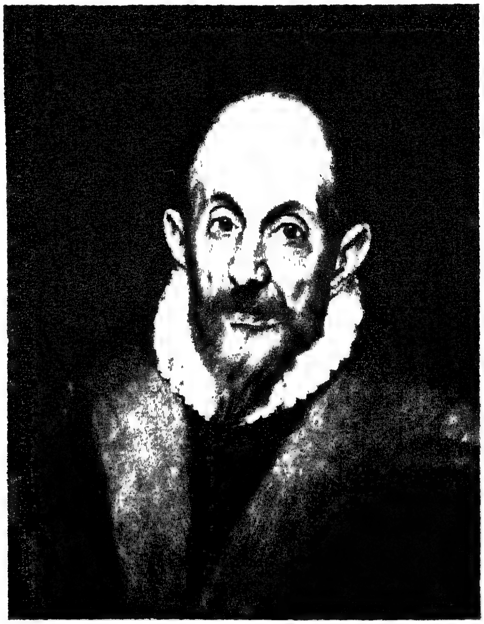


Figure 37. El Greco. *

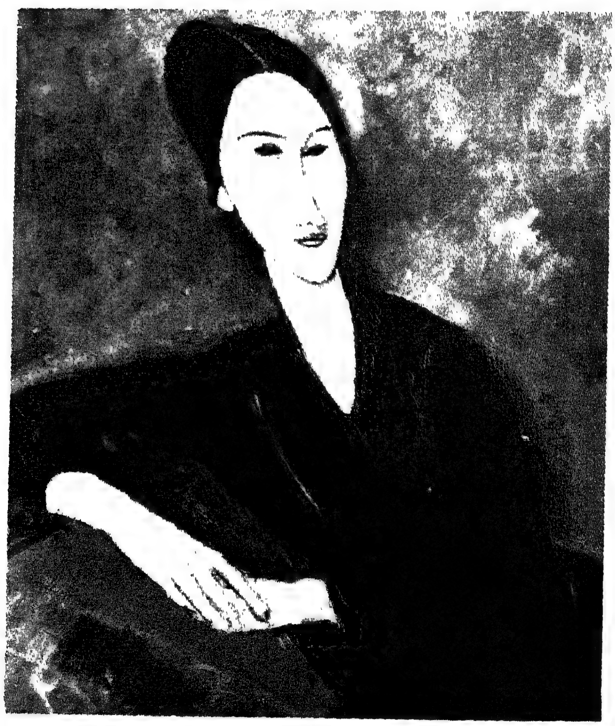


Figure 38. Modigliani. **

*Courtesy of The Metropolitan Museum of Art. **Collection, Museum of Modern Art.

As each coefficient of expansion is applied to music, the original is translated into a different style, a style often separated by centuries. It is sufficient to translate music written in the 18th century by the coefficient 2 in order to obtain music of greater consistency than an original of the early 20th century style. For example, a higher quality Debussy-like music may be derived by translation of Bach or Handel into the coefficient 2.

The coefficient 3 is characteristic of any music based on $\sqrt[4]{2}$ (i.e., the "diminished 7th" chord). Any high-quality piece of music of the past exhibits, *under such projection*, a greater versatility than any of the known samples that would stylistically correspond to it in the past. For the sake of comparing the intuitive patterns with the corresponding forms of geometrical projection, it is advisable to analyze such works as J. S. Bach's *Chromatic Fantasy and Fugue*, Liszt's *B Minor Sonata*, L. Van Beethoven's *Moonlight Sonata*, first movement.

The coefficient 4, being a multiple of 2, gives too many recurrences of the same pitch-units since it is actually confined to but 3 units in an octave. Naturally, such music is thereby deprived of flexibility.

But the 5p expansion is characteristic of the modern school which utilizes the interval of the 4th—such as Hindemith, Berg, Krenek, etc. Music corresponding to further expansions, such as 7p, has some resemblance to the music written by Anton von Webern. Drawing comparisons between the music of Chopin and Hindemith, under the same coefficient of expansion, i.e., either by expanding Chopin into the coefficient 5, or by contracting Hindemith into the coefficient 1, we find that the versatility of Chopin is much greater then that of Hindemith. Such a comparison may be made between any waltz of Chopin and the waltz written by Hindemith from his piano suite, *1922*.

Comparative study of music under various coefficients of expansion reveals that often we are more impressed by the raw material of intonation than by the actual quality of the composition.

The opposite of this procedure of expansion of pitch is *contraction* of pitch. Any pitch interval-unit may be contracted twice, three times, etc., which is expressible in $\sqrt[24]{2}$, $\sqrt[36]{2}$, etc., providing that instruments with corresponding tuning are devised. Those esthetes who usually love to talk about the "economy of material" and "maximum of expression" will perhaps be delighted to learn that an entire 4-part fugue of Bach occupying a range of $3\frac{1}{2}$ octaves would require only one whole tone if the pitch interval-unit were $\frac{1}{18}$ of a tone ($\sqrt[216]{2}$). Applying the same principle to the contraction of the absolute time duration-unit, we could hear this fugue in a few seconds instead of several minutes!

The natural pitch-scale, i.e., the series of harmonics, does not produce uniform ratios but gives a natural logarithmic contraction. The intervals between the pitch-units decrease, while the absolute frequencies increase. This phenomenon is analogous to the *perspective* contraction in space *as we see it*. If music were devised on natural harmonic series, the relative group-coefficients of expansion and contraction could be used. But it seems that the natural harmonic series does not, in fact, provide any flexible material for musical intonation but merely for building up various tone qualities—for the fact is that a group of harmonics sounded at the same intensity produces one saturated unison rather

than harmony. This phenomenon is somewhat similar to that of *white light*, in which all spectral hues merge—becoming noticeable only when the beam is broken up. Logarithmic contraction of pitch combined with the logarithmic contraction of time *may* come into existence in the remote future in connection with the development of automatic instruments for composition and execution of music.

The technique of pitch-expansion may be executed directly from a graph or from a corresponding chromatic scale of expansion. In such a case, 2p will produce a whole tone scale progressing through 2 octaves instead of a full chromatic scale progressing through one octave (when p = 1.) While expansion of time extends the graph along the abscissa, the increase of the absolute time unit is not noticeable unless compared with the original. When we hear a musical continuity, we do not know (unless it is extremely exaggerated) whether it is the original velocity or a derivative thereof. The difference becomes apparent only when different coefficients of velocity of the same musical continuity are brought close together. Thus, time extension produces a different pattern on a graph without producing a difference detectable in the absence of comparison.

Pitch expansion works under the same conditions. It is only through comparison that we can learn that a certain musical continuity has been expanded or contracted from its original. This is apparent in the process of *tonal* expansion (which preserves all the pitch-units while the range increases) as it was described in the *Theory of Pitch Scales.**

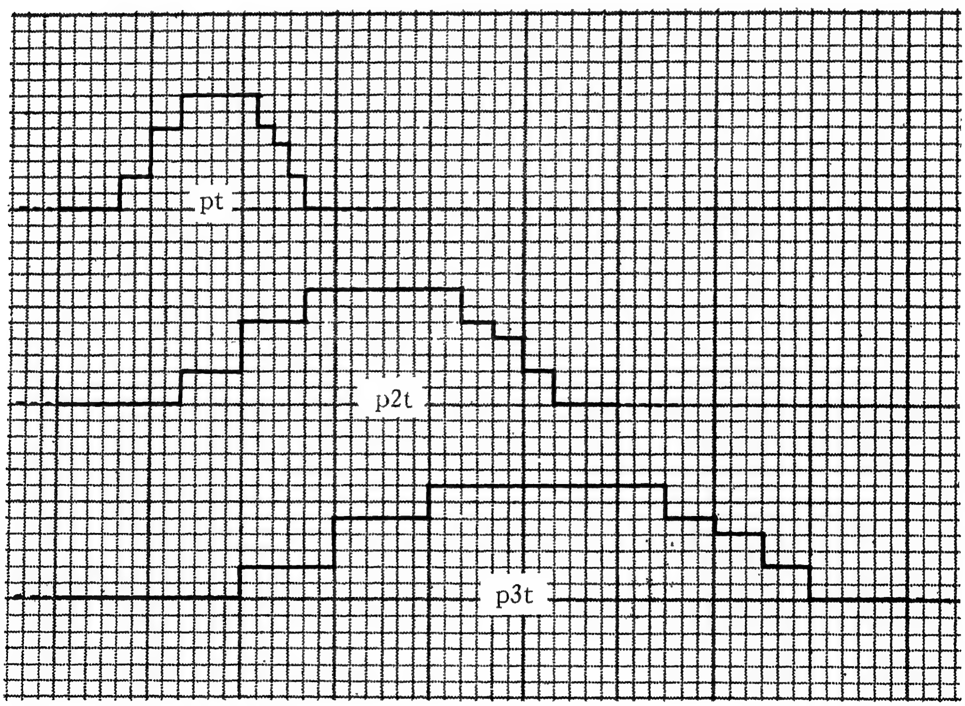


Figure 39. Time and pitch expansions (continued).

*See Book II.

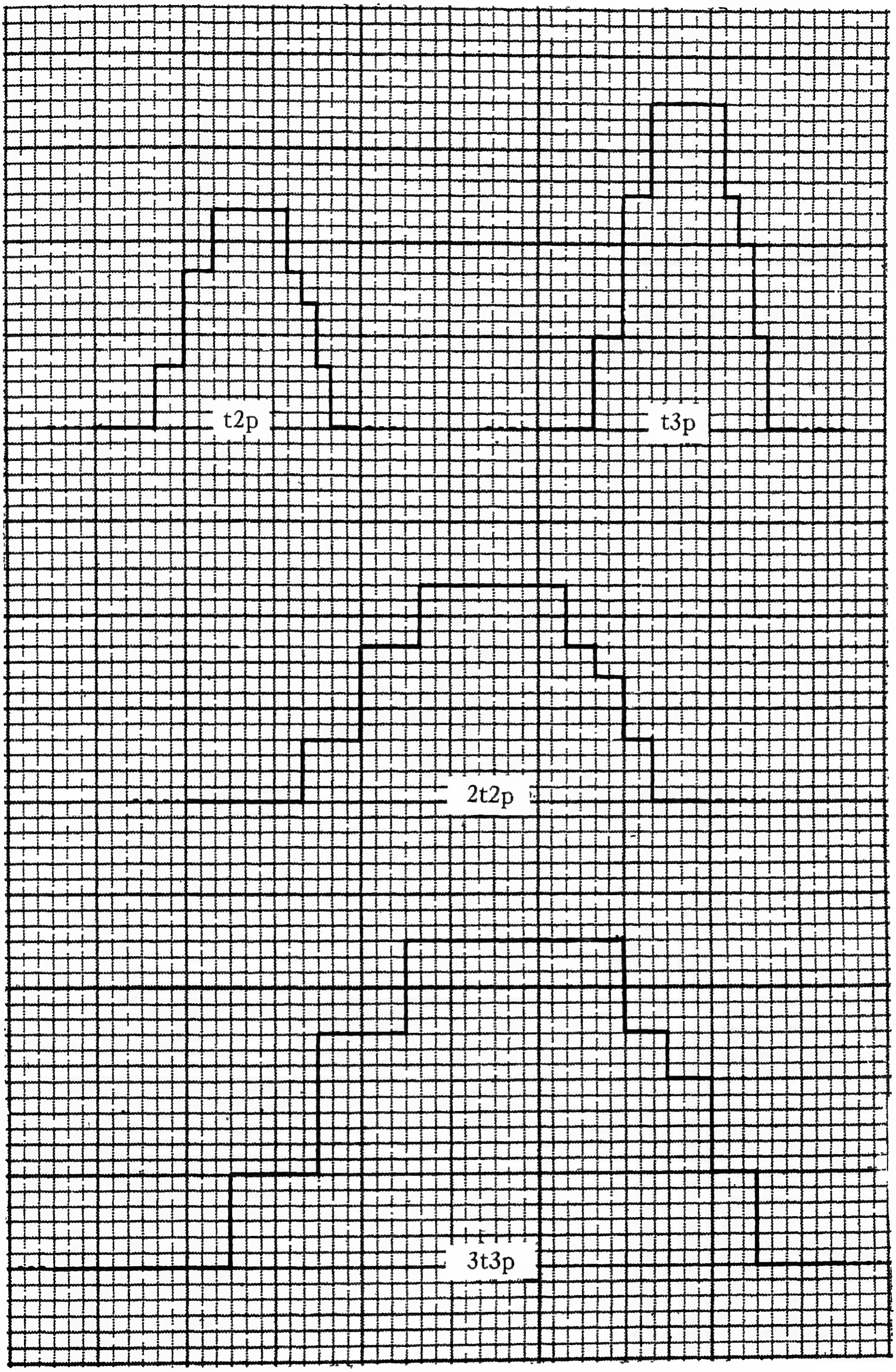


Figure 39. Time and pitch expansions (concluded).

If pt represents the original, 2t and 3t produce the corresponding time expansions. Likewise, 2p and 3p produce the corresponding pitch expansions. The expansion through two coordinates preserves the absolute form of the configuration, merely magnifying it (2p2t and 3t3p).

It might seem at first that the ordinary enlargement or reduction of an original image—such as that effected by any natural optical projection (lantern slide projector, motion picture projector, magnifying glass, etc.)—does not change the appearance of the image. Yet when carried to an extreme, it does in fact transform the image to a great extent. For example, an ordinary close-up of a human head seen on the screen does not change our impression of the image. But when a human head is subjected to a several hundred power magnification, the original image is changed beyond recognition. A photograph of the skin surface of the human arm occupying only 1/100th of a square inch produces an image which is not easily associated with the human arm.

Thus, the difference in the actual sound of music (like the magnification of Haydn into von Webern) is only quantitatively different from the enlarging of visual images. Even with coefficients as low as 5, a melody is transformed beyond recognition. But the magnification of visual images requires at least one-hundred power magnification in order to achieve a similar effect.

It is interesting to note that bizarre effects of optical magnification are often due to the fact that such images are merely *hypothetic* and have no actual correspondences in the physical world of our planet. An image of a chicken can be magnified to the size of the Empire State building (for example, by being projected on an outdoor smoke screen), yet no real live chicken could exist on this planet even the size of an ostrich, because—as the volume grows in cubes—the legs of such a chicken could not support the weight of its body.

The following chart represents pitch expansions of the melody graphed in Figure 39.

Figure 40. Pitch expansions of the melody of Figure 39. (*continued*)

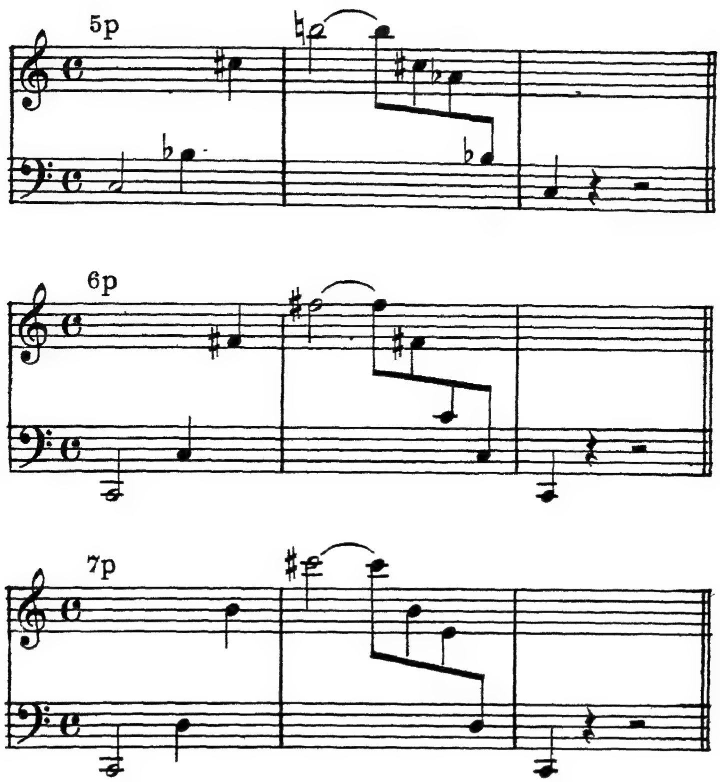


Figure 40. Pitch expansions of the melody of figure 39 (concluded).

For most purposes the lower coefficients are the most practical ones. Examples of geometrical expansion may be seen in the following excerpts from J. S. Bach:

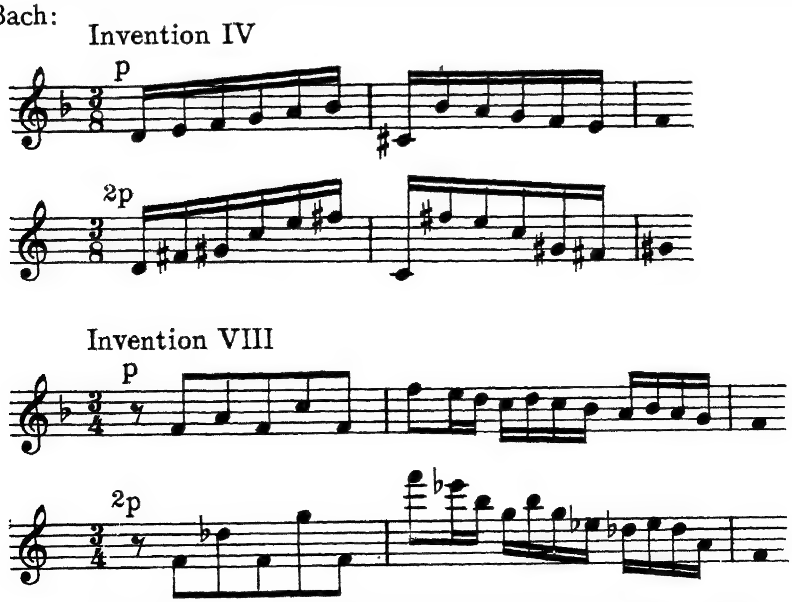


Figure 41. Tonal expansions of J. S. Bach, Two-Part Inventions.

Figure 42. Tonal expansions of J. S. Bach, Well-Tempered Clavichord.

Figure 43. More tonal expansions of J. S. Bach, Well-Tempered Clavichord

Different geometrical expansions may follow one another as elements in a continuity *only* when used in *very short portions*—in order to enable memory to retain the original pattern. When the ear accommodates itself to one coefficient of expansion for a considerable period of time, then a sudden change to a new coefficient produces such a surprising effect that the desirability of the use of the device in one continuity becomes questionable. For this effect is equivalent to a sudden change of style; it may be described as music beginning somewhat like Debussy, suddenly changing to Bach, and then again to Hindemith. Yet tests with various listeners show that in immediately following fragmentary sequences, the device sounds perfectly acceptable.

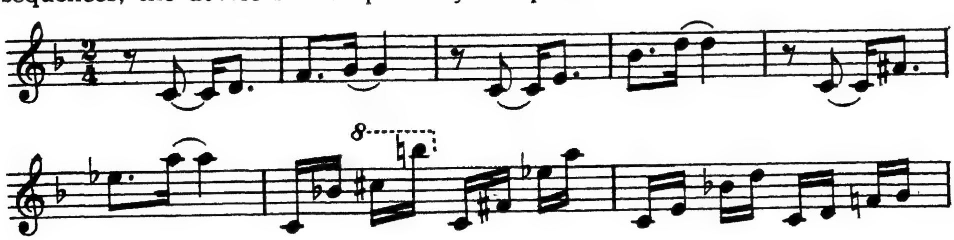


Figure 44. Tonal expansions of George Gershwin's I Got Rhythm*.

Geometrical expansions of melody may also serve the purpose of modifying motifs through the method of geometrical projection. The original melodic pattern becomes entirely modified—yet the system of pitch-units is the outcome of a consistent translation from one system of pitch relations to another. The technique of such modification is equivalent to the contraction of the general pitch range emphasized by the geometrically expanded form. Some melodies, especially those with big coefficients of expansion, permit several different versions (degrees) of contraction.

The following example presents the exact geometrical expansions with the respective contractions of their ranges:

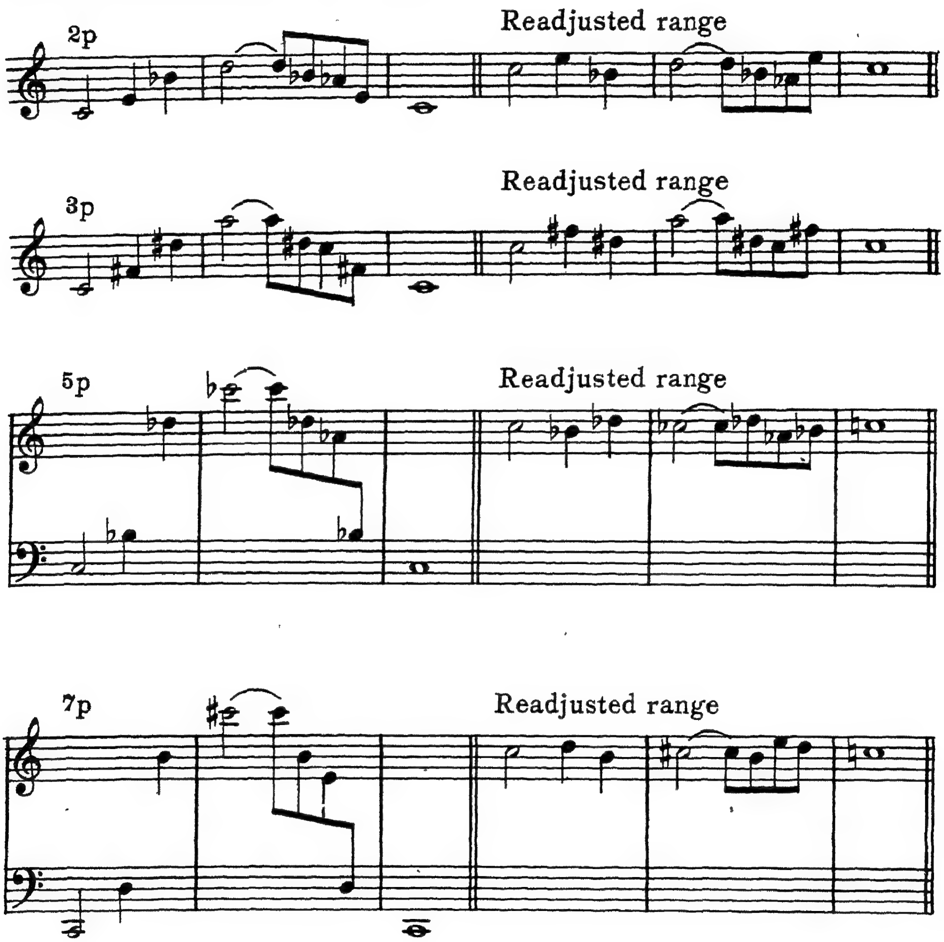


Figure 45. Geometrical expansions with readjusted (contracted) range.

The process of *range-contraction* often introduces new characteristics into geometrically expanded forms. For example, in the case of 5p in the preceding example: in its readjusted form, it seems to be more "conservative" than in its respective geometrical expansion. In the case of 7p, the contracted form is reminiscent of the music of Prokofief rather than that of von Webern.

Geometrical expansion of the harmony which accompanies melody expanded through the same coefficient (whether with readjusted range or not), must be performed from the pitch axis of the entire system (usually the root-tone).

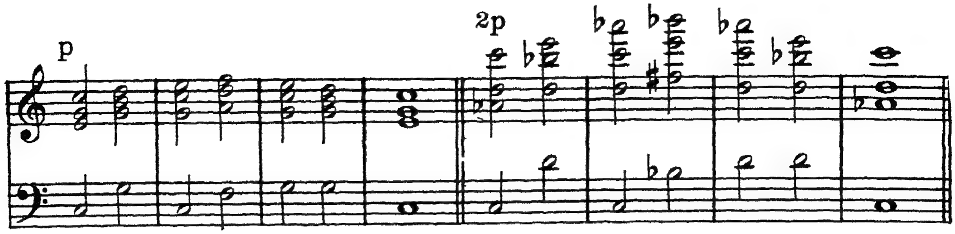
Figure 46. Geometrical expansion of a harmonized melody.

This translation of harmony may be accomplished either through transcription of a graph or through step by step translation from the original. One may also prepare in advance chromatic scales from the respective pitch axes where all the pitch-units may be found directly in the corresponding expansions.

Figure 47. Scale of pitch units and their corresponding expansions.

Further expansions may be evolved in a similar way. When harmony is to be translated into a geometrical expansion, it is sufficient to find the first chord of its original setting and to proceed horizontally with each voice as melody, thus performing the voice-leading of the original. If after such translation the range seems to be too extreme for any instrumental applications, the above-described range-readjustment may be applied.

Here is an example of a conventional harmonic continuity first translated into 2p and then readjusted into two further contracted forms. In such a case, the extreme upper voice may become one of the inner voices by being placed one octave below.



First Readjustment

Second Readjustment

Figure 48. Harmonic continuity projected into 2 p and readjusted.

Translation of polyphonic continuity into geometrical expansions must be carried out on the same principle. The pitch intervals between the theme and the reply must be doubled or tripled in relation to the pitch axis. For example, if the pitch axis is c and the reply in the original starts on g, when p = 2, the beginning of the reply will be on d, i.e.. the interval 7 becomes 14.

Fugue IV- Vol. 1

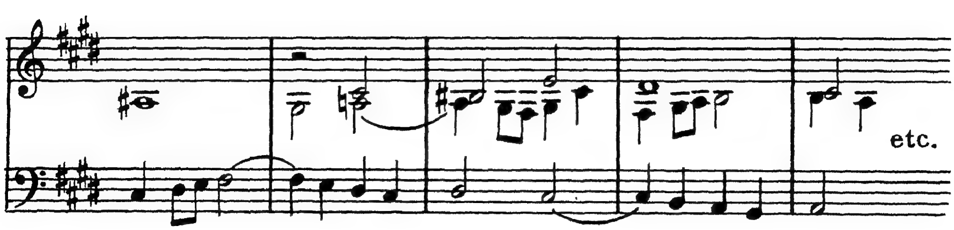


Figure 49. 2p expansion of J. S. Bach, Well-Tempered Clavichord (continued)

Figure 49. 2p expansion of J. S. Bach, Well-Tempered Clavichord. (concluded).

All geometrical *expansions* are subject to geometrical *inversions* as well. A consistent musical continuity may be evolved through the variation of inversions under the same coefficient of expansion. Thus the two methods of mathematical variation of music, based on geometrical projection, bring an effective solution to two very important technical problems:

1. Composition of infinite melodic or harmonic continuity containing organically related contrasts.
2. Translation of music of one epoch into another, "modernization" and "antiquation."

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK IV
THEORY OF MELODY

BOOK FOUR

THEORY OF MELODY

CHAPTER 1

INTRODUCTION

IN ORDER successfully to produce anything out of given material, it is necessary to know the properties of such material as well as all the processes involved. Any material which is to be dealt with must consist of a number of components. Unless all the components required for structural realization are known, the result of such a procedure will be failure. The components of a structural process specify different individual procedures to be coordinated in the whole.

There exists a great deal of scepticism as to the possibility of constructing musical melodies rationally—but no such scepticism exists as to the evolution of chord progressions. This is because it happens that the musical study of harmony is based on a quite developed tradition describing such procedures, while no workable theories of melodic composition have thus far been offered in the civilized world.

Although we hear about such theories existing in Oriental civilizations, such as those of the ancient Hindus and the ancient Chinese, these theories are not available in any form other than the original and therefore are not accessible to anyone not familiar with the respective languages. I may say that there is no evidence among scientific musicological documents which would offer any positive proof of the existence of such theories. Perhaps, it is one of those "Oriental mysteries," like the rejuvenation of an old person or the resurrection of someone buried alive. Naturally, we cannot use such methods—yet we are surrounded by the sceptical attitude of musicians brought up in the romantic traditions of the 18th and 19th centuries, and the intuitive approach to the art of musical composition.* To one associated with the method of engineering musical melodies, however, the possibility of such a creative process is beyond doubt. A buffalo is almost a zoological myth in Europe, but a common reality in America. A zoologist dealing with some rare specimen on the African continent would have to face the same scepticism from people whose scientific criterion is "seeing is believing."

*Although Schillinger refers merely in passing to this development in the history of ideas, it is extremely important to us. Living as we do in the years following the 19th century rise of romanticism, we are heirs of the over-emphasis on intuition and inspiration in the arts. Prior to the romantic reaction, there was no jealous dichotomy between the arts and sciences. In fact, during the 17th and 18th centuries, the role of reason in the arts was widely recognized and accepted. The romanticists revived the Platonic conception of the artist as an inspired madman. "Gie me ae spark o' Nature's fire, That's a' the learning I desire," chanted Robert Burns, the Scotch romanticist. This false dichotomy between knowledge and inspiration has persisted into our time. It has permeated the thinking of composers and the public to such an extent that a scientific approach to the arts is sometimes condemned without thought or investigation. Nevertheless, such an approach is grounded on the best experience of composers through the ages as well as on the most recent discoveries of modern psychology. Schillinger's discoveries were made possible because he began with the idea that the arts could be rationalized and that the process of musical composition was subject, as he demonstrates, to scientific analysis. (Ed.)

Technical experience shows that failure of realization in constructing any instrument is often due to *inefficient engineering* and not to any error in the theoretical assumptions behind it. With all the extraordinary progress that engineering technique has made recently, some people still doubt that a musical instrument for automatic performance may be entirely different in quality from the automatic piano of the present. The difference between the two may be much greater than is the difference between the first biplane of the Wright brothers and the latest air superliner for 120 passengers by Sikorsky. Or consider the first magnifying glass as compared to the new 200-inch telescopic mirror. We have already witnessed an obvious example of the progress which scientific technique has brought into the field of music, for electronic sound production offers any desirable tone quality, intensity and form of attack. Very soon, when performers are relieved of the struggle for tone quality, they will have time to be confronted with the major problem of musical interpretation. This problem of interpretation will be concerned with relative tone colors, dynamics, forms of attack, group distributions, etc. When this becomes a reality, (and that day may come very soon) the composer will lose his present dependence upon the performer. If and when an automatic instrument can carry out the composer's intentions to any desirable degree of subtlety, the composer can celebrate the arrival of a new era that will liberate him from the centuries of slavery imposed upon him by the performer. This, in turn, will call for much greater precision of creative intention on the part of the composer. It will not be sufficient to write the pitch and the time components. It will be necessary to include *all* the technical forms of execution. As our reaction to music changes with different eras and civilizations, with different historic periods, and often varies in trends during one decade, a new profession will emerge in the near future: the members of this new profession will be called *readjusters* of music (not to use the word "arrangers", which would convey a misleading meaning due to its previous connotation). Their duty will be to make the music of primitive civilizations or of any of the remote periods of our civilization comprehensible to the listeners of their own time. This will require various forms of technical readjustment and rejuvenation of the media which have lost their expressiveness a long time ago. The phenomenon of the transcription of music has been known since time immemorial. However, the new method will be much more radical than any of those used in transcriptions of the past; it will require definite engineering techniques instead of vague intentions.

By studying different aspects of music in different civilizations and by mathematical analysis of various procedures involved in the making of a musical composition, a group of engineering routines may be evolved.

The most important stages in evolving a theory of melody are:

1. Study of the general properties of melody with respect to its convertibility and other forms of geometrical projection (expansions-contractions).

2. Comparative study of the patterns appearing in natural configurations (crystal, vegetable and animal forms).

3. Study of the properties of curves and of statistical records specifically (technology of events).
4. Recording and analysis of the reflex patterns (respiratory, muscular, nervous, etc.)
5. Study of the trajectorial curves evolving linear design in the visual arts.
6. Study of graphs expressing intuitive musical compositions in terms of pitch, time, intensity and tone quality.
7. A comparative study of all the above-mentioned patterns.
8. Deduction of a system of patterns to serve as stimuli of reaction of definite character and intensity.
9. Development of a group of routines leading to efficient artistic creation and providing the testing criteria.
10. Elaboration of a scientific theory of production of musical melodies.

A. Semantics

Music in general—and melody in particular—has been considered, since time immemorial, a supernatural, magical medium. Many great philosophers in different civilizations have given their attention and directed their thoughts toward this elusive phenomenon. The more definitions of music you know, the more you wonder what music really is. It seems to fall into the category of life itself. It seems to have too many "x's"

People did not know much about lightning even ten thousand years ago, and ten millennia make only a one-hundredth in the range of *human* evolution. We tend to ascribe supernatural powers to any phenomenon we cannot explain. Today, we are surrounded by things more miraculous than any of the products of ancient imagination—and when you think of the achievements of modern technique, it seems to be incredible that a toy—as simple as melody—should still remain in the category of the irrational.

Following our method of analysis, however, we may assume that *any phenomenon can be interpreted and reconstructed.* To accomplish this, it is necessary to detect *all* the components and to determine the exact form of their correlation.

There are two sides to the problem of melody: one deals with the sound wave itself and its physical components and with physiological reactions to it. The other deals with the structure of melody as a whole, and esthetic reactions to it.

Further analysis will show this *dualism* is an illusion and is due to considerable quantitative differences. The shore-line of North America, for example, may be measured in astronomical, or in topographical, or in microscopic values. The difference between melody from a physical or musical standpoint is a *quantitative* difference. The differentials of the physical analysis become negligible values for purposes of musical (esthetic) analysis.

Melody is a complex phenomenon and may be analyzed from various standpoints. Physically, it can be measured and analyzed from an objective record, such as a sound track, a phonograph groove, an oscillogram or the like. Melody when recorded has the appearance of a curve. There are various families of

curves, and the curves of one family have general characteristics. Melodic curve is a trajectory, i.e., a path left by a moving body or a point. Variation of pitch in time continuity forms a melodic trajectory.

Melody from a *physical* standpoint is a compound trajectory of frequency and intensity. Melody from a *musical* viewpoint is a compound trajectory of pitch, quality, and volume. The components of quality are timbre, attack, and vibrato.

Physically, pitch is an accelerated periodic attack. Physically, the difference between rhythm and melody is purely quantitative. Therefore, time-rhythm in a melody may have two forms.

1. Through periodicity of attacks of low frequency, which is unavoidable when the pitch-frequency is constant;
2. Through variation of frequencies, i.e., through changes in pitch itself.

Frequency constitutes musical *pitch*. Any sound wave of a given frequency (constant or variable) generates its own frequency *subcomponents* (known as "partials" or "harmonics") resulting from purely physical causes. The latter are disturbances which convert a simple wave (known as a sine wave) into a complex one. The sound of a simple wave may be heard on specially made tuning forks and electronic musical instruments.

The intensity of a sound wave is one of the factors of disturbance, and the duration of intensity and its stability in time continuity are others. The latter are musical factors: depend on *form of attack* (or *accentuation*). Finally, the resultant of both components and all the subcomponents, i.e., the interaction of all component frequencies and intensities in a sound wave, constitutes the musical component of *timbre and character* (quality) of sound.

The relative importance of musical components and subcomponents has already been measured, so to speak, by agreement among musicians and music lovers. The conclusion has been reached that two melodies are *identical* if their main components (time and pitch) are identical. For instance, a melody played on the piano, or sung, or played loud or soft, or with vibrato or without it, would be considered "the same" melody if rhythm (time) and intonation (pitch) are identical. The subcomponents and the sub-subcomponents pertain to execution, i.e., to the performance of melody, not to its own structural actuality. The very neglect of subcomponents, on the one hand, relieves the composer of a certain amount of responsibility; on the other hand, it leads to loss in esthetic value of melody when the melody is wrongly executed by the performer. For then the performer has to supply the subcomponents without the benefit of any exact indication by the composer and therefore he acts at his own discretion, whether rightly or wrongly.

At this point we may adopt Helmholtz' definition of melody (which satisfies the musical aspect): *melody is a variation of pitch in time.** Is *any* variation of pitch in time a melody? An attempt to answer this question leads into the *semantics of melody*.

*Hermann Ludwig Ferdinand von Helmholtz (1821-1894), the great German physicist and physiologist, sought to devise rules of musical science based on the physical nature of musical sounds. His most significant work is *On the Sensations of Tone as a Physiological Basis for the Theory of Music* published in 1863. (Ed.)

B. Semantics of Melody

The fundamental semantic requirements are that melody must "make sense," it must have (like words) associative power, i.e., it must be able to convey an *idea* or *mood*, to "express something."

But these are also the requirements of language, and yet there is a distinct difference between *word* and *melody* as symbols of expression. The function of *words* is to express the *concept* of actuality, to find its verbal symbol. The function of melody is to express the *structural scheme* of actuality. Words have their origin in thought; melody has its origin in feeling, i.e., originally in the reflexes. *Words generate concepts which may or may not stimulate feelings.* Melody, on the contrary, *stimulates feelings* (emotions, moods) as spontaneous reactions, *which may or may not generate concepts.* Melody expresses actuality *before* the concept is formed for that actuality. This is why, in listening to a melody, one is satisfied with its expression to such an extent that the quest for the concept, "What does it actually express," is never aroused. But, on the contrary, when a melody *does not convey sufficient associative power* (to stimulate reflexes, reactions or moods), then the listener looks for a verbal description of it, or, at least, for a title, a "label," a concept. Melody is *insufficient* whenever it calls for a verbal explanation. When a word does not convince through its own associative power, or in order to increase the latter, one resorts to intonation and gestures.

Words or melody may or may not be self-sufficient. Words that are not self-sufficient call for a specific form of intonation in order to acquire the necessary associative power. We may also state, reciprocally, that melody which is not self-sufficient as intonational form calls for word and often for a symbol in the form of a verbal concept. These two statements can be verified by simply studying the facts.

Here we arrive at the idea that although, in their developed forms, both word and melody are self-sufficient—in their early periods of formation they produce hybrid forms: an intonational form that calls for a concept—and a conceptual form that calls for intonation.

Here are a few of many references. According to the statements of George Herzog, Columbia anthropologist who made some pertinent recordings and demonstrated the phonograms, there are certain Central African tribes whose verbal language is just such a hybrid. A word of the same etymological constitution (spelling) has at least ten different forms of intonation, each attributing a different meaning to the word. In this case *intonation* is an *idiomatic factor.*

In other cases, as in some instances of Chinese music,* melody or even the single units of a scale become symbolic of a concept—i.e., they assume the function of words.

The Stony Indians of Alberta, Canada, try in their songs to express the sound of a brook, the murmur of leaves, etc. Yet as a descriptive means it is not self-sufficient; it calls at least for a title. This is a case in which melody is a bad competitor of poetry.

* See Karl Stork, *History of Music.* (J.S.)

Out of many hypotheses as to the origin of music and word, I select the *reflexological* one.* Sound reflexes (of the vocal cords), before they crystallized into relatively distinguishable forms of word and melody, were spontaneous expressions of satisfaction or lack of satisfaction in an animal organism. Any cause of actual or potential disturbance that endangered an organism became a stimulus for the defensive reflex. This is probably the original form of the intonational *signal.* If such a form was at first an improvised reflex movement of the vocal cords expressing fear—a spontaneous reaction to danger—it may have crystallized later into the etymological form of the concept of "danger."

When an organism is on the verge of struggle for its own survival, it usually resorts to *intonational signaling* rather than to an etymological one. Even in our own time, a drowning man does not say: "I am drowning!" He generally shouts: "Help!"

The amount of semantic and acoustical elements in words or melodies varies greatly. There are all gradations from an exclamation to a polyconceptual polysyllabic word of the German language with the relative decrease of the acoustical (intonational) and the relative increase of the semantic element. In many undeveloped forms of speech, an outsider may in fact mistake such speech for melody.**

Melody always contains well-defined acoustical elements, although it may be alien to an ear trained to different systems of intonation. Melody offers also a scale of semantic gradations from imitative descriptive intonations, through symbolic abstractions, to the expression of mechanical forms.

Both imitative and symbolic functions of music tie it closely to verbal semantics. In this stage, melody is the language of a given community only. Tests show that even such commonplace moods as "gayety" and "sadness" cannot be expressed by means of melody that will mean the same thing to all nations. Melody is a *national* language or a language of a *given epoch* with regard to descriptive or symbolic qualities.

Arabian funeral music sounds anything but "sad" to us because of our association with major scales—which mean gayety, heroism, happiness and satisfaction to us. Gay Arabian dance-songs sound "sad" to us because of our association with harmonic minor scales, which mean exactly the opposite to us. It is similar with the forms of musical harmony. Through previous associations we react to major chords as we react to major scales. Yet we have the curious phenomenon of the Negro-American "blues," which is supposed to express depression, but which, nevertheless, has the richest scale of major chords.

All the controversies ascribing this or that semantic connotation (descriptive or symbolic) to music will vanish when we penetrate the real meaning of music, namely, *the expression of the forms of movement.* The objectification of

*Here begins, in a partial form, Schillinger's exposition of his theory of the correspondences between music—melody, in particular—and the objective world of life. As such, his theory offers us the means whereby esthetic phenomena can be correlated in a scientific and materialistic way with the rest of human experience; in consequence, even this partial exposition is of the utmost philosophical importance. (Ed.)

**"Program music is a curious hybrid, that is, music posing as an unsatisfactory kind of poetry"—*Oxford History of Music,* Volume 6, Page 3. (J.S.)

this meaning requires only one premise: *biomechanical, physiological experience, combined with a highly developed sensory system.* The requirement may be satisfied by any normal specimen of the higher animal forms.*

Though commonly unknown and generally repudiated when brought into a discussion, this fundamental form of musical semantics had already been known to Aristotle. Here is his definition: "Rhythms and melodious sequences are movements quite as much as they are actions." This is the first historical instance of penetration into the true nature of musical language.

The meaning of music evolves in terms of physico-physiological correspondences. These correspondences are *quantitative* and the quantities express form. This can be easily illustrated by the following example.

A sound of constant frequency and intensity and made up of a simple wave affects the eardrum and the hearing centers of the brain as an excitor of a simple pattern. Such a pattern may be projected by various means so that its structure becomes apparent to another more developed, and therefore more critical organ of sensation, that of sight. The complexity of reaction (i.e., its form) is equivalent to the complexity of the form of the excitor. The number of components in a wave affects a corresponding number of the arches of the inner ear's Corti's organ, putting them into oscillatory motion. If a sine wave has one component, it will affect only that arch which reacts on the frequency corresponding to that transmitted through the air medium in the form of periodic compressions. When a wave of greater complexity affects the same organ, the reaction becomes more complicated.

It is a known fact that the ear can be trained. Therefore, the pattern of reaction is equivalent to the pattern of excitation with various degrees of approximation. All the components of sound work in similar patterns because these patterns are similar in all sensory experiences. Formation of the patterns is due to (1) *configuration* and (2) *periodicity*. The *configuration* may be simple or complicated in a mathematical sense, i.e., its simplicity or complexity can be expressed in terms of components and their relations. This emphasizes both frequency and intensity in a sound wave, as well as the character of sound which is the resultant of the relations of the two components. *Periodicity* defines the form of recurrence and may be also of different degrees of complexity—for example, the periodicity of recurring monomials as compared to the periodicity of permutable groups.

Our physiological experience, combined with our awareness of that experience through our sensory and mental apparatus, makes it possible for us to understand the meaning of music in terms of "actions." Thus, *regularity* means *stability*, and *simplicity* means *relaxation*. Thus, the satisfied organism at rest is comparable to simple harmonic motion. The loss of stability is caused by powerful excitors affecting the very existence of the organism. Sex and danger are the excitors, and love and fear are the expressions of instability.

*Compare Plato's ideas on the meaning of music in his *Republic* and Ivan Pavlov's experiments with the pitch discrimination of dogs in his *Conditioned Reflexes*. (J.S.)

The awareness of such instability comes through variations in blood circulation sensed through the heart-beat and variations in blood-pressure, resulting in respiratory movements. The whole existence of an organism is a variation of degrees of stability, fluctuating between certain extremes of restfulness and restlessness. The constitution of melody is equivalent to that of an organism. It is a variation of stability in frequency and intensity. Melody expresses those actions we know and feel through our very existence in terms of sound waves.

C. Intentional Biomechanical Processes

We come now to a consideration of *intentional* biomechanical processes. Efficiency of action in relation to its goal is the foundation of evolution. The forms of action by which living organisms adapt themelves to the goal of survival in the existing medium may serve as a fundamental illustration. This efficiency comes about through "instinct" among the lower species, but through the conscious utilization of previous experiences leading to deliberate efficiency among the higher animals. Muscular tension and relaxation constitute the first instruments of such intentional action.

The mechanical constitution of melody varies with times and places, yet its patterns are familiar to us from our own biomechanical experiences.

The "contemplative" and the "dramatic" become two poles of our esthetic reactions. They grow out of the same biomechanical diads: restfulness-restlessness, and stability-instability.

Dramatic patterns themselves evolve out of two sources: the first is fear (defense—dispersed energy) and is caused by danger or aggression; it results in *contraction patterns*. The second is aggression (attack—concentrated energy) and is caused by an impulse or resistance; it results in *expansion patterns*. Confusion of patterns of compression with those of expansion (aberration of perception caused by instability) explains why the very same music sounds "passionate" to one listener but "weary" to another. This is a typical confusion observed by Professor Douglas Moore of Columbia in tests performed on students of non-musical departments at various universities, using Wagner's *Isolde's Love-Death* as material.

All the technical specifications for melodic pattern-making will be given later. The immediate question is: *how does it happen that the physiological patterns are identical with the esthetic patterns?* We can answer this question only hypothetically for we know very little about the technique of pattern formation at present. But as science progresses, we notice more and more correspondences in different fields. We find identical series in such seemingly remote fields as crystal formation, ratios of curvatures in the celestial trajectories, musical rhythms, design patterns, and, finally, in the very molecular structure of matter itself. Modern chemistry shows how by *geometrical* variation of mutual positions of the same group of electrons, entirely different substances are produced. Little as we know for the present about the electro-chemistry of brain-functioning, we may well suspect that all our pattern conception and pattern-making are merely the *geometrical* projection of electro-chemical processes, in the making, that occur in our brain. This geometrical projection is thought itself.

D. Definition of Melody

The summary definition of *melody*:

(1) *Physiological definition:* Melody is an excitor existing in the form of a sound-wave which affects the organ of hearing. The latter being a receiver and a transmitter transfers it to the biomechanical pattern-making center of the brain.

(2) *Semantic definition:* Melody is an expression of biomechanical experiences in the sound medium.

(3) *Musical definition:* Melody is a variation of pitch in time, wherein pitch units follow a preselected scale of frequencies and express a relative stability of each individual unit.

The summary definition of *word*:

(1) *Physiological definition:* Word is an excitor existing in the form of a sound-wave which affects the organ of hearing. The latter, a receiver and a transmitter, transfers it to the concept-making center of the brain.

(2) *Semantic definition:* Word is an abstraction of biomechanical experiences in the sound medium. "Poetic image" is a variation of the original biomechanical abstraction.

(3) *Musical (tonal) definition:* Word is a variation of pitch in time, wherein pitch units express a relative instability of each individual unit and do not necessarily follow a preselected scale of frequencies.

It follows from these definitions: (1) that in symbolic notation (though different patterns are used)—printed letters or musical notes—both word and melody are identical; (2) a poem recited in a foreign or unknown language becomes an undeveloped form of music.

CHAPTER 2

PRELIMINARY DISCUSSION OF NOTATION

BEFORE ENTERING upon the subject of actual computation and construction of melody, there are a number of questions surrounding notational systems that require clarification.

A. History of Musical Notation

The historical evolution of musical notation starts with alphabetical systems of notation of musical pitch. We find this system in ancient Greek notation; the Greeks utilized the characters of the alphabet to indicate intonations. For rhythm they used, among other devices, the rhythmic groups of poetry (*i.e.*, the "foot").

The second step in this evolution brought the use of *neumes—i.e.*, indications of musical pitch in the form of graphic symbols. We find evidence of this second step in the Middle Ages. A number of hypotheses have been advanced regarding the source of early medieval notation.*

Byzantine notation from the 10th century to the 15th century evolved a system of *interval* indications. This notation, when fully developed, included symbols for an ascending second, a descending second, an ascending third, a descending third, a descending fourth, an ascending fifth and a descending fifth. In the course of a few centuries the symbols gradually modified their appearance, and a new system of representation was evolved.

The first use of horizontal lines (a staff) was devised in the west by Hucbald in the 10th century—see his *De Harmonica Institutione.* The steps on the staff were indicated by the letters "ton" for tone and "sem" for semitone. The words of the text were placed directly on the staff. Only the spaces between the lines of the staff were used.

Guido of Arezzo (who died in 1050) invented the four-line musical staff. Through him we learn also about the origin of most of the present names of musical pitches, these being derived from a hymn to St. John, which the students of a certain monk, Michael, had to sing so that each line would sound one step higher; the first syllables of the lines of the hymn became names for six of the steps

*According to one theory Latin notation was taken from the Hebrew cantillation signs, according to another, from the Byzantine ekphonetic notation. But the most convincing evidence points to the hypothesis that Latin notation derived from the transposition of the signs used for accentuation and punctuation (i.e., grammatical accent-signs) from the vocal text to the melody itself. These grammatical accent-signs may be said to be the source of the Byzantine-ekphonetic notation also. The latter did not show the size of the musical intervals, and therefore was useful chiefly as a mnemonic guide. A system of neumes originated in Byzantine notation in the 10th century, but this at first offered little improvement over the ekphonetic notation since it still denoted intervals only approximately.

in our present system of solmisation.* Guido of Arezzo also developed a very complicated system of tone nomenclature for the purposes of solmisation (*i.e.*, "Guido's hand").

Hermannus Contractus, who died in 1054, offered a mixed system of Greek and Latin characters indicating, in terms of tones and semitones, all the intervals of the octave except the augmented fourth and the major and minor seventh.

Another important step in this evolution was the development of a system of notation of musical durations.** The first indications of musical duration represented only two relative durations ("long" and "breve"), to which others were soon added. The 13th century classified music into measured and unmeasured music (*musica mensurata* and *musica plana*). The notation of rests also goes back to this early period. Ternary time-signatures originated in the 13th century, binary time-signatures in the 14th.

Our present form of "white" musical notation goes as far back as the 15th century. The evolution of the chromatic signs now in use (sharps, double-sharps, flats, double-flats and naturals) occupied eight centuries, from the 11th century to the 18th. Key signatures did not make frequent use of sharps or flats, except in the one-flat key signature, until the late 15th century.

The system whereby we notate dynamics, attacks and phrasing in graphic symbols and words begins to appear gradually towards the end of the 16th century. This system of notation is seen in such indications as legato, staccato, portamento, crescendo, diminuendo, forte, piano.

Indications of speed and character of motion in words (tempo) came into use (except for an isolated 16th-century example) in the early 17th century. Indications of this sort that are now common include largo, andante, moderato, allegro, presto. We also now have metronomic indications. For example: "MM ♩ = 96" means that by Maelzel's metronome there will be 96 quarter notes per minute.

Clefs had a gradual evolution from the days when one or more lines were used as part of a rudimentary form of staff notation, during which stage each line was preceded by a letter indicating the pitch. The F, C and G clefs, which are now standard, gradually evolved from the corresponding Latin characters.

Observing the evolution of the notation systems of the past in different musical civilizations, we notice the *casual* character of this evolution. Through continuous trial-and-error attempts, certain forms were improved as compared to their original state; the forms grew more practical. The general trend of this improvement lies in the direction of greater precision of notation.

Early forms of intonation dealt with large groups symbolized by one sign which could be deciphered only by the performers familiar with the conditions

*Ut, re, me, fa, sol and la. (*Ut* is still used in many countries instead of *Do.*)

**Although various tentative methods had been designed to remedy the rhythmic indefiniteness of the neumes, possibly as early as the 8th century, it was not until shortly before 1200 that rhythmic values were definitely established in notation.

and conventionalities of a given musical locality or style. The final forms used today offer an abominable conglomeration of languages, systems and symbols, often mutually exclusive and contradictory. Various innovations which piled up on top of the old forms, symbols and systems, produce extreme confusion.

The evolution of musical notation started on the wrong track from the very beginning. As music was closely associated with words, linguistic forms became the more influential in determining the system of musical notation. While the *neumes* have a directional association with a pitch line the early specimens, unfortunately, are too vague to define such a line with the precision necessary if the line is to be universally deciphered. Music written in such *neumes* could be read only by people who knew their exact intonational content in advance.

The forms of musical notation in Europe developed in correspondence with ecclesiastic forms of music and not with the secular music of a dance-song; the notation of durations evolved in reference to music of the utmost rhythmic simplicity. We are even today handicapped by the arithmetical crudeness and inefficiency of our notation of durations. Any music which does not derive from the ecclesiastic forms of European music looks forbiddingly complicated on our musical staff.

It is easy to demonstrate the inefficiency of our musical notation whenever we have to perform a rhythmic group new to our ear. Few performers can read it by sight; only by discovering a *familiar* rhythmic pattern in the musical notation can one execute it without delay. Since we face difficulties in overcoming the conservatism of our muscular and sensory habits, the best we can do is to use an efficient system of notation.

Our present system of notation is entirely fictitious with regard to pitch. The only fields in which it gives the true correspondence of the intonation of the music performed is when the music is for piano or organ. The actual differences of intonation in *a cappella* singing or in instrumental chamber music playing *leave no trace* whatever in our musical notation. The pitch discrimination ability of humanity has always been in advance of the present system of musical notation.

With the development of greater refinement in the execution of music as to intonation, rhythm and other forms of expression, and with the development of greater precision of thought by composers as to all the detailed specifications for the performance of music—a reliable, precise and versatile system of notation becomes an utmost necessity. In musicological research and comparative musicology, study is hindered and made difficult by the system of musical notation now in use. An efficient system of musical notation must be universal enough to express *any* time duration in *any* form and relationship, *any* form of intonation pertaining to *any* type of tuning system, *any* form of relative intensity, attack and variation of tone quality. Such a system may be developed only on a strictly geometrical basis, the foundation of which lies in the graphic projection of physical phenomena.

The scientific system of recording known as *nomography* deals with different methods of graph notation. While various forms of recording events scientifically exist in all statistical fields, music continues its semi-happy existence in a state of affairs in which nothing can be too wrong—and nothing can be too right! Centuries of the isolation of music from science brought about this unfortunate and chaotic situation. It is about time to acknowledge the inefficiency of our system of musical notation and take a grown-up attitude towards a field which is now unfortunately a back-yard of human thought.

B. Mathematical Notation, General Component

1. Notation of Time.

Measuring musical time from a minimum standard unit was known both to the Greeks (*chronos protos* = primary time) and the Romans. For the reasons presented in the preceding section, this usual way of direct measurement adopted in various other fields did not survive in musical notation.

Assuming that the *shortest* duration of any given musical continuity is to be the standard unit of measurement, any degree of precision may be achieved. In terms of musical notation, this means that if musical continuity includes musical halves, quarters, eighths and sixteenths, one should then express *all* the durations as sixteenths. The standard unit of measurement is the common denominator of various durations occurring in one musical context. We shall express such a unit as "t". When still greater subtlety is required, we shall use "τ" (tau) to mean a unit of deviation. This symbol will be useful in expressing somewhat unaccountable durations, such as individual grace-notes and groups of grace-notes. There also may be a need for applying τ to various forms of unbalancing—groups customarily designated by means of so-called "rubato." Every bar representing a group will always correspond to unity and will be expressed through

$$T = \frac{t}{t} = 1$$

If we want to achieve musical performances that possess in reality the subtlety they claim to have, the expression of the bar would require the inclusion of τ. The latter corresponds to the infinitesimal (dx) of the calculus.

In the new system of notation every time value becomes a rational fraction, the numerator of which expresses the period of duration, and the denominator of which expresses the standard unit of measurement ($\frac{1}{t}$). As problems of musical duration involve not only the values but also their mutual position and distribution in time continuity, it is necessary to introduce a nomenclature which will also take care of the distributive characteristics of durations. For example: ♩. ♫♪ $= 3 + 1 + 1 + 1$. As the common denominator in this case is $\frac{1}{6}$, the values acquire the following expression: $\frac{3}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. If we wish to refer to the third term of this group, such a quantitative indication is not entirely sufficient. The last three terms have uniform values, so they must acquire a special system of indications as to their *place* in the time continuity. As each "t" represents a term, the consecutive enumeration of such terms will be expressed by $t_1, t_2 \ldots t_n$ to indicate their relative position, and the coefficients

will express the actual time values. The preceding example will thus acquire the following appearance:

$$\frac{6}{6} = \frac{3t_1}{6} + \frac{t_2}{6} + \frac{t_3}{6} + \frac{t_4}{6}$$

This system of notation permits any form of distributive variations in a given continuity without running the risk of losing any corresponding placement of any given term. For example, $\frac{t_3}{6}$ may be placed in the first position in a new variation group. The previous continuity will acquire the following appearance:

$$\frac{6}{6} = \frac{t_3}{6} + \frac{3t_1}{6} + \frac{t_2}{6} + \frac{t_4}{6}$$

This means that the original third term now appears in the first place in the variation group.

The study of durations and the composition of continuity from the latter is, of course, the subject of my theory of rhythm.*

C. SPECIAL COMPONENTS

2. Notation of Pitch.

Systems of intonation used as material for music constitute the *primary selective systems.* Such systems may be uniform or non-uniform. Non-uniform systems are characterized by variable ratios between the adjacent pitch units. The series of natural harmonics is a non-uniform selective system; the intervals between the pitches contract progressively, producing a natural logarithmic series. This corresponds to the perspective contraction in optics. The systems of uniform ratio in music are known as *equal temperament,* and they express different forms of symmetry in the range of one octave. One octave is merely the *simplest* ratio; analogous systems of pitch symmetry might be evolved from any other ratio conceived as a range of emphasis. Our musical civilization deals with pre-arranged selective system of pitch known as the equal temperament of twelve ($\sqrt[12]{2}$).

The *secondary selective systems* constitute all the distributive scales *within* a primary system. The standard unit of pitch measurement, "p", becomes the symbol of a pitch unit within the equal temperament of twelve. Thus, "p" expresses a semi-tone and acquires the value of the unit expressing the logarithm to the base $\sqrt[12]{2}$. The limit integer within the octave (12p) is 11. All other intervals are expressed through respective integers. Any initial pitch represents a starting point of zero. Thus, a semi-tone from 0 equals 1, a whole tone equals 2, etc.

As this does not specify the direction or pitch from the 0 (zero) point, an additional system of indications is required. In view of this need, two methods may be offered: one, to consider all points above 0 as positive and all points below 0 as negative; two, to introduce a system of axes so that with the specification of an axis a certain direction becomes positive or negative. Moving in one direction produces either positive or negative values only. Movement in both

*See Book I.

directions produces an alternation of positives and negatives. For example, the progression c — d — f — g acquires the following notation: 2p + 3p + 2p. The figure c — f — d — g — c has the following notation: 5p — 3p + 5p — 7p.

As T expresses a time group-unit in relation to t, which is the common denominator of the group, P expresses a pitch group-unit (pitch range) in relation to p, the standard unit of pitch measurement in a given primary selective system.

Pitch ranges become important when they are treated as sections of the total range emphasis of a given musical continuity. In such a case, each pitch range corresponds to a certain axis, and the total value of the pitch units within one axis depends on the total value of all axes within the entire range. For example, if a melody evolves in a range of 15p ($c - e^1\flat$) and three axes are required, then each axis will emphasize $\frac{15p}{3} = 5p$, i.e., the partial ranges of the total range will be $P_1 = 5p$, $P_2 = 5p$, $P_3 = 5p$ (c — f; f — b♭, b — e♭).

3. Notation of intensity.

In order to establish any system of notation for intensity, we must consider a fundamental fact basic to the psycho-physiological law of Weber-Fechner: that the intensity of reaction does not vary as the intensity of stimulus; for it grows with an increase of about 15 percent in relation to the physical intensity of the stimulus. For example, when we double the amplitude of a sound wave we obtain a reaction in the ear that is only 15 percent and not 100 percent greater.

This means that the difference between very low and medium intensities appears to be much greater to our ear than does the difference between medium and high intensities. For instance, the difference between 5 and 40 decibels seems to be much greater to our ear than the difference between 40 and 75 decibels. This, obviously, is one of the psycho-physiological limitations developed for the protection of the species.

In the future, with the appearance of instruments performing music automatically, any precise mathematical specification will be possible and could be offered in any desirable type of physical correspondence.

For the present, our system of notation has to be limited in exactly the same fashion as it is limited for the expression of durations and pitch. We have to establish a certain *range* of loudness, as conceived musically, (in a given epoch and locality), and assume the lowest degree of it as one unit of intensity. Thus, if we would like to establish three important points of intensity and enumerate them as i_1, i_2 and i_3, their respective values of intensity will be i; 2i, 3i. $i_0 = 0i$. The rests (periods of silence) will be expressed by $i_0 = 0i$. These three degrees of intensity may correspond to *piano*, *mezzo-forte* and *forte* respectively.

The method of denoting intensity by minimum units is more precise, for we can establish a scale of dynamic marks of greater subtlety and precision than by using the method which expresses all this in Italian words. Thus, a selective scale of 2 degrees of intensity ($i_1 = i$, $i_2 = 2i$) may correspond to *piano* and *forte*. A scale of 3 degrees of intensity ($i_1 = i$, $i_2 = 2i$, $i_3 = 3i$) may correspond to *piano*, *mezzo-forte* and *forte*. A scale of 5 degrees of intensity ($i_1 = i$, $i_2 = 2i$, $i_3 = 3i$, $i_4 = 4i$, $i_5 = 6i$) may correspond to *pianissimo*, *piano*, *mezzo-forte*, *forte*, *fortissimo*.

One may devise scales with many more degrees of intensity when the complexity of indications through i remains the same. If, however, we used the Italian words or their musical abbreviations, it would all become quite confusing. The main reason for this confusion is not only the quantity of words employed to indicate the various degrees of intensity, but the lack of an objective scale of intensity. For instance, it is very far from obvious just what relation of intensity *mezzo-forte* has to pianissimo; but in the scale of 5 intensity units, it conveys purely quantitative $i_3 = 3i$ through $i_1 = i$ associations.

3. Notation of Quality.

Musical conception of tone *quality* emphasizes physically such different factors as harmonic saturation (density), duration of tone, form and intensity of attack, etc. In denoting quality we shall consider only the first factor. A one-component wave is the minimum limit of harmonic saturation—and the oboe-like quality (with predominant 5th, 7th and 13th harmonics) is the maximum limit of harmonic saturation, as it appears to our ear. The entire range emphasizes qualities from the flute stops of an organ ("tibia clausa") up to pure reed stops (like the "English Horn"). The intermediate quality zone embraces such tone qualities as clarinet and violin. Tone quality may also be illustrated by means of vowels: the minimum limit is "oo", the maximum, "ee", with the intermediate zone around "o" and "a".

Using the same system of notation as previously (time, pitch and intensity) and indicating quality through "q", we obtain scales with a different number of quality points. Thus, a 2 unit scale—$q_1 = q$, and $q_2 = 2q$—can express relative harmonic saturation in relation to the limits selected. A 3 unit scale consists of $q_1 = q$, $q_2 = 2q$, $q_3 = 3q$. A 5 unit scale: $q_1 = q$, $q_2 = 2q$, $q_3 = 3q$, $q_4 = 4q$, $q_5 = 5q$.

These instrumental media for achieving variation of tone color are the subject of my later discussion of the acoustical basis of orchestration.

D. Relative and Absolute Standards

Notation methods similar to the method applied to the differential deviations in recording musical time must be used when *subtle* deviations from the established scales of pitch, intensity and quality are to be handled.

It is impractical, in the present state of music, to deal with differential equations so long as performers are human beings confined in their interpretation to crude arithmetical limitations. When greater subtlety of performance is required, the respective differential values for the time, pitch, intensity and quality components will be: dt, dp, di and dq. Thus, the existing state of music and of musical notation, and the conception of any recording of a musical composition by means of notation—all this presupposes both scalewise (discontinuous) and differential (continuous) constitution of music. For example, in a "gradual" increase of intensity on the piano, the physical reality of it is a group of intensified points with quick drops, with every succeeding attack greater than

the preceding one and every attack having the characteristic of a constant form of fading (decrease of intensity). On the contrary, when they deal with pitch—which in ordinary musical notation is always discontinuous—the performers of our civilization as well as of the Oriental cultures, actually produce a *differential curve* of frequencies very often.

The actual difference between Chinese singing and Hungarian violin playing is the quantitative difference of time that elapses between the stabilized pitch points of the scale—the time period is longer and the attack is stronger with the Chinese. Minute variations of tone quality are often beyond the control of a performer. Often even a very experienced performer, in trying to produce one tone quality, actually obtains another. How many *unintentional* harmonics break through on account of a wrong angle or wrong pressure of the bow over the strings in violin playing! How many performers on stringed bow instruments produce the jumping effect (saltando) instead of the intended smoothly repeated attacks (portamento) because of mere nervousness! The adoption of the manifold of mathematical resources for musical notation would seem ridiculous for the present when the requirements are so low that deviation from a proper set of time durations and proper intonation is a very common sin.

All forms of musical notation must deal with the expression of *relative* quantities only. Absolute standards of pitch, intensity and quality vary during different epochs. Even the absolute speed of time is somewhat affected. The general tendency toward faster *tempi*, as compared with those used in the 17th century, becomes quite apparent. This is most probably influenced by the general acceleration of vehicular motion and general development of engineering technique. Many performers now interpret some of the classical music of the 18th century and 19th century at much faster *tempi* than would have been desirable or even possible at the time this music was written.

Pitch standards have also had a tendency to accelerate. At the time of Haydn and Mozart, *a* of the middle octave generally had 422 vibrations per second, while the concert tuning of the corresponding tone of today ("American concert pitch") is 440.6, a change of more than one and a half semitones. These variations of frequencies, with regard to absolute pitch, are decided at various international conferences of acousticians and manufacturers of musical instruments. The application of pitch *ranges* also grows. Take, for example, the range of the violin, where g of the small octave being the lower limit, we notice a constant extension of the upper limit: it was c of the third octave during Beethoven's time, e above it with the early Wagner, g♯ above it with the later Wagner, and b above it with Rimsky-Korsakov (as in *Kitezh*).

One century produced a gain of one complete octave in extending the range of the violin. The desire to obtain greater *tension* effects leads to the employment of higher frequencies; it implies a growth of virtuosity in playing musical instruments. Paganini was about the only person in his time who was capable of playing some of his most difficult works for the violin. Nowadays, however, any capable student of a violin department in the conservatories is able to play them. Today in America, we witness an extraordinary virtuosity in extending the range of such instruments as trumpet and trombone; sometimes the gain is a whole octave beyond the standard range.

The range of intensity also grows because of a desire for production of greater dynamic contrasts and a desire to obtain extreme intensities. One of the causes may be the amount of noise in the big cities of today; it is necessary to be loud in order to stand out amidst noisy surroundings. At the time of Bach few dynamic marks existed. At the time of Beethoven the dynamic expression had to be guided by the conductor's discrimination—the dynamic marks referred *not to the performers*, but to the listeners. When Beethoven used one or another dynamic mark (*piano* or *forte*), he meant that the corresponding degree of intensity would be *heard by the audience*. Nowadays, however, dynamic marks are used *for the performer*. The composer now assumes the responsibility of producing the *total of dynamic balance* by marking the individual parts in a score in a different way. For example, in order to balance trumpets with clarinets in music which must sound quite *loud*, the trumpets may have an indication of *mezzo-forte*—while the clarinets are marked *fortissimo*. This trend toward producing very loud sounds, so fashionable with the Italian *bel canto* in the 19th century, was transferred to the instrumental field in the 20th century.

As to tone quality, there is also a noticeable tendency towards the increase of the quality range. There are many new mutes devised for the brass, producing a considerable variety of tone quality variations; various semi-mechanical instruments (organs) have been built with a tremendous number of stops, which provide different tone qualities. The development of electronic sound production leads to a variety of tone qualities that would have been beyond the imagination of any composer of the past. Some of the models of today place *more than a billion* different tone qualities at the disposal of the composer and the performer.

Thus we notice a definite tendency towards the expansion of the range within different sound components, as well as a definite tendency of acceleration. As the absolute standards for any established average tempo, average range for a melody, average range for intensity and tone color are not invariants, all this necessarily affects the system of relations within the above-mentioned component. The notation of relative, not absolute, values is the more important one for the purposes of composition and reproduction of music.

E. Geometrical (Graph) Notation

The adoption of the *graph* method for the recording of musical composition and performance has obvious advantages over the present system of musical notation. In the first place, it offers as much precision as is desired; in the second, it stimulates direct associations with the pattern of a given component.

A physical record of what is audible, such as an oscillogram or a photogram of a sound track, is too complicated to be used as musical notation. But the geometrical notation offered in this theory is the general method of graphs, the same as that commonly used for the statistical recording of events, i.e., a record of the variation of special components in time continuity (general component).

Graph notation records individual components through individual curves. The special components of sound are frequency, intensity, and quality—and they may be recorded through the corresponding individual graphs. By means of such notation the composer can define his intentions with the utmost precision; the performer can then decipher the desires of the composer to the latter's full satisfaction.

In the future, with the elimination of the living performer, the graph method will still be valid for use with automatically performing musical instruments. Curves of composition and curves of execution will then merge into one.

The horizontal direction (the *abscissa*, read from left to right) expresses time in all graphs; the vertical direction (*ordinate*) expresses variation of some special component: pitch, intensity, quality. The graph method is an objective one and is therefore a general method. Any wave motion records itself automatically.

The units on cross section paper to be used for a graph recording of music represent the standard units of measurement with respect to the units of preselected pitch, intensity and quality scales. The best graph paper to use is that ruled 12 x 12 per square inch; the reason for this is the versatility of the number 12 with respect to divisibility and the definition of an octave of the equal temperament of twelve for the pitch.

The scales referring to different components may be different in quantity. For example, a scale of pitch may conform to 7 units while the scale of intensity in the same music may conform to 3 units, and the scale of quality to 2 units. This will be reflected respectively in the complexity of the corresponding graphs.

Graduality or suddenness of transition from one stabilized point to another is expressible by a definite degree of curvature. Variation of pitch in the asymmetric tuning system may be recorded on logarithmic graph paper. The logarithmic contractions of abscissa and ordinate were described in Book Three.* Notation of pitch variation of actual violin playing assumes hyperbolic curvatures. The pitch-graphs of piano or organ are rectangular.

The customary conception of melody in the Western World is based on a *rectangular* conception of pitch, i.e., all the sliding between the stabilized tempered pitch-points is left to the discrimination of the performer. Because of this fact, different styles of interpretation reveal different degrees of curvature of a melodic line. Continuous uniform sliding, without any stabilized pitch-points, may be observed in the sound of a siren or of a fire alarm signal; extreme abruptness (rectangular graph) is found on the piano or organ; intermediate forms (hyperbolic graph) are found on stringed bow instruments, woodwind instruments, in the human voice and in the space-control theremin.

In the following exposition only rectangular graphs are used, as the different degrees of curvature refer to the performance and not to the composition of music; such curvatures are to be discussed in my theory of interpretation.

One vertical segment on the graph paper expresses a unit in the corresponding equal temperament, a $\sqrt[12]{2}$ in the case of the present-day system. The intensity curves may be used either as continuously sliding curves or as rectangular curves with the stabilized levels expressing definite predetermined degrees of intensity. The same refers to tone quality graphs: for those instruments which are capable of producing continuous quality variation and are controlled by a graduated scale (such as some of the electronic instruments), the graph is curvilinear; but for those instruments capable only of abrupt (discontinuous) transitions, only a rectangular graph is necessary.

*See pp. 211-2.

CHAPTER 3

THE AXES OF MELODY

WITH THE conclusion of the foregoing discussion of the philosophical setting of the problem of melody and of the notational problems of all music, we are now in a position to approach the actual technology of making melodies.

We are concerned first with two kinds of axes: primary axis and secondary axis.

A. Primary Axis of Melody

Definition: Primary axis is a pitch-time maximum.*

In order to determine what is the pitch-time maximum in any given melodic continuity, sum up all the pitch levels occurring in the continuity; then establish the pitch which has the greatest number value as the *primary axis of the melody.*

Our ear and our auditory consciousness apprehend music in *portions* of continuity. When the preceding portion fades out we hear the new one evolving. The time values absorbed by the memory while one is listening to music varies with the structural constitution of the melody. Rests, ties, accents, and other signs of musical punctuation—all dissociate the portions of musical continuity in our consciousness.

The location of the *primary axis* is therefore relative to the amount of continuity retained by our memory. While concentrating our attention on melody-in-the-making within, let us say, two seconds of emphasis, we detect one primary axis; but during twenty seconds of the same continuity, our attention may be directed towards an entirely different pitch center.

Our musical memory selects the primary axis through its reactions to a quantity of repeated excitations produced by certain frequencies.

Musical orientation is based on the relations of a melodic configuration to its primary axis, without which melody does not produce any musical reality. A melody without an axis seems "not to hold together," to have no comprehensible structural constitution. When some of our contemporary composers reject the idea of the primary axis (consciously or unconsciously), they revolt not only against musical traditions but against the laws of nature as well. So-

*As with other Schillinger concepts, the idea of the primary axis serves a double function. From the standpoint of musical composition, it is the point of reference around which the construction of melodies fluctuates. Without the concept of a primary axis as a starting point, no scientific approach to melody making would be possible. In addition to the technological function, the concept of the primary axis also serves a critical function. It can be used in the analysis of music. As Schillinger indicates, melodies lacking a clearly defined primary axis lack certain qualities, just as melodies containing rapidly changing axes possess others. Thus, the concept of the primary axis is of practical value, and not merely a verbalism. This is true of all Schillinger concepts, such as the secondary axes of melody, forms of trajectorial motion, etc. (Ed.)

called "atonality", i.e., the *neutral* distribution of pitch units within a given tuning system in various arrangements in order to produce a melody, does not make any "musical"; i.e., *organic*, sense. Listeners usually object to such music and they are perfectly justified in doing so.

Inasmuch as counting pitch units in their time continuity does not present any difficulties, no properly constructed melody will ever leave room for any doubt as to where its primary axis is located. In analyzing music on a geometrical basis, one comes across a number of inconsistencies even in the well-known themes of important composers of the past. If they had known the mechanical specifications of melody, their intentions would have been clearer both to themselves and to their listeners. Such partly deficient melodies create certain difficulties when analyzed.

Another case—which may seem doubtful only at first (when a student has not acquired enough analytical experience)—is that in which the primary axis is *variable*, i.e., when a melody, after being centered on a definite primary axis, deviates from it for a considerable period of time and establishes a *new* primary axis from which it may proceed further on in a similar fashion. Such a case involves modulation. As a winding plant, such as ivy, stretches from one branch to another, winds around its coils—and, when it grows out the length of the respective branch, stretches to the new one, so in music an analogous case would be the use of modulation as an outcome of excessive tonal stability.

In geometrical notation, primary axis is the abscissa itself, i.e., a continuous horizontal extension. *The primary axis is the only axis which actually sounds.* All the secondary axes merely represent *directional* lines.

We shall analyze now the three characteristic types of melodic structures with respect to their primary axes. The three melodic themes are taken from Ludwig van Beethoven's Piano Sonata No. 8, the *Pathetique.*

B. Analysis of Three Examples

1. The first 8-bar melody is the beginning of the First Allegro. The graph of this melody, as shown in figure 1 on the following page, has its primary axis on the c located on the third space in the treble, where it accumulates a total of 18t. All the other levels do not withstand the competition, as the greatest number value on them does not exceed 6t. Musical analysis does not provide such precision. In the case of this melody, there would be three competing pitch levels: middle c, third space c, and c on the second leger line. Geometrically the lower one accumulates 6t and the upper one 4t, leaving the middle one (18t) without doubt. Musically, all the three c's are the official tonics of the scale. According to the key signature, the melody is written in c minor. Thus, the importance of the axis is greater than that of the tonic.

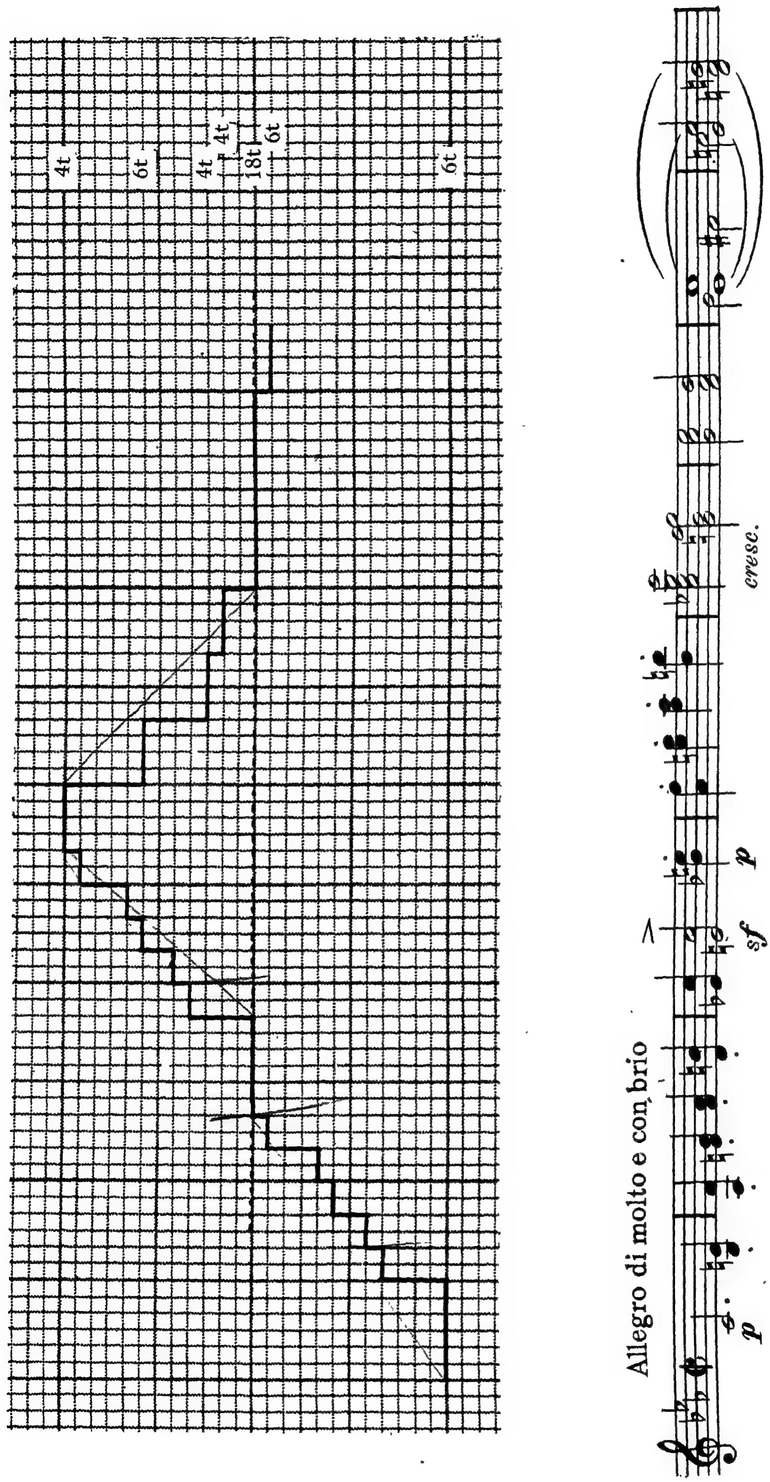


Figure 1. Graph of Beethoven's Pathetique, 1st Movement (Allegro).

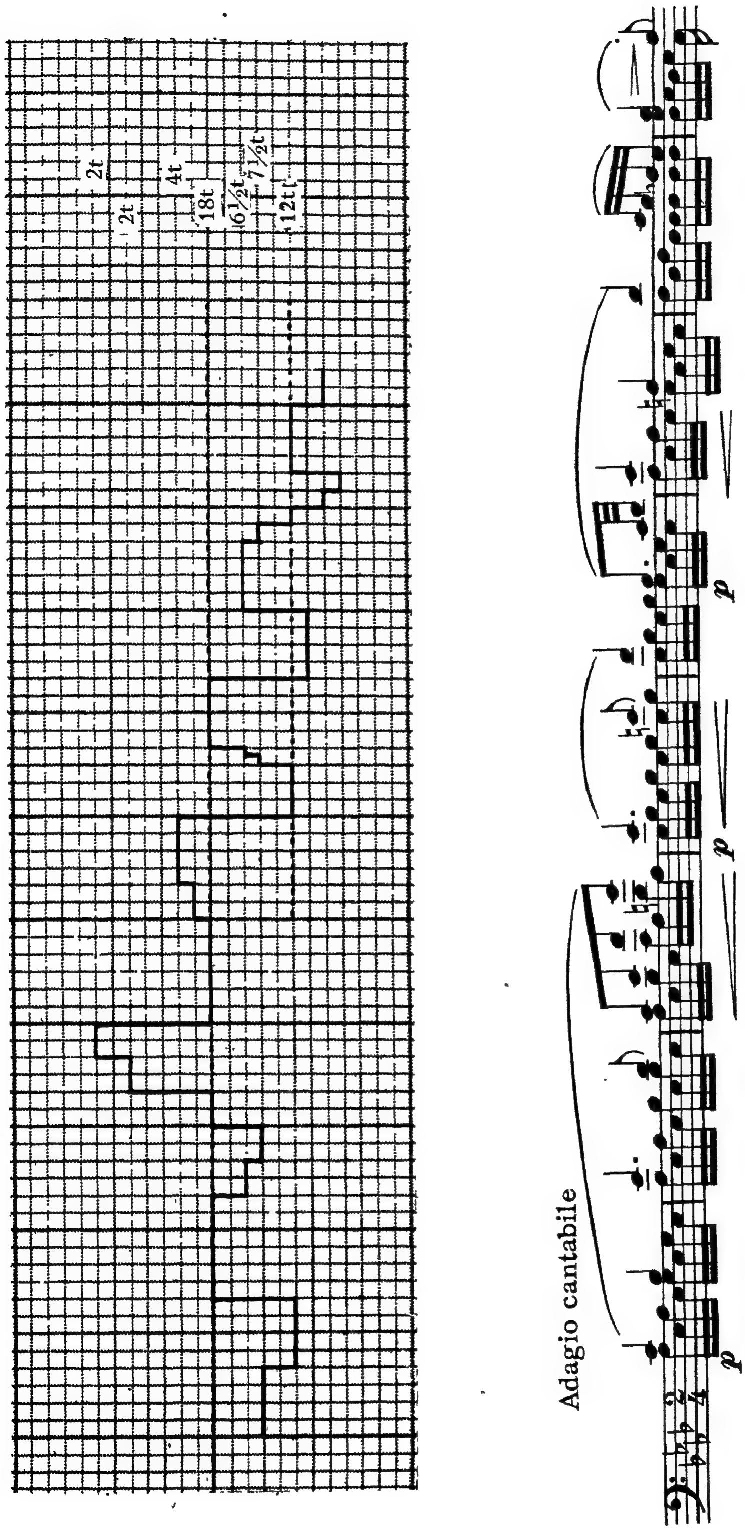


Figure 2. Beethoven's Pathetique, 2nd Movement (Adagio).

2. The first 8-bar melody of the Second Movement of the same Sonata (see figure 2 on preceding page) serves as an illustration of modulation. This melody evolves along the two consecutive primary axes. The first one has 18t, the second—12t. As the melody does not return to the first level, modulation and the establishment of a new primary axis become necessary. If we subtract from the first primary axis the total duration which appears on it after the melody makes a transition to the second primary axis, we obtain 18t − 4t = 14t for the first primary axis. Subtracting the total duration of the second primary axis while the melody adheres to the first primary axis, we obtain 12t − 4t = 8t for the second primary axis. Thus, the first axis level amounts to a total of 14t, and the second primary axis amounts to a total of 8t. From a musical viewpoint this melody, according to its key signature, is written in A major. After detection of the two primary axes, we find that in reality this melody evolves in the Mixolydian scale in its first portion, then modulates and establishes itself in the Dorian scale. (See page 249)

3. The first 8-bar melody of the Final Movement of the same Sonata (see figure 3 on following page), illustrates a case of wrong proportions, which may look doubtful to the beginner. There are two competing pitch levels, each amounting individually to 15t. Observing more closely the form of this melody, one finds that the lower axis deviates a number of times from its level, appearing near the beginning and at the very end (besides the four other points). The upper axis appears on the same level in two portions, both near the center of the entire graph. This construction reveals that the melody is actually "attached" to the lower axis, which thus becomes the primary axis. The upper axis being equally important but not important enough to retain the melody centered around it, represents an hypertrophied climax. This melody can be improved by being deprived of part of its overgrown climax. By shifting the entire graph 4t to the right, we would acquire the beginning of the climax on the downbeat of the 4th bar; by taking out the last 4t of the same level, we would improve this melody and still let it end at the same time moment as in the original, i.e., on the downbeat of the 8th bar. (See page 251)

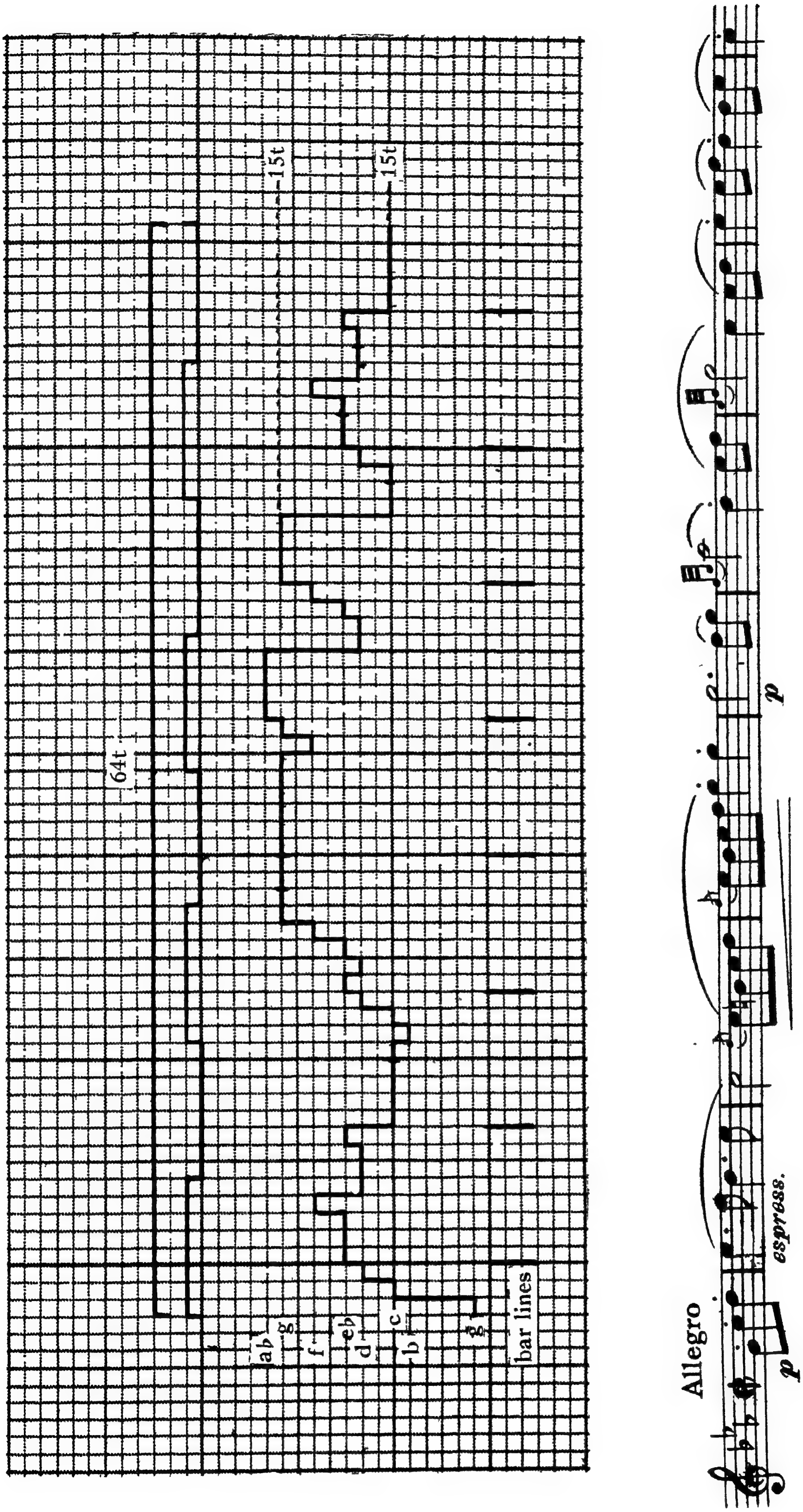


Figure 3. Beethoven's Pathetique, 3rd Movement (Rondo).

C. Secondary Axes

Definition: Secondary axes are the directional axes with respect to the primary axis.

1. The zero axis (0)
2. The "a" axis (a)
3. The "b" axis (b)
4. The "c" axis (c)
5. The "d" axis (d)

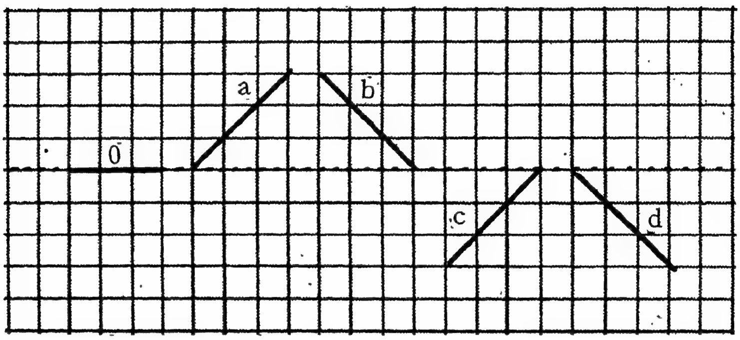


Figure 4. Secondary axes.

The zero axis is the direction of motion along abscissa. The a axis is the ascending direction from the primary axis. The b axis is the descending direction toward the primary axis. The c axis is the ascending direction toward the primary axis. The d axis is the descending direction from the primary axis. The a, b, c and d axes are mutual geometrical inversions obtained by revolving the a axis through the quadrants around the ordinate and the abscissa in an 180° angle. Thus, b represents the backward motion of a; c the backward upside-down of a; d the forward upside-down of a.

The zero axis represents an absolute balance. The balancing axes (leading toward balance) are b and c. The unbalancing axes (leading away from balance) are a and d.

Balancing Axes *Unbalancing Axes*

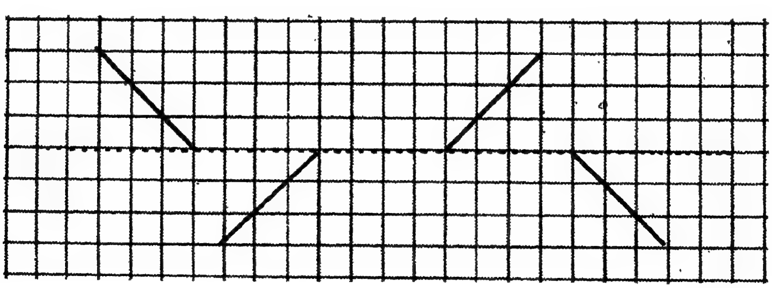


Figure 5. Balancing and unbalancing axes.

The unbalancing axes are characteristic of beginnings. The balancing axes are characteristic of endings. The zero axis is characteristic of the beginning before the motion acquires inertia, or of the ending when all the inertia is exhausted.

Every melody represents a combination of different directions as expressed by 0, a, b, c and d axes. Various combinations of axes produce various forms of melodic continuity. The unbalancing axes produce the effect of tension, the balancing axes produce the effect of release. As the zero level represents zero tensions, the increase of tension grows with the increase of distance from the primary axis.

Composition of melodic continuity, with respect to pitch and time, may be based on monomial, binomial, trinomial and polynomial combinations of the secondary axes.

D. Examples of Axial Combinations

1. Monomials

0 + . . .
a + . . .
b + . . .
c + . . .
d + . . .

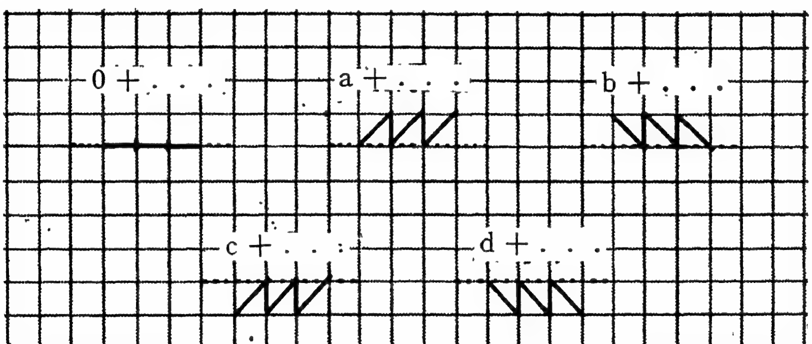


Figure 6. Monomial axial combinations.

2. Binomial Combinations

0 + a
0 + b
0 + c
0 + d
a + b
a + c
a + d
b + c
b + d
c + d

10 combinations, 2 permutations each.
Total number of cases: 10 × 2 = 20.

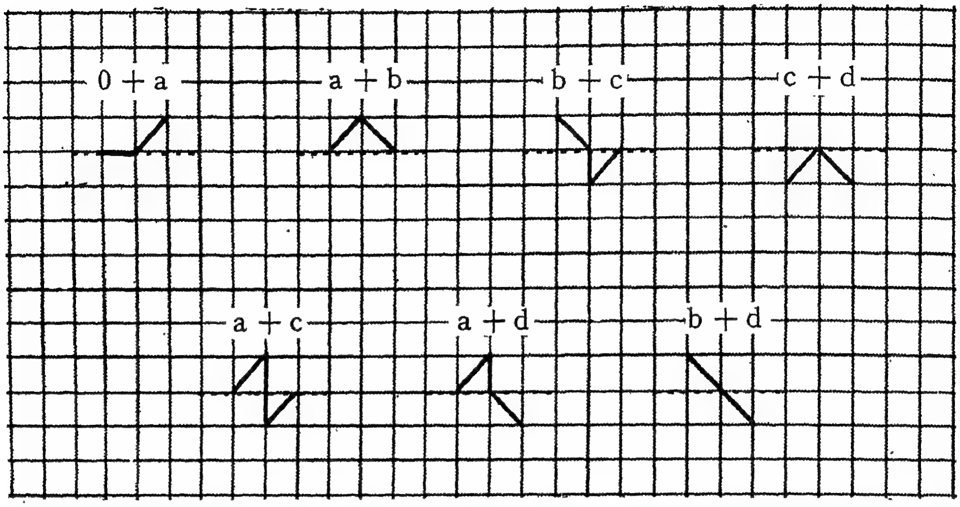


Figure 7. Binomial axial combinations.

3. Trinomial Combinations. Two identical terms.

0 + 0 + a
0 + 0 + b
0 + 0 + c
0 + 0 + d

a + a + 0
a + a + b
a + a + c
a + a + d

b + b + 0
b + b + a
b + b + c
b + b + d

c + c + 0
c + c + a
c + c + b
c + c + d

d + d + 0
d + d + a
d + d + b
d + d + c

20 combinations, 3 permutations each.
Total number of cases: 20 × 3 = 60.

3 different terms:

0 + a + b	a + b + c
0 + a + c	a + b + d
0 + a + d	a + c + d
0 + b + c	b + c + d
0 + b + d	
0 + c + d	

10 combinations, 6 permutations each.
Total number of cases: 10 × 6 = 60.

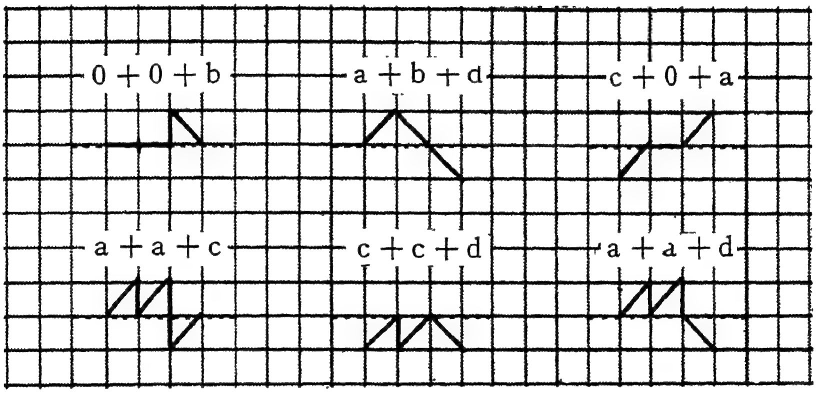


Figure 8. Trinomial axial combinations.

4. Quadrinomial Combinations

4 places with 3 identical terms:

0 + 0 + 0 + a
0 + 0 + 0 + b
0 + 0 + 0 + c
0 + 0 + 0 + d

a + a + a + 0
a + a + a + b
a + a + a + c
a + a + a + d

b + b + b + 0
b + b + b + a
b + b + b + c
b + b + b + d

c + c + c + 0
c + c + c + a
c + c + c + b
c + c + c + d

(continued)

d + d + d + 0
d + d + d + a
d + d + d + b
d + d + d + c

20 combinations, 4 permutations each.
Total number of cases: 20 × 4 = 80.

4 Places with 2 identical pairs:

0 + 0 + a + a
0 + 0 + b + b
0 + 0 + c + c
0 + 0 + d + d

a + a + b + b
a + a + c + c
a + a + d + d

b + b + c + c
b + b + d + d

c + c + d + d

10 combinations, 6 permutations each.
Total number of cases: 10 × 6 = 60.

4 Places with 2 identical terms:

0 + 0 + a + b
0 + a + a + b
0 + a + b + b
0 + 0 + a + c
0 + a + a + c
0 + a + c + c
0 + 0 + a + d
0 + a + a + d
0 + a + d + d

0 + 0 + b + c
0 + b + b + c
0 + b + c + c
0 + 0 + b + d
0 + b + b + d
0 + b + d + d

0 + 0 + c + d
0 + c + c + d
0 + c + d + d

a + a + b + c
a + b + b + c
a + b + c + c
a + a + b + d
a + b + b + d
a + b + d + d
b + b + c + d
b + c + c + d
b + c + d + d

a + a + c + d
a + c + c + d
a + c + d + d

30 combinations, 12 permutations each.
Total number of cases: 30 × 12 = 360.

4 different terms:

0 + a + b + c   0 + b + c + d
0 + a + b + d
0 + a + c + d

a + b + c + d
5 combinations, 24 permutations each.
Total number of cases: 5 × 24 = 120.

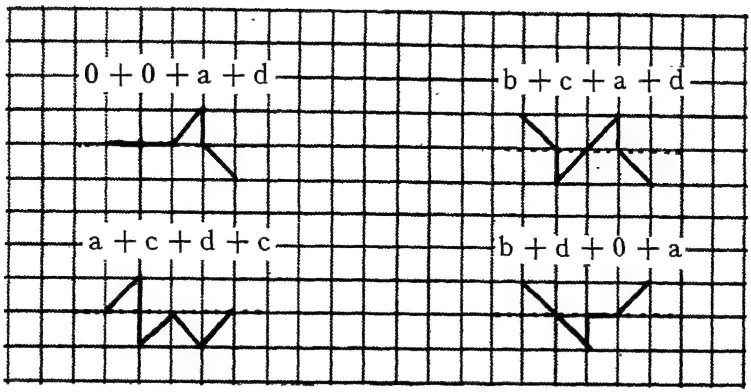


Figure 9. Quadrinomial axial combinations.

5. Quintinomial Combinations

5 Places with 4 identical terms:

0+0+0+0+a	a+a+a+a+0	b+b+b+b+0	c+c+c+c+0
0+0+0+0+b	a+a+a+a+b	b+b+b+b+a	c+c+c+c+a
0+0+0+0+c	a+a+a+a+c	b+b+b+b+c	c+c+c+c+b
0+0+0+0+d	a+a+a+a+d	b+b+b+b+d	c+c+c+c+d

d+d+d+d+0
d+d+d+d+a
d+d+d+d+b
d+d+d+d+c

20 combinations, 5 *permutations each.*
Total number of cases: 20 × 5 *= 100.*

5 places with 3 identical terms and 2 identical terms:

0+0+0+a+a	a+a+a+0+0	b+b+b+0+0	c+c+c+0+0
0+0+0+b+b	a+a+a+b+b	b+b+b+a+a	c+c+c+a+a
0+0+0+c+c	a+a+a+c+c	b+b+b+c+c	c+c+c+b+b
0+0+0+d+d	a+a+a+d+d	b+b+b+d+d	c+c+c+d+d

d+d+d+0+0
d+d+d+a+a
d+d+d+b+b
d+d+d+c+c

20 combinations, 10 permutations each.
Total number of cases: 20 × 10 = 200.

5 Places with 2 identical pairs:

0+0+a+a+b	0+0+b+b+c	0+0+c+c+d
0+0+b+b+a	0+0+c+c+b	0+0+d+d+c
a+a+b+b+0	b+b+c+c+0	c+c+d+d+0

0+0+a+a+c	0+0+b+b+d
0+0+c+c+a	0+0+d+d+b
a+a+c+c+0	b+b+d+d+0

0+0+a+a+d
0+0+d+d+a
a+a+d+d+0

a+a+b+b+c	a+a+c+c+d
a+a+c+c+b	a+a+d+d+c
b+b+c+c+a	c+c+d+d+a

a+a+b+b+d
a+a+d+d+b
b+b+d+d+a

b+b+c+c+d
b+b+d+d+c
c+c+d+d+b

30 combinations, 30 permutations.
Total number of cases: 30 × 30 = 900.

5 Places with 2 identical terms:

0+0+a+b+c	a+a+b+c+d
0+a+a+b+c	a+b+b+c+d
0+a+b+b+c	a+b+c+c+d
0+a+b+c+c	a+b+c+d+d

0+0+a+b+d
0+a+a+b+d
0+a+b+b+d
0+a+b+d+d

0+0+b+c+d
0+b+b+c+d
0+b+c+c+d
0+b+c+d+d

16 combinations, 60 permutations each.
Total number of cases: 16 × 60 = 960.

5 different terms:

0 + a + b + c + d

1 combination, 120 permutations.
Total number of cases: 1 × 120 = 120.

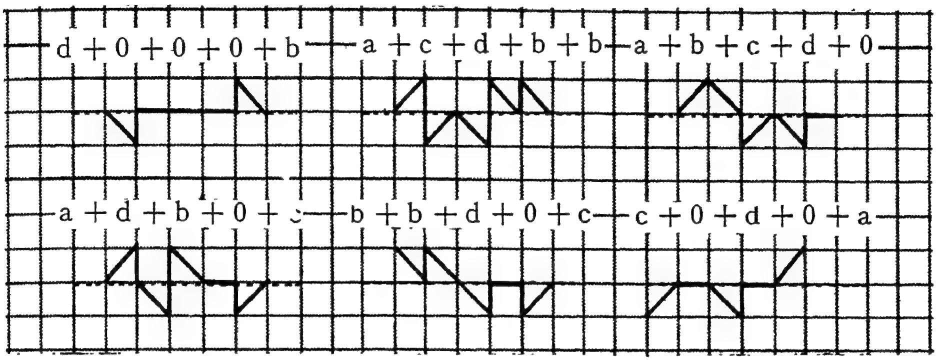


Figure 10. Quintinomial axial combinations.

E. Selective Continuity of the Axial Combinations.

In order to make a preferential selection of recurrence of the secondary axes in composing continuity, coefficients must be used. The following cases are possible.

1. Monomial axis, monomial coefficient

Example:
2a; 3a; 5a; . . .

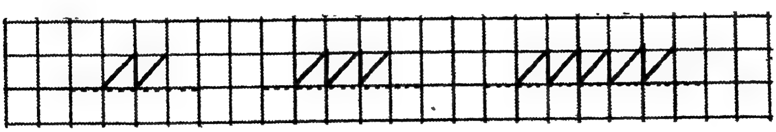


Figure 11. Monomial axis, monomial coefficient.

2. Binomial axial combination, binomial coefficient.
Binomial axial combination, quadrinomial coefficient.
Binomial axial combination, polynomial group-coefficient with even number of terms.

Example:

2a + b; 3a + 2b; 2a + b + a + 2b; 3a + b + 2a + 2b + a + 3b

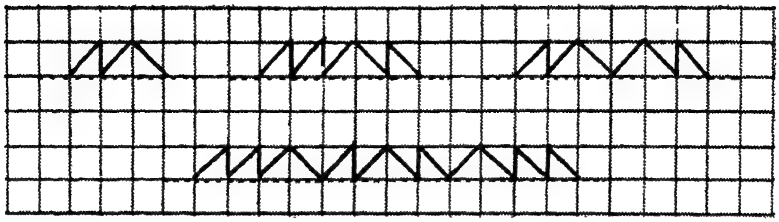


Figure 12. Binomial axial combinations.

3. Trinomial axial combination, trinomial coefficient.
Trinomial axial combination. Polynomial group-coefficient with the number of terms divisible by 3.

Example:

3a + b + c; 3a + b + 2c + 2a + b + 3c

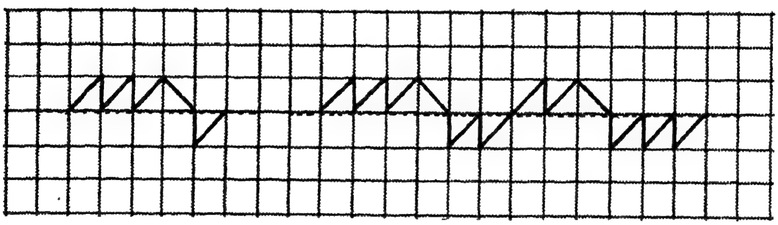


Figure 13. Trinomial axial combinations.

4. Quadrinomial axial combination, quadrinomial coefficient.
Quadrinomial axial combination. Polynomial group-coefficient with the number of terms divisible by 4.

Example:

3a + b + 2c + 2d; 4a + b + 3c + 2d + 2a + 3b + c + 4d

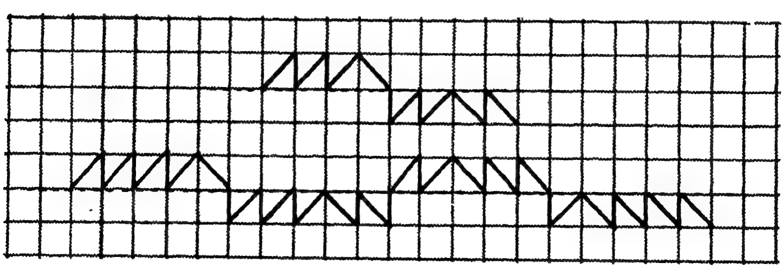


Figure 14. Quadrinomial axial combinations.

5. Quintinomial axial combination, quintinomial coefficient.
Quintinomial axial combination. Polynomial coefficient with the number of terms divisible by 5.

Example:

5 (0) + a + 4b + 2c + 3d;
5 (0) + a + 4b + 2c + 3d + 3 (0) + 2a + 4b + c + 5d.

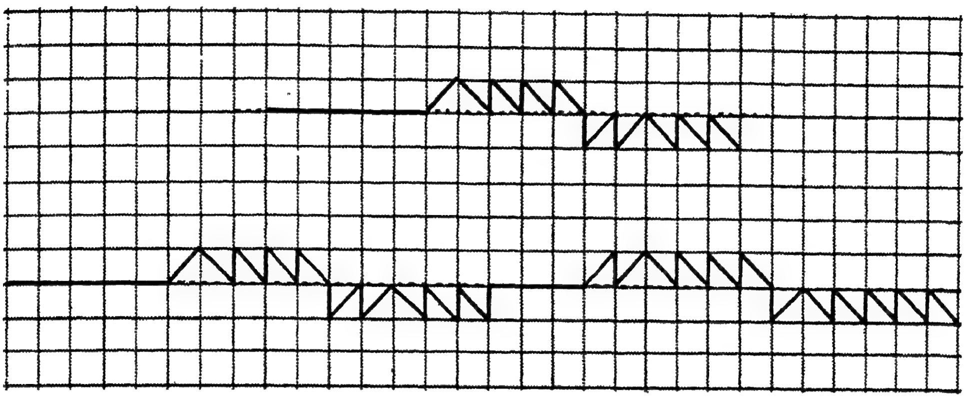


Figure 15. Quintinomial axial combinations.

When the number of terms in a coefficient-group does not coincide with the number of terms in the axial-group, or does not offer common divisors, then interference between the number of places in both groups will occur.

Example:

Binomial axial combination: a + b
Trinomial coefficient group: 3 + 2 + 1
The product: $3 \times 2 = 6$
The complementary factor: 2(3), 3(2)
The resultant of interference: 3a + 2b + a + 3b + 2a + b

F. Time Ratios of the Secondary Axes

Various axial combinations assume various time ratios. Identical axial combinations produce an infinite variety of patterns through different time-ratios. A melody derived from one or another axial pattern is influenced in different degrees by the mutual relations of the balancing and unbalancing axes. An effect of gradual deviation from balance with quick return to balance is entirely different when the time-ratio is inverse. Deviation from balance on one side of the primary axis produces a different degree of tension when the balancing axis appears on the opposite side of the primary axis than when each combination occurs on one side of the primary axis. With these facts in view, the selection of time coefficients for the secondary axes must be guided by the type of melody, with respect to its tranquility or lack of it. Detailed information on *tension relations* produced by means of axes will be presented at a later point.

It would be correct in most cases to assume a time group unit (T) to be the unit of duration for the secondary axes. Naturally any multiple thereof may serve as a unit; but one bar gives a clear association to minds accustomed to musical ways of thinking.

Here are a few illustrations of the typical axial combinations in relation to time ratios:

1. Monomial Axial Combination

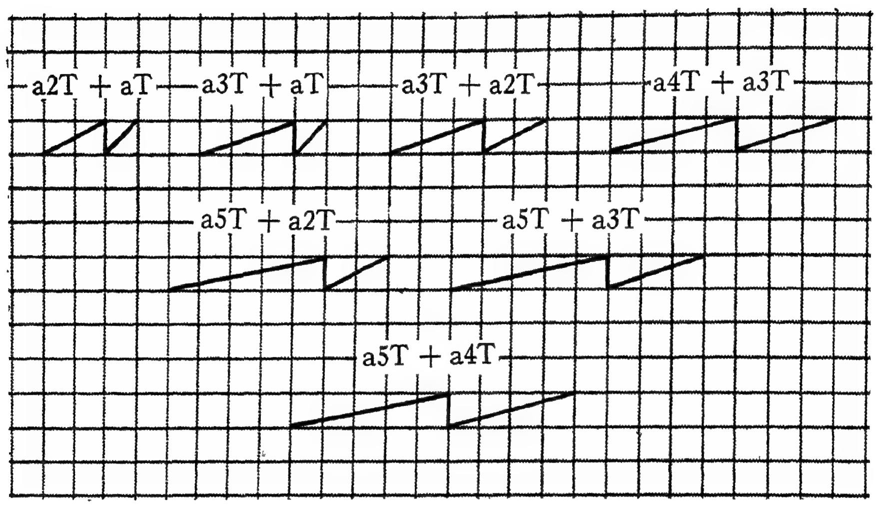


Figure 16. Binomial time-ratio.

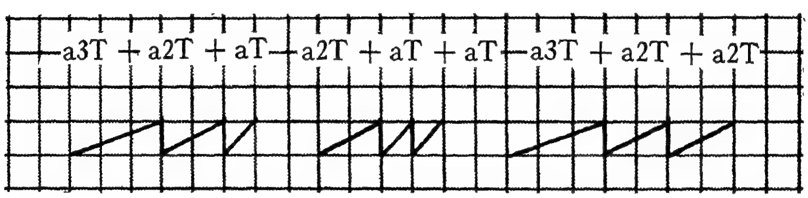


Figure 17. Trinomial time-ratio.

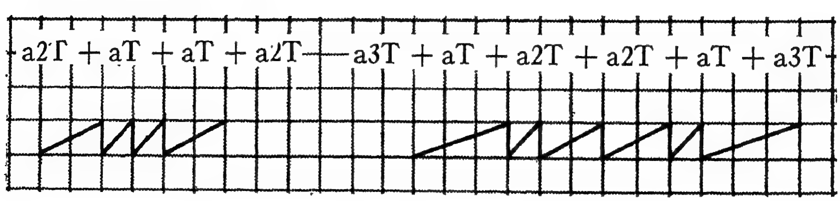


Figure 18. Polynomial time-ratio.

2. Binomial Axial Combination

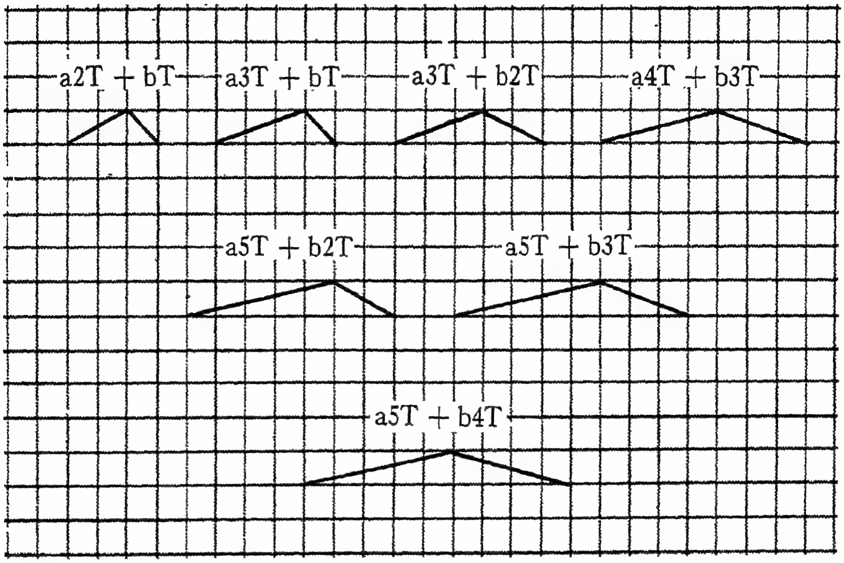


Figure 19. Binomial time-ratio.

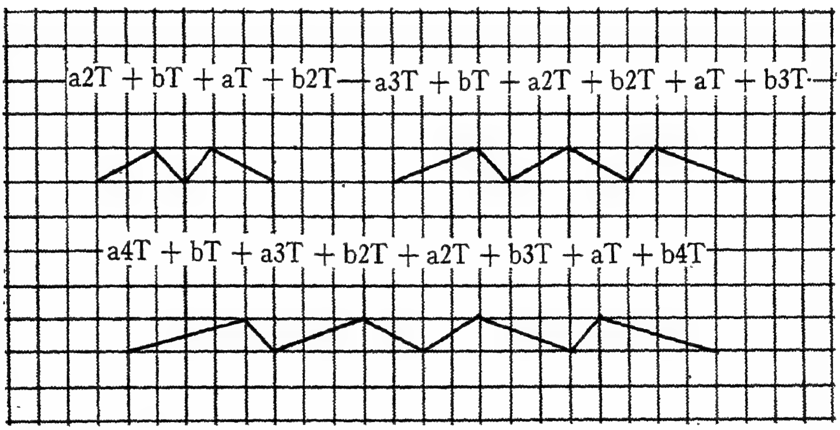


Figure 20. Polynomial time-ratio with the number of terms divisible by 2.

Axes: a, b Time ratio: 3 ÷ 2 ÷ 1

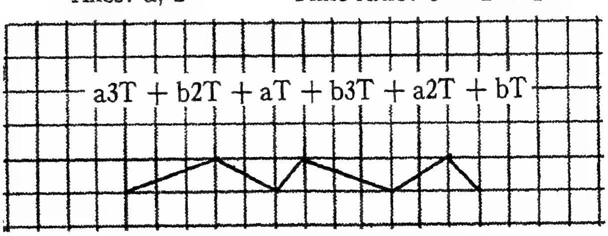
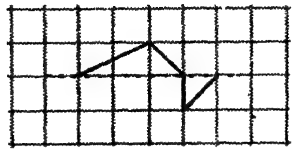


Figure 21. Interference time-ratio.

3. Trinomial Axial Combination

Axes: a, b, c Time ratio: $2 \div 1 \div 1$

$a2T + bT + cT$



Axes: a, b, c Time ratio: $3 \div 3 \div 2$

$a3T + b3T + c2T$

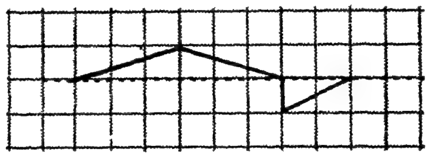
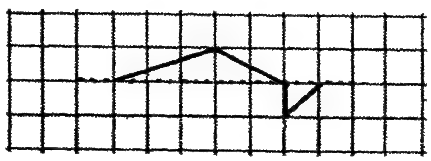


Figure 22. Trinomial time-ratio with two identical terms.

Axes: a, b, c Time ratio: $3 \div 2 \div 1$

$a3T + b2T + cT$



Axes: a, b, c Time ratio: $4 \div 1 \div 3$

$a4T + bT + c3T$

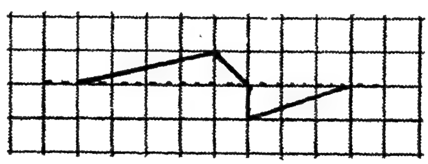


Figure 23. Trinomial time-ratio with three different terms.

Axes: a, b, c Time ratio: $r_{4 \div 3}$

$a3T + bT + c2T + a2T + bT + c3T$

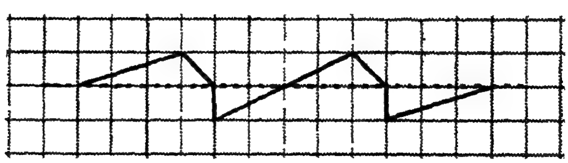


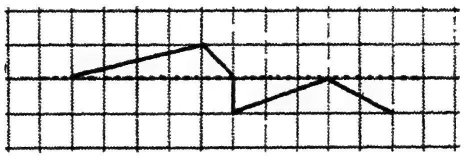
Figure 24. Polynomial time-ratio with the number of terms divisible by 3.

4. Polynomial Axial Combination

Polynomial time-ratio with the number of terms corresponding to the number of terms in the axial group or any multiple thereof.

Axes: a, b, c, d Time ratio: $4 \div 1 \div 3 \div 2$

$a4T + bT + c3T + d2T$



Axes: a, b, c, d Time ratio: $r_{5 \div 4}$

$a4T + bT + c3T + d2T + a2T + b3T + cT + d4T$

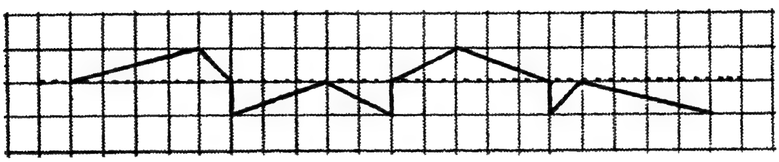


Figure 25. Polynomial time-ratio.

The number of variations for each axial combination, with a selected time-ratio, depends on the number of terms in the axial group and the number of terms in the time-ratio.

A *monomial axial combination* with a binomial time-ratio produces 2 variations:

$a2T + aT$ $aT + a2T$

A monomial axial combination with trinomial time-ratio having 2 identical terms produces 3 variations:

$$a2T + aT + aT$$
$$aT + a2T + aT$$
$$aT + aT + a2T$$

A monomial axial combination with trinomial time-ratio having all 3 terms different:

$$a3T + a2T + aT$$
$$a3T + aT + a2T$$
$$aT + a3T + a2T$$
$$a2T + a3T + aT$$
$$a2T + aT + a3T$$
$$aT + a2T + a3T$$

A monomial axial combination with polynomial time-ratio produces a number of variations equivalent to the number of permutations of terms in the time-ratio:

$$a3T + aT + a2T + a2T + aT + a3T$$

6 elements, 90 permutations.

A *binomial axial combination* with binomial time-ratio. The number of variations equals $2^2 = 4$.

$$\begin{aligned} &a2T + bT \\ &aT + b2T \\ &bT + a2T \\ &b2T + aT \end{aligned}$$

A binomial axial combination with polynomial time-ratio produces a number of variations equivalent to the product of the number of permutations in the axial group by the number of permutations in the time-ratio.

$$a2T + bT + aT + b2T$$

4 terms with 2 identical pairs produce 6 permutations. In this case the axial group and the time-ratio have identical structure. The number of variations: $6^2 = 36$.

Generalization

In order to compute the total number of permutations for any axial combination and any time ratio, it is necessary to synchronize the numbers of terms of both groups.

Let T be the original time ratio, or duration-group, and let Ax be the original axial combination, or axial group. Then let T' and Ax' be the synchronized forms of the respective groups. Then synchronization (S) occurs as follows:

$$\begin{aligned} S &= \frac{T}{Ax} \\ T' &= Ax\,(T) \\ Ax' &= T\,(Ax) \end{aligned}$$

The fraction $\frac{T}{Ax}$ must be reduced, if reducible. T' expresses the total number of terms in the synchronized duration-group and Ax', the total number of terms in the synchronized axial group.

Let the number of permutations be P and P' for each respective group. Then *the final number of permutations (P'') equals the product of the permutations of both groups in synchronization.*

$$P'' = P \cdot P'$$

Example:

Binomial axial combination $= a + c$	Trinomial time-ratio: $3 + 2 + 1$
$Ax = 2$	$T = 3$

Synchronization:

$$\begin{aligned} S &= \tfrac{3}{2} \\ T' &= 2(3) \\ Ax' &= 3(2) \\ S &= 3a + 2c + a + 3c + 2a + c \end{aligned}$$

T' has 6 terms with three identical pairs, (two three's, two two's and two ones). The number of permutations (P) in such a group equals: factorial six (6!) divided by factorial two (2!), by factorial two (2!), by factorial two (2!).

$$P = \frac{6!}{2!\ 2!\ 2!} = \frac{720}{8} = 90.$$

Ax' has 6 terms with two groups of three identical elements (three a's and three c's). The number of permutations (P') in such a group equals: factorial six (6!) divided by factorial three (3!), by factorial three (3!).

$$P_1 = \frac{6!}{3!\ 3!} = \frac{720}{36} = 20$$

The total number of permutations (P'') equals P by P_1.

$$P'' = P \cdot P' \text{ or}$$
$$P'' = 90 \cdot 20 = 1800$$

Time ratios for the axial combinations must be selected according to rhythm families (*factorial continuity*). In classical music of the 18th century type, the family is $\frac{4}{4}$ series. The binomial ratios of this family are $3 + 1$ and $1 + 3$. Any axial combination selected will assume such ratio when a binomial variation of time (factorial periodicity) is required. For example, a trinomial axis, a, b, c, combined with one of the above binomials produces the following combinations:

$$a3T + bT + c3T + aT + b3T + cT$$

The trinomials of this series are:

$$2 + 1 + 1,\ 1 + 2 + 1,\ 1 + 1 + 2$$

With the same selection of axes it would give:

$$a2T + bT + cT$$

Each of these cases offers a corresponding number of variations.

Melody evolving in $\frac{8}{8}$ series will assume the factorial forms of the $\frac{8}{8}$ series. For example, in order to construct a trinomial axial combination for 8-bar continuity, we may choose a, d, b combination of the axes and the time-ratio of $3 \div 3 \div 2$. This will result in:

$$a3T + d3T + b2T$$

This method permits the construction of factorial continuity by means of the secondary axes with any desirable consistency of style. By conforming the selection of the time-ratios to one series of continuity, we achieve the utmost unity of style.

When a *hybrid* style is required, any of the non-corresponding series may be chosen. Such a case would be music evolved in $\frac{8}{8}$ series in its fractional continuity and in $\frac{4}{4}$ series in its factorial continuity.*

*Fractional and factorial continuity are discussed fully in Book I, Chapter 12. (Ed.)

G. Pitch Ratios of the Secondary Axes

As T expresses a time group unit in relation to t, which is the common denominator of the group, so does P express a pitch group unit (pitch range) in relation to p, which is the standard unit of pitch measurement in a given primary selective system.

Pitch ranges become important when they are treated as sections of the total range emphasis of a given musical continuity. In such a case each pitch range corresponds to a certain axis and the total value of the pitch units within one axis depends on the total value of all axes within the entire range. For example, if a melody evolves in a range of 15p ($c - e'\flat$) and three axes are required, each axis will emphasize $\frac{15p}{3} = 5p$, i.e., the partial ranges of the total range will be $P_1 = 5p$, $P_2 = 5p$, $P_3 = 5p$ ($c - f$; $f - b\flat$; $b\flat - e\flat$).

When the quotient has a remainder, the nearest integer must be taken. For example, if the entire range equals 12p and 2 pitch ranges are required, each of the 2P's equals $\frac{12}{2} = 6$. Diatonic scales not containing such intervals will offer the nearest points to 6. For example, in major or minor scales the nearest points produce 7p or 5p, thus offering 2 pitch ranges.

$$P_1 = 5\ (c - f)$$
$$P_2 = 7\ (f - c')$$

or

$$P_1 = 7\ (c - g)$$
$$P_2 = 5\ (g - c')$$

In symmetric systems of pitch, pitch ranges are between the adjacent tonics. Any scale of the third or the fourth group, whether used in its original or contracted form, produces a number of pitch ranges corresponding to the number of tonics. For example, a scale of the fourth group with 3 tonics in its original form offers 3 pitch ranges:

$$P_1 = 8\ (c - a\flat)$$
$$P_2 = 8\ (a\flat - e')$$
$$P_3 = 8\ (e_1 - c'')$$

As time is infinite and pitch is limited to a few thousand cycles per second, the general conception of coefficients pertaining to P must be handled with a certain amount of discrimination. The type of melody in its general range depends upon the pitch scale from which it is evolved. Thus, scales with one octave range naturally require P's which consist of a very limited number of p's, while scales belonging to the second or the fourth group* naturally lend themselves to widespread P's. The classical average on major and minor scale is about one-half of one octave, the usual binomials being 5 + 7 or 7 + 5. Modern composers have a tendency to write melodies with an enormous pitch range emphasis. In such cases P's may include as many as 11p.

In the following description of various axial combinations in relation to various pitch-ratios, we shall assume a uniform time-ratio.

*See Book II, Chapters 5 and 8. (Ed.)

1. Monomial Axial Combinations

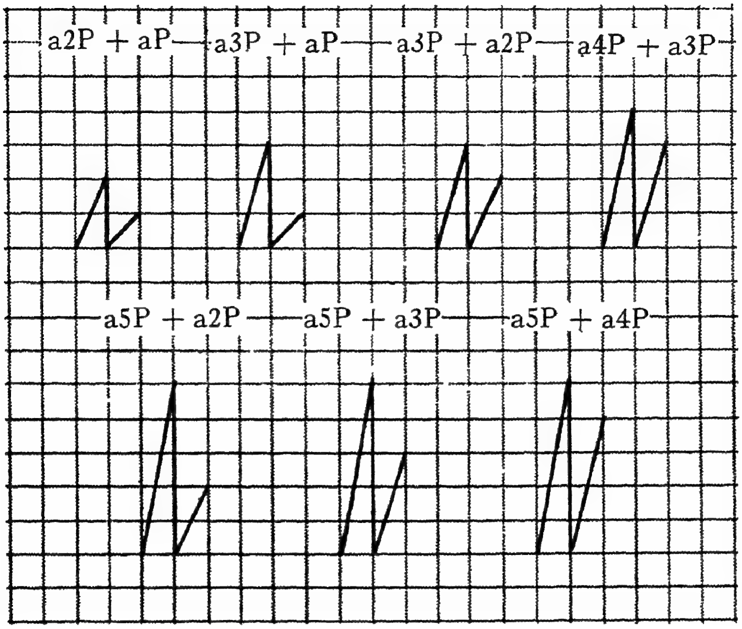


Figure 26. Binomial pitch-ratio.

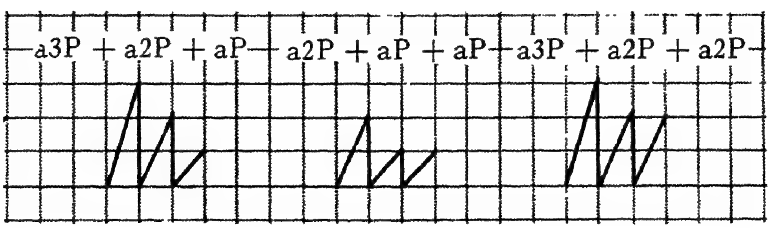


Figure 27. Trinomial pitch-ratio.

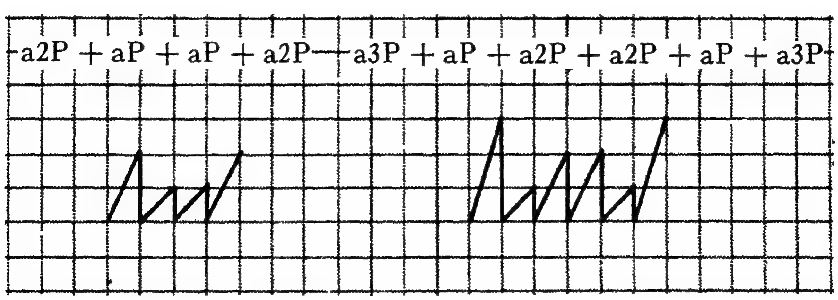


Figure 28. Polynomial pitch-ratio.

2. Binomial Axial Combination

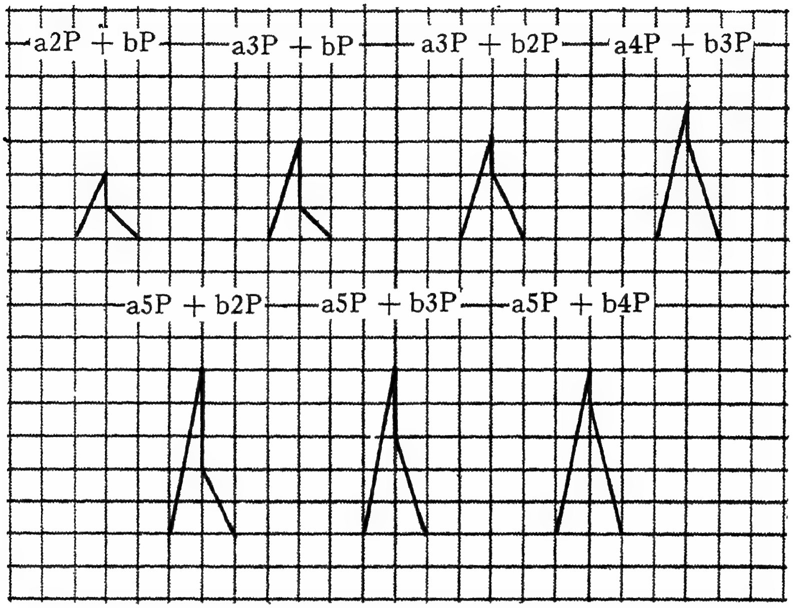


Figure 29. Binomial pitch-ratio.

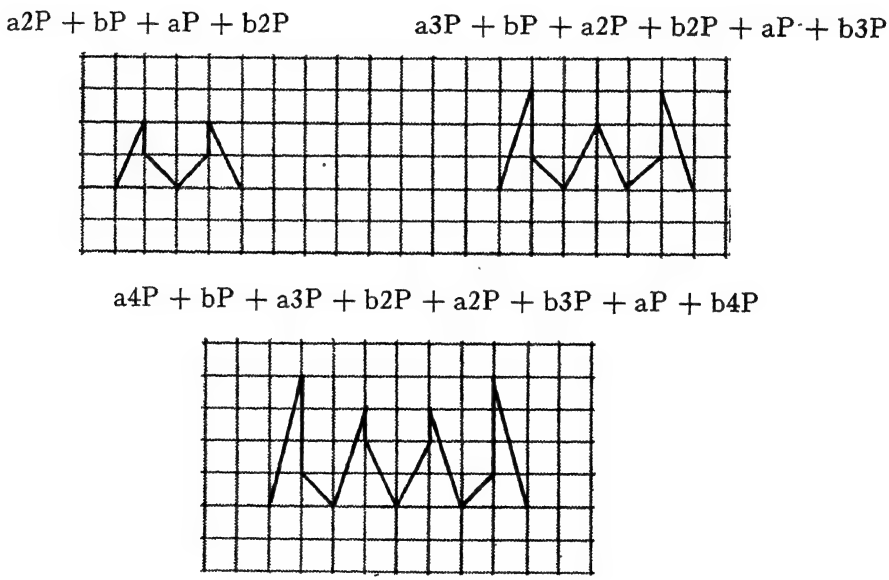


Figure 30. Polynomial pitch-ratio.

Axes: a, b   Pitch-ratio: $3 \div 2 \div 1$

$$a3P + b2P + aP + b3P + a2P + bP$$

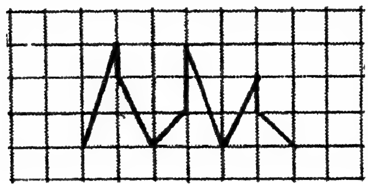


Figure 31. Interference pitch-ratio.

3. Trinomial Axial Combination

Axes: a, b, c   Pitch-ratio: $2 \div 1 \div 1$

$$a2P + bP + cP$$

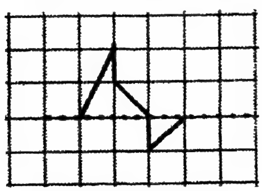


Figure 32. Trinomial pitch-ratio with 2 identical terms.

Axes: a, b, c   Pitch-ratio: $3 \div 3 \div 2$

$$a3P + b3P + c2P$$

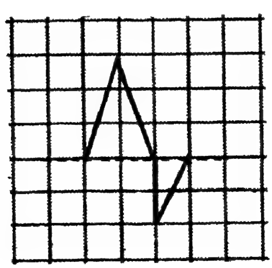
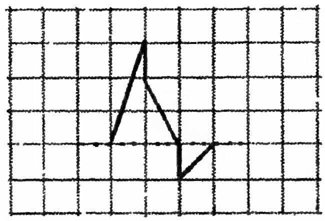


Figure 33. Trinomial pitch-ratio with 2 identical terms.

Axes: a, b, c Pitch-ratio: $3 \div 2 \div 1$

$$a3P + b2P + cP$$



Axes: a, b, c Pitch-ratio: $4 \div 1 \div 3$

$$a4P + bP + c3P$$

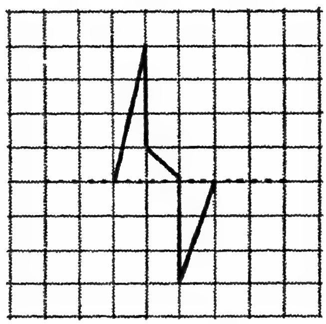


Figure 34. Trinomial pitch-ratio with 3 different terms.

Axes: a, b, c Pitch-ratio: $r_{4 \div 3}$

$$a3P + bP + c2P + a2P + bP + c3P$$

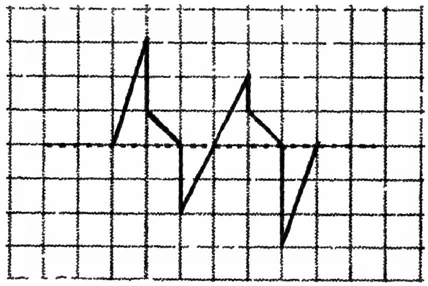


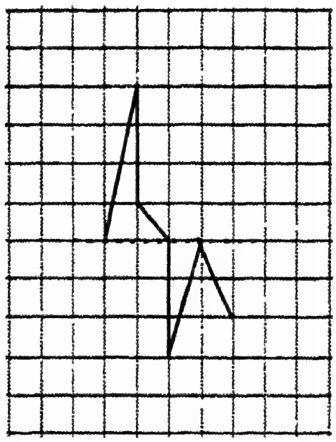
Figure 35. Polynomial pitch-ratio with the number of terms divisible by 3.

4. Polynominal Axial Combination

(a) Polynomial pitch-ratio with (1) the number of terms corresponding to the number of terms in the axial group, or (2) any multiple thereof.

Axes: a, b, c, d Pitch-ratio: $4 \div 1 \div 3 \div 2$

$$a4P + bP + c3P + d2P$$



Axes: a, b, c, d Pitch-ratio: $r_{5 \div 4}$

$$a4P + bP + c3P + d2P + a2P + b3P + cP + d4P$$

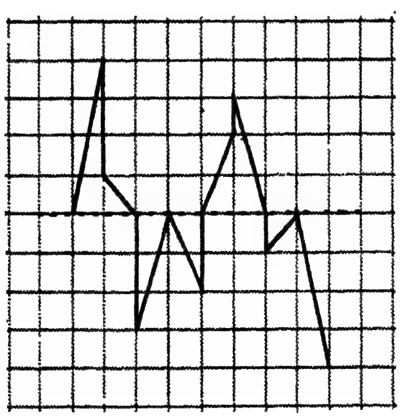


Figure 36. Polynomial pitch-ratio.

(b) Polynomial pitch-ratio with a number of terms which does not correspond to the number of terms in the axial group (interference pitch-ratio).

Axes: a, b Pitch-ratio: $3 \div 2 \div 1$

$$a3P + b2P + aP + b3P + a2P + bP$$

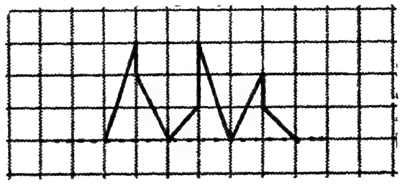


Figure 37. Polynomial pitch-ratio.

The number of variations of each axial combination with a selected pitch-ratio depends on the number of terms in the axial group and the number of terms in the pitch-ratio.

A *monomial axial combination* with a binomial pitch-ratio produces 2 variations:

a2P + aP   Var. aP + a2P

A monomial axial combination with trinomial pitch-ratio having 2 identical terms produces 3 variations:

a2P + aP + aP
aP + a2P + aP
aP + aP + a2P

A monomial axial combination with trinomial pitch-ratio having all 3 terms different:

a3P + a2P + aP
a3P + aP + a2P
aP + a3P + a2P
a2P + a3P + aP
a2P + aP + a3P
aP + a2P + a3P

A monomial axial combination with polynomial pitch-ratio produces a number of variations equivalent to the number of permutations of terms in the pitch-ratio:

a3P + aP + a2P + a2P + aP + a3P
6 elements, 90 permutations.

A *binomial axial combination* with binomial pitch-ratio. The number of variations equals $2^2 = 4$:

a2P + bP
aP + b2P
bP + a2P
b2P + aP

A binomial axial combination with polynomial pitch-ratio produces a number of variations equivalent to the product of the number of permutations in the axial group by the number of permutations in the pitch-ratio:

$$a2P + bP + aP + b2P$$

Four terms with 2 identical pairs produce 6 permutations. In this case the axial group and the pitch-ratio have identical structure.

The number of variations: $6^2 = 36$.

Computation of the total number of permutations for the synchronized axial combinations (axial groups) and the synchronized pitch-ratios (pitch-range groups) follows the same formulae as in the case of axial groups synchronized with duration groups.

Pitch ratios for the axial combinations must be selected according to the total range to be emphasized and the type of pitch-scale to be used.

H. Correlation of Time and Pitch Ratios of the Secondary Axes

The correspondence of the number values expressing pitch and time-ratios of the secondary axes is entirely immaterial for reasons mentioned above, i.e., the different nature of the limitations pertaining to time and pitch in our sensory continuum.

But the form of relations between time and pitch that is essential refers to different forms of correspondences between the two, with only a certain amount of influence on the actual ratios—and, in most cases, with no influence at all on the actual number values. The forms of correspondence are:

(1) Parallel (direct)
(2) Contrary (inverse)
(3) Oblique (indirect)

There are two forms of oblique correspondences:

(a) circumstantial
(b) intentional

(1) Parallel Correspondences

a2T2P + bTP aTP + b2T2P

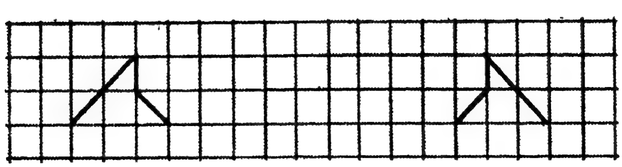


Figure 38. Parallel Correspondences (continued).

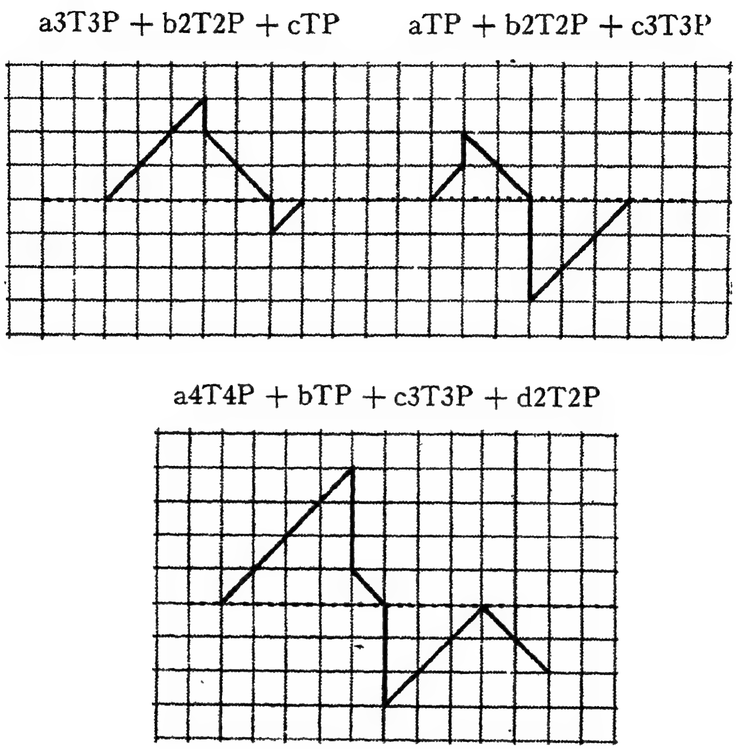


Figure 38. Parallel correspondences. (concluded).

(2) Contrary Correspondences.

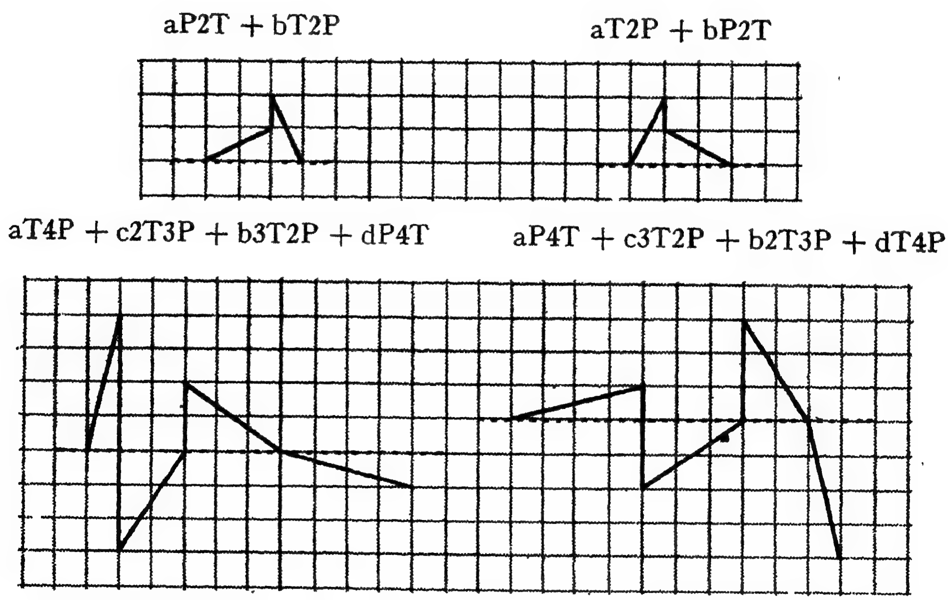
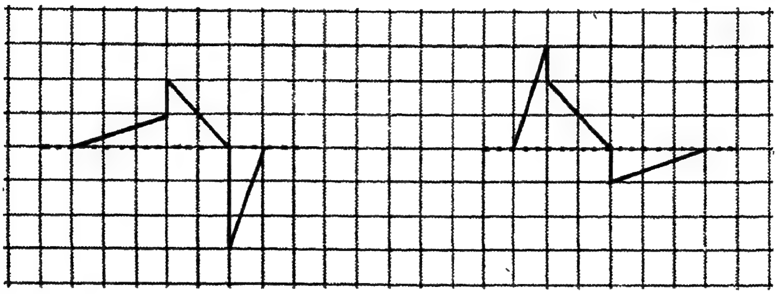


Figure 39. Contrary correspondences.

(3) Oblique Correspondences.

(a) Circumstantial: when the axial combination has an uneven number of terms (this produces a coincidence of both coefficients on the middle term).

aP3T + b2T2P + cT3P   aT3P + b2T2P + cP3T



aP5T + b4T2P + c3T3P + b2T4P + aT5P

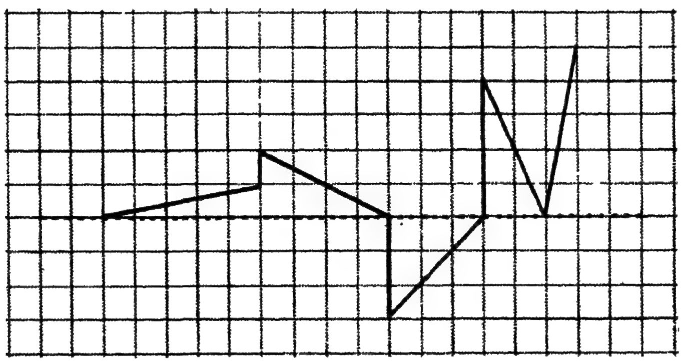


Figure 40. Oblique correspondences. Circumstantial.

(b) Intentional: when partial coincidence is desired regardless of the number of terms.

Axes:	a	b	c	d
T	4	1	3	2
P	2	3	1	2

a4T2P + bT3P + cP3T + d2T2P

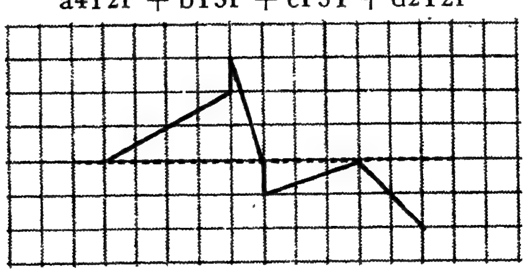


Figure 41. Oblique correspondences. Intentional.

The general effect of parallel correspondences is one which is expected; it may be associated with stability and common sense. The path of melody through time and pitch appears under the conventional mechanical conditions, i.e., the greater the pitch range to be covered, the greater the time required. The smaller the pitch range to be covered, the less the time required.

The contrary correlation produces an effect of tension or surprise. It has an attractive and often dramatic quality. Greater pitch-ranges are achieved in shorter time (greater velocity) and smaller pitch-ranges are covered in a longer period of time (smaller velocity, resistance, delays).

The oblique correlation produces intermediate effects offering more of the surprise element when the coefficients are different, and bringing it back to more conventional effects when there is such a coincidence.

All the problems of the actual relationship of patterns through the character of reaction, resulting as a response to such patterns, are discussed in the following chapters.

CHAPTER 4

MELODY: CLIMAX AND RESISTANCE

THE PROJECTION of melody is a mechanical trajectory. Its kinetic components are balance, impetus and inertia. Resistance produces impetus, leading either towards the *climax*, which is a *pt* (pitch-time) *maximum with respect to the primary axis*, or towards balance. The impetus is caused by resistance which results from rotation. The geometrical projection of rotation is a circle which extends itself in time projection into a cylindrical or spherical spiral, or ultimately (through time extension) into wave motion (plane projection).

The kinetic result of rotary motion is *centrifugal* energy. The discharge of accumulated centrifugal energy is equivalent to a *climax.* A heavy object attached to a string and put into rotary motion about an axis-point develops considerable energy—enough to move it a long distance when detached from the string.

Overcoming inertia increases mechanical efficiency (gain of kinetic energy). Any body set in motion acquires its ultimate possible speed in a certain period of time. The shorter the period from the moment of the application of the initial force (impetus) till the moment when the body acquires its ultimate speed, the greater is the mechanical efficiency of such motion.

Motion is expressible in wave amplitudes; the projection of kinetic climax is the maximum amplitude. Inert matter does not acquire its maximum amplitude instantaneously when starting from balance just as the maximum cannot recede to balance (rest) instantaneously. This is true both of velocities (frequencies) and amplitudes.

Mechanical experiences, whether instinctive or intentional, are known to all types of zoological species and are inherited and perfected in the course of evolution. A grown animal has a perfect judgment of distances, of directions, and of the amount of muscular energy necessary in leaps or flights, without any theoretical knowledge of the law of gravity or mechanics in general. There is no misjudgment in the monkey's flights from tree to tree; there is none for a gazelle leaping over a creek, or for an eagle falling on its prey. A certain amount of intentional mechanical efficiency and psycho-physiologic coordination is inherent with every surviving species of the animal world. The relativity of the standards of mechanical efficiency corresponds to the relativity of reflexes, reactions and judgments.

The leap of a human being over a 14 foot rod was the highest achievement in the International Olympics for 1936, and this with the aid of a pole. The mechanical efficiency of an ordinary flea is fifty times greater. The leap of a human being over a rod 50 feet high would seem supernatural, while the same kind of leap by a flea would be far below the standards of flea efficiency—the flea leaps about one hundred times its own size.

Standards of mechanical efficiency vary with ages and places, even among human beings. They also vary with different races as well as with different ages. The development of athletic qualities and forms of locomotion implies the raising of the requirements necessary for mechanical efficiency.

The geometrical conception of mechanical and bio-mechanical trajectories necessitates the analysis of the corresponding trajectories of nervous impulses and muscular reactions. There are correspondences between the two, and the knowledge of such correspondences leads to scientific production of excitors capable of stimulating the intended reactions (in this case, esthetic excitors: music in general, or melody in particular). Simple reflexes and reactions project themselves into simple trajectorial patterns; on the other hand, excitors having the form of simple trajectories stimulate reactions of a corresponding simplicity. Likewise, this correspondence occurs with complex patterns.

The intensity-interdependence between the excitor and the reaction was formulated in Weber's and Fechner's psycho-physiological law. Both as to configurations (patterns) and as to amplitudes (intensities), there are correspondences between the excitors and the reactions. Judgment based on mechanical experience and mechanical orientation leads higher animals and human beings to certain expectations. In the case of an absolute correspondence between the realization of a mechanical process and the expectation, the resulting reaction is balance (normal satisfaction). A result above expectation stimulates the intensification of activity (positive reaction) and at its extreme, ecstasy. On the other hand, the result of a mechanical process which is below expectation stimulates passivity (negative reaction) and at its extreme, depression. The two opposite poles of reactions, brought to their absolute limit, stimulate astonishment (irrational or zero reaction).

Geometrical projection of a scale of psychological adjectives on a circumference produces the poles of the two rectangular coordinates (the diameters of the circle): 1. normal—absurd; 2. depressing—ecstatic. Producing four new poles on the intermediate arcs of the circumference through the addition of another pair of rectangular coordinates (under 45° to the original pair) we obtain nine poles altogether (including both 0° and 360°). These nine poles, through the application of the method of evolving concept series, become expressible in adjectives standing for the psychological categories.

Scale of psychological categories as represented through geometrical projection on a circumference. (See Figure 41A on next page).

The circumference is divided, by the poles of the coordinates, into 8 arcs, 45° each. The geometrical poles correspond to the psychological poles. Arcs represent the transition zones, and the poles—their absolute expression.

Zero or 360° — abnormal	180° — normal
90° — infranormal	270° — ultranormal
135° — subnormal	315° — supernatural
45° — subnatural	225° — supernormal

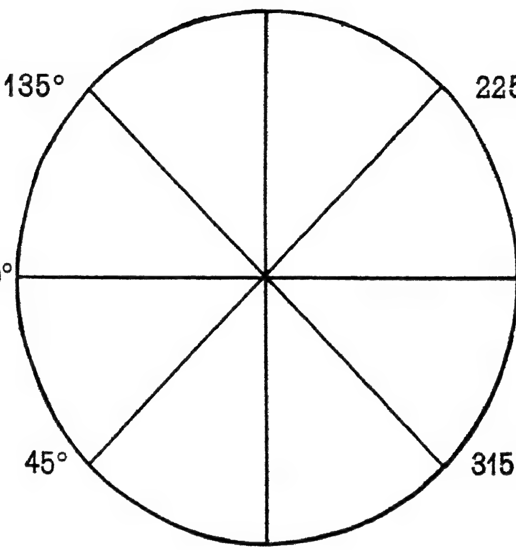


Figure 41A. Scale of psychological categories.

The zone around 0° or 360° stimulates astonishment (zero reaction or delayed reaction). The zone around 45° stimulates either pity or humor. The zone around 90° stimulates depression (pessimism). The zone around 135° stimulates the sense of lyricism (regret, melancholy, pleasant sadness, joyful sadness, controllable, self-imposed sadness; close to positive zone: joy of self-destruction, self-sacrifice.) The zone around 180° stimulates the sense of quiet contemplation, full psychological balance and satisfaction. The zone around 225° stimulates the sense of heroism and admiration. The zone around 270° stimulates the sense of exaltation, ecstasy and worshipping. The zone around 315° stimulates either the sense of the fantastic or the sense of fear (unfavorable surroundings, uncontrollable, unaccountable forces, fear for existence, struggle for survival).

A discus thrower participating in the Olympics and reaching the previous year's record would stimulate a reaction corresponding to 180° point. The actual reflexes of the spectators would be polite applause. Throwing beyond the expected range would stimulate a reaction corresponding to the zone between 180° and 225°, culminating in ultimate ecstasy when it reached 270°—and this would be evidenced in the audience by shouting, stamping and whistling, the reactions increasing not only in intensity, but in quantity as well—i.e., the maximum conceivable limit. The clapping reflexes would grow accordingly from 180° to 270°. If the disc does not reach the range expected, the reaction would be disappointment, increasing toward 135° with the sympathetic spectators.

With the range reaching only 90°, it would lead ultimately toward depression. The spectators will not applaud when the effect of the disc throwing is near the 90° point. It is natural to assume that certain groups of spectators, influenced by their sympathy for opponents of the first discus thrower, would display exactly opposite reactions. These considerations cover the semicircle above the horizon.

The lower zone, on the negative side, i.e., between the 0° and 90°, stimulates the reaction of laughter. In the case of the discus thrower, it would amount to a range of perhaps only a few yards from his position after a long and arduous preparation for the throw. When the spectators see a husky, muscular athlete deprived of mechanical efficiency, they unquestionably react to it as if the episode seemed decidedly humorous.

On the positive side of the lower semicircle, between 315° and 360°, lies the zone of the supernatural, where the range of throw of a disc would be beyond any biomechanical possibility. For example, if the range of throw amounted to three miles. In such cases the presence of a trick or a supernatural force would be a necessary ingredient for the logical comprehension of the phenomenon. The usual reaction would be that of a smile or laughter moving toward astonishment in the direction of the zero point.

The 360° point when reached from the positive side would amount to the absurd caused by an impossible mechanical over-efficiency. Such would be the case when the disc being thrown would never come back, would never fall anywhere on the ground, but vanish in interstellar space, thus overcoming the law of gravity.

When zero is reached from the negative side, it would mean an impossible mechanical inefficiency. In the case of a disc thrower, it would happen if the disc were to slip out of the athlete's hands before he actually threw it.

A trajectory expressing a mechanically efficient kinetic process, whether that of a pendulum or a musical melody, will have mechanical fundamentals in common. *A pendulum cannot start instantaneously at its maximum amplitude; neither can a melody.* A pendulum cannot stop instantaneously from its maximum amplitude; neither can a melody. The corresponding effects in both cases will be either *supernatural* or *humorous.*

The actual quantitative specifications serving different purposes and expressing different forms of mechanical efficiency vary with times and places. To satisfy any esthetic requirement, one has to know the style in which such requirements have to be carried out—also beyond what specifications the entire kinetic process, whether efficient or not, will become meaningless. As standards vary, the coordinates on the circle described above change their absolute positions, i.e., the zero point may move with the entire system, either clockwise or counter-clockwise. If we would assume, with regard to athletic standards, 180° to be a limit of certain mechanical operations—when the achievement of the succeeding epoch increases the quantitative value of normal, placing the point of normality on what is 225° on our diagram, the opposite pole of the coordinate will occupy respectively the 45° position.

Referring to music in general and melody in particular, we find that certain standards become old-fashioned and we begin to feel that although they may be charming yet they are entirely inadequate for the purposes of a more mechanically efficient epoch. We feel it in every field concerned with motion,* i.e., mechanics.

One has a humorous or a pitying reaction toward the 1900 "horseless carriage"—and it becomes still more humorous when there is an accumulation of quantities of the symbols of inadequacy, such as the prerequisites of travel required by a horseless carriage: dusters, goggles, safety belts. We have exactly the same picture (i.e., if we are people representing our epoch rather than living anachronisms), in melodies composed by a Verdi or a Bellini; the mechanical efficiency is so low that it makes us smile, if not laugh. The same melodies stimulate entirely different reactions among octogenarians surviving in our epoch of 400 miles per hour.

In order to achieve an efficient climax, it is necessary to accumulate energy that will be effectively discharged in such a climax. The means for accumulating energy, as was described above, are achieved through rotary motion developing centrifugal energy. Trajectories expressing musical pitches of various frequencies are heard by listeners in relation to the entire trajectory. It is possible not only to show the range of frequencies (such as a form of direct transition from one frequency to another), but also to show in what way this variation of frequency was achieved.

The portion of a melodic trajectory leading toward the climax, without resistance preceding such a climax, does not produce any dramatic effect. It is resistance that makes the climax appear dramatic. A portion of melodic trajectory leading from a climax (maximum amplitude) towards balance (minimum amplitude) must be performed in accordance with natural mechanical laws, i.e., it must contain resistance before it reaches the balance (compare with pendulum). Inefficiency, or excess of the forms of resistance (rotary motion), leads to a mechanical abnormality. Abnormal melody stimulates the sense of dissatisfaction or humor. The forms of resistance leading toward climax acquire centrifugal form (increasing amplitude). The forms of resistance leading toward balance acquire centripetal form (decreasing amplitude). The relative period of rotary motion and amplitudes produces various forms and gradations of resistances. For example, the period of rotation may be long, with the amplitude remaining constant; or the period of rotation may be short with rapidly increasing amplitudes. The period of rotation may be short with correspondingly increasing amplitude. The duration of the rotary period may be in inverse proportion to the amplitude—and often the law of squares takes its place.

*The practical value of Schillinger's work in correlating music and motion—sound and the mathematical laws of motion—appears in each section of his System. In *Theory of Melody* this correlation takes a most interesting form, and yields insights of inestimable value to the composer. The effort to relate psychological categories and music is almost as old as music itself. But most such efforts have been impressionistic, critical and intuitive—and therefore so subjective as to be useless both to the composer and critic. Schillinger's procedure in projecting a melody on a graph and correlating this melodic trajectory with the scale of psychological categories offers a scientific and objective approach to the problem. With intelligent application composers now can unerringly evolve a melody to produce a given psychological effect. Music critics likewise have an instrument—which does not depend on how they feel at a given concert—for judging the success of the composer's effort. (Ed.)

A. Forms of Resistance Applied to Melodic Trajectories

The corresponding forms of resistance as applied to melodic trajectories are:

1. **Repetition** (correspondences: aiming, rotary motion with infinitesimal amplitudes, affirmation of the axis level as a starting point). Musical form: repeated attacks of the same pitch discontinued by rests or following each other continuously.

Physical Form *Musical Form*

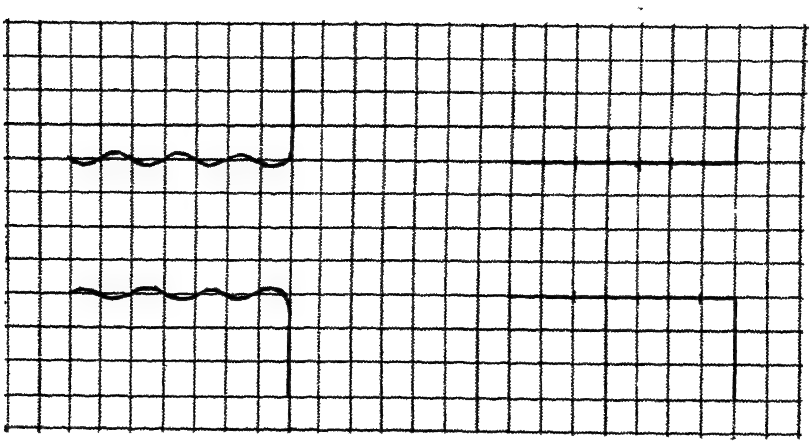


Figure 42. Repetition as form of resistance.

2. **One phase rotation** (correspondences: preliminary contrary motion, initial impulse in archery, artillery, springboard diving, baseball pitching, tennis service, etc.) Musical form: a movement or a group of movements in the direction opposite to the succeeding leap.

Physical Form

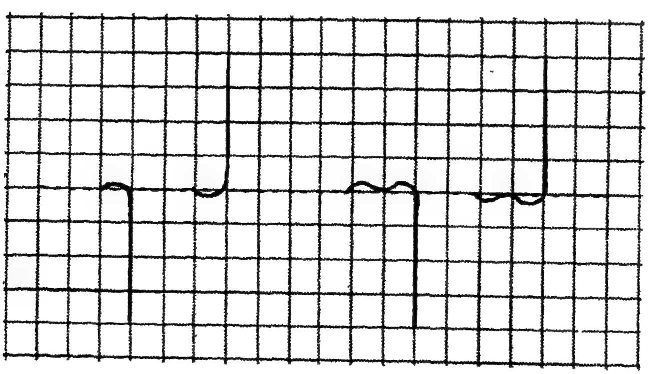


Figure 43. One phase rotation (continued).

Musical Form

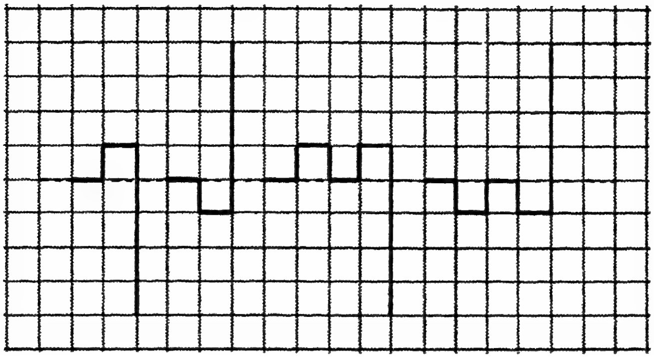
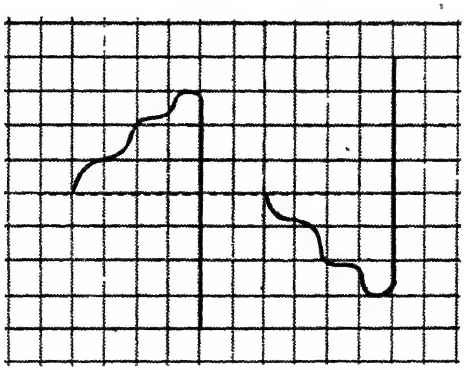


Figure 43. One phase rotation (concluded).

This form often acquires more than one phase following in one direction which intensifies the resistance.

Physical Form



Musical Form

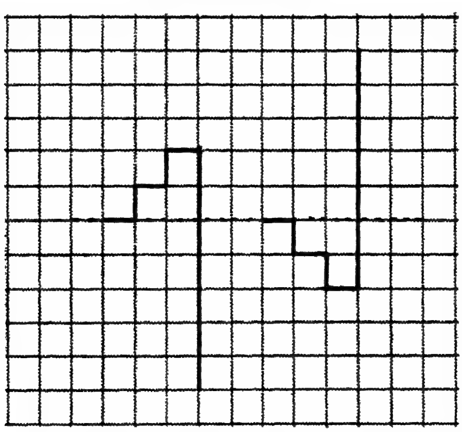


Figure 44. More than one phase rotation.

3. **Full periodic rotation** (one or more periods).

a. Constant amplitude (correspondences: rotation around a stationary point, a top, somersaults—with diving and without—lasso, axis and orbit rotation of the planets, Dervish dances).

Musical Form: mordents, trill, tied tremolo, gruppetto.

Physical Form *Musical Form*

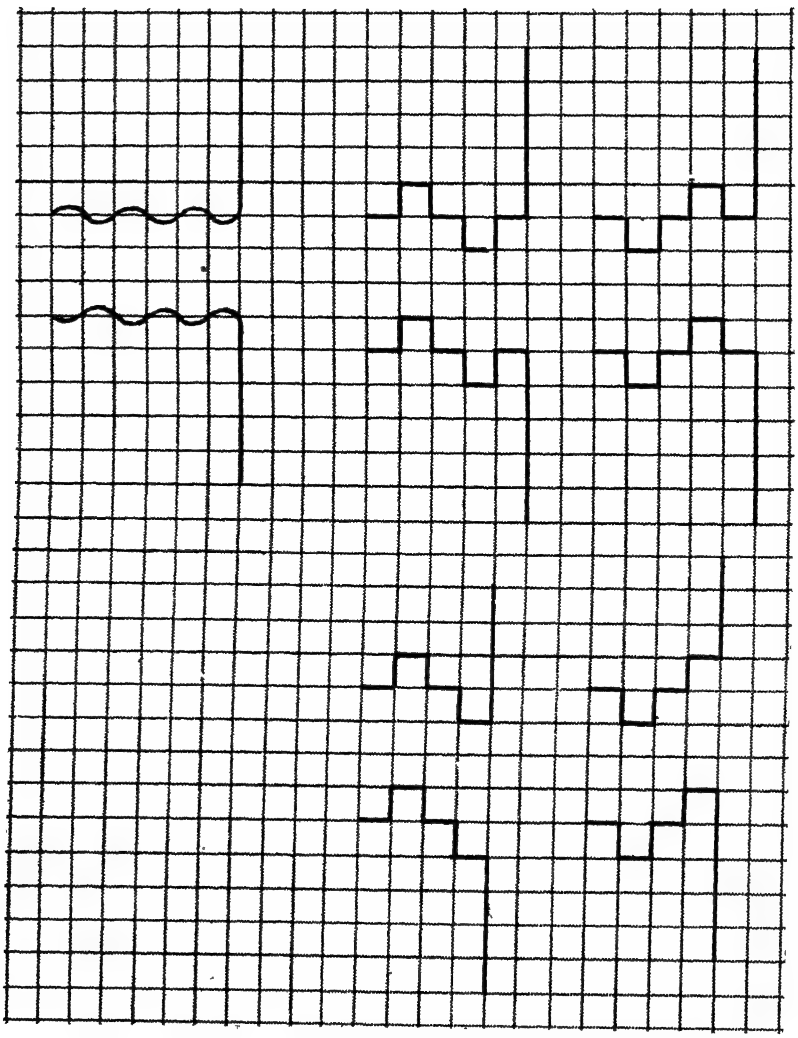


Figure 45. Full periodic rotation: constant amplitude.

Variable amplitude (correspondences: gyroscope, spiral motion, tornado, expansion, contraction). Musical form: expanding and contracting, simple and compound motion.

Whereas the preceding forms of resistance require only one of the secondary axes, the variable amplitude rotation requires a simultaneous combination of two or three secondary axes. In this case the axis leading towards climax or balance will be considered *fundamental* and the other axes—*complementary.*

Simultaneous combinations of two axes:

(a) Centrifugal (expanding):

$\frac{a}{o}$; $\frac{o}{a}$; $\frac{d}{o}$; $\frac{o}{d}$;

Physical Form *Musical Form*

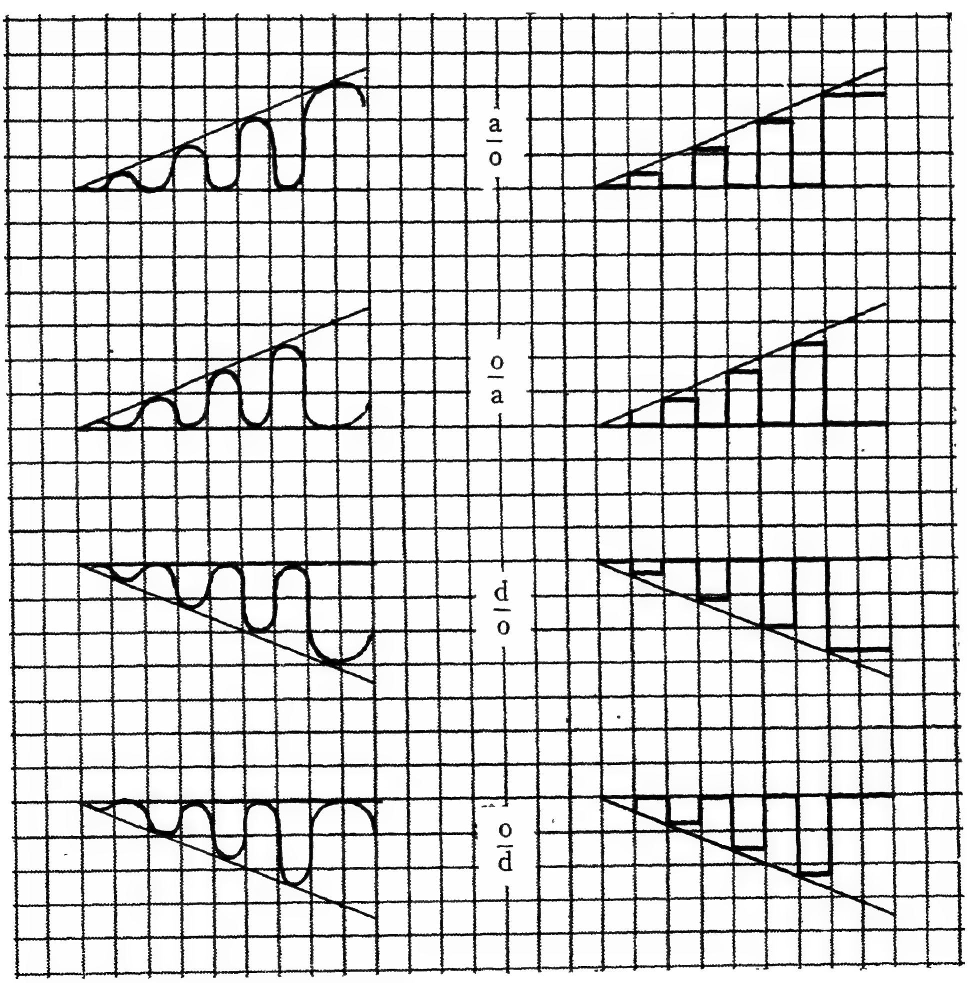


Figure 46. Centrifugal combination of two axes.

(b) Centripetal (contracting)

$\frac{b}{o}$; $\frac{o}{b}$; $\frac{c}{o}$; $\frac{o}{c}$;

Physical Form *Musical Form*

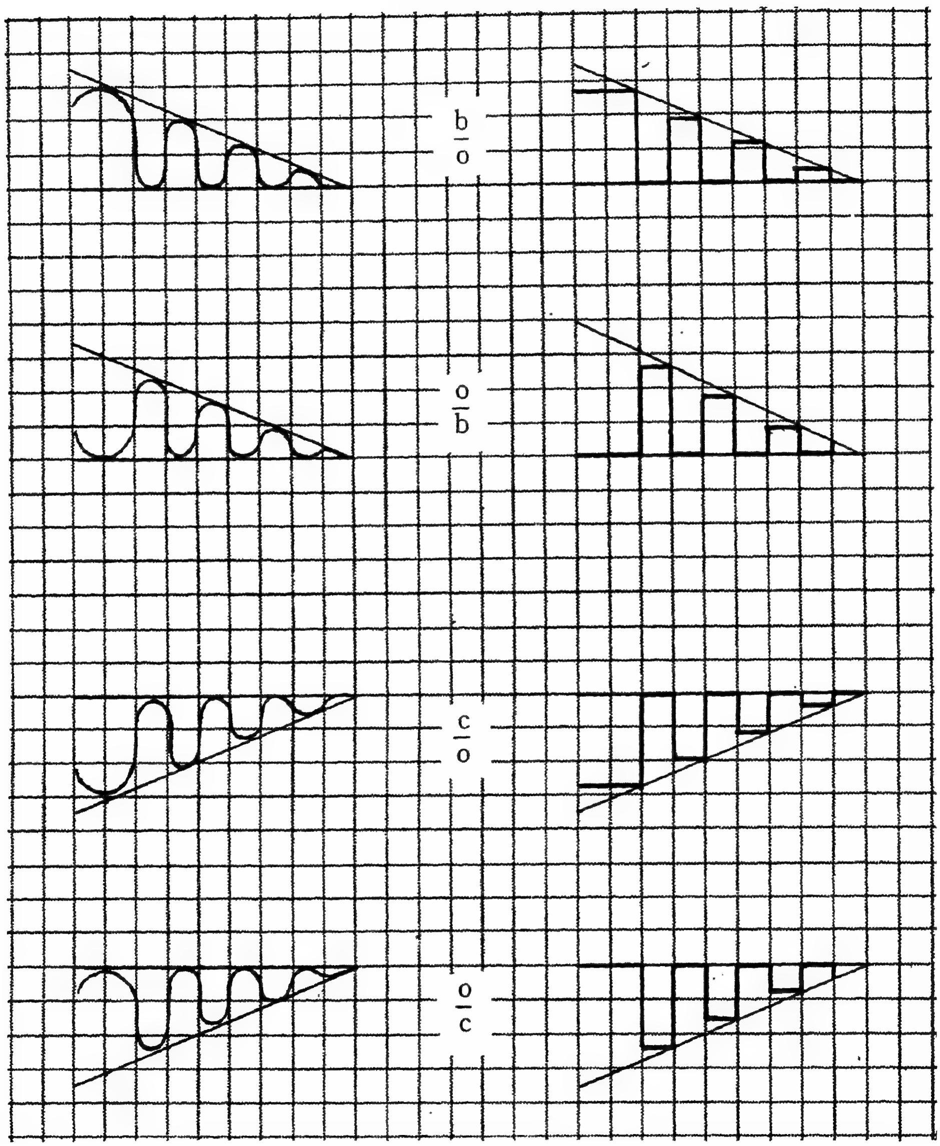


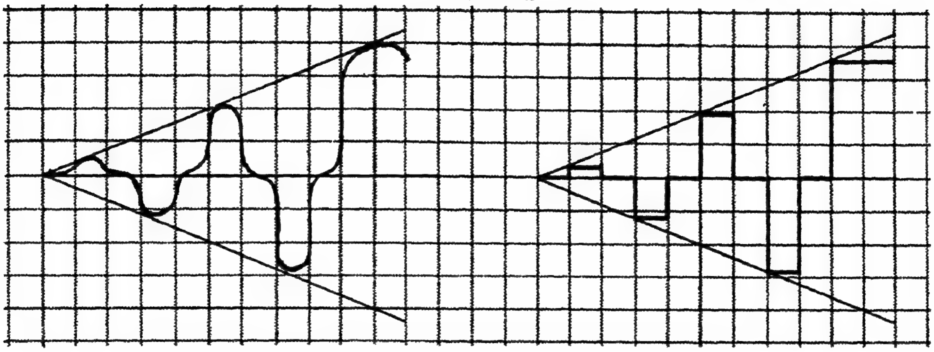
Figure 47. Centripetal combination of two axes.

Simultaneous combinations of three axes:

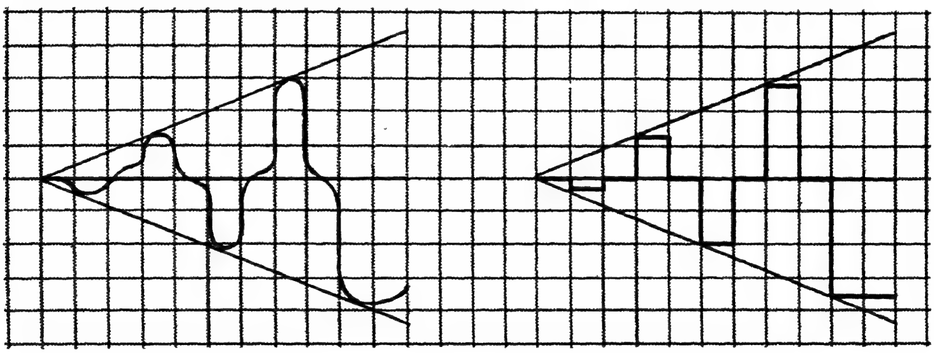
(a) Centrifugal (expanding):
a ÷ 0 ÷ d; d ÷ 0 ÷ a
(b) Centripetal (contracting):
b ÷ 0 ÷ c; c ÷ 0 ÷ b

Physical Form *Musical Form*

a ÷ 0 ÷ d



d ÷ 0 ÷ a



b ÷ 0 ÷ c

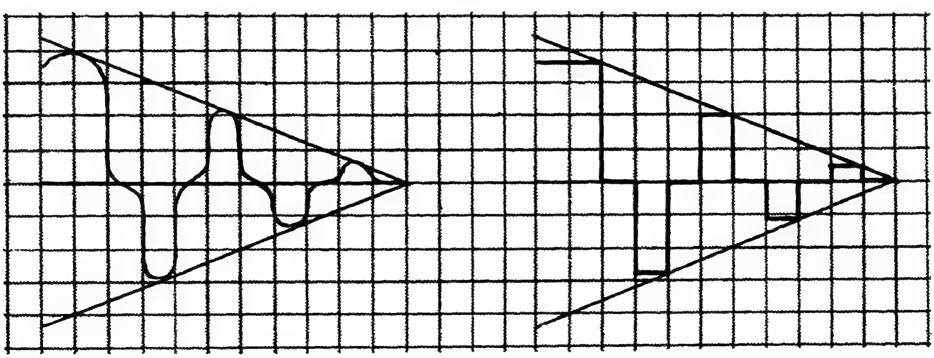


Figure 48. Simultaneous combination of three axes (continued).

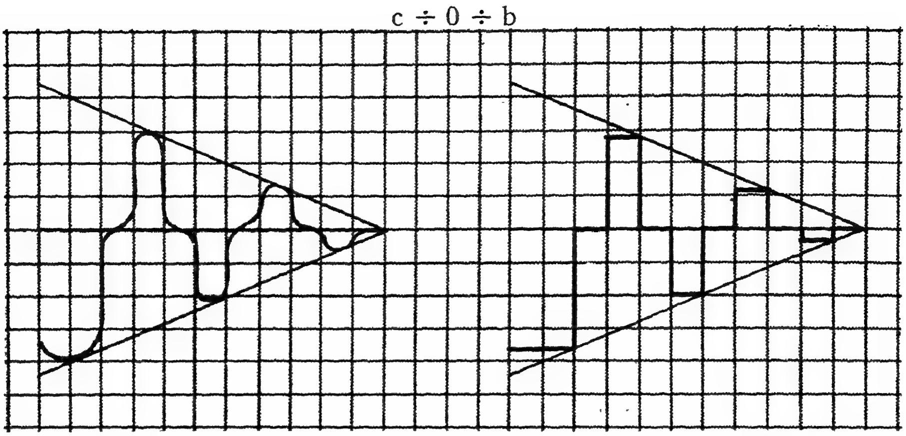


Figure 48. Simultaneous combination of three axes (concluded).

As the interval of a pitch level from the primary axis affects tension (gravity effect where P.A. is a gravitational field), resistance may also result from two parallel secondary axes. The complementary parallel axis may be placed either above or below the fundamental axis. The effect of motion through a pair of parallel axes is that of an extended trajectory (delayed, forced inefficiency). In reality it is the usual rotary motion only evolving between the two axis-boundaries.

The correspondences of such motion are: rising and falling, zigzag ascending and descending. Musical form: revolving around alternately progressing points (ascending or descending).

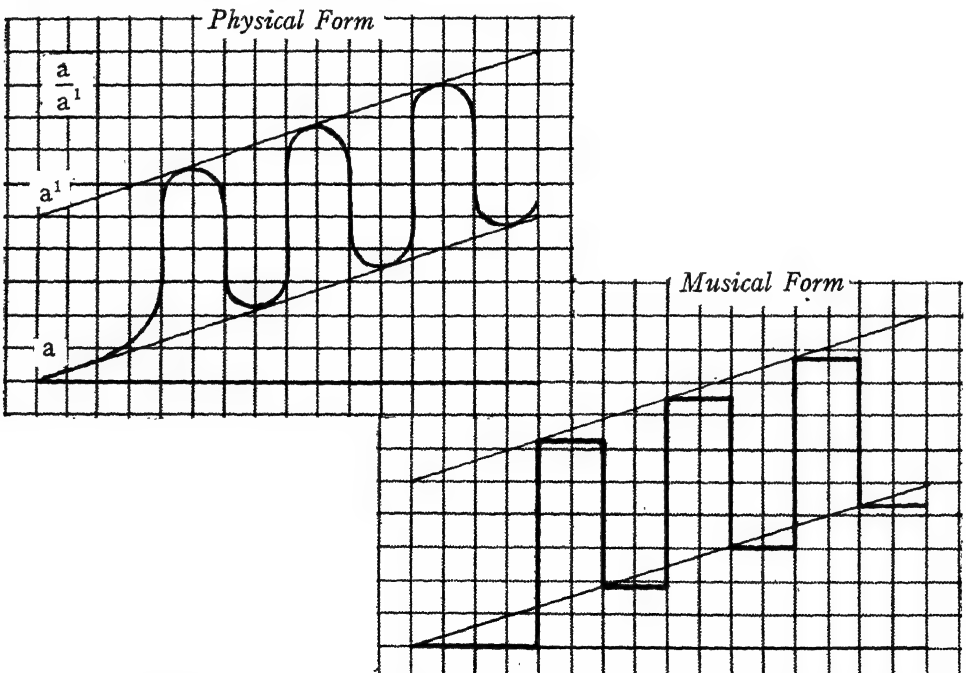


Figure 49. Two parallel secondary axes $\frac{a}{a_1}$ *(continued).*

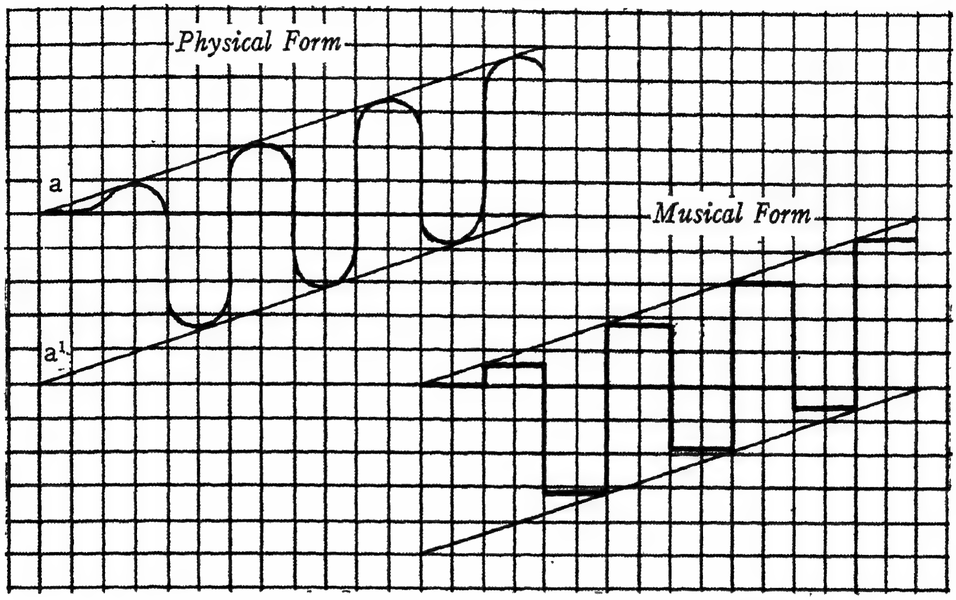


Figure 49. Two parallel secondary axes $\frac{a}{a^1}$ (concluded).

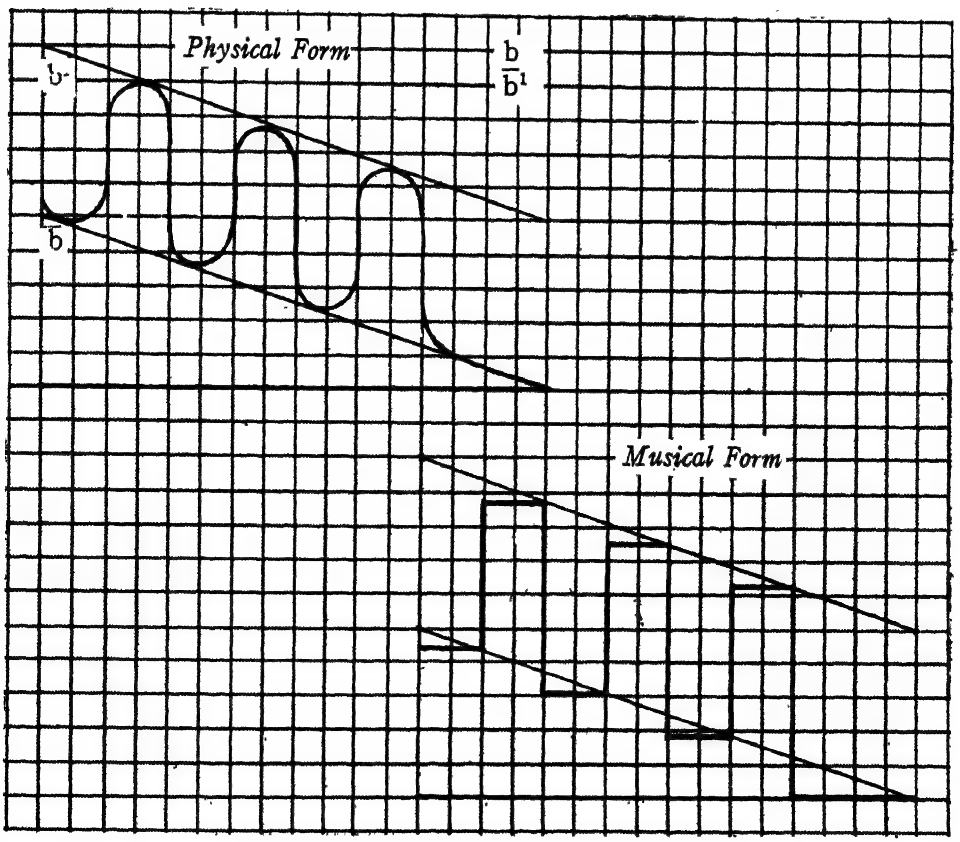
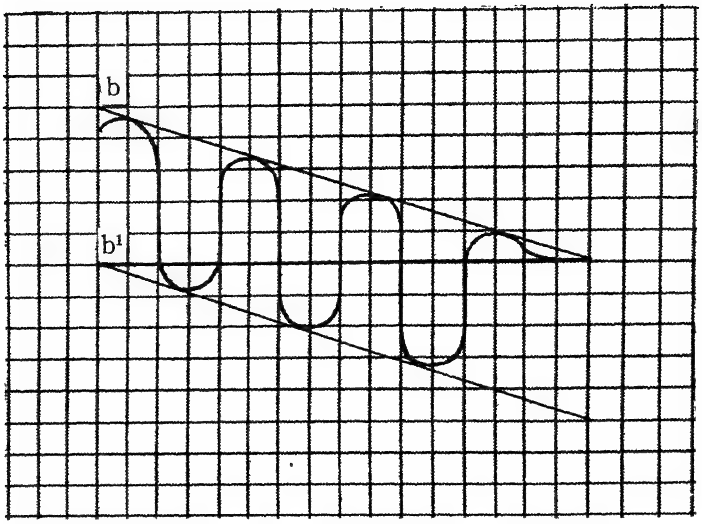


Figure 50. Two parallel secondary axes $\frac{b}{b^1}$ (continued).

Physical Form



Musical Form

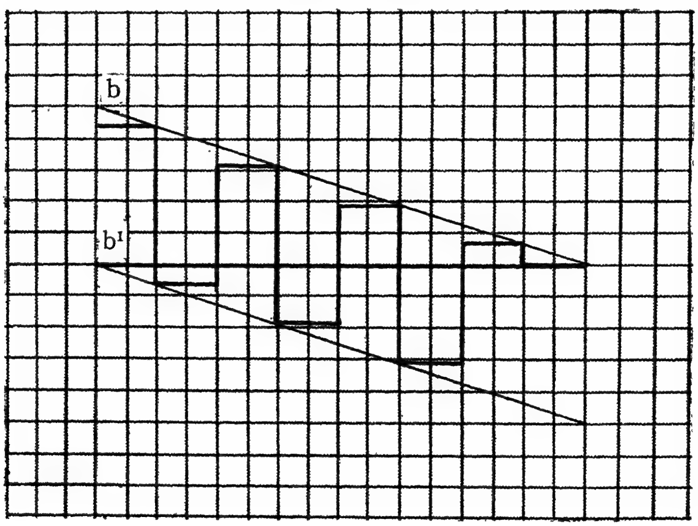


Figure 50. Two parallel secondary axes $\frac{b}{b^1}$ *(concluded).*

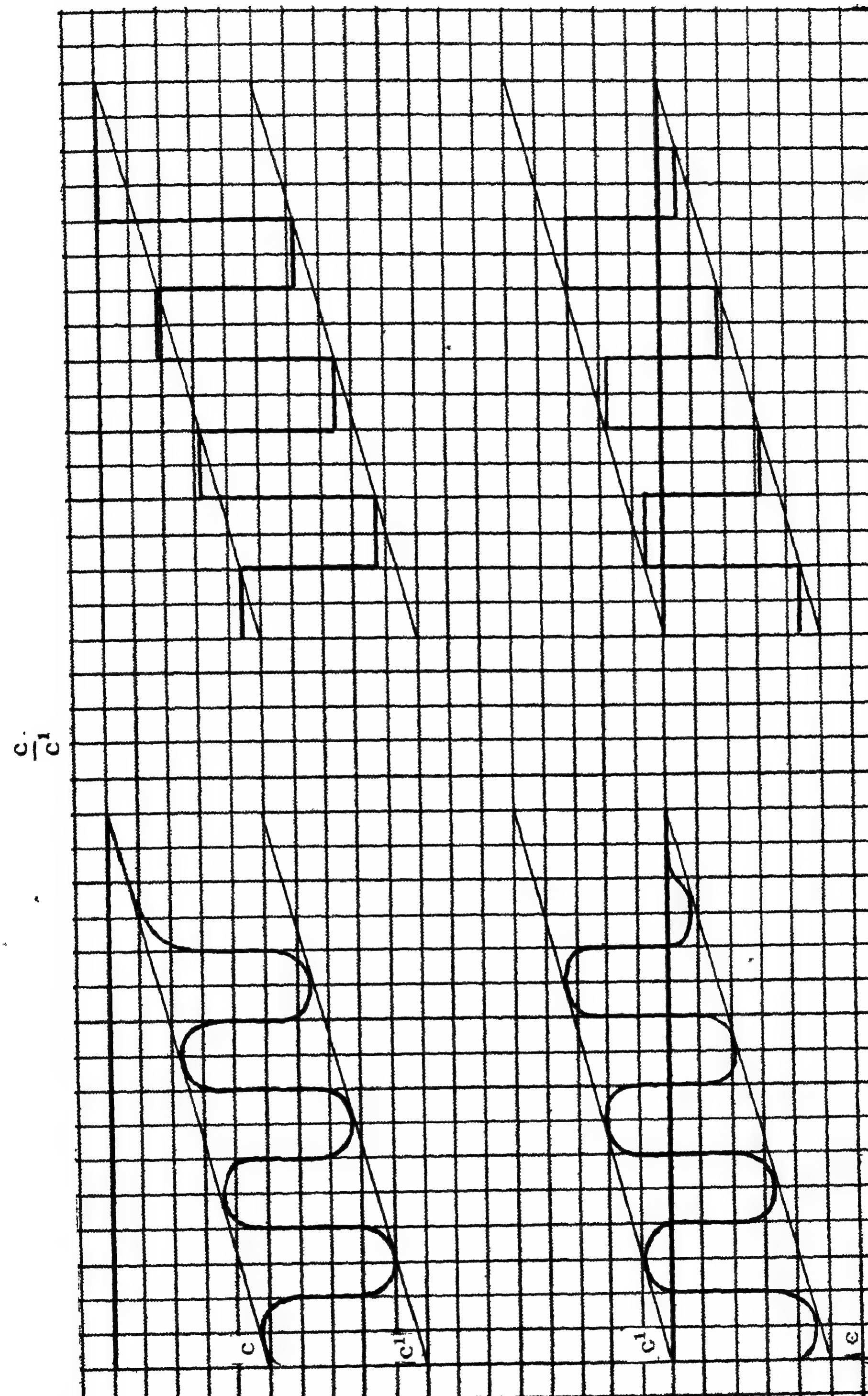


Figure 51. Two parallel secondary axes $\frac{c}{c^1}$.

$\frac{d}{d^1}$

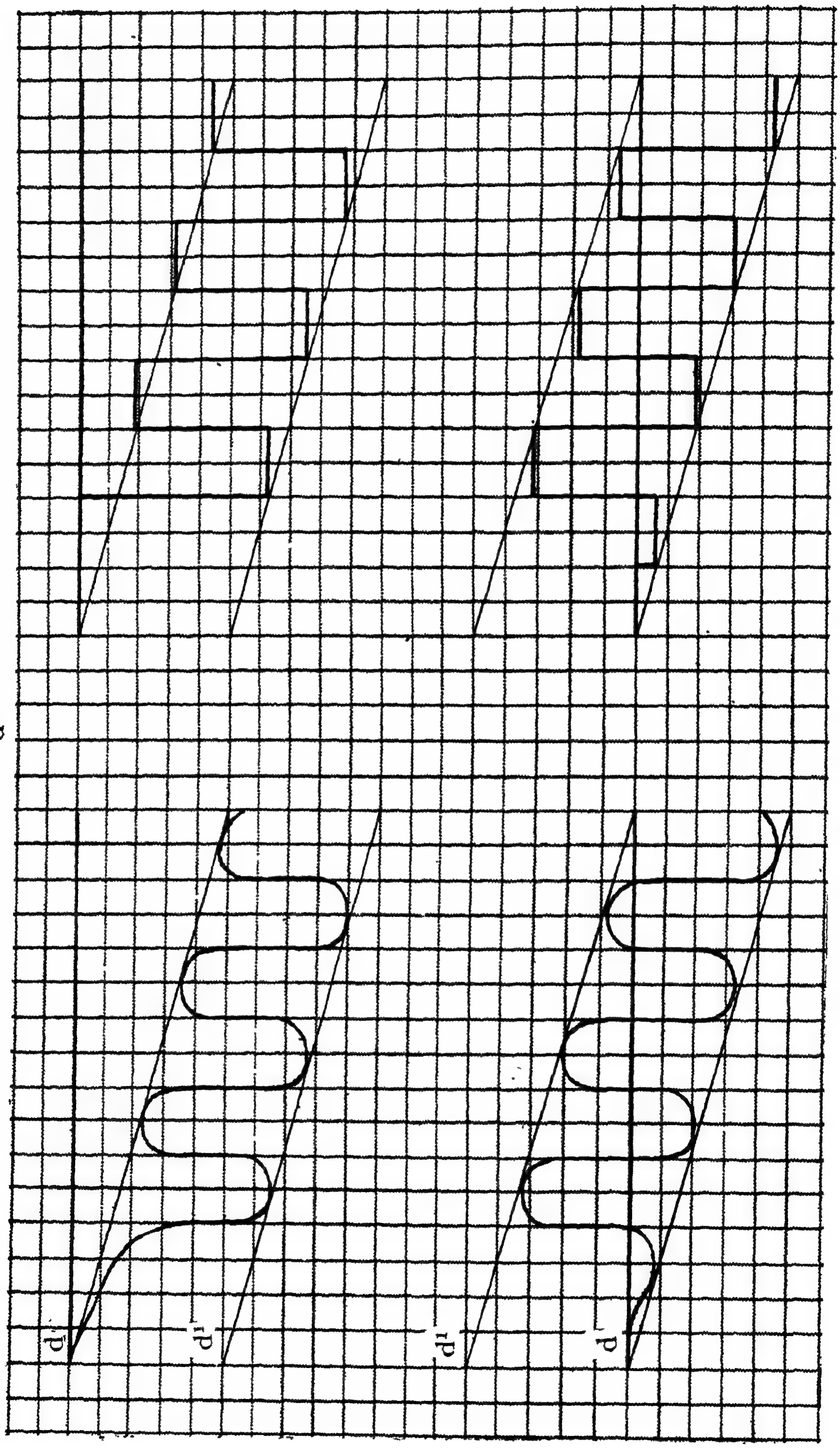


Figure 52. Two parallel secondary axes $\frac{d}{d^1}$.

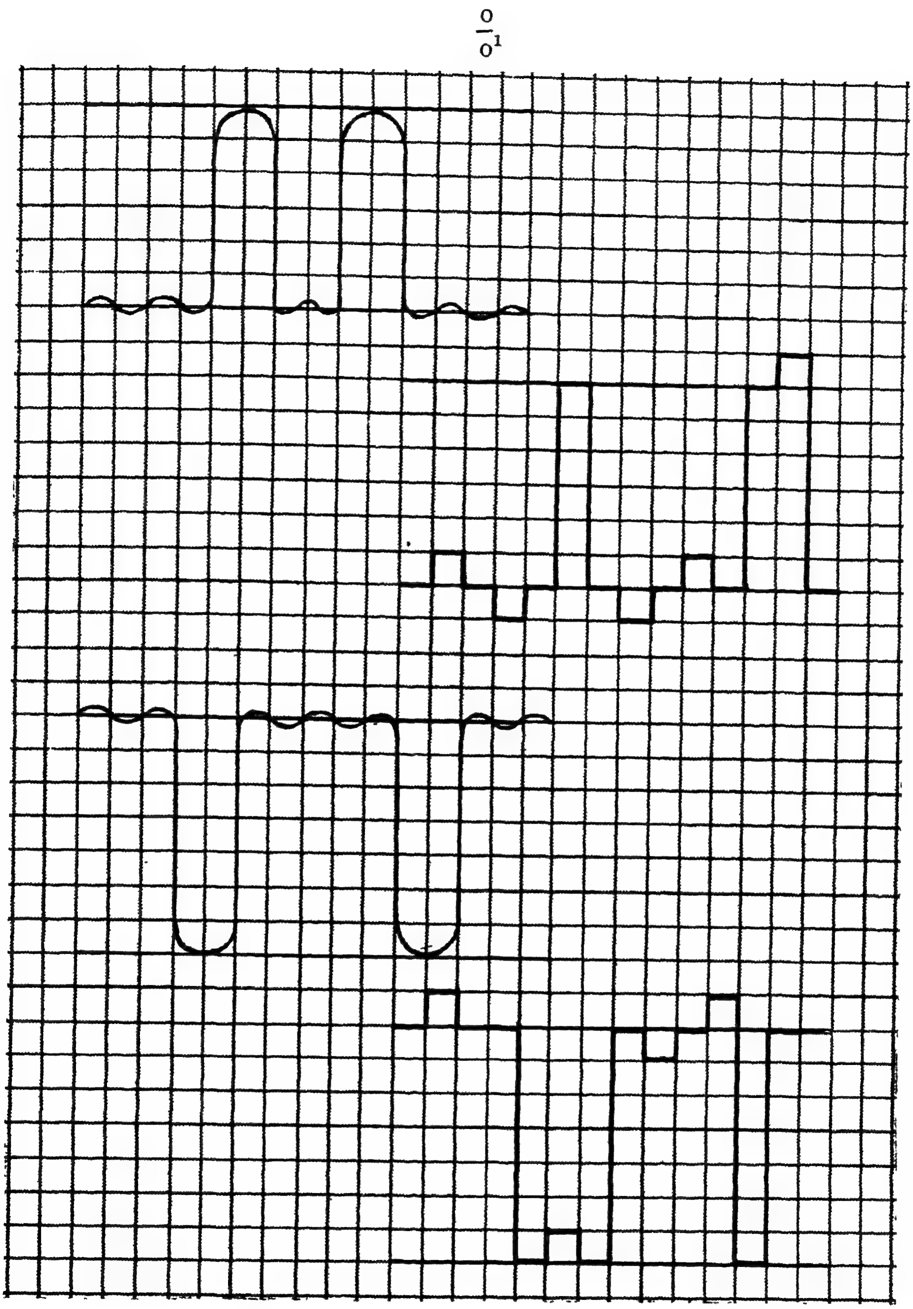


Figure 53. Two parallel secondary axes $\frac{0}{0^1}$.

The 1, 2 and 3 forms of resistance produce the respective degrees of resistance. When more than one form is used in successive portions of melodic continuity, they must follow one another in *increasing* degrees. The opposite arrangement is mechanically inefficient and therefore produces an effect of weakness.

Resistances lead either toward climax or toward balance.

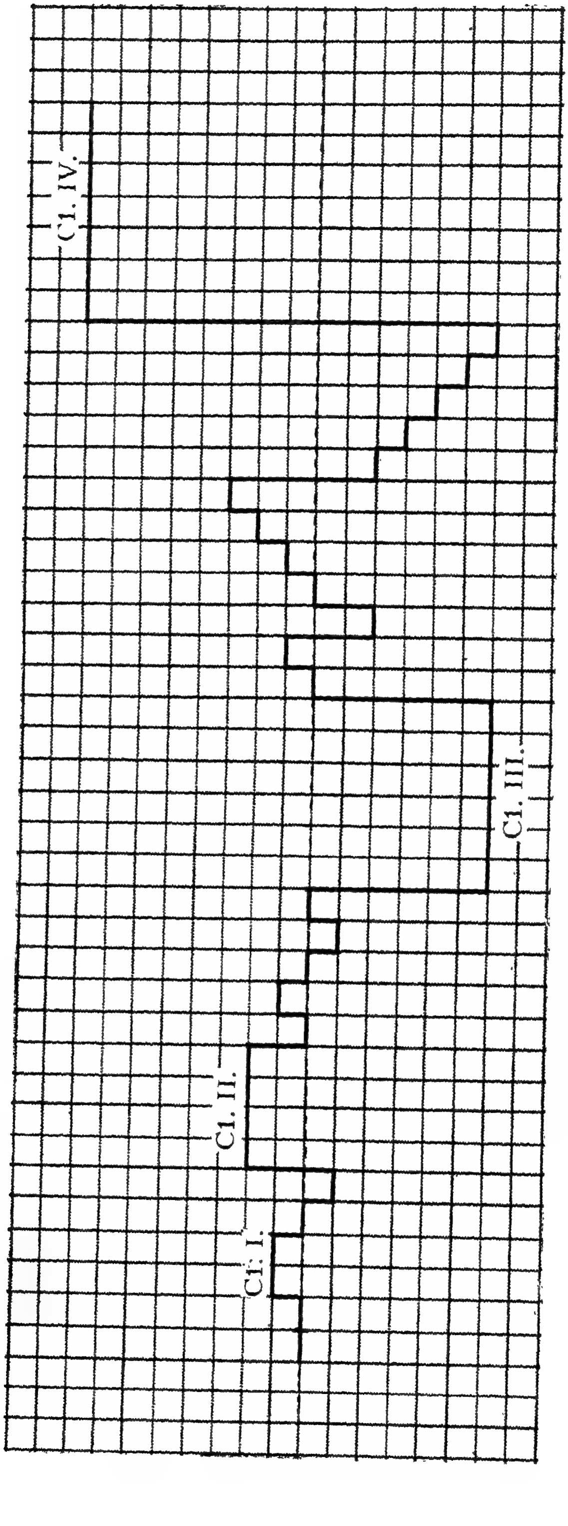


Figure 54. From balance.

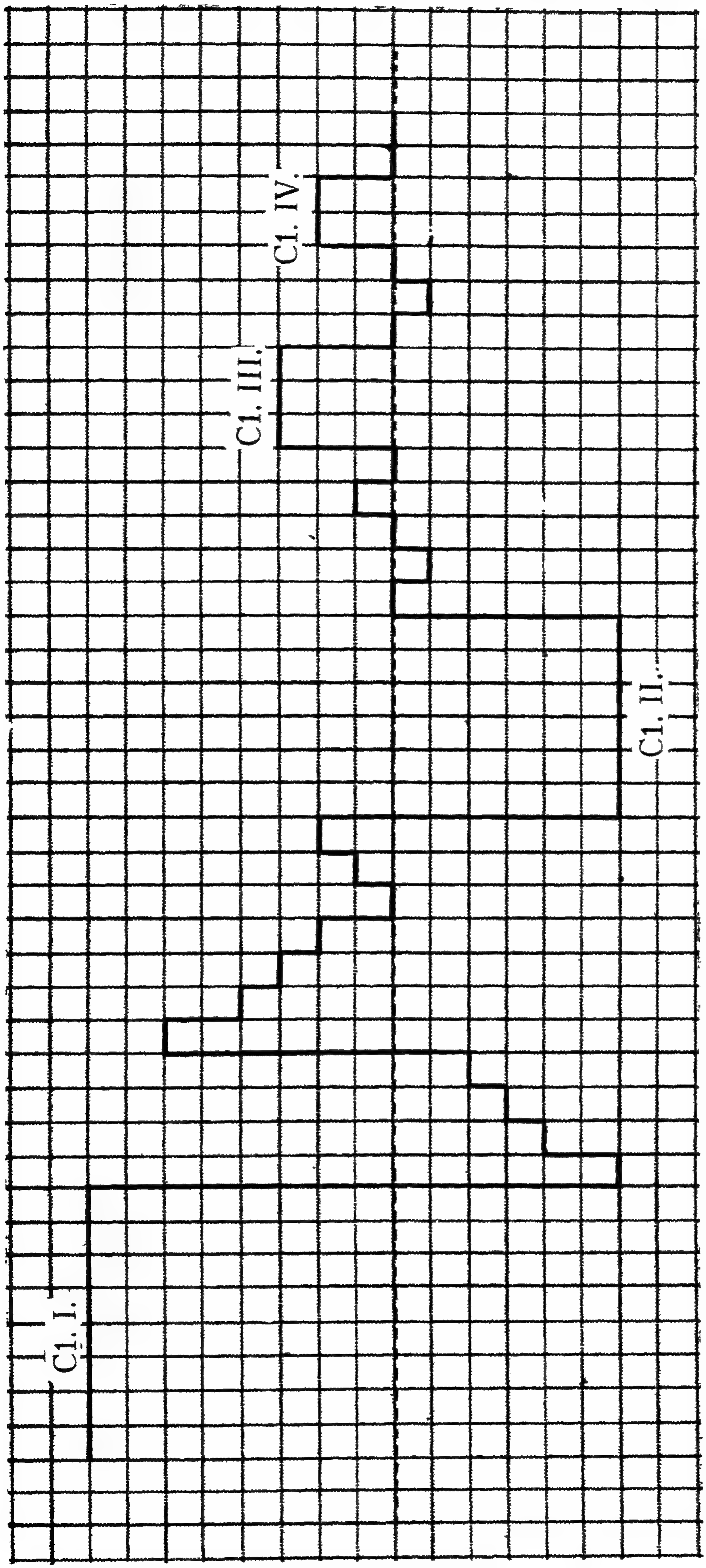


Figure 55. Toward balance.

B. Distribution of Climaxes in Melodic Continuity

The distribution of climaxes in melodic continuity must be arranged with respect to the total duration of such continuity. The relative intensity of climaxes depends on both time and pitch ratios leading toward the respective climaxes. A natural tendency is the expansion of pitch and the contraction of time. These two components mutually compensate each other.

The climactic gain between the two adjacent climaxes takes place when:

1. The pitch-ratio is *increasing* and the time-ratio is *constant;*
2. The time-ratio is *decreasing* and the pitch-ratio is constant.

The climactic gain reaches its mechanical maximum when both forms are combined (*increasing* pitch-ratio and *decreasing* time-ratio).

It is practical to save the last effect for the main climax of the entire melodic continuity; use it only when the extreme of exuberance has to be attained.

As a decreasing time-ratio is characteristic of continuity with a group of climaxes, rhythmic material which would appropriately distribute the climaxes must belong to the decreasing series of growth, such as the summation or power series. Smaller number values and in inverse correlation serve as material for the distribution of the pitch ratios for a group of successive climaxes.

This description refers to a trajectory moving towards the main climax and must be inverted for a trajectory moving in the opposite direction.

CHAPTER 5

SUPERIMPOSITION OF PITCH AND TIME ON THE AXES

WHAT IS called "beauty" is the resultant of harmonic relations. In order to obtain a "beautiful" (esthetically efficient) melody, it is necessary to establish harmonic relations between its factorial and its fractional rhythms. This may be achieved by means of a homogeneous series of factorial-fractional continuities.

Rhythm of time durations occurring within the bars must belong to the same series as rhythm of the secondary axes. Naturally, there are hybrid melodies in which fractional and factorial rhythm belong to different series; a homogeneous series is merely an expression of stylistic consistency.

Melodies with structural consistency may be found in nearly every folklore, as well as in the works of composers who synthesized and crystallized the efforts of their predecessors. Beethoven crystallized the melodic style of the "Viennese" school, which at its time was a revolt against counterpoint and polyphonic writing. Bach, in his melodic themes, (in many cases with an odd number of bars), crystallized the efforts of several centuries.

Different styles have different evolutionary velocities. "Jazz" has a very high one, like some specimens of Alpine flora with a very short life-span; jazz has already crystallized its homogeneity. Examples are numerous and may be found more in "swing" *playing* than in the printed copies of the songs.

After the series has been selected, the actual composition of the fractional continuity may be accomplished in two ways:

(1) by using the resultants or the power groups,
(2) by composing freely from the monomials, binomials, trinomials and quintinomials of a given family.

Here is an example of composing fractional continuity in $\frac{4}{4}$ series:

Suppose we have a trinomial of the secondary axes, $a2T + bT + cT$. In this case, $4T = 16t$. To satisfy $16t$ we may use $r_{4\div3}$, or $\left(\frac{2+1+1}{4}\right)^2$, or any of the variations, i.e., the permutations or the resultants.

A free composition according to (2) may give results identical with some of the variations.

The groups of the $\frac{4}{4}$ series are:

monomial . . . 4
binomials . . . 3 + 1 and 1 + 3
trinomials 2 + 1 + 1, 1 + 2 + 1 and 1 + 1 + 2
the uniform quadrinomial . . 1 + 1 + 1 + 1

Having decided on a2T as (3+1) + (2+1+1), bT as 1+1+2 and cT as 1+3, we obtain $r_{\underline{4 \div 3}}$. By selecting freely various recurrences of the same binomial, like 3+1, we obtain: a2T = (3+1) + (3+1), bT = 3+1, cT = 3+1, or various recurrences of the same trinomial with variations like: a2T = (2+1+1) + (2+1+1), bT = 1+2+1, cT = 1+1+2, we obtain groups that are not identical with the resultants or the power groups.

When a climax is desired, the maximum time value must be placed at the corresponding point of a secondary axis (in *a* at the end, in *b* at the beginning, in *c* at the beginning and in *d* at the end). For instance, if a climax is desired on a2T, it must be the last term of a rhythmic group of this axis. In the $\frac{4}{4}$ series it would be:

a2T = (2+1+1) + (1+3)
or (2+1+1) + (1+1+2)
or (2+1+1) + 4, and the like.

To superimpose a fractional rhythmic group on a factorial group of the secondary axes, means to distribute the points of attack on a pitch trajectory (the path of a moving point).

Let us assume that a group of secondary axes has been constructed with no reference to any particular logarithmic (tuning) system. Placing the pre-selected fractional group above the axes and dropping perpendiculars from the points of attack, we accomplish the distribution of the points of attack (which become the moments of attack) along the pitch trajectory of a hypothetic tuning system.

Example

a2T + bt + ct = (2+1+1) + (1+1+2) + (1+2+1) + (1+3)

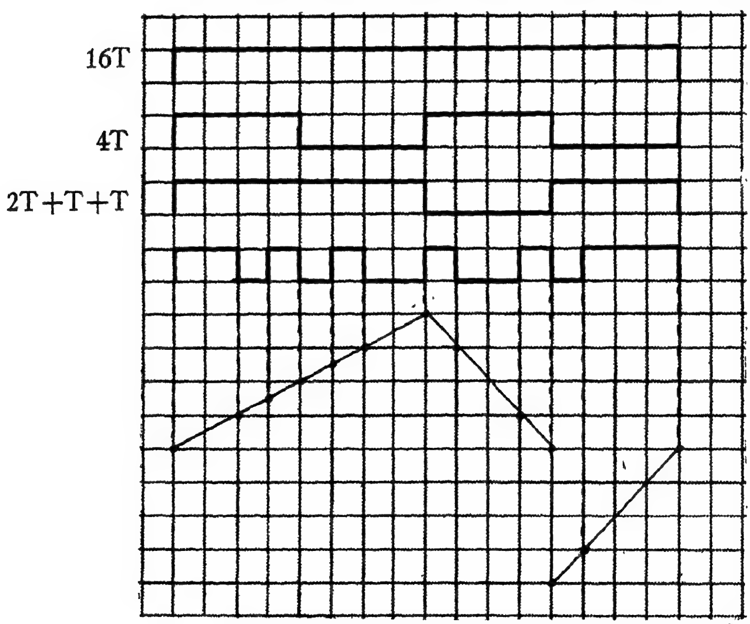


Figure 56. Superimposing a fractional rhythmic group on a factorial group.

Thus, the intersections of dotted lines with secondary axes are the moments of attack on this pitch trajectory.

Here we arrive at the following definition of melody: *melody is the resultant trajectory of the axis-group moving through the points of attack.* Melody, in the academic sense, i.e., with sudden pitch variations within a tuning system, is a *rectangular* trajectory. Melody, in the Oriental conception as well as in any musical actuality, is a *curvilinear* trajectory, i.e., contains a certain amount of pitch-sliding. We shall deal with composition of a melody in the academic sense, as our musical culture leaves the bending of a rectangular trajectory to the instrumental performer.

As the secondary axes form triangles (with respect to the primary axis), two forms of rectangular motion through the points of attack are possible:

(1) ascribed (sine phases).
(2) inscribed (cosine phases).

Although in composing melody a free choice of the two may take place, in balancing melody at its end on *b* or *c* axes, the *ascribed* motion produces an incomplete (i.e., unbalanced) cadence, while the *inscribed* motion produces a complete (i.e., balanced) one. The first one is a device for deviating from balance, i.e., for accumulating tension, *a stimulus for the new recapitulation.*

Examples of rectangular trajectories evolved through the axes of the previous example:

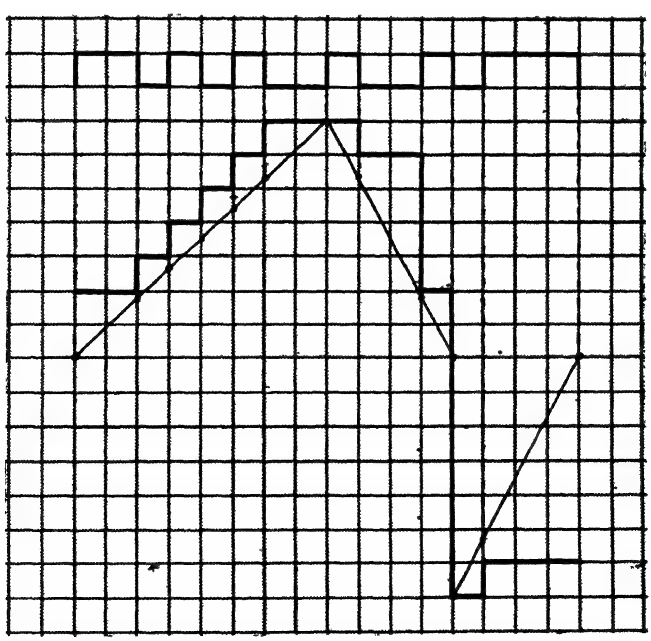


Figure 57. Ascribed motion.

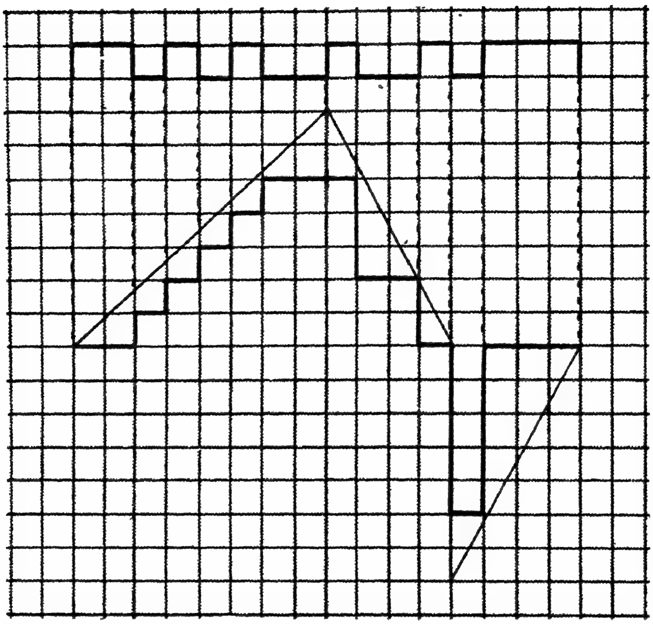


Figure 58. Inscribed motion.

These two potential melodies are totally different as to their pitch progressions. The usual, commonplace composition of pairs varies with respect to the cadence only. Such pairs may be either inscribed or ascribed, but must be identical otherwise; the ending of the first one is ascribed, while the ending of the second is inscribed.

A. Superimposition of Pitch-Rhythm (Pitch-Scale) on the Secondary Axes

Uniform time-intervals (durations) when geometrically projected produce space-intervals, (extensions). Such uniform time scales are *primary selective* systems when $T = r_{a \div 1}$. When $b \neq 1$ (*i.e.*, is not equal to 1) they become *secondary selective systems* (rhythm-scales).

Uniform pitch-intervals of our tuning system produce logarithms to the base of $\sqrt[12]{2}$ (semitones). The chromatic scale is the primary selective system of pitch in our intonation. Geometrical projection of such a scale is uniformity along the ordinate. Any other pitch-scale within the same tuning system is a *secondary selective system*, (i.e., a derivative of the primary selective system).

It is easy to see that a pitch-time trajectory moving in either ascribed or inscribed form of motion through the *points of intersection* of time (abscissa) and pitch (ordinate) uniformities (primary selective systems), is structurally the simplest form of melody, i.e., a chromatic scale in uniform rhythm.

Here we arrive at the following definition of melody: *melody is a pitch-time trajectory resulting from the intersection of the points of intonation (pitch-units) with the points of attack (time-points) in a specified axis-system.*

When the geometrical points of intersection do not coincide with the pitch-units of a scale, pitch-units nearest to the coincidence-points must be used.

Let us superimpose an Aeolian scale (2+1+2+2+1+2) on the axis-group illustrated in the preceding chapter. Let us assume a2P + bP + cP, i.e., a parallel PT correlation. And let P = 5, which in this case gives a symmetric distribution. Further, let pitch *c* be the primary axis. Then a2P extends from *c* to *b♭*, bP from *f* to *c*, and cP from *g* to *c*.

Here is the final construction of the axis group:

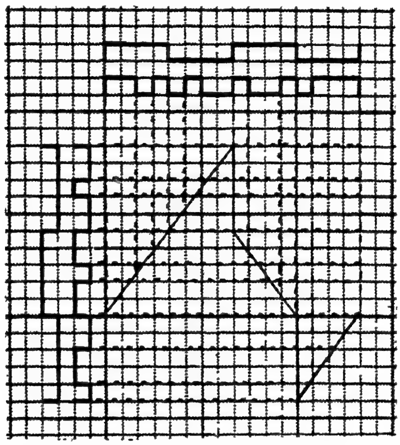


Figure 59. Scheme of the points of geometrical intersection.

This diagram produces a slight deviation from the description given in the text, because of the fact that the scale is so small that it gives deviations. However, this is not essential, as further adjustments follow the scale.

The next step is to adjust the points of intersection to the Aeolian scale. Let us analyze point by point.

If the first point of intersection is *c*, the nearest pitch-unit to the second point of intersection on the Aeolian scale is *d*. Next, we select *e♭* as the nearest to the third intersection-point. The fourth falls exactly on *f*. The fifth falls on *f♯* which is not in the scale. In this case either the repetition of *f*, or *g* is available. The next point is nearest to *g*. Through ascribed motion the entire axis *a* would start on *d* and end on *b♭*.

As in inscribed motion, *pitch-levels* move *toward* the points of intersection; the first pitch-unit on *b*- axis will be either *f* or *e♭*, as the geometrical intersection coincides with *e♮*. The next intersection-point is nearer to *d*. In order to complete *b*– axis through inscribed motion, it will be necessary to consider *c* as the last intersection point. C– axis through the inscribed motion gives points of intersection at *a♭* and *c*.

We shall reconstruct now the axis-group with respect to the Aeolian scale, as just described, and draw an inscribed trajectory. This trajectory is the most elementary form of an *actual melody.*

Figure 60. Trajectory of an actual melody.

It would not be difficult to find all other versions, i.e., the ascribed trajectory and the trajectories where either axis may be realized in ascribed or inscribed motion.

Here is a chart of combinations:

Axes:	a	b	c
	ascribed	ascribed	ascribed
	ascribed	ascribed	inscribed
	ascribed	inscribed	ascribed
	inscribed	ascribed	ascribed
	inscribed	inscribed	inscribed
	inscribed	inscribed	ascribed
	inscribed	ascribed	inscribed
	ascribed	inscribed	inscribed

There are eight versions altogether. After obtaining an actual melody, such melody becomes subject to scale variation, tonal and geometrical expansions and inversions. For instance, the same melody in a "blue" scale would sound:

Figure 61. Same melody in a "blue" scale.

Or in a Chinese (2+3+2+2) scale (through translation of the corresponding degrees):

Figure 62. Same melody in a Chinese scale.

Here an allowance has to be made on the first note of the last bar, as the VI does not exist in the Chinese scale (the last degree of the scale, i.e., V, which is *a* substituted).

B. Forms of Trajectorial Motion

The trajectory obtained above was called "the most elementary form of an actual melody" because its form of motion is *simple harmonic* (*i.e.*, motion within the scale). As noted earlier, such a melody cannot be too expressive or dramatic. In order to obtain an expressive melody, it is necessary to build resistances. This cannot be realized without introducing more complex forms of motion.

We shall present now all the trajectorial forms with respect to the zero axis.

(1) Sin (sine) motion with constant amplitude:

Figure 63.

(2) Cos (cosine) motion with constant amplitude:

Figure 64.

(3) Combined sin + cos motion with constant amplitude:

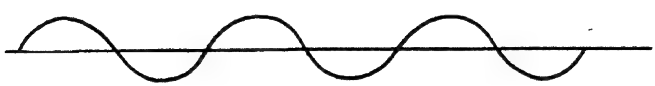


Figure 65.

(4) Combined cos + sin motion with constant amplitude:

Figure 66.

(5) Sin motion with increasing amplitude:

Figure 67.

(6) Sin motion with decreasing amplitude:

Figure 68.

(7) Sin motion with combined increasing-decreasing amplitude:

Figure 69.

(8) Sin motion with combined decreasing-increasing amplitude:

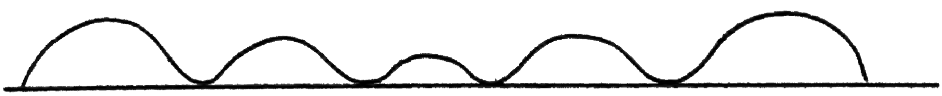


Figure 70.

(9) Cos motion as (5):

Figure 71.

(10) Cos motion as (6):

Figure 72.

(11) Cos motion as (7):

Figure 73.

(12) Cos motion as (8):

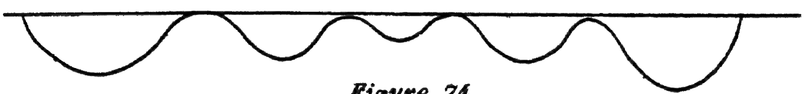


Figure 74.

(13) Combined sin + cos motion with combined amplitude as (5):

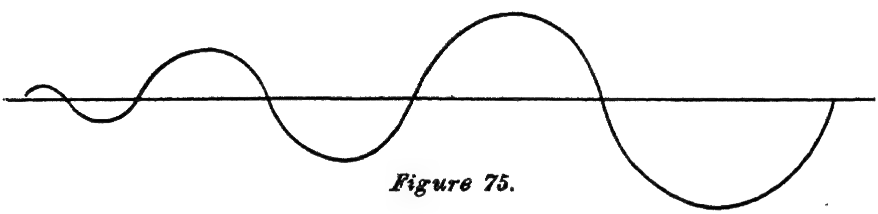


Figure 75.

(14) Combined sin + cos motion with combined amplitude as (6):

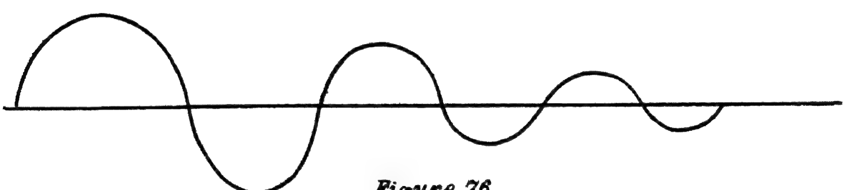


Figure 76.

(15) Combined sin + cos motion with combined amplitude as (7):

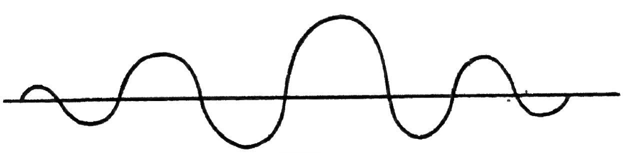


Figure 77.

(16) Combined sin + cos motion with combined amplitude as (8):

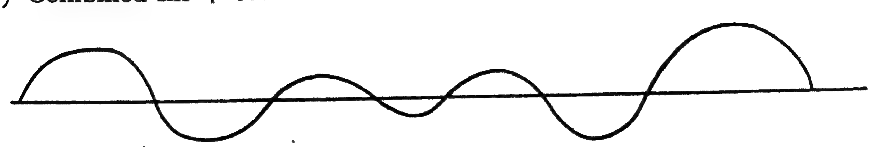


Figure 78

(17) Combined cos + sin motion with combined amplitude as (13):

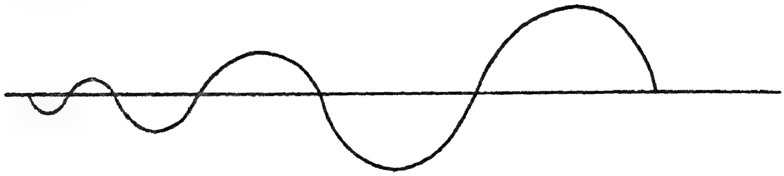


Figure 79.

(18) Combined cos + sin motion with combined amplitude as (14):

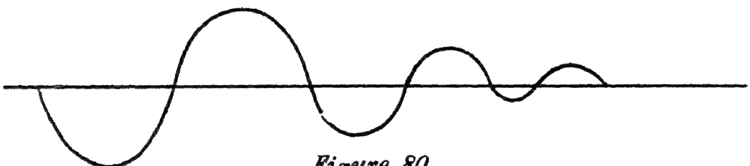


Figure 80.

(19) Combined cos + sin motion with combined amplitude as (15):

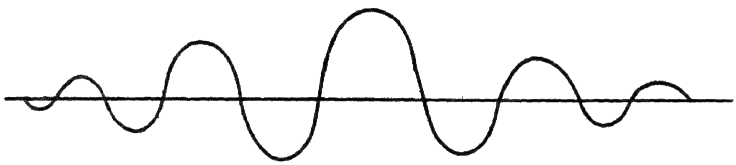


Figure 81.

(20) Combined cos + sin motion with combined amplitude as (16):

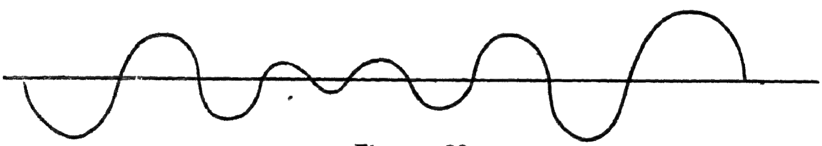


Figure 82.

These twenty versions are merely variations of the two original forms, i.e., (1) and (5). Every cos is ⓓ* of the sin and every decreasing amplitude is ⓑ** of the increasing amplitude.

Further development of these trajectorial forms may be obtained through application of the coefficients of recurrence of the sin, the cos and the growth of amplitudes. For instance, 3 sin + cos + 2 sin + 2 cos + sin + 3 cos on constant amplitude:

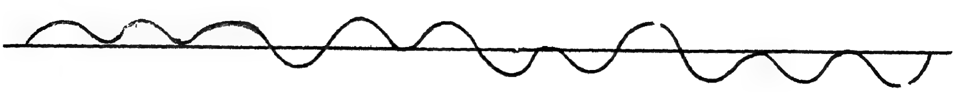


Figure 83.

*The reference is to the fourth position in geometric inversion: the forward-upside down inversion with regard to the original. See Book III. (Ed.)

**The reference is to the second position in geometric inversion: the backward inversion with regard to the original. See Book III. (Ed.)

The same case on increasing amplitude:

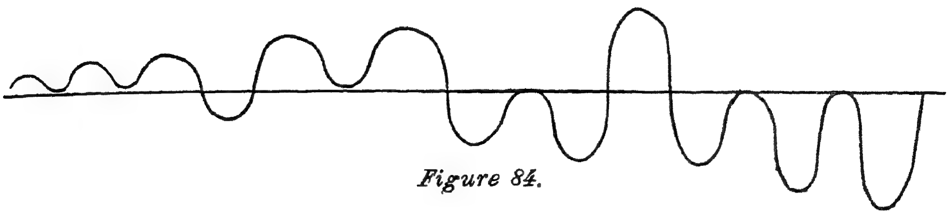


Figure 84.

All these forms being transformed into rectangular trajectories, with respect to a definite intonation (tuning) system, become actual intonation-groups, i.e., melodic forms. For example, a *gruppetto* is sin + cos with constant amplitude.

Including the zero of pitch variation, (absolute zero-axis trajectory), we have the following forms of trajectorial motion:

(1) constant pitch trajectories (repetition on extension).
(2) sin or cos trajectories (one phase motion).
(3) combined trajectories (full period motion or rotation).

Application of various trajectorial forms to *a*, *b*, *c* and *d* axes gives the following correspondences: All the sin of 0 remain sin on all other axes. All the cos of 0 remain cos on all other axes. All the combined forms of 0 with respect to sin, cos and the constancy of amplitude remain respectively the same on all other axes. Zero axis is the only one to be heard. The rest are merely hypothetic lines.

Here are examples of the corresponding translations of a curvilinear sin trajectory into rectangular trajectories of the *0*, *a*, *b*, *c* and *d* axes:

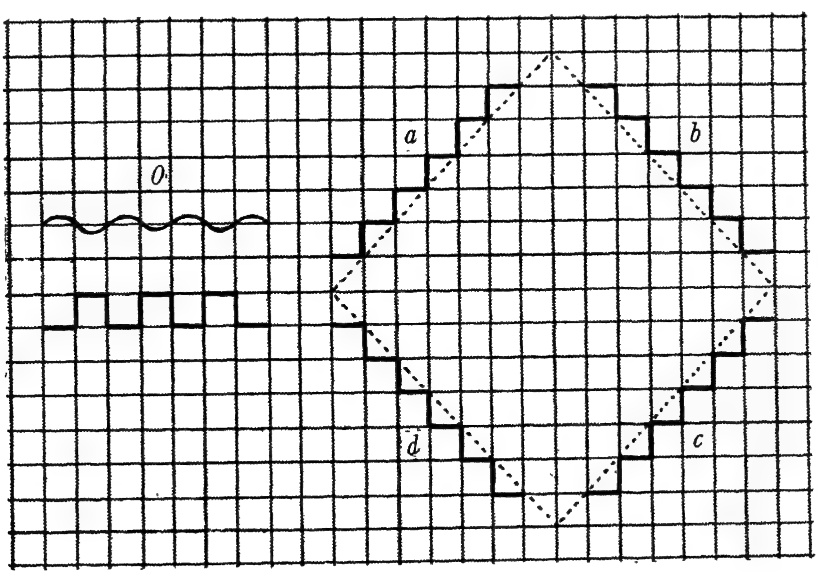


Figure 85. Translation of a curvilinear sin trajectory into rectangular trajectories.

Translations of the cos trajectory:

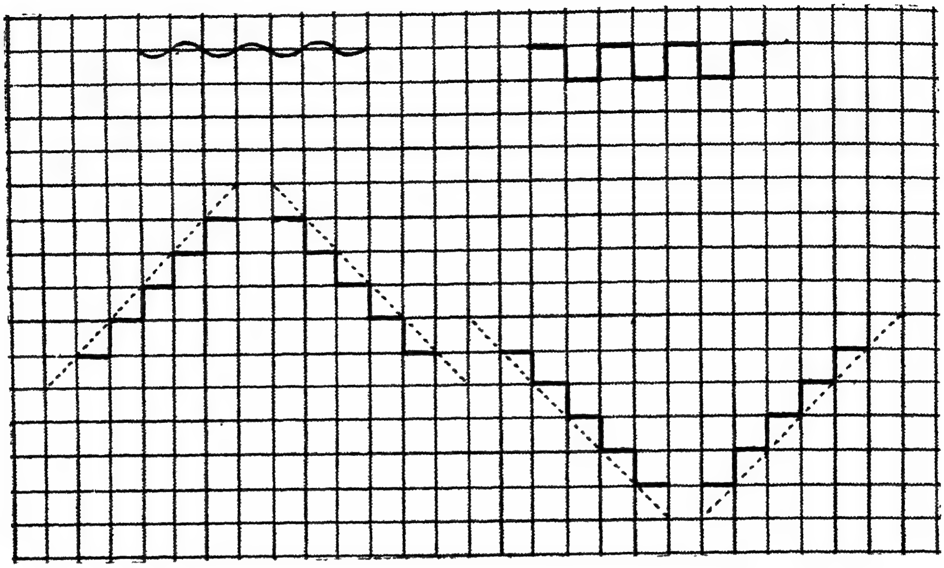


Figure 86. Translation of cos trajectory into rectangular trajectories.

Translations of the combined trajectory:

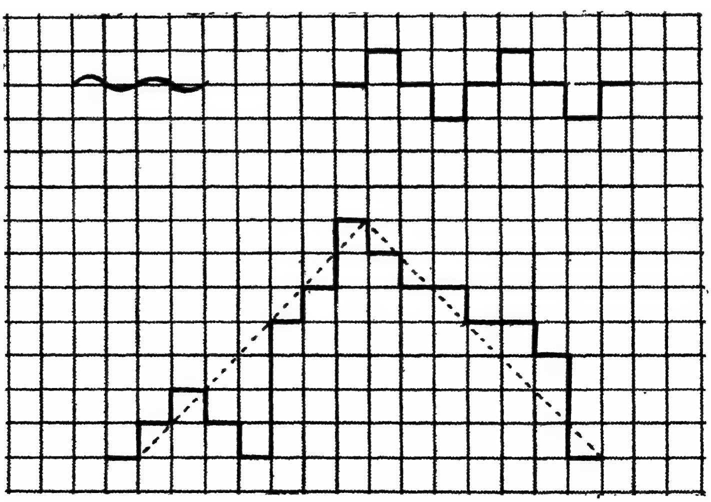


Figure 87. (a) With continuous tangency.

Translation of the combined trajectory:

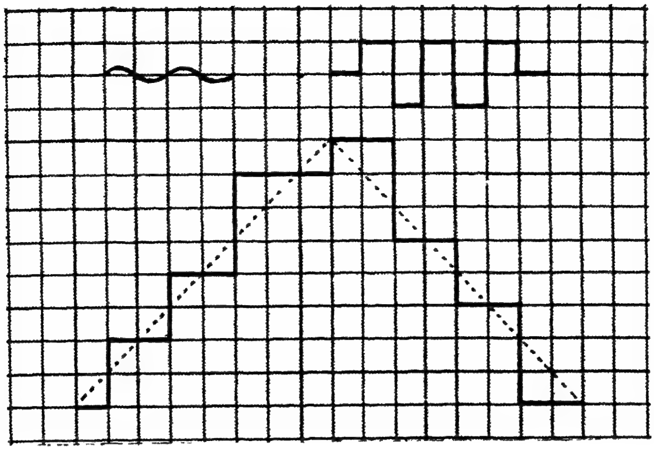


Figure 88. (b) Without continuous tangency.

Figure 88. (a) may be called revolving trajectories.
Figure 88. (b) may be called crossing trajectories.

Deviation of a rectangular trajectory from its corresponding axis signifies inconsistency and lowers the esthetic value of a melody.

An esthetically efficient melody must display, besides consistency, a variety of the forms of motion.

When a trajectory is controlled by the two simultaneous axes (fundamental and complementary), the points of attack may fall on either axis according to the form of alternation.

Example:

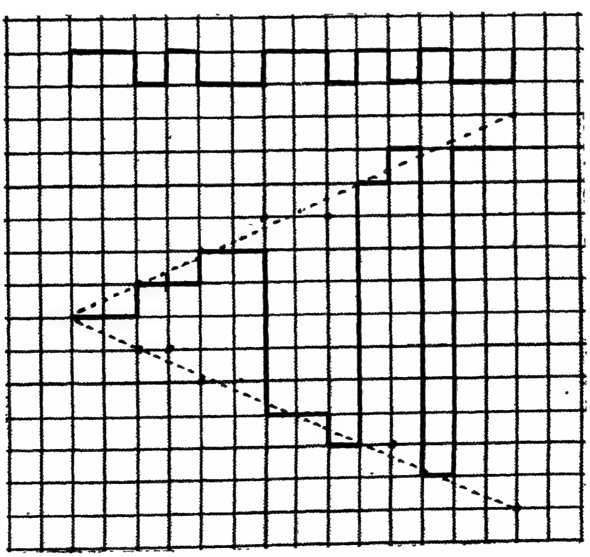


Figure 89. Trajectory controlled by two simultaneous axes.

The form of alternation is subject to distribution, i.e., rhythm.

An example of analysis of the trajectorial motion in J. S. Bach's Two-Part Invention, No. 8:

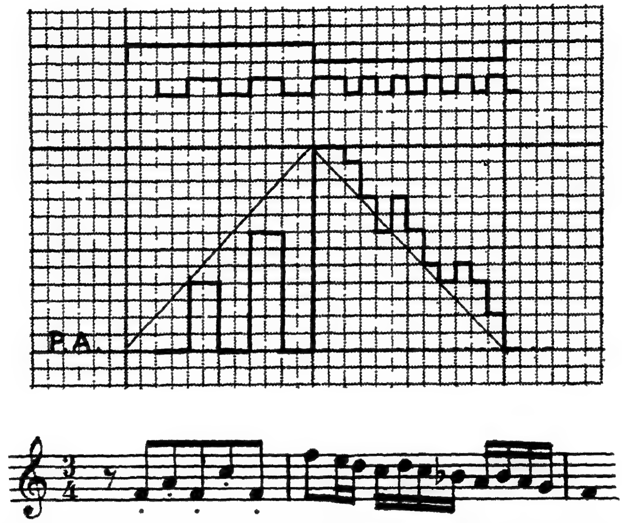


Figure 90. Trajectorial motion in Bach's Two-Part Invention, No. 8.

The staccato eighths are expressed on the graph as sixteenths.

This trajectory has a primary axis defined by its first, last and two intermediate attacks. The group of the secondary axes is: $\frac{a}{o} + b$. The pitch and time ratios are uniform, ie., $\frac{a}{o}PT + bPT$. The first attack of *b* is a climax. The form of motion on $\frac{a}{o}$ is sin motion with increasing, (centrifugal), amplitude. The alternation of the points of attack on the two conjugated axes is uniform. The form of motion on *b* is combined (sin + cos) and has a constant amplitude. It is ascribed with respect to *b*. The effect of revolving due to the combined form produces a resistance and delays the balance. This melody would lose most of its esthetic value if the o-axis were eliminated (loss of resistance moving toward the climax), and the b-axis were to have one-phase motion.

At this point it would be very advisable for the reader to make a thorough analysis of the outstanding, as well as of the deficient, themes taken from existing music. This procedure must follow all sections of the theory of melody. A precise statement must be made on each item regarding the form and measurement.

Although a theme of any dimension (duration) may be constructed to full satisfaction, it is more practical in most cases to compose continuity out of a short original structure. Memory is very limited and the latter will produce an effect of greater unity.

After having acquired enough experience in analysis, one may start composing melodies according to this theory. Success depends upon thorough knowledge of all the preceding material—and the ability to think!

CHAPTER 6

COMPOSITION OF MELODIC CONTINUITY

WHEN MELODIES are constructed, i.e., plotted, according to the techniques described in earlier chapters of this discussion of the theory of melody, the melodies will be found to have such properties as render them susceptible to the following treatments and techniques:

1. Permutability of the secondary axes with their respective melodies in time continuity.
2. Permutability of the individual pitch-units (preferably through circular permutations) pertaining to one individual secondary axis.
3. Geometrical convertibility of the entire melody.
4. Geometrical convertibility of portions of melody pertaining to the individual secondary axes or any groups thereof.
5. Tonal expansion of the entire melody.
6. Tonal expansion of portions of melody pertaining to any individual secondary axis or portions thereof. In this case different axes may appear with different coefficients of expansion.
7. Combined variations of geometrical inversions and tonal expansions applied to the entire melody.
8. Combined application of geometrical inversions and tonal expansions applied to the portions of melody pertaining to individual secondary axes or any combinations thereof. In this case different coefficients of expansion may be combined with different geometrical inversions.

Consequently, melodic continuity may be composed through any of the above-mentioned forms of variation or any combination thereof.

Here is an example of the quantitative development of melodic continuity from the original theme:

Let us take a trinomial axial combination, a, b, c. Each of the individual axes has four geometrical inversions. Thus, the number of combinations of the three axes that may be used in identical or different geometrical inversions equals $4^3 = 64$. This number refers to one constant E. If any of the individual axes appears in three forms of tonal expansion, the entire quantity will be $64^3 =$ $= 262{,}144$.

The following is a method of indicating a secondary axis where the geometrical positions and the coefficients of expansion are specified. For example, an axis *a* in position ⓒ in the second expansion (E_2) may be expressed like this:

$$a ⓒ E_2$$

A trinomial axial-combination consisting of a, b and c axes, with specified time and pitch ratios, and the geometrical positions and coefficients of expansion,

assumes the following appearance:

Time ratios: $2 + 1 + 1$
Pitch ratios: $1 + 2 + 3$
Geometrical positions: ⓓ, ⓐ, ⓑ
Coefficients of expansion: E_0, E_2, E_1
$aP2T_{ⓓ}E_0 + bT2P_{ⓐ}E_2 + cT3P_{ⓑ}E_1$

This method of indication emphasizes not the axial structure alone, but the pitch-units (intonation) as well. For example, a melody in its third displacement, on axis a, in position ⓓ, in the third expansion, may be expressed as follows:

$$a_{ⓓ}E_3d_3$$

When this method is systematically applied, the sequence of the different displacement phases, with regard to consecutive secondary axes, may assume different forms of distribution. For example, it may start with the first phase within the first axis, with the second phase within the second axis, with the third phase within the third axis, etc. It may follow a rhythm of any resultant or any of the series of growth:

(a) $d_3 + d_1 + d_2 + d_2 + d_1 + d_3$
(b) $d_0 + d_1 + d_3 + d_6 + d_{11} + \ldots$

Naturally, the rhythm for such variations of motif depends upon the number of pitch-units within the motif.

Ability to produce expressive melodies (themes) does *not* make a great composer, but ability to produce an organic continuity out of original thematic material *does!*

Going as far back as the strict style of counterpoint written to a *cantus firmus*, we find that the composition of continuity is based on *uniform factorial periodicity:* the theme regularly appears in different voices and that keeps the music moving.

In all elementary homophonic forms, continuity is based on a composition of *biners*, $(a_1 + a_2)$, usually similar structures with different endings, consisting of $4 + 4$ or $8 + 8$ bars. Next comes the method of *terners*, i.e., $a_1 + b + a_2$, involving the introduction of new material in the center term.

The most advanced forms in the past were offered first by J. S. Bach; he used a sequence of biners in contracting geometrical progressions (see Fugue V, Vol. II, *The Well-Tempered Clavichord*). In his case, it meant that a greater overlapping (stretto between the theme and the reply) occurred with each succeeding announcement. In Beethoven's case, it meant a continuous breaking up of the original biner.

All these forms of continuity, (in the past), are rigidly attached to the $\frac{2}{2}$ series.

Richard Wagner built his continuity according to the script, i.e., the operatic libretto. Although he wrote these libretti himself and although he was quite skilful at it, his musical continuity suffered greatly from this syntactic dominance.

Wagner's faults were then adopted as virtues by Scriabine and by others. Literary influence, together with linguistic, logic and syntactic (propositional) technique were the factors that delayed, if they did not prevent, the sound development of the forms of musical continuity.*

Forms of musical continuity are purely quantitative and pertain to motion. They are biomechanical, i.e., they are forms of growth. When they grow normally, they survive better. It is like pure Darwinism: the struggle for existence, the survival of the fittest. A star-fish is not "just a pretty pentagon" but an organic form evolved through the necessity of efficient functioning.

Many an unpretentious melody is appealing, i.e., esthetically efficient, due to the fact that within the eight-bar structure certain processes evolve in a very consistent manner. It happens quite often that the efficiency of structure is greater in smaller portions and smaller in greater portions.

These bio-mechanical forms are primarily concerned with three basic factors:

(1) Symmetric development, i.e., the axis-inversion.
(2) The ratio of growth, such as summation.
(3) Movement with respect to tension and release resulting in balance, i.e., an arithmetical or a geometrical mean.

Growth along the axis of symmetry (compare the case of the human body with its growth along the spinal cord) is a continuity formed by geometrical inversions of the original structure or of its portions (melody) along the primary axis. The regularity of recurrence of the different inversions is subjected to rhythm. Pitch expansions (tonal and geometrical), combined with their geometrical inversions, may be used as components of musical continuity.

The most fluent form of continuity results from symmetric growth along the time-axis. This is the most complete form of continuity as it exemplifies birth, growth, maturity, decline and death—all in one process. To accomplish this in melody it is necessary to split the original structure into a number of elements (such as bars or secondary axes), to show these elements in their gradual addition, and then in their gradual subtraction.

Suppose we have a three-bar structure and split it into a, b, c elements. Gradual addition of the elements will give: a + ab + abc. Gradual subtraction of the elements will give: abc + bc + c. The combination of the two forms offers—the time-axis on abc. The entire continuity will be this:

a + ab + abc + bc + c

Examples:

The original structure split into three elements:

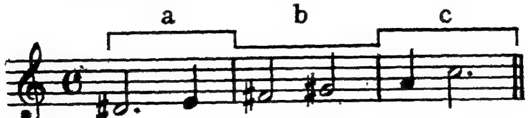


Figure 91.

*See the definition of program music in the *Oxford History of Music*, Vol. 6, page 3, which reads: "Program music is a curious hybrid, that is, music posing as an unsatisfactory kind of poetry." (J.S.)

Continuity composed through the time-axis.

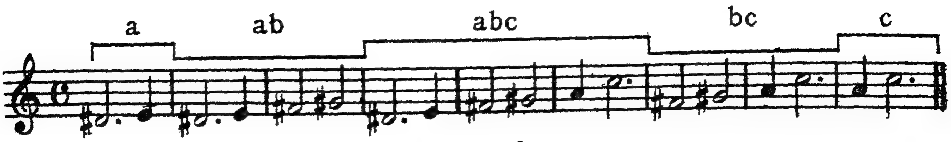


Figure 92.

The process of summation may pertain to the preceding procedure, as well as to factorial ratios of the secondary axes, or the number of individual attacks.

As an example of summation through the first summation series based on the time axis, let us take an eight-bar structure and split it into *a b c d e f g h* elements. The continuity will have this form:

$$a + ab + abc + abcde + abcdefgh + defgh + fgh + gh + h.$$

The entire structure moves across itself through its own axis, while time goes on.

The next point is obvious. Using the same series for the T of the secondary axes, we obtain:

$$T + 2T + 3T + 5T + 8T + 5T + 3T + 2T + T$$

whatever axis (o, a, b, c or d) each term may represent.

Summation through the number of individual attacks may be found in many melodies. Take the popular song by Arthur Johnston *One, Two, Button Your Shoe** for instance.

Figure 92A.

The first eight bars give the following summation of attacks: $2 + 4 + 6 + 12$, i.e., $2 + 4 = 6$, and $6 + 6 = 12$. It means that there are four distinct substructures, each containing the number of notes in this particular summation, carried out with absolute precision.

It is important not to confuse the *rhythm of attacks* with the *rhythm of durations*.

The method of summation is very flexible, and with a little initiative one may secure a great deal of variety.

In the song, *But I Only Have Eyes for You*, you find the following scheme of attacks: $6 + 9 + 6 + 3$. This is an *incomplete* form of $3 + 6 + 9 + 6 + 3$, where the central term is the result of summation $3 + 6 = 9$. At the same time, the central term becomes an axis of time symmetry.

With respect to tension and release, movement resulting in balance may refer to factorial or fractional time-rhythm as well as to the rhythm of the number of individual attacks. Use of the arithmetical mean is the most common device in this case.

An *arithmetical mean* is the quotient of the division of the sum by the number of elements. With two elements, a and b for example, it equals $\frac{a+b}{2}$. Musical intuition has a certain amount of precision, and in some cases these arithmetical means come out with a very good approximation. For example, in the first $3\frac{1}{4}$ bar structure of the song *Stormy Weather*, the first sub-structure has three attacks, the second has seven, and the third has four. The *exact* number for the last sub-structure would be $\frac{3+7}{2} = 5$, not 4. This is a very good approximation, for there is only 20 percent of error; yet you get a greater satisfaction by adding one more attack. Try it by making a triplet out of the two eighths at the beginning of the third bar.

This procedure is analogous mechanically to underbalancing-overbalancing-balancing; or to overbalancing-underbalancing-balancing.

The following graphs and music serve as examples of composition of melodic continuity. Each example given is a complete musical composition written for an unaccompanied instrument. This art has been greatly neglected today. In the 17th and 18th centuries, composers possessed enough technique to accomplish such tasks. J. S. Bach wrote many outstanding works, even sonatas, for violin or viola da gamba alone. Today only a very few high-ranking composers—such as Paul Hindemith (*Suite for Viola* alone) or Wallingford Riegger, an American, (*Suite for Flute* alone, in seven movements)—have dared to write a whole opus for an unaccompanied instrument.

The three compositions I offer here are constructed from the scales of the first group. Each graph represents a theme originally plotted. Musical examples are complete compositions developed by means of variation.

The notation is as follows:

M—the entire melody

a, b, c, d—portions of melody pertaining to the respective axes

$\frac{a^1}{a}, \frac{b^1}{b}, \frac{c^1}{c}, \frac{d^1}{d}$

or

$\frac{a}{a^1}, \frac{b}{b^1}, \frac{c}{c^1}, \frac{d}{d^1}$ —parallel binary axes

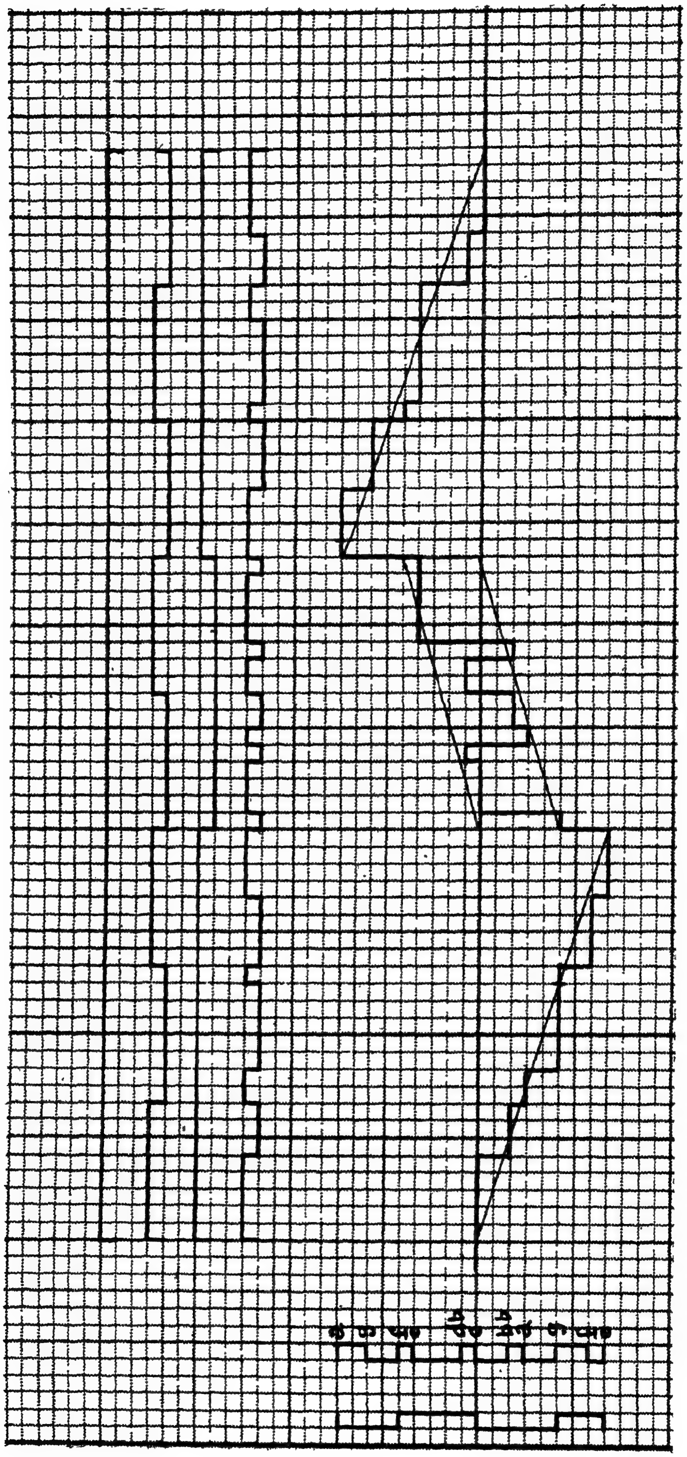
ⓐ, ⓑ, ⓒ, ⓓ —geometrical positions of M or of the respective axes

$p_0, p_1, p_2, \ldots$—permutations of pitch-units of M or of the respective axes

$E_0, E_1, E_2, \ldots$—tonal expansions of M or of the respective axes

In this form of notation, each original melody (the theme of the composition) appears as $M_{ⓐ}\ p_0E_0$.

It is advisable to be conservative in planning a complete melodic continuity, as application of too many variations at a time (i.e., p, E and the geometrical positions) may increase the complexity of the entire composition beyond the listener's grasp.



c, d♭, e, f, g, a, b♭, c, d♭, e, f, g, a, b♭, c,

Figure 93. Plotted melody.

*

$$\left[M_{\textcircled{a}}\ E_0p_0\right] + \left[\tfrac{C^1}{C}\textcircled{a}\ E_0p_0\right] + \left[D_{\textcircled{a}}\ E_0p_0\right] + \left[B_{\textcircled{a}}\ E_0p_0\right] + \left[D_{\textcircled{a}}\ E_0p_1\right] +$$

$$\left[B_{\textcircled{a}}\ E_0p_2\right] + \left[\tfrac{C^1}{C}\textcircled{a}\ E_0p_3\right] + \left[M_{\textcircled{d}}\ E_0p_0\right] + \left[\tfrac{C^1}{C}\textcircled{d}\ E_0p_4\right] + \left[D_{\textcircled{d}}\ E_0p_5\right] +$$

$$\left[D_{\textcircled{a}}\ E_1p_0\right] + \left[M_{\textcircled{a}}\ E_0p_0\right]$$

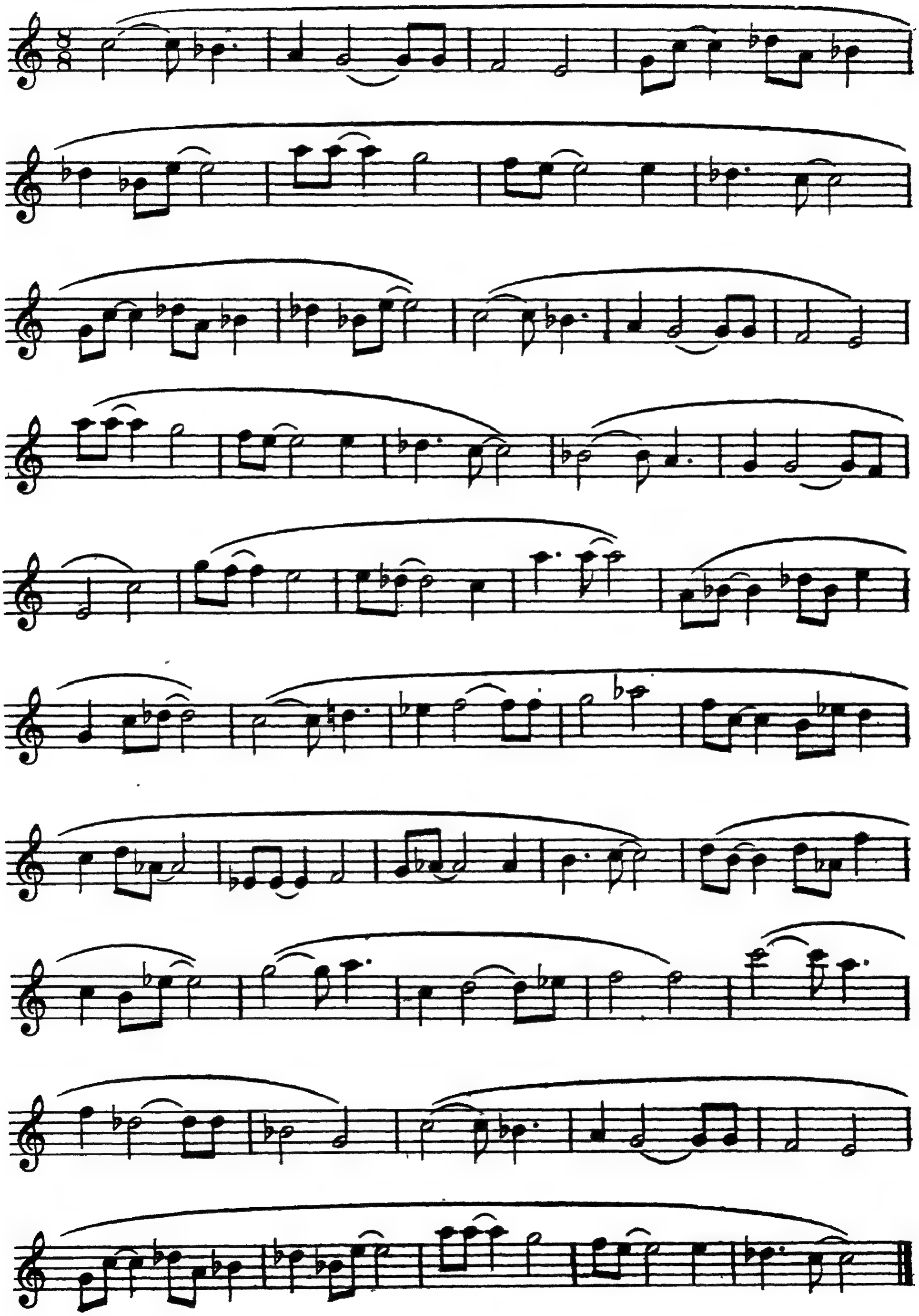


Figure 94. The melody of Figure 93.

c, d, e♭, f♯, g, a, b♭, c, d, e♭, f♯, g, a, b♭, c, d, e♭, f♯, g

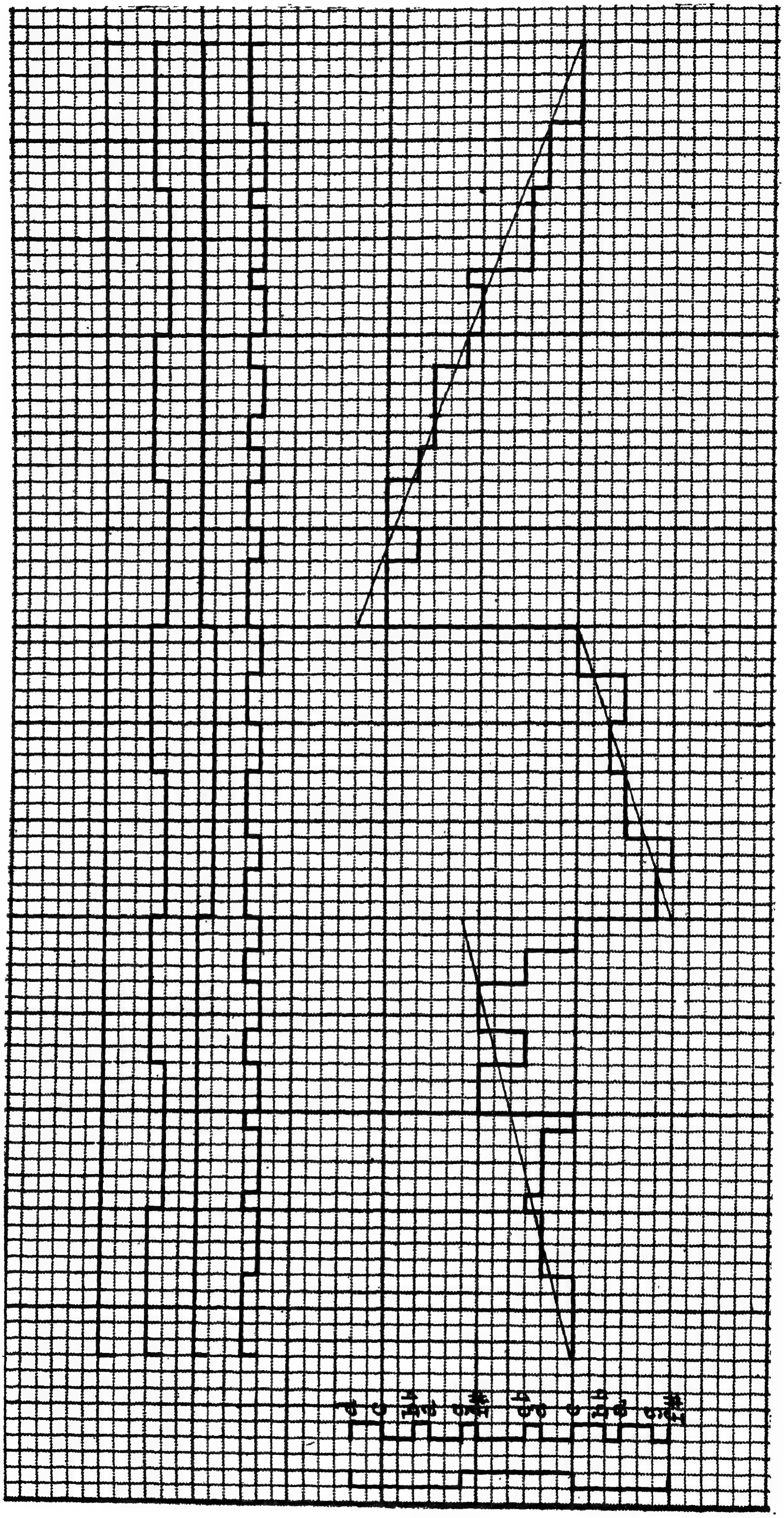


Figure 95. Plotted melody.

$[M\textcircled{a}\ E_0p_0]+[M\textcircled{d}\ E_0p_0]+[C\textcircled{a}\ E_1p_1]+[B\textcircled{h}\ E_0p_2]+[A\textcircled{a}\ E_1p_2]+$
$[B\textcircled{c}\ E_0p_4]+[A\textcircled{a}\ E_0p_5]+[B\textcircled{a}\ E_0p_0]$

Figure 96. The melody of Figure 95.

CHAPTER 7

ADDITIONAL MELODIC TECHNIQUES

IN THIS chapter I have grouped a number of brief discussions of other facets of the process of building melodies, facets which will be of use to the practical composer.

To begin with, there is the question of the use of symmetric scales in melody-making.

A. USE OF SYMMETRIC SCALES

First: the intervals between the tonics in all settings (i.e., the original, the first contraction and the final contraction, the latter being an equivalent of the scales of the third group) determine the pitch-ranges. The first tonic corresponds to the primary axis.

Secondly: in using the first contraction, we acquire an overlapping of the secondary axes.

Thirdly: the following correspondence of the secondary axes takes place in the original setting.

The a-axis of the lower section is the c-axis of the adjacent upper section. The b-axis of the lower section is the d-axis of the upper. The c-axis of the lower section is the a-axis of the section below the lower section. The d-axis of the lower section is the b-axis of the section below the lower section.

Considering this, it is practical to conceive the axial group in such an arrangement that the first tonic is flanked by other tonics (still referring to the original setting). This permits a unified reading of the axes.
For example:

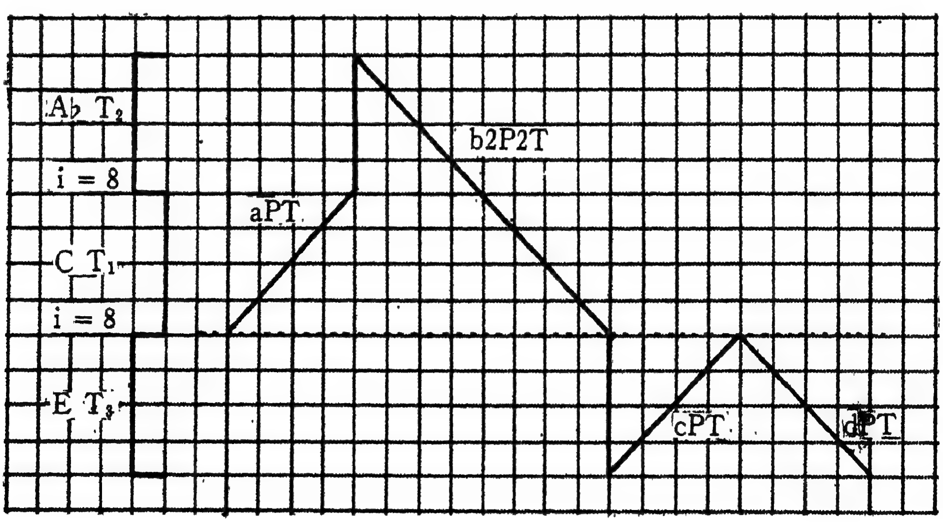


Figure 97. Surrounding first tonic with other tonics.

For the same reason it is practical to surround the first tonic by other tonics (making the first tonic a primary axis) in the settings of the first contraction. For example:

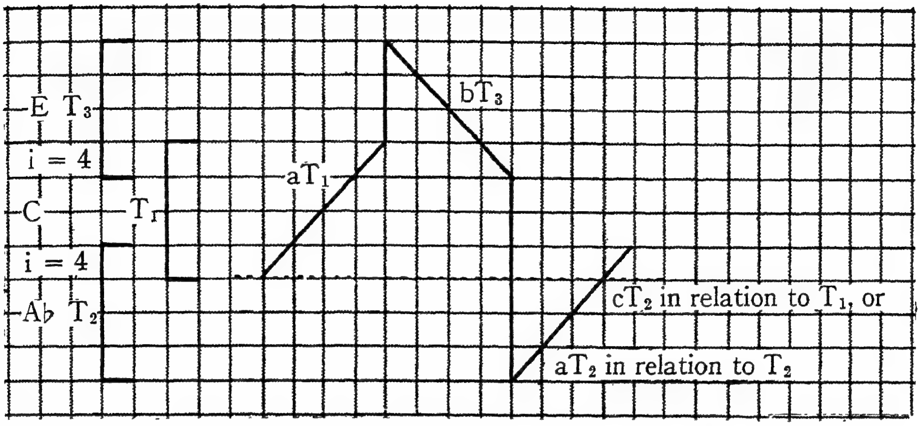


Figure 98. Another illustration of first tonic surrounded with other tonics.

Figure 99. Geometric projection of preceding figure (continued).

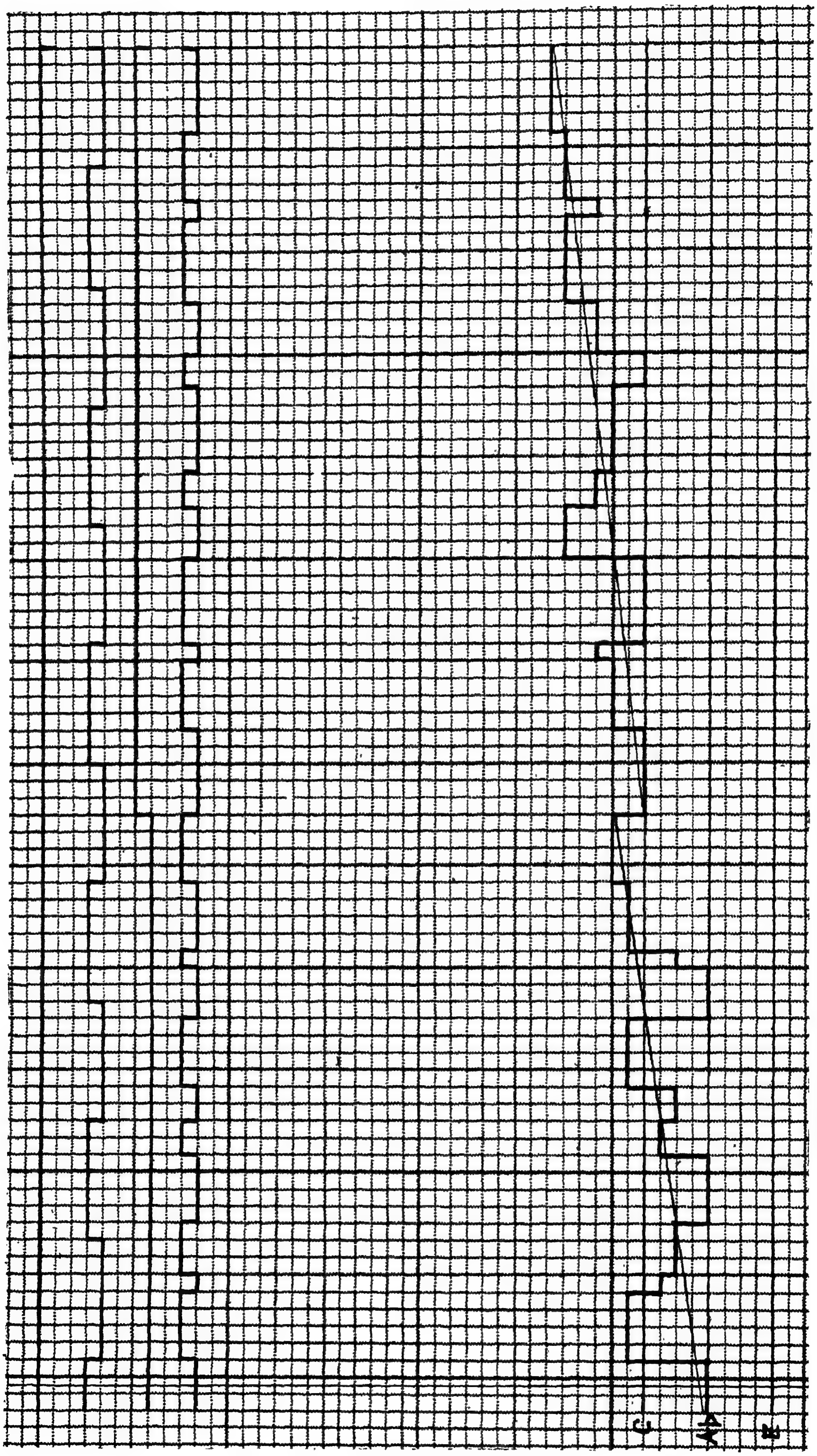


Figure 99. Geometric projection of preceding figure (concluded).

$[M\,a\ E_0p_0]+[A\,a\ E_0p_1]+[B\,a\ E_0p_2]+[C\,a\ E_0p_3]+[B\,b\ E_0p_4]+$
$[C\,b\ E_0p_5]+[M\,d\ E_0p_0]+[A\,a\ E_1p_0]+[M\,a\ E_0p_0]$

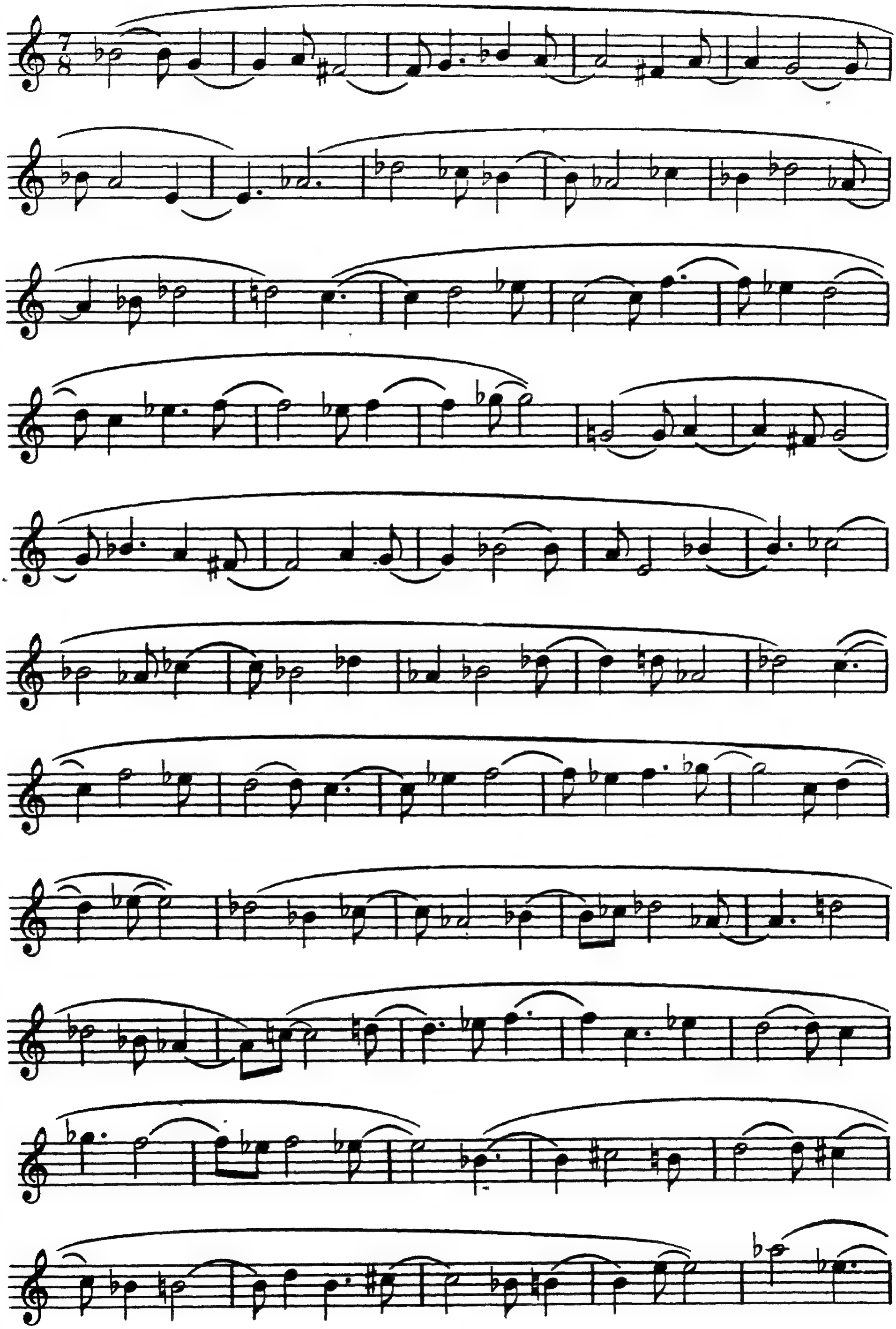


Figure 100. The melody of Figure 99 (continued).

Figure 100. The melody of Figure 99 (concluded).

B. Technique of Plotting Modulations

First: a modulation through common tones. Plot the scales of all keys in which the melody will appear on the left side of the graph. Draw all the pitch-levels which appear in common for any two of the adjacent keys at the corresponding period of time assigned for such modulation. Composition of durations for the modulation must be made in advance, i.e., at the time the entire continuity of time rhythm is planned. The final step is to drop perpendiculars from the points of attack upon the common pitch levels.

No selection of secondary axes for the period of modulation is necessary. You are free to choose the trajectorial phases. If the portion of melody appearing in the succeeding key starts on the primary axis, it is desirable to select the phases which will permit the use of leading tones. As modulation means a transition from one primary axis to another, it is necessary to plan the axial schemes for the adjacent keys *before plotting the modulation.*

A modulation through *chromatic alterations* must be plotted first rhythmically (i.e., by using long durations) and then by dropping perpendiculars on adjacent uncommon tones.

The technique of *identical motifs* requires first a rhythmic identity of adjacent groups and, secondly, imitation of the first configuration (motif belonging to the preceding key) carried out through pitch-levels of the following key. Both configurations must be in the same pitch-range.

Here is an example of melody plotted with all three types of modulations:

Melody in $\frac{8}{8}$ Series

Theme: $(5+3) + (3+2+3)$; Theme (factorial): aTP + bT2P

First Modulation (common tones): $(2 + 1 + 2 + 1 + 2)$

Second Modulation: (chromatic alterations): $(3 + 3 + 3 + 3 + 4)$

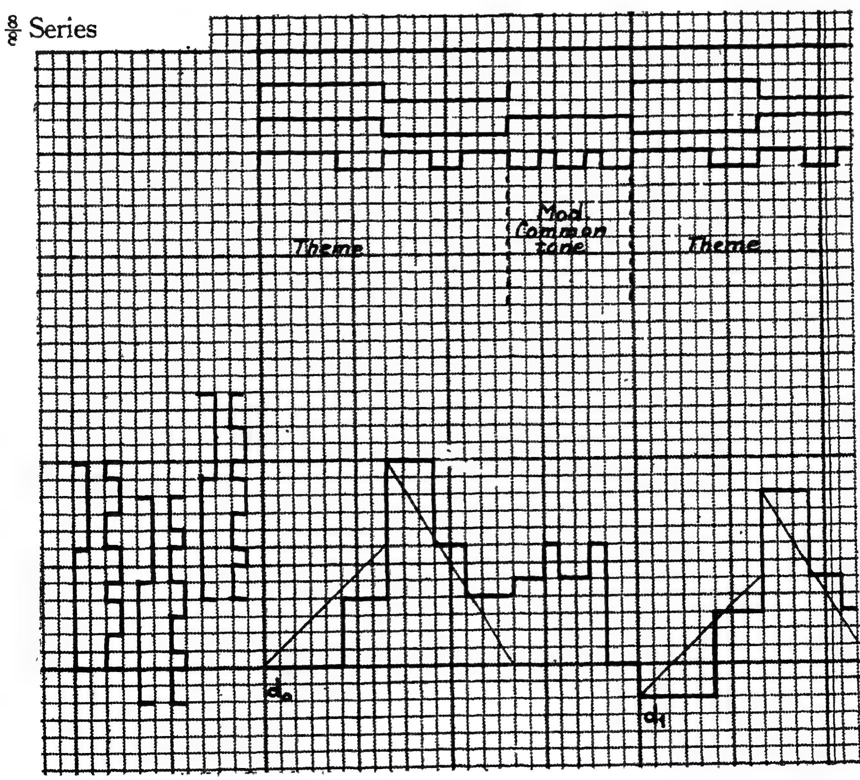
Third Modulation (identical motifs):

$$(3 + 2 + 1 + 1 + 1) + (3 + 2 + 1 + 1 + 1)$$

The Sequence of Keys and Scales:

(1) C	maj. natural:		P.A. d_0
(2) A♭	"	"	P.A. d_1
(3) G	"	"	P.A. d_5
(4) C	"	"	P.A. d_0

Modulating Melody Graphically Composed



Graph continued on following page

Figure 101. Plotted melody with three modulations (continued).

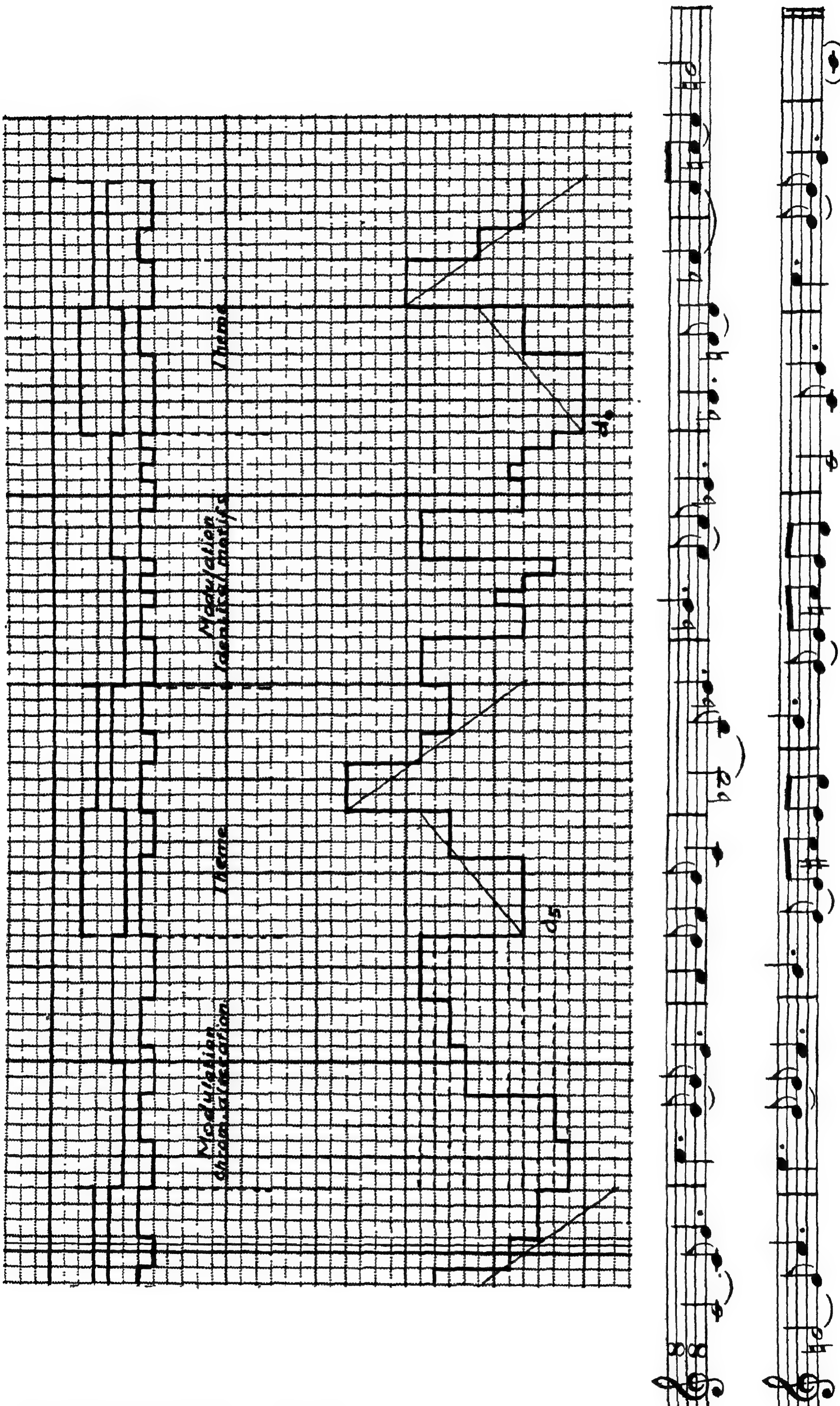


Figure 101. Plotted melody with three modulations (concluded).

Graph concluded from preceding page.

CHAPTER 8

USE OF ORGANIC FORMS IN MELODY

THE TERM *organic* is usually associated with living matter. The most obvious forms of organic existence manifest themselves in growth. Different rates of growth have been observed in different fields. Even the ancient Egyptians and Greeks had stumbled upon different forms of regularity, which they discovered as geometrical proportions of a rectangle. This discovery led to the development of a system of proportions expressing a harmonic relation between the preceding and the succeeding link. Numerical values arranged in an increasing order on the basis of this form of proportional growth became known in the 13th Century as *summation series.* It was formulated by the Italian mathematician, Fibonacci, and became known as the *Fibonacci series.*

This is how summation series were deduced on a purely geometrical basis. Take a square, and use the diagonal of it as a radius. From any of the four possible points of origin, draw an arc. Extend one of the sides which does not intersect the arc until the arc intersects it. Erect a perpendicular at this point of intersection and extend the opposite side of the square until it intersects the perpendicular. This newly formed rectangle possesses proportions which develop the Fibonacci series.

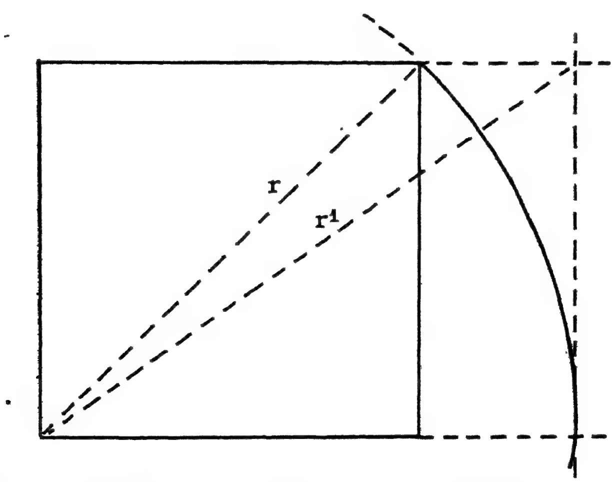


Figure 102. Deducing the Fibonacci series on a geometrical basis.

The Fibonacci series is based on the principle of adding every two consecutive numbers in a series to obtain the third.* Thus, starting with 1, we obtain 1, 2. By adding 1 and 2 we acquire the third number of this row: 1 +2 = 3.

*The Fibonacci series is a series in which the first term is one, the second term is two, and every term thereafter is the sum of the two immediately preceding terms. Other related series can, of course, be obtained by using some other number than two as the second term, thereafter proceeding to arrive at each term by adding the two immediately preceding terms. (Ed.)

The following numerical values are obtained in exactly the same way. This summation series developed through eleven terms acquires the following appearance: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144. These numerical values can be obtained purely geometrically, i.e., without computation and directly from the rectangle in Figure 102.

By drawing the diagonal of a rectangle, indicated in Figure 102 as r^1, we subdivide the entire area into two triangles known as "pyramid triangles". Let us consider the lower pyramid triangle for the development of the proportions representing the summation series.

Consider point V in Figure 103 a vertex of the triangle. Drop the perpendicular from point V on the base of the triangle. This produces the line p_1V. Now we have acquired a new triangle, Vp_1p_n. Dropping a perpendicular from the vertex p_1 on the base Vp_n, we acquire a new triangle, $p_1p'p_n$. Continuing this procedure further, we obtain a group of triangles which become partials of the original pyramid triangle. The lines p_1p' p_2p'' etc., produce the extensions which in turn represent the numerical values of the summation series.

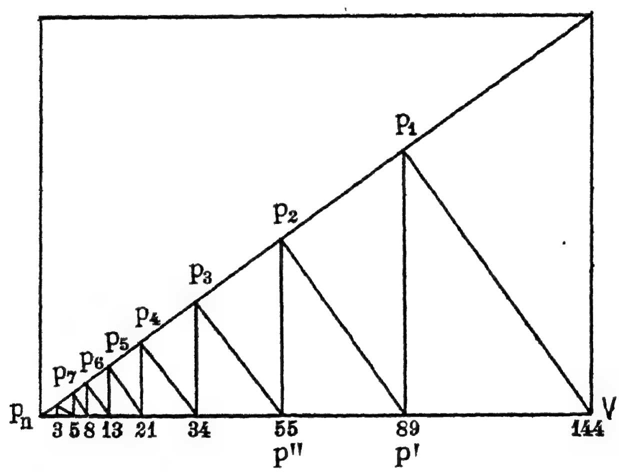


Figure 103. Partials of the pyramid triangles.

A clear realization of the principle of summation series as a foundation of beautiful proportions was presented by Luca Pacioli in his treatise, *De Divina Proportione* (1509). The principle of the "divine proportion" is derived from the ratios of the summation series. It is also known as "Gold Section," "Gold Cut" and "Golden Mean."

This particular proportion is $\frac{b}{a} = \frac{a}{a+b}$. This expression can be read: the short segment is related to the long segment as the long segment is related to the sum of both segments. The usual presentation is in the form of a subdivision of a given line through the "Golden Cut," that is, dissecting a line into two

segments so that the short segment is related to the long segment as the long segment is related to the whole original line. Michelangelo, a friend of Pacioli, applied the "Gold Section" ratio to proportions of the human body.

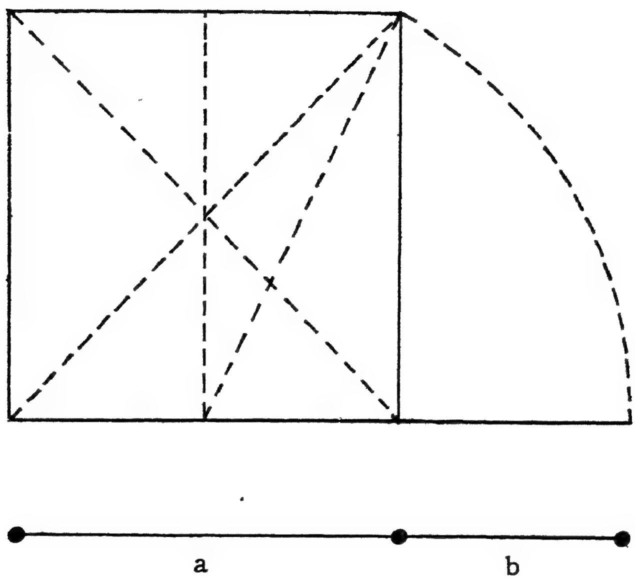


Figure 104. The Golden Mean.

Later, Leonardo da Vinci, while studying plant structures, discovered that the arrangement of leaves on a stem, or of various members of a plant, follows the spiral whose radii grow through the summation series. This study was followed up in the 20th century by A. H. Church in his *Principles of Phyllotaxis* (Oxford University Press).

Artists, and more particularly sculptors since ancient Greece, have devoted themselves to the subject of applying the ratios of the summation series to bodily symmetry. The first known contributor to this analysis was Polykleitos (5th century B.C., Greece). Professor Church has demonstrated that the formation of seeds in a sun flower, the tangent to a maple leaf, and other botanical patterns of growth follow the summation series. Artists and art theorists have tried to develop these principles to serve their purpose. An exhaustive study of spiral formations as they appear in plant and animal life was completed by Theodore A. Cook in his *Curves of Life.**

Renewed interest in summation series was stimulated by the publication of Jay Hambidge's *Dynamic Symmetry,*** in which he tried to apply this principle to pictorial composition. Jay Hambidge and Howard Giles have developed and applied these principles in their teaching of art in New York City. It met with great success and today has become so common that the principle of dynamic symmetry is applied even in the construction of such articles as radio cabinets.

*New York, 1914. **New York, 1920.

A thorough survey and analysis of the whole problem has been accomplished in very extensive research by Wilford S. Conrow, a New York artist, in his *The Ratios of Bodily Symmetry and Growth in Relation to Sculpture and Medical Science.** Some further developments of the Hambidge theory were made by one of his collaborators, Edward B. Edwards, in his *Dynamarhythmic Design.*

A property of the summation series known as Fibonacci series is that it contains symmetry throughout. The word *symmetry* emphasizes the equality of two measured ratios, according to an authority on the subject, Dr. William Churchill. Thus the adjacent portions of any structure following the summation series produces equality of ratios.

Summation series spirals can be constructed through a group of 90° arcs so that the value of the radius grows after every 90° through the summation series.

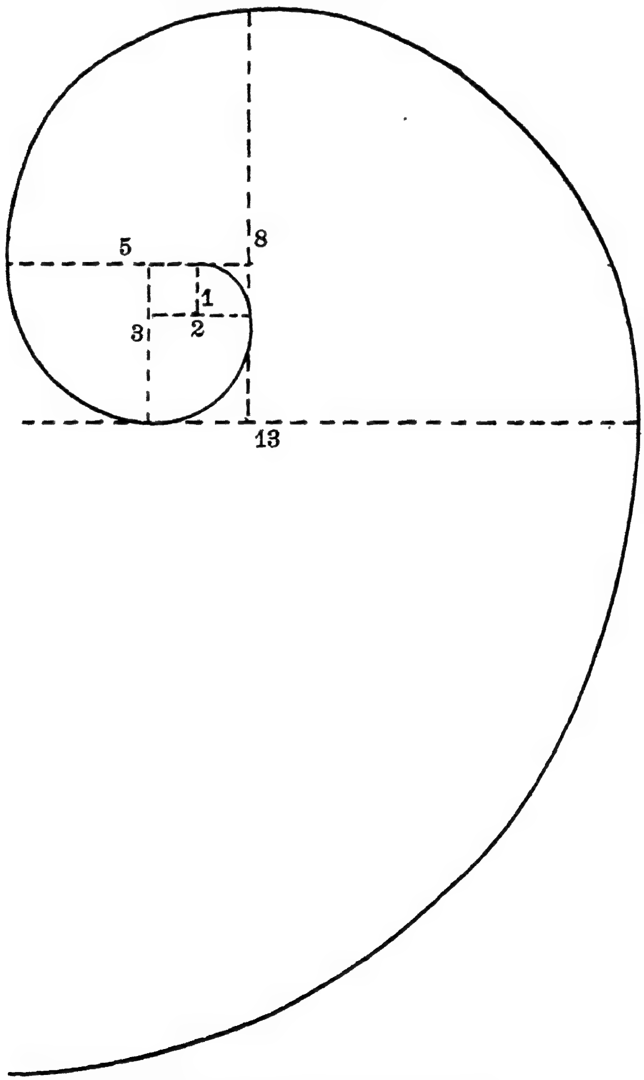


Figure 105. Summation series spirals.

*New York, 1937.

The values of the summation series may be applied to intonations as well. Portions of musical melody appealing to us as organic are based on identical principles of expanding intervals. In music, the unit of measurement for the intervals between the pitch units of an octave is expressed in semitones. The growth of semitones through the summation series in unilateral and bilateral symmetry develops motifs, i.e., melodic forms, which are truly organic as they exhibit the processes of growth of intervals. Such melodic forms can be often found as the outstanding themes of recognized composers as well as in folklore.

Historians and musicologists have an accepted term for such motifs, calling them "traveling" or "wandering" motifs. These motifs have such a universal appeal that, whether they appear in folk music or in the work of an individual composer, they become universally accepted as definite crystallized symbols of musical expression. It is interesting to note that "tonality" is an outcome of organically related number values and is not a "musical" quality a priori.

The unilateral symmetry of the Fibonacci series, applied to semitones, produces the following sequence:

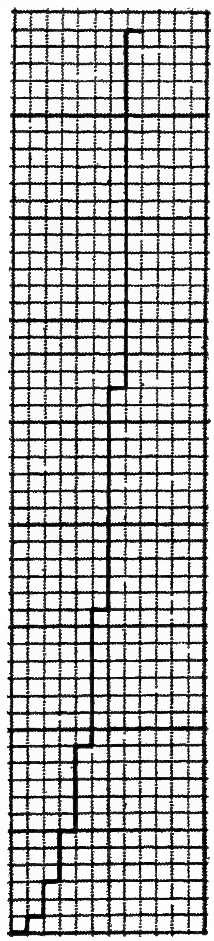


Figure 106. Unilateral symmetry of Fibonacci series (continued).

Unilateral Summation

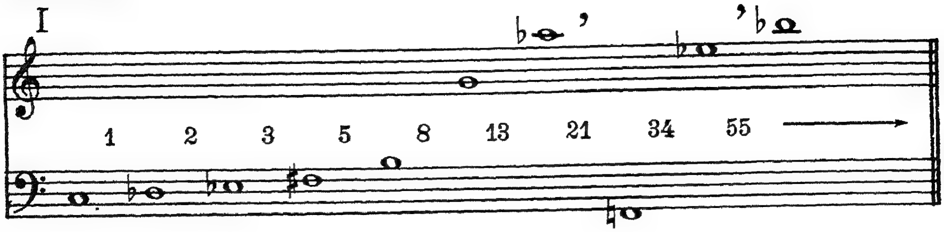


Figure 106. Unilateral symmetry of Fibonacci series (concluded).

As in every spiral, it is only in using a few successive links that we can achieve what we term "beauty." Beyond this the form becomes too extreme; and the same is true in music, too. Thus a melody seems more melodious if it emphasizes only the first few steps of the summation series. Beyond this point the intervals become so great that our conditioned perception of melody, as melody of a vocal type, is disturbed by such extreme dimensions. Some contemporary composers, however, use such intervals, being guided by a purely intuitive urge. Their ears are pleased and satisfied by such wide intervals. The most representative extremist in this field is the Austrian composer, Anton von Webern.

After a melody is constructed through summation series in the unilateral form, it is possible to produce any number of derivative melodies through *readjustment of the range*, i.e., by means of octave transposition of the corresponding pitch units. In such a case any spiral may be confined to a very limited range, yet produce intonations which originally were organically related. The following is range readjustment of the scale in Figure 106.

Range readjustment:

Figure 107. Range readjustment of scale in figure 106.

In addition to the Fibonacci series, a number of *other* summation series of the same class can be developed. We shall call the Fibonacci series the *first* summation series. In order to obtain the *second* summation series of the same class, i.e., by the addition of every two consecutive numbers, we have to start with 1 and add 3 instead of 2; thus we obtain: 1, 3, 4, 7, 11, 18, 29..... ... The third summation series introduces 4 after 1; thus we obtain 1, 4, 5, 9, 14, 23.........

It is easy to see that the number of summation series in this class is infinite. Other classes of summation series can be developed by obtaining every *fourth* value as the sum of the *three* preceding values. For example: 1, 2, 3, 6, 11, 20...... Further classes represent the addition of a greater quantity of numbers, and there is always an infinite number of series in each class.

My own applications of the various summation series to *design* as well as *music* (not only to pitch, but also to the development of durations) show that such groups of lines or durations or pitches affect us as *organic formations*.

Application of the second summation series to melody produces the following scale:

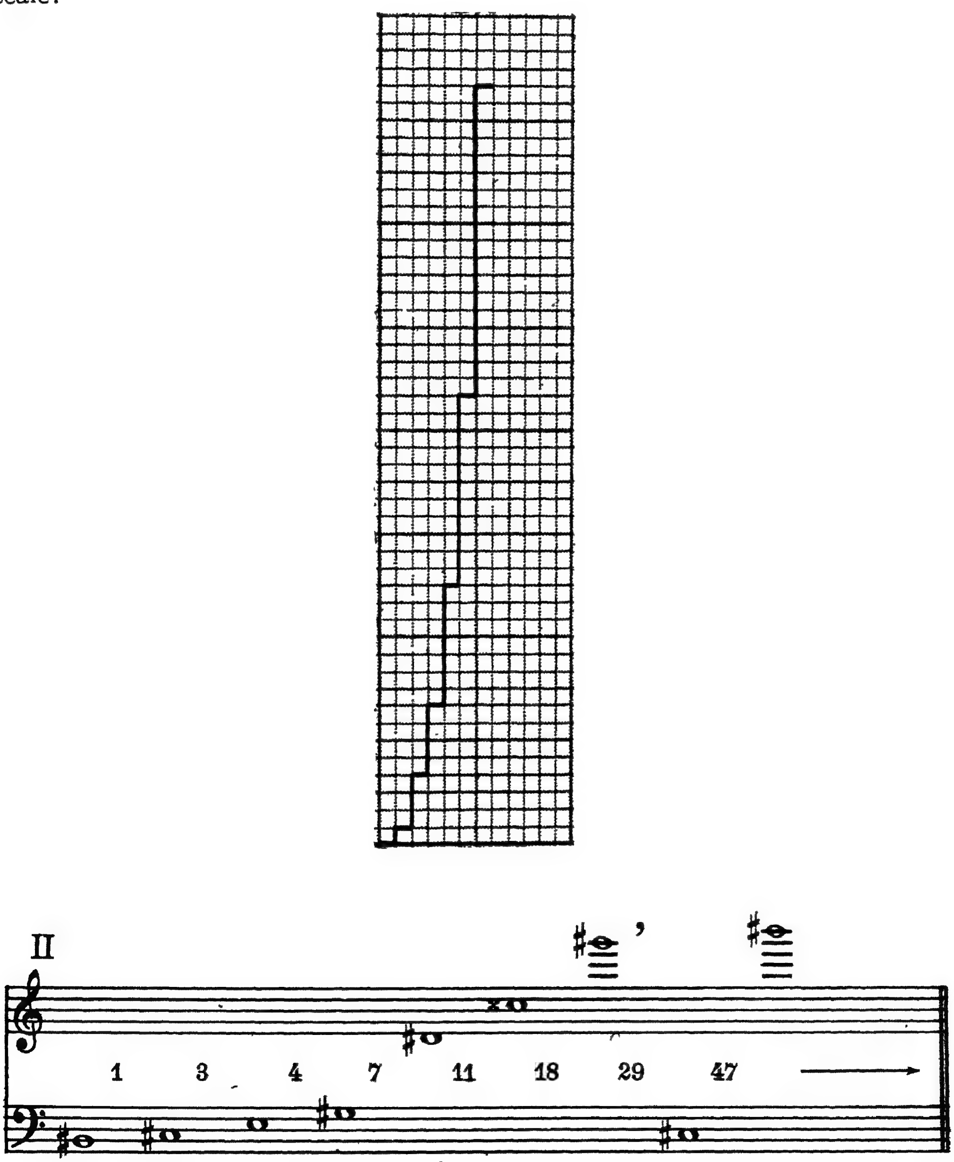


Figure 108. Second summation series applied to melody.

After readjustment of pitch ranges through octave inversion we obtain the following melodic forms:

Figure 109. Re-adjustment of pitch ranges in figure 108.

The above melodic forms are naturally only a few of the basic ones. The following figure represents the third summation series in unilateral symmetry and is followed by examples of readjusted ranges.

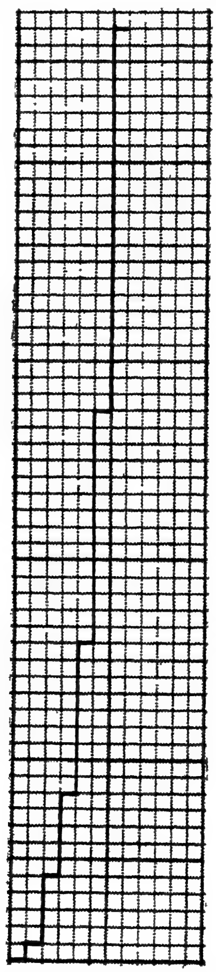


Figure 110. Third summation series in unilateral symmetry (continued).

III

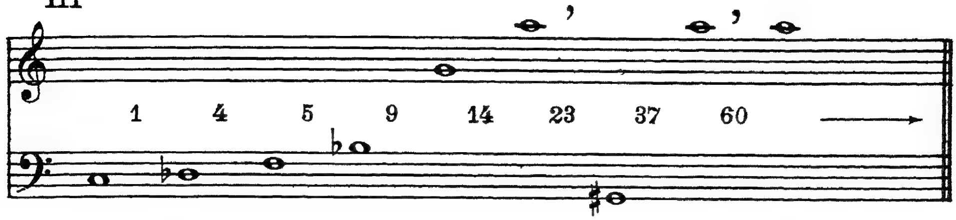


Figure 110. Third summation series in unilateral symmetry.

Range readjustment

Figure 111. Readjusted range in figure 110.

Forms of bilateral symmetry can be devised from summation series in a similar fashion. The values of a summation series follow the directions of an alternating spiral. Thus, if the first number represents an ascending interval, the second number represents a descending interval from the origin. Using the three summation series we obtain the following two fundamental forms.

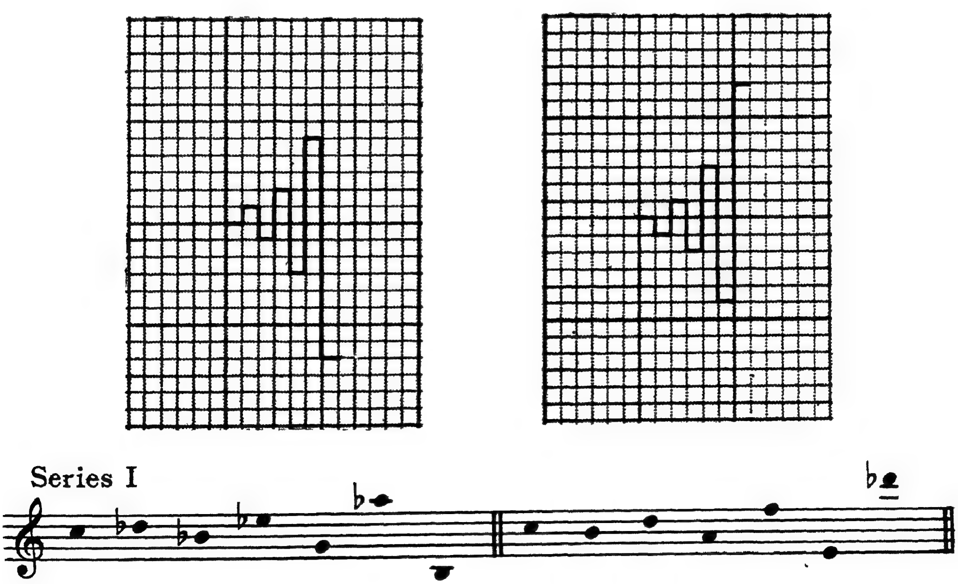


Figure 112. Bilateral symmetry in first summation series.

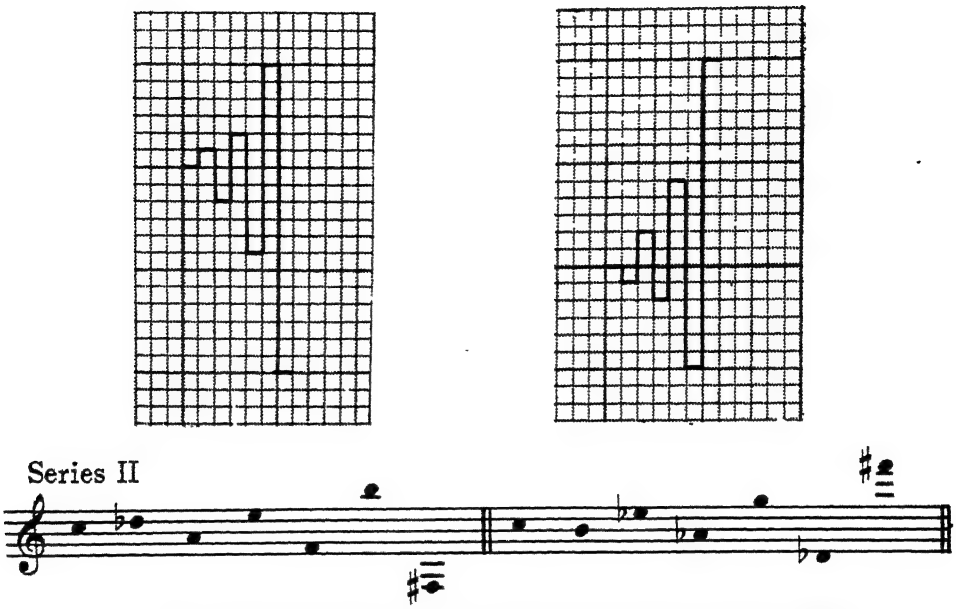


Figure 113. Bilateral symmetry in second summation series.

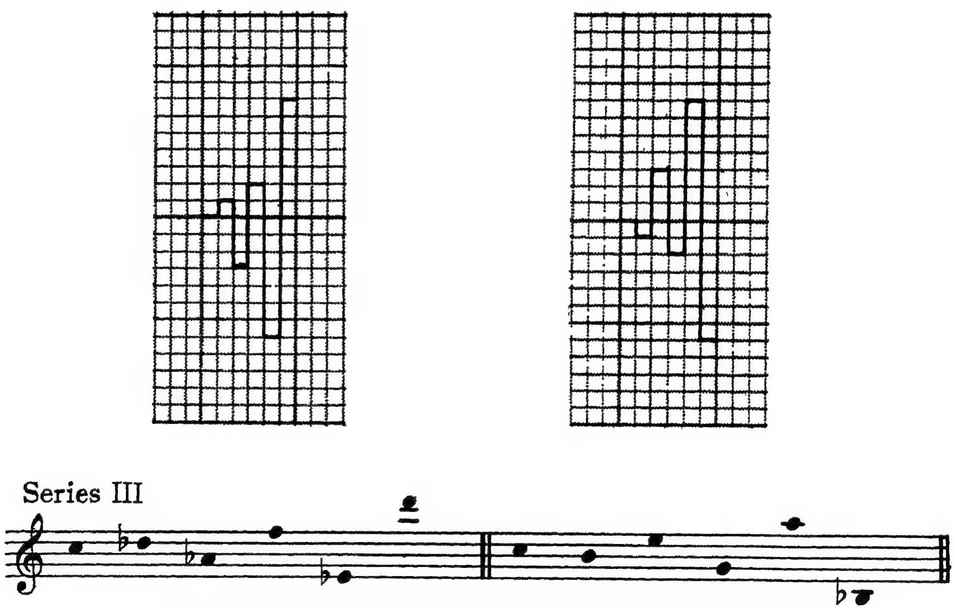
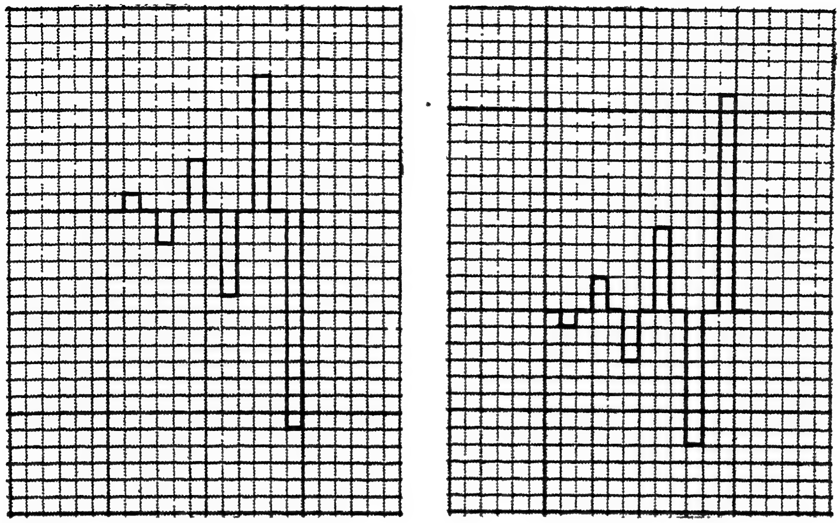


Figure 114. Bilateral symmetry in third summation series.

The readjustment of range is not necessary in the case of alternate spirals as it is more limited than in the case of unilateral spirals.

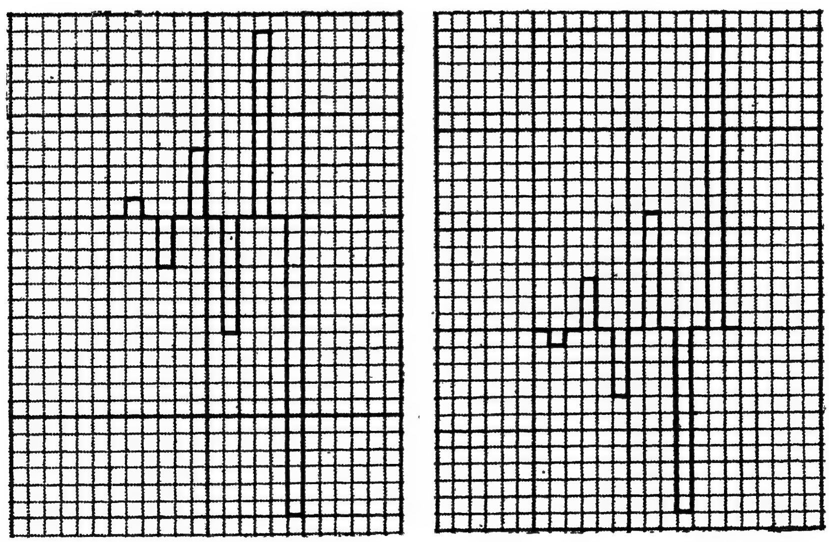
Another group of spirals can be developed with the use of bilateral structures and the inclusion of the axes. Such melodic forms being played at a relatively great speed produce effects of three parts moving in rapid alternation. From a musical standpoint, it is similar to a rapid arpeggio with one alternately repeated tone in the center.



Series I

1-1-2+2+3-3-5+5+8-8-13+13 -1+1+2-2-3+3+5-5-8+8+13-13

Figure 115. First summation series and alternating axes.



Series II

1-1-3+3+4-4-7+7+11-11-18+18 -1+1+3-3-4+4+7-7-11+11+18-18

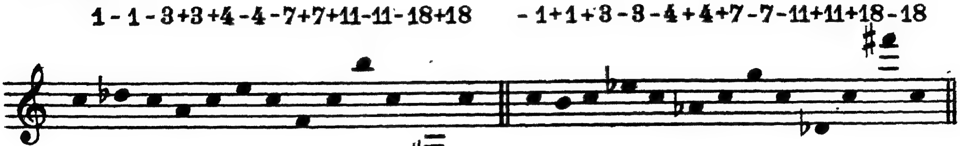


Figure 116. Second summation series and alternating axes.

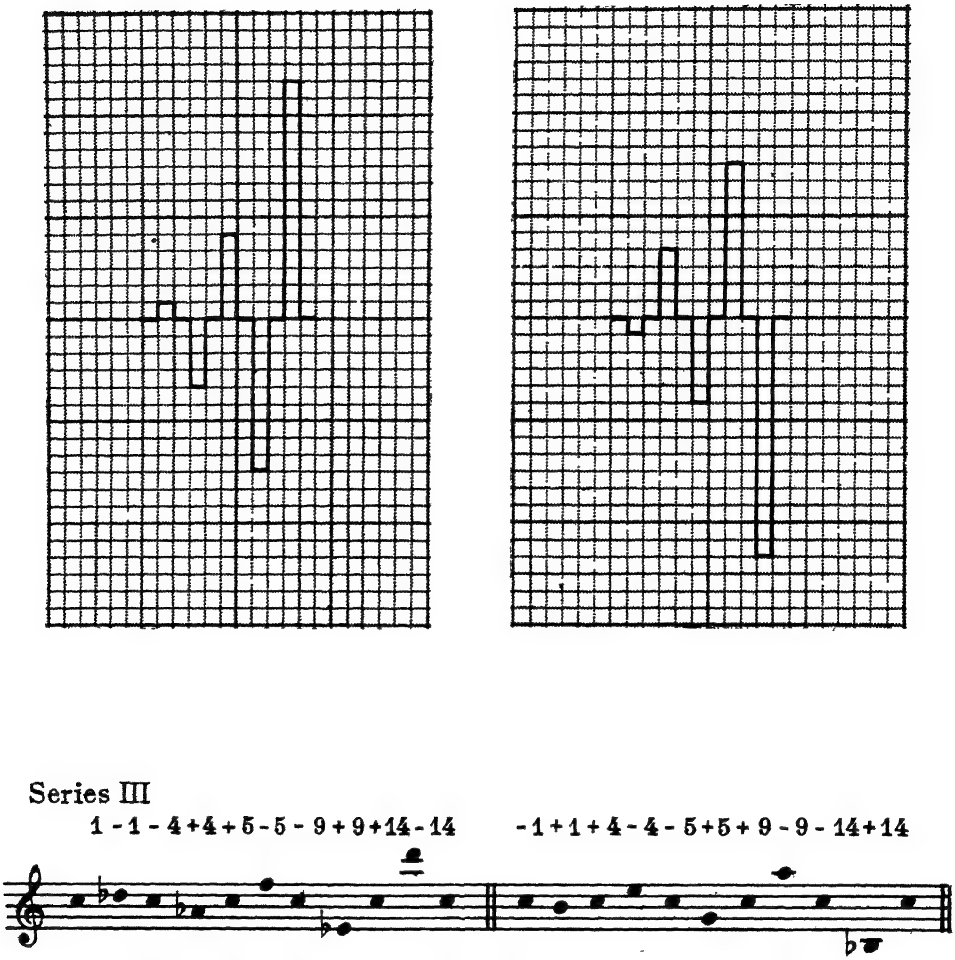


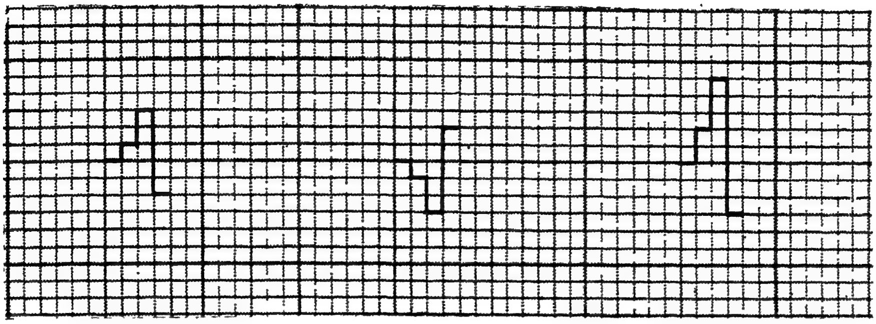
Figure 117. Third summation series and alternating axes.

The most satisfactory melodies from the viewpoint of their organic development are the spirals whose successive links involve movement in the opposite direction. The most common type of crystallized melodic forms usually corresponds to the following formula:

$$S^{\rightarrow} = t_1 + t_2 - t_4.$$

Assuming the ascending steps as the positive and the descending as the negative, we can transcribe the above formula as follows: the spiral sequence consists of the following terms of a summation series: the first term (t_1), followed by the second term (t_2), the *omission* of the third term and the appearance of the following term (t_4) with the opposite sign. The following organic forms of

melody may start at different points of one summation series and be carried out to any desirable limit. The following represents the application of this principle to the three summation series:



Summation Series

I

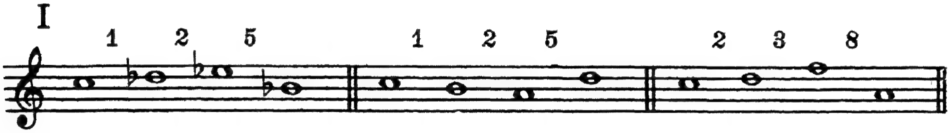
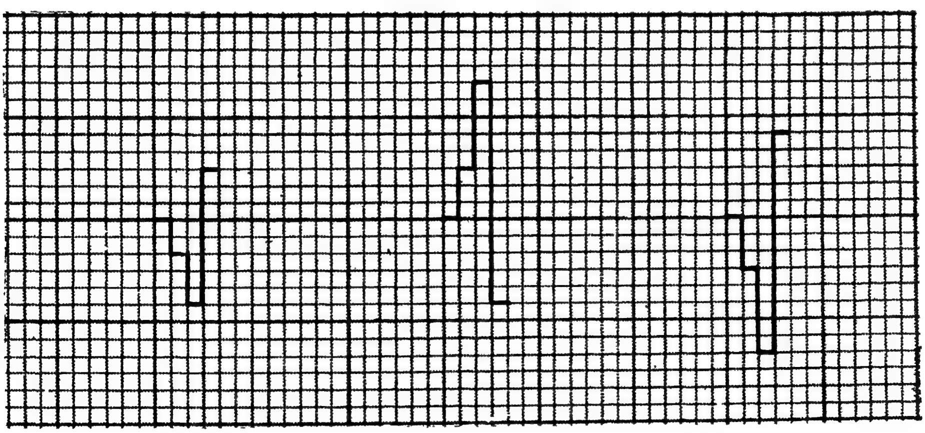


Figure 118. Spiral sequence of first summation series.



II

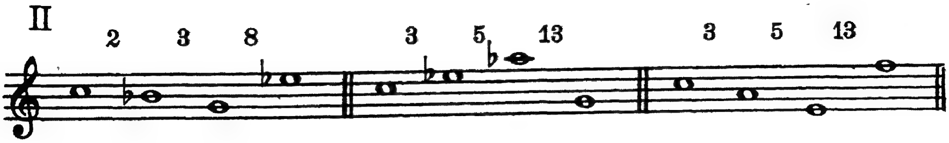
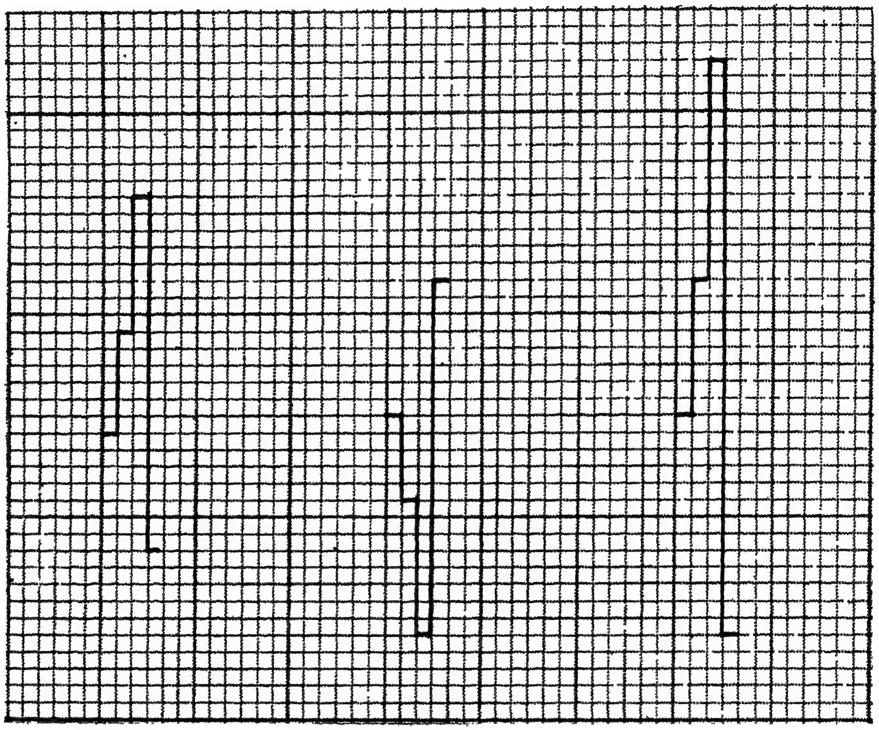


Figure 119. Spiral sequence of first summation series.



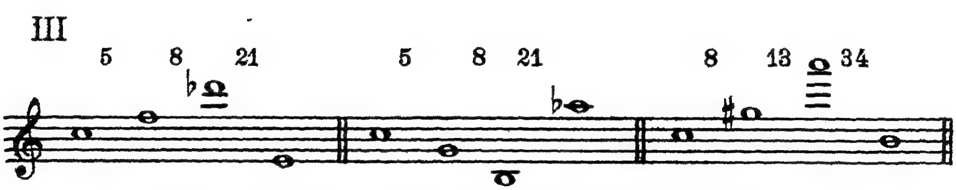


Figure 120. Spiral sequence of first summation series.

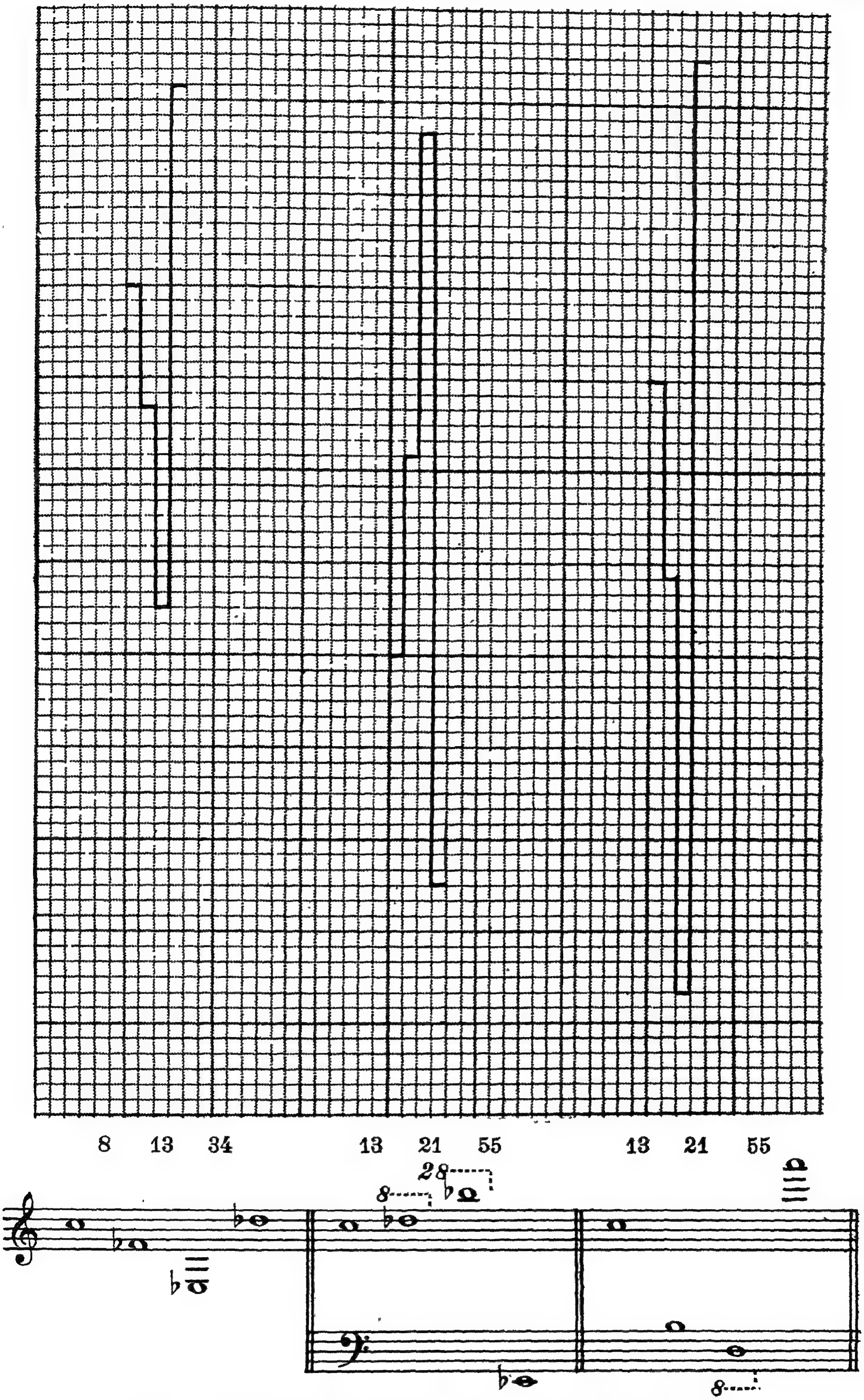


Figure 121. Spiral sequence of first summation series.

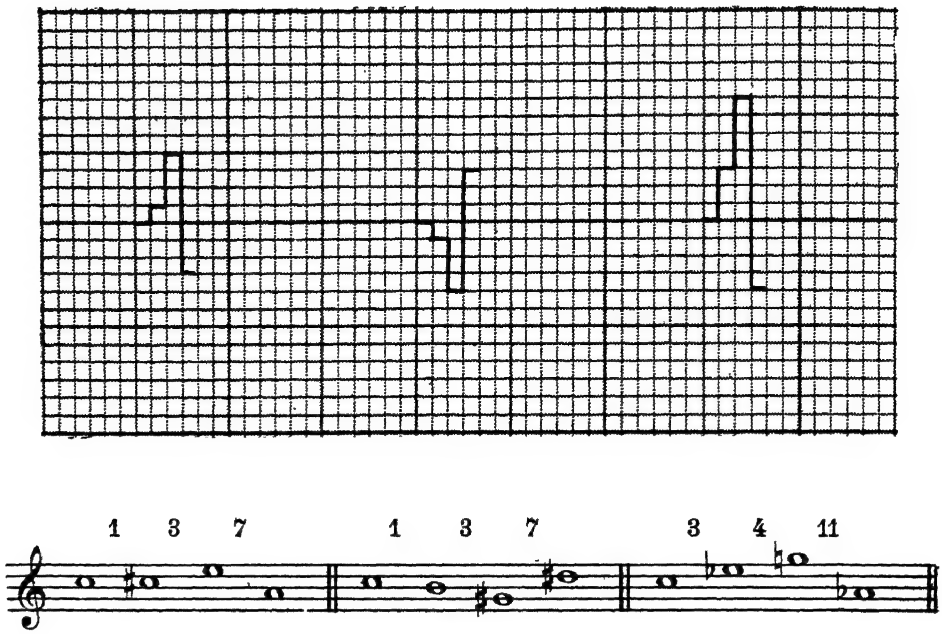


Figure 122. Spiral sequence of second summation series.

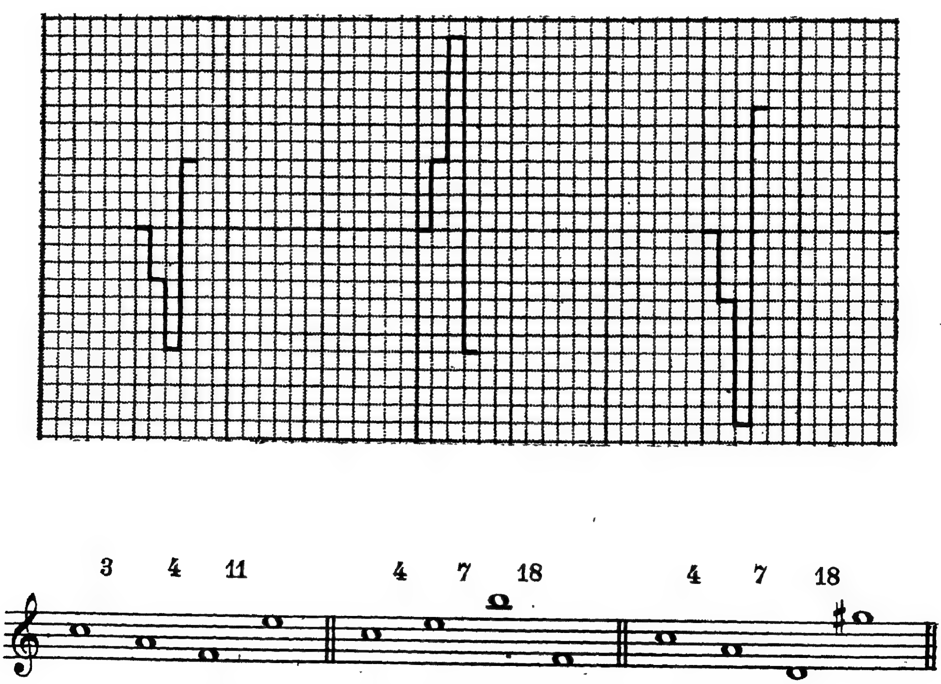
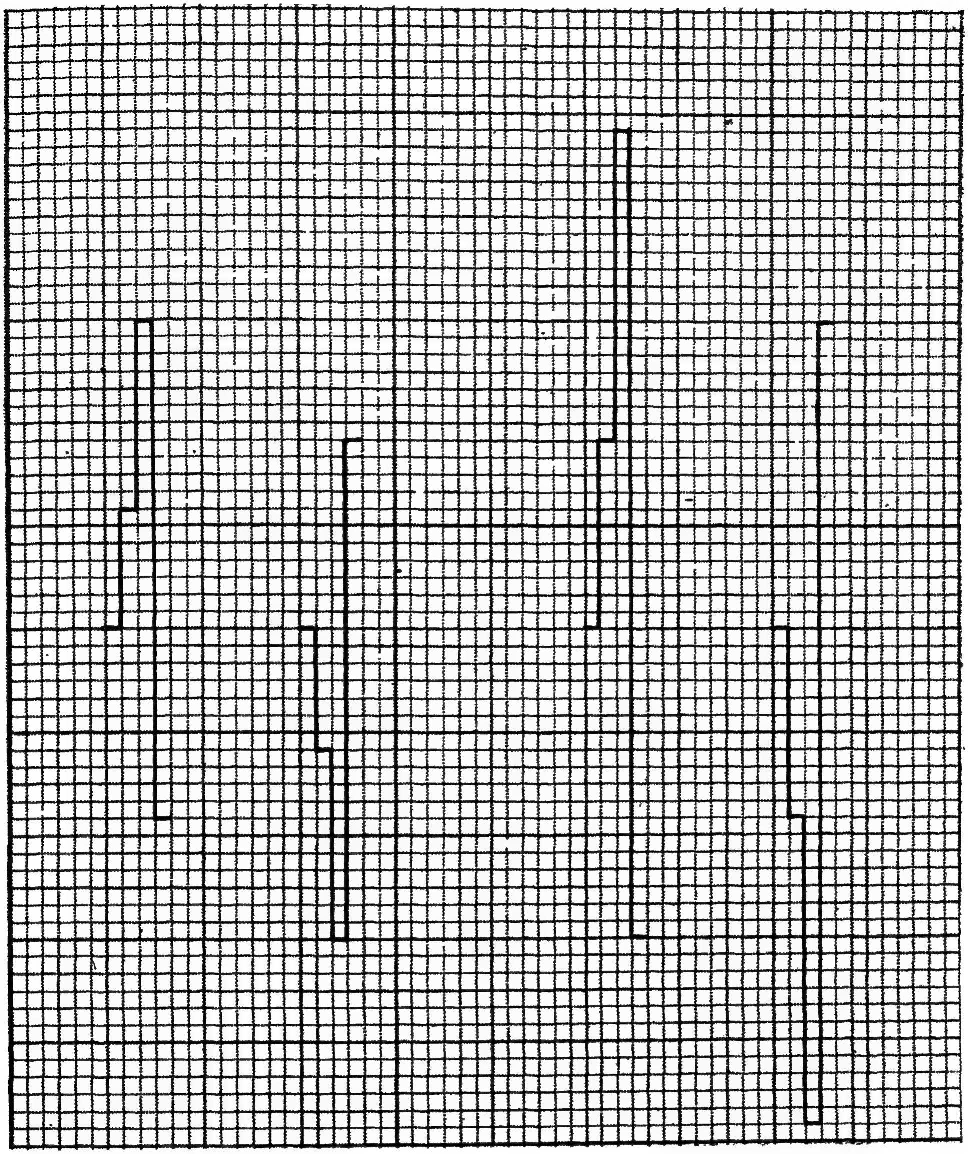


Figure 123. Spiral sequence of second summation series.



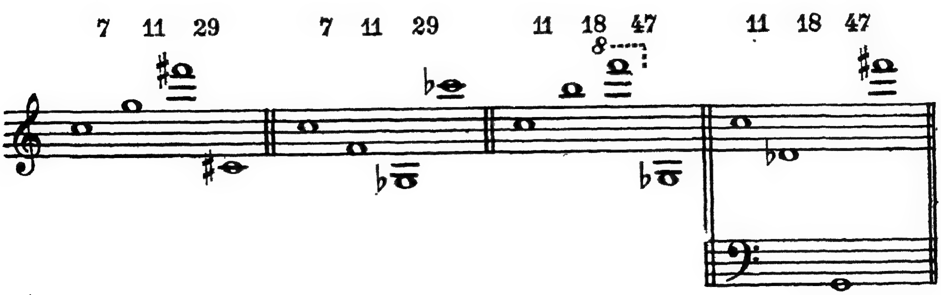


Figure 124. Spiral sequence of second summation series.

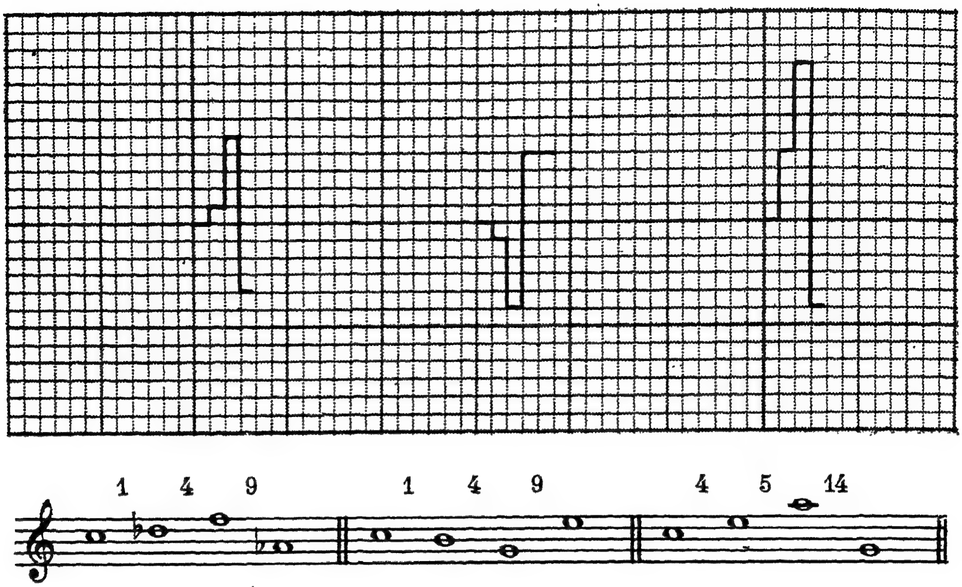


Figure 125. Spiral sequence of third summation series.

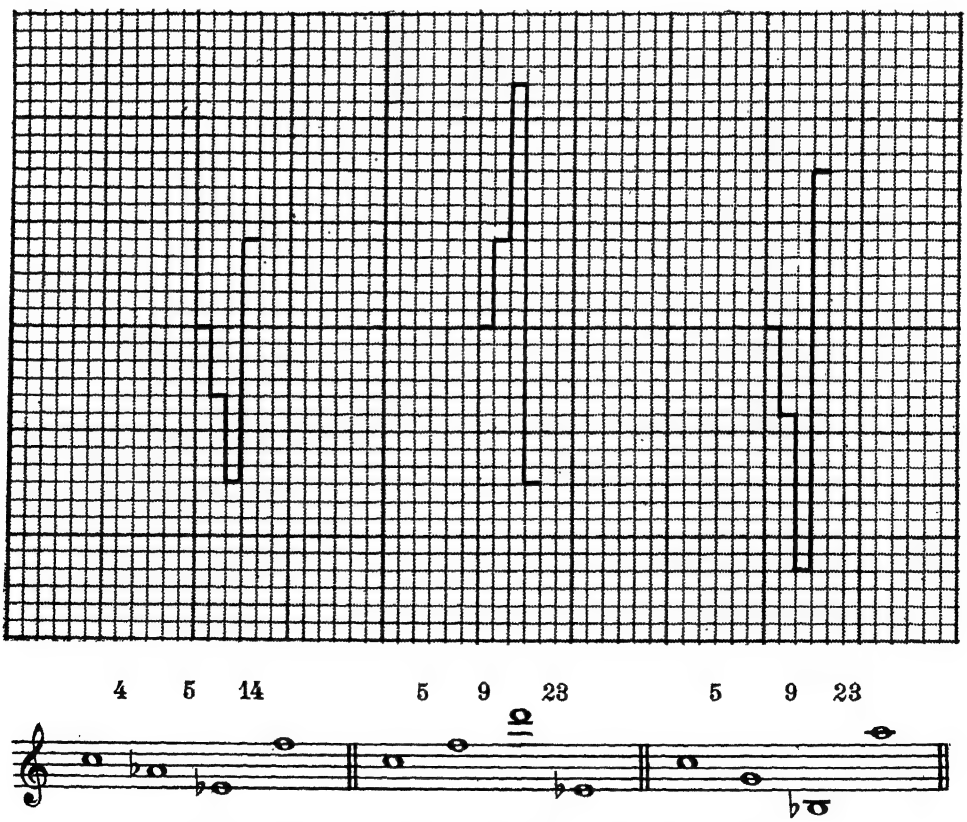


Figure 126. Spiral sequence of third summation series.

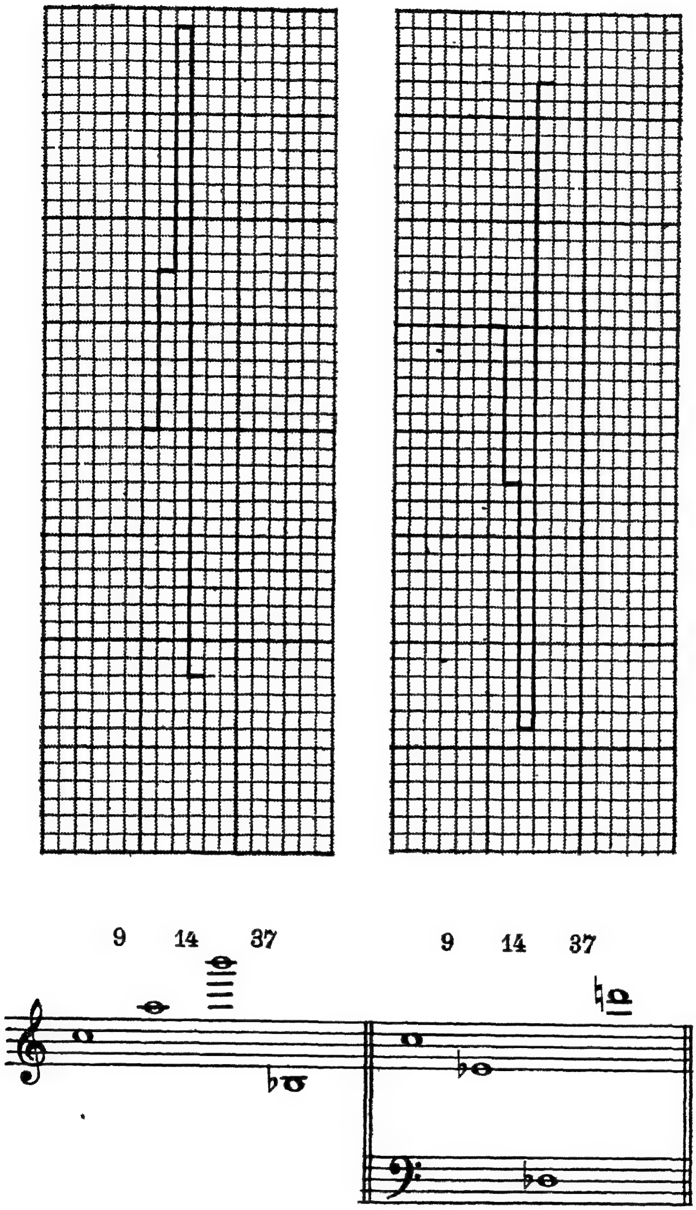


Figure 127. Spiral sequence of third summation series.

All the above forms contain four pitch units and three intervals. More developed forms of organic motifs can be obtained through the addition of three successive terms, the *omission* of one term, and the addition of the next term with the opposite sign:

$$S^{\rightarrow} = t_1 + t_2 + t_3 - t_5.$$

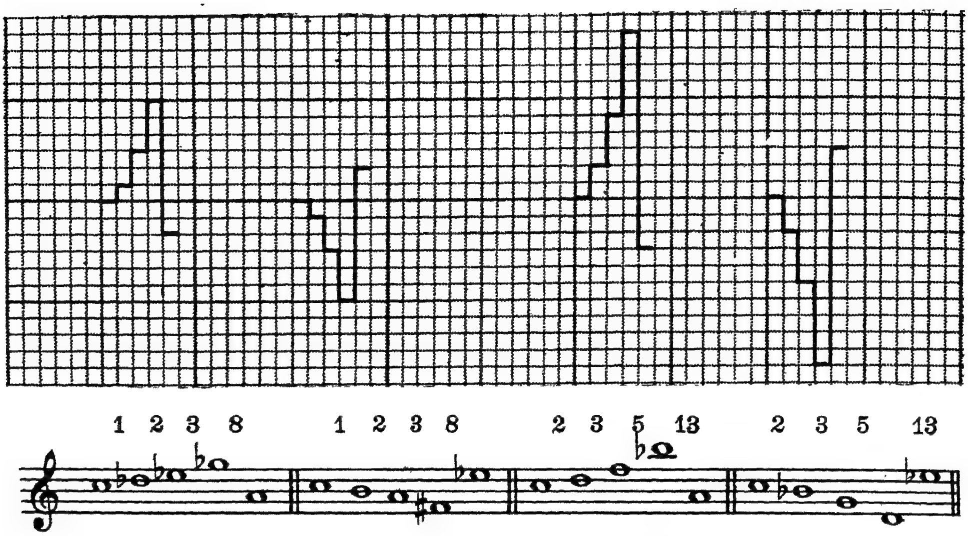


Figure 128. Spiral sequence of five pitch units in first summation series.

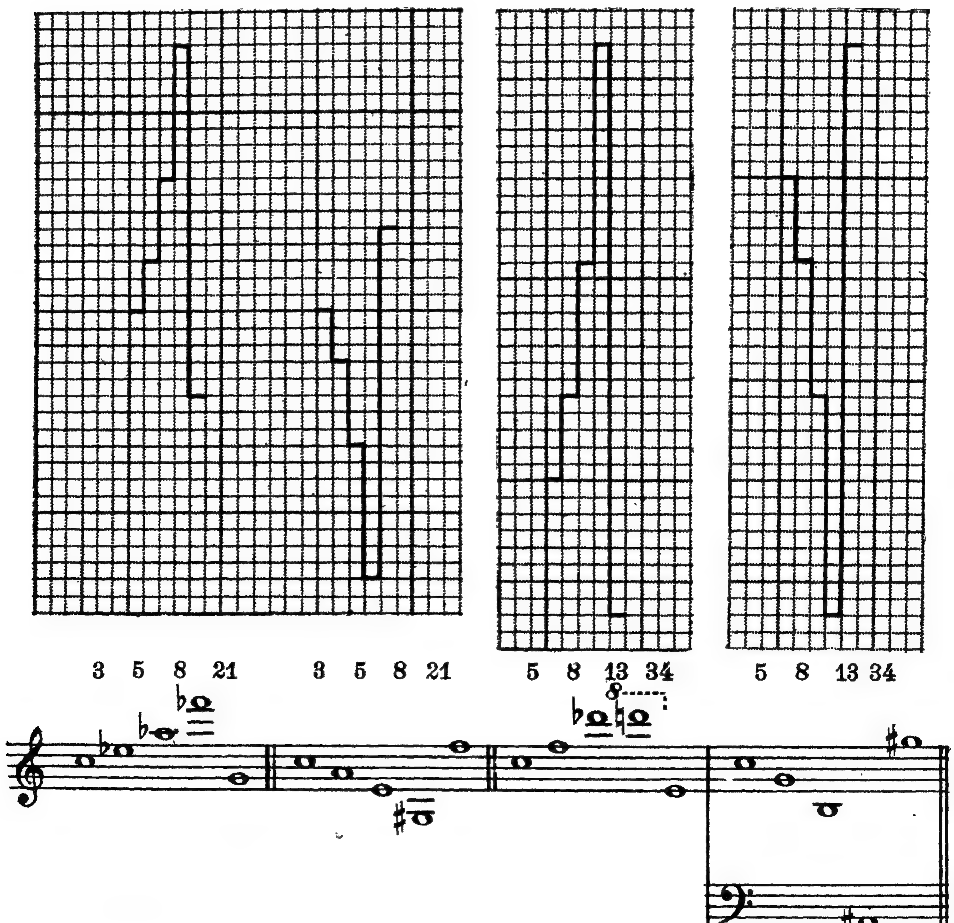


Figure 129. Spiral sequence of five pitch units in first summation series.

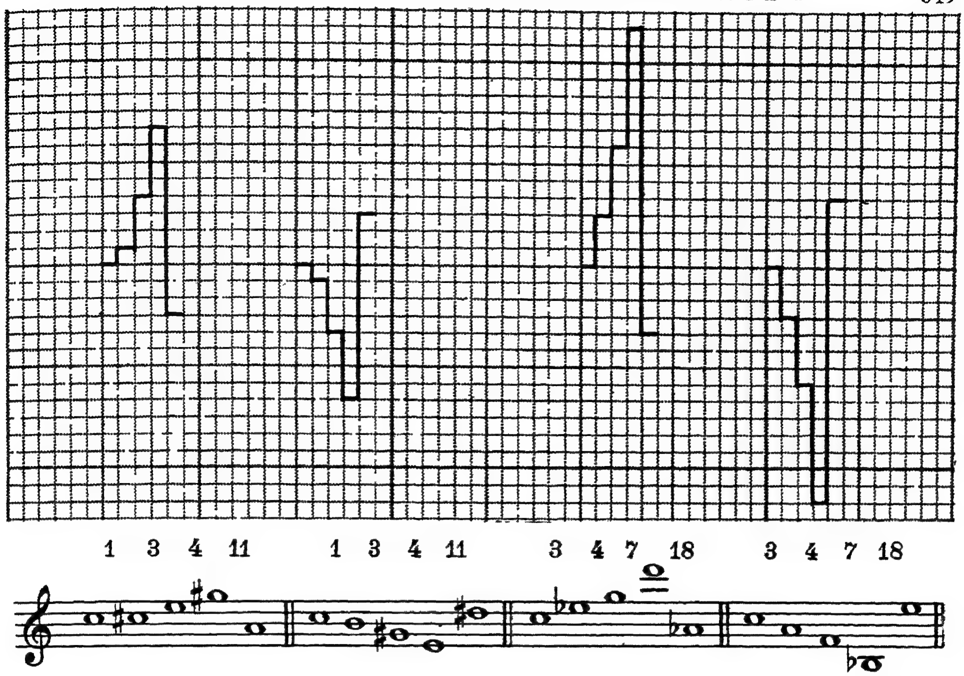


Figure 130. Spiral sequence of five pitch units in second summation series.

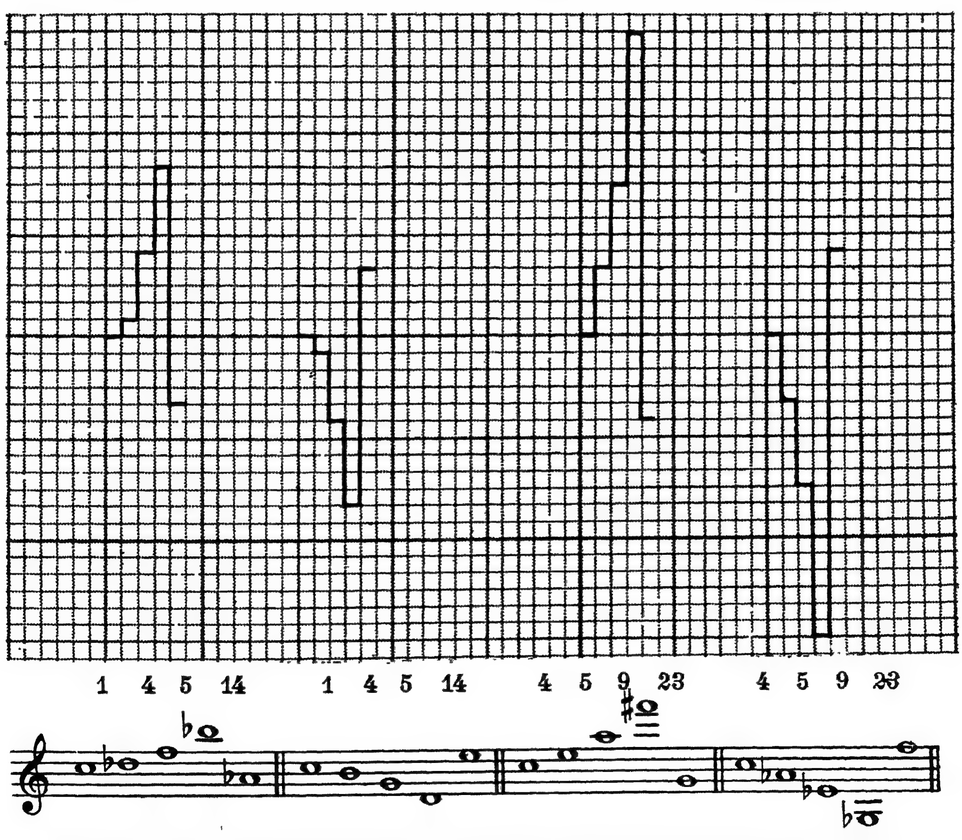


Figure 131. Spiral sequence of five pitch units in second summation series.

Another form of melodic spiral without the change of the original direction can be obtained through the omission of two terms after the summation of three terms and the appearance of the last term with the opposite sign: $S^{\rightarrow} + t_1 + + t_2 + t_3 - t_6$.

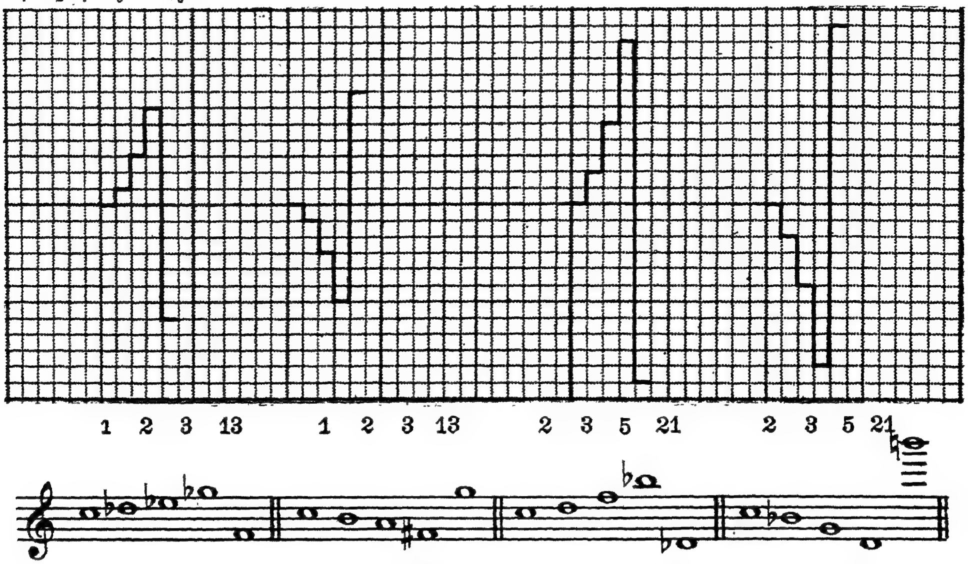


Figure 132. Another type of spiral sequence in first summation series.

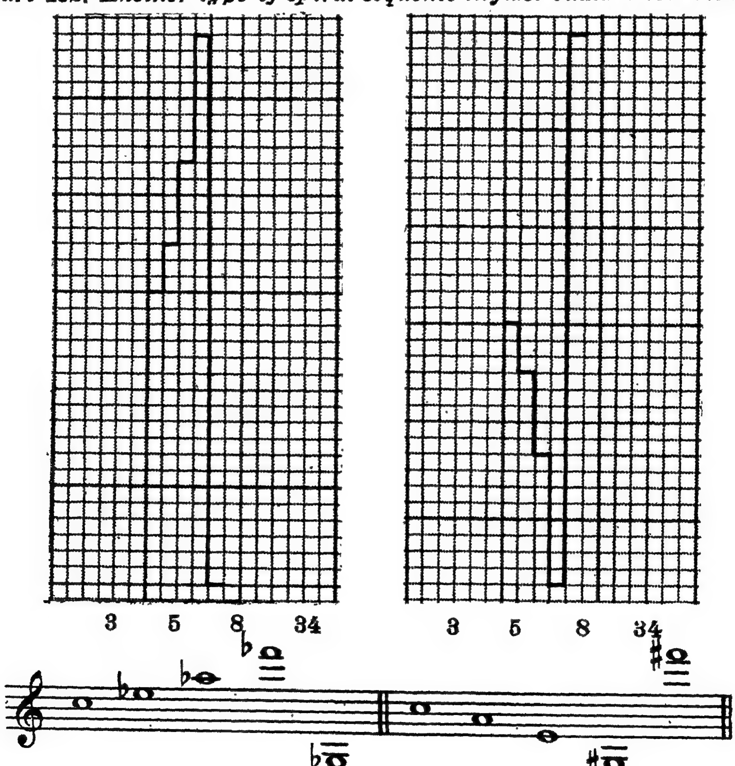


Figure 133. Another type of spiral sequence in first summation series.

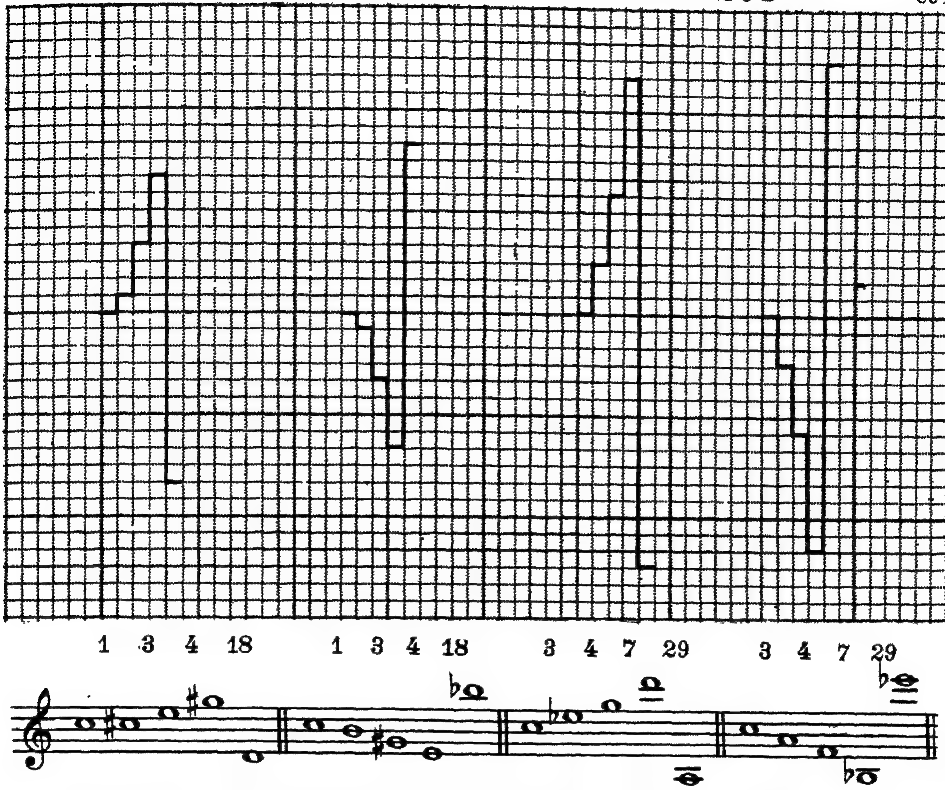


Figure 134. Another type of spiral sequence in second summation series.

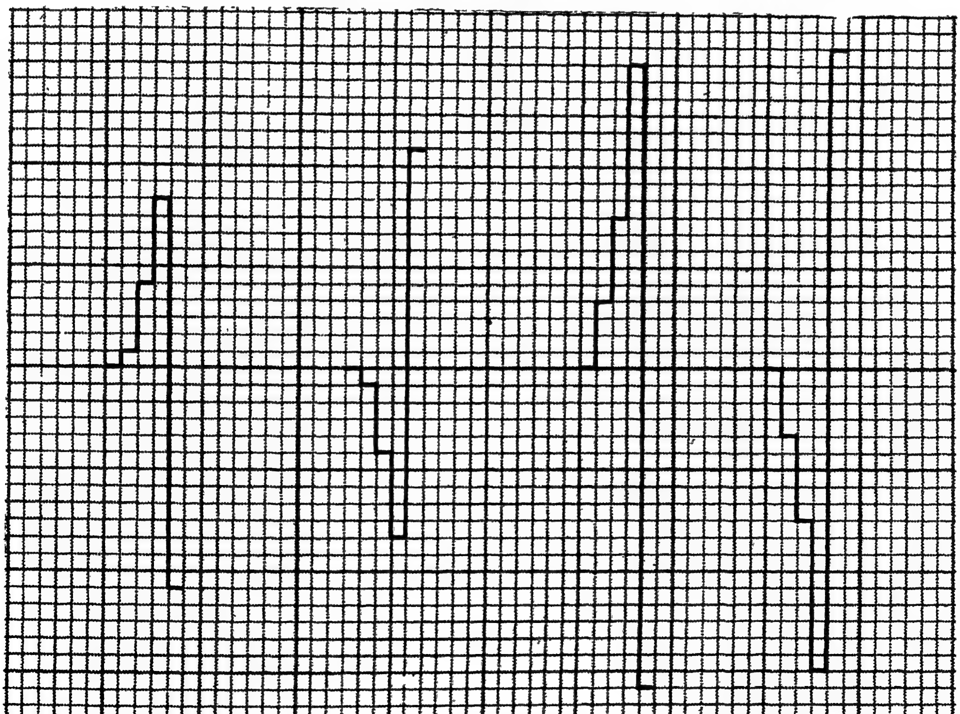


Figure 135. Another type of spiral sequence in second summation series (continued).

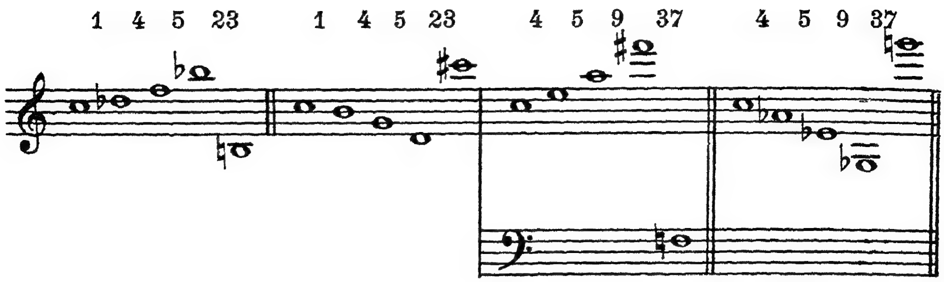


Figure 135. Another type of spiral sequence in second summation series.

Many other forms of the harmonic arrangement of numbers produce an *organic effect* upon the listener when such harmonic relations underlie the structure of melodic intervals.

Among such harmonic relations I will mention only the most fundamental ones:

1. Natural harmonic series.
2. Arithmetical progressions.
3. Geometrical progressions.
4. Involution series.
5. Various logarithmic series.
6. Progressive additive series.
7. Prime number series.
8. Arithmetical mean.
9. Geometrical mean.

These series of constant or variable ratios with harmonic arrangement of number values, when translated into an art medium, produce organic or nearly organic effects. Spiral formation as revealed through Summation Series affects us as being organic because there is *an intuitive interdependence of man and surrounding nature.* The patterns of growth stimulate in human beings a definite response which is more powerful than many other similar but casual formations.

Thus we see that the *forms of organic growth associated with life, well-being, self-preservation and evolution appeal to us as a form of beauty when expressed through an art medium.* Intuitive artists of great merit are usually endowed with great sensitiveness and intuitive knowledge of the underlying scheme of things. This is why a composer like Wagner is capable of projecting spiral formations through the medium of musical intonations without any analytical knowledge of the process involved. On the other hand, scientific analysis shows that the efforts of greatly endowed and creative persons could have been accomplished without any waste of time, introspection, or over-sensitiveness. Once the laws underlying certain structures have been disclosed, anyone can develop any number of structures in a class through the use of a formula. This does not prevent an artist, who makes an individual selection (whatever the value of such selection may be), from operating under the illusion of as great a freedom as that he imagines he possesses when he creates through the channels of vague intuition and nebulous notions.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK V

SPECIAL THEORY OF HARMONY

BOOK FIVE

SPECIAL THEORY OF HARMONY

CHAPTER 1

INTRODUCTION

MY SPECIAL theory of harmony is confined to E_1 of the first group of scales, which contains all musical names (c, d, e, f, g, a, b) and without repetition. There are 36 such scales in all. The total number of seven-unit scales equals 462.*

The uses of E_1 refer both to structures and progressions in the diatonic system of harmony. The latter can be defined as a system which borrows all its pitch units for both structures and progressions from any one of the 36 scales. When the structures are limited to the above scales but the progressions develop through all the semitonal relations of equal temperament, the latter comprises all the symmetric systems of pitch, *i.e.*, the third and the fourth group.

Chord-structures, contrary to common notion, do *not* derive from harmonics. If the evolution of chord-structures in musical harmony had paralleled the evolution of harmonics, we would never have acquired the developed forms of harmony we now possess.

To begin with, a group of harmonics when simultaneously produced at equal amplitudes, sounds like a saturated unison, not like a chord. In other words, a perfect harmony of frequencies and intensities does not result in musical harmony but rather in a unison. This means that through the use of harmonics, we would never have arrived at musical harmony. But actually, we do get harmony and for exactly the opposite reason. The relations of the sounds we use in equal temperament are not simple ratios (harmonic ratios).

When acousticians and music theorists advocate "just intonation", that is, the intonation of harmonic ratios, they are not aware of the actual situation. On the other hand, the ratios they give for certain familiar chords, like the major triad ($4 \div 5 \div 6$), the minor triad ($5 \div 6 \div 15$), the dominant seventh-chord ($4 \div 5 \div 6 \div 7$), do not correspond to the actual intonations of equal temperament. Some of these ratios, like $\frac{7}{4}$, deviate so much from the nearest intonation, like the minor seventh which we have adopted through habit, that it sounds to us out of tune.

Habits in music, as well as in all manifestations of life, are more important than natural phenomena. If the problem of chord-structures in harmony were confined to the ratios nearest to equal temperament, we could have offered $16 \div 19 \div 24$ for the minor triad, for example, as that ratio in fact approaches the tempered minor triad much more closely than $5 \div 6 \div 15$. But, if accepted, this would discredit the approach commonly used in all textbooks on harmony, for

*Special theory is used, of course, in distinction to general theory. Schillinger's special theory confines itself to structures built in thirds, whereas in the general theory—which is set forth at a later point in the work—the possibilities of harmonies which construct chords in fourths, fifths, etc., are discussed. (Ed.)

the following reason: if such high harmonics as the 19th are necessary for the construction of a minor triad, what would chords of superior complexity, which are in use today, look like when expressed through ratios? When a violinist plays b as a leading tone to c and raises the pitch of b above the tempered b, his claims for higher acoustical perfection are nonsense, as the nearest harmonic in that region is the 135th.

Facing facts, we have to admit that all the *acoustical* explanations of chord-structures—to the effect that they are developed from the simple ratios—are pseudo-scientific attempts to rehabilitate musical harmony and to give the latter a greater prestige. Though the original reasoning in this field resulted from the honest spirit of investigation of Jean Philippe Rameau (*Generation Harmonique*, Paris, 1737), his successors overlooked the development of acoustical science. Their inspiration was Rameau—plus their own mental laziness and cowardice.

The whole misunderstanding in the field of musical harmony is due to two main factors:

(1) underrating habit;

(2) confusion of the term "harmonic" in its mathematical connotation—i.e., pertaining to simple ratios—with "harmony" in its musical connotation—i.e., simultaneous pitch-assemblages varied in time sequence.

Thus, musical harmony is not a "natural phenomenon," but a highly conditioned and specialized field. It is the material of musical expression, for which we, in our civilization, have an inborn inclination and need. This need is cultivated and furthered by existing trends in our music and musical education.

CHAPTER 2

THE DIATONIC SYSTEM OF HARMONY

CHORD structures and chord progressions in the diatonic system of harmony have a definite interdependence: *chord-structures develop in a direction opposite to their progressions.*

This statement brings about the practical classification of the diatonic system into two forms: the *positive* and the *negative.*

As the term *diatonic* implies, *all pitch-units of a given scale* constitute both structures and progressions, without the use of any other pitch-units (those not existing in a given scale) whatsoever.

In the form which we shall call *positive*, all chord structures (S) are the component parts of the entire structure (Σ) emphasizing all pitch-units of a given scale in their *first tonal expansion* (E_1) and in position ⓐ. In the same form, chord progressions derive from the same tonal expansion but in position ⓑ.

In the *negative* form of the diatonic system, it works in the opposite manner. Chord-structures derive from the scale in E_1 and in position ⓑ, while the progressions develop from E_1ⓐ.

By reason of the personal qualities we have inherited and developed, the positive form produces an effect of greater tonal stability upon us. It is chronologically true that the negative form is an *earlier* one. It predominates in the works where the effect of tonality, as we know and feel it today, is rather vague. Such is 14th and 15th century ecclesiastic music, developed on contrapuntal, not harmonic, foundations.

Many theorists confuse the negative form of the diatonic system with "modal" harmony. Since to them diatonic tonality generally means natural major or harmonic minor scales moving in the positive form, they notice the lack of tonal stability when harmony moves backwards. Losing tonal orientation, they mistake such *progressions* for modes—and modes are merely derivative scales, and may also have the positive, as well as the negative, form. But—as we have seen in the *Theory of Pitch Scales**—modes can be acquired from any original scale through the introduction of accidentals (sharps and flats).

In the following table, MS represents "melody scale" (pitch-scale), and HS represents "harmony scale" (i.e., the fundamental sequence of chord progressions).

Diatonic System

Positive Form	*Negative Form*
$\Sigma = MS_{E_1ⓐ}$	$\Sigma = MS_{E_1ⓑ}$
$HS = MS_{E_1ⓑ}$	$HS = MS_{E_1ⓐ}$

*See Book II, Chapter 3.

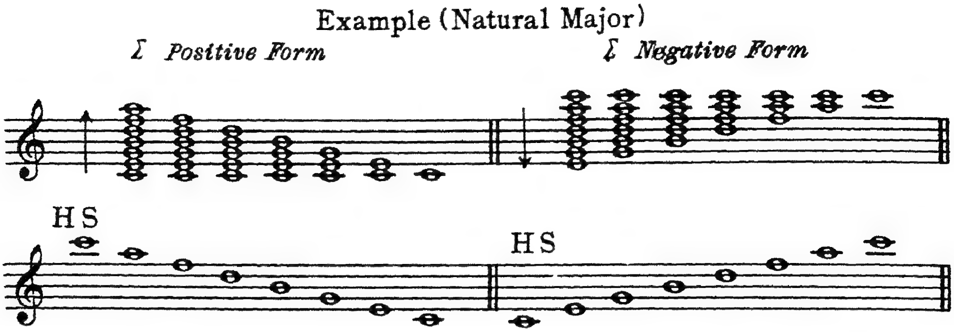


Figure 1. Diatonic system, negative and positive forms.

In the positive form, chords are constructed upward; in the negative, on the contrary, downward. The matter is greatly simplified by the fact that any progression, originally written as positive, becomes negative when read backwards. *All the principles of structures and motion involved are therefore reversible.* No properly constructed harmonic continuity can be wrong in backward motion.

Some composers without training in harmony (for example, Modest Moussorgsky)—as well as beginners because of inadequate study—confuse the positive and the negative forms in writing their harmonic progressions. The resulting effect of such music is a vague tonality. The admirers of Moussorgsky consider such style a virtue (in Moussorgsky's case it is about half-and-half positive and negative) and do not realize that all the incompetent students of a harmony course incompetently taught possess full command over such a style.

A. Diatonic Progressions (Positive Form)

Expansions of the original harmony scale produce the derivative harmony scales. The original HS and its expansions form the *diatonic cycles.* Diatonic (or tonal) cycles represent all the fundamental chord progressions.

There are three Tonal Cycles in the positive form for the seven-unit scales. The first cycle, or *cycle of the third* (C_3), corresponds to HS_{E_0}; the second cycle, or *cycle of the fifth* (C_5), corresponds to HS_{E_1}; the third cycle, or the *cycle of the seventh* (C_7), corresponds to HS_{E_2}. Beyond these expansions of HS lie the negative forms of the diatonic progression.

In addition to both forms of progressions, there may be changes in a chord pertaining to the same root (axis). Connections of an S with its modified S of the same root will be considered a *zero cycle* (C_0).

In the following table, notes are used merely for convenience; they indicate the sequence of roots; their octave position is dictated by purely melodic considerations and by the necessity of moderating the range.

The respective intervals representing cycles must be *constructed* downward for the positive form regardless of their actual position on the musical staff.

Diatonic Cycles (Positive Form)

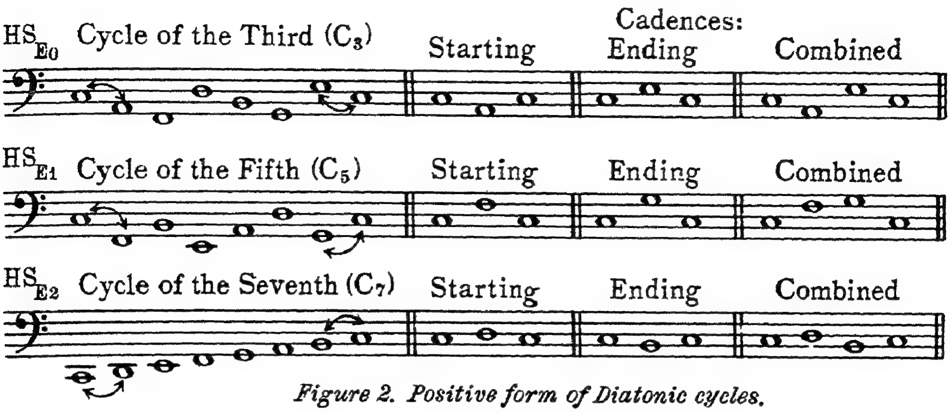


Figure 2. Positive form of Diatonic cycles.

In the above table, arrows indicate *cadences* of the respective cycles. Cadences consist of the axis-chord moving into its adjacent chord and back. It is interesting to note that what are usually known as *plagal cadences* are the *starting cadences* of the cycle, and that *cadences* known as *authentic* are the *ending cadences.* The immediate sequence of starting and endings cadences produces *combined cadences* (the axis-chord is omitted in the middle).

Progressions of constant tonal cycles (C_3, or C_5, or C_7 const.) produce a sequence of seven chords each appearing once and none repeating itself. The repetition of the axis-chord either completes the cycle or starts a new one. The addition of cadences to the cycles is optional as cycles are self-sufficient.

Considering constant cycles as a form of *monomial progression,* we can devise *binomial* and *trinomial progressions* by assigning a sequence of two or three cycles at a time.

In binomial progressions *each chord* appears *twice* and in a different combination with the preceding and the following chord. Thus, a complete *binomial cycle* in a seven-unit scale consists of $(2 \times 7 =)$ 14 chords.

Binomial Cycles

$C_3 + C_5$	$C_5 + C_3$	$C_7 + C_3$
$C_3 + C_7$	$C_5 + C_7$	$C_7 + C_5$

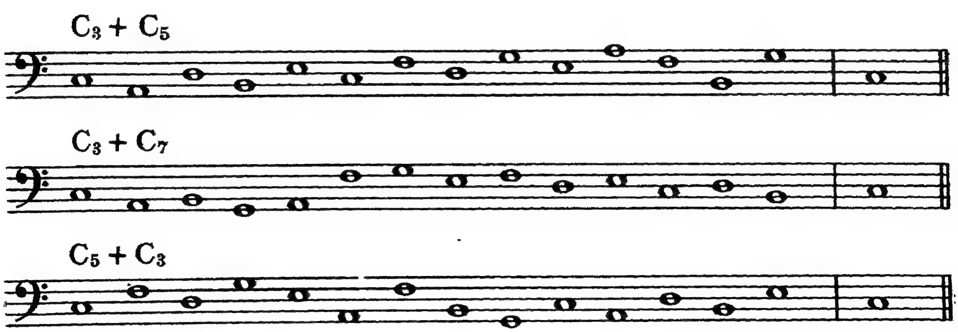


Figure 3. Binomial cycles (continued).

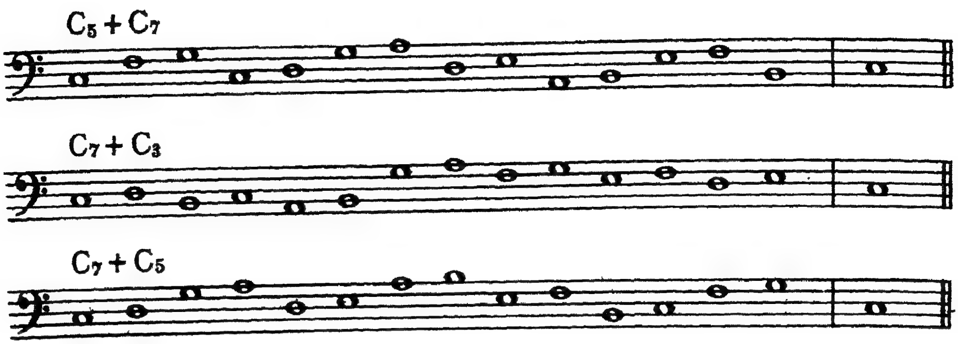


Figure 3. Binomial cycles (concluded).

In trinomial progressions *each chord* appears *three times* and in a different combination from the preceding and the following chord. Thus, a complete *trinomial cycle* in a seven-unit scale consists of $(3 \times 7 =)$ 21 chords.

Trinomial Cycles

$C_3+C_5+C_7$	$C_7+C_3+C_5$	$C_5+C_7+C_3$
$C_3+C_7+C_5$	$C_5+C_3+C_7$	$C_7+C_5+C_3$

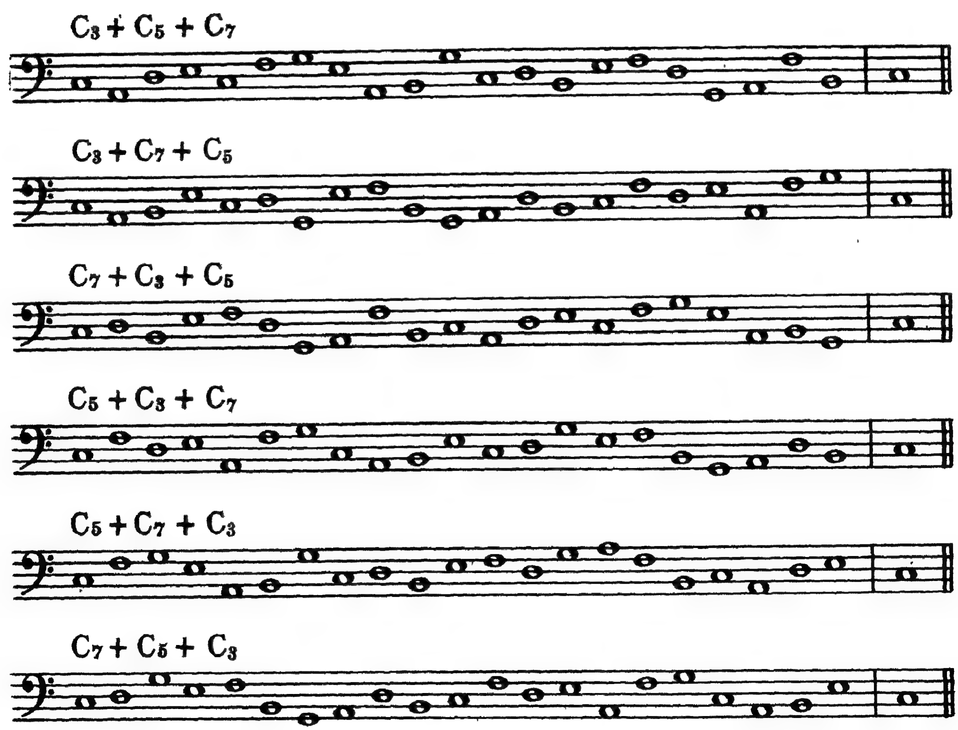


Figure 4. Trinomial cycles.

Both binomial and trinomial cycles produce marked *variety* combined with absolute *consistency* of character (style) of harmonic progression. Being *perfect* in this respect they are of little use when a personal selection of character becomes a paramount factor.

In order to produce an individual style of harmonic progression, it is necessary to use a *selective continuity* of cycles. This can be accomplished by means of the *coefficients of recurrence* applied to a selected combination of cycles. A combination of cycles can be either *binomial* or a *trinomial*. Groups producing coefficients of recurrence can be *binomial*, *trinomial* or *polynomial*. The materials for these are presented in the *Theory of Rhythm.** Rhythmic resultants of different types and their variations provide various groups which can be used as coefficients of recurrence. Distributive power-groups, as well as the different series of growth and acceleration,** can be used for the same purpose.

Binomial Cycles, Binomial Coefficients

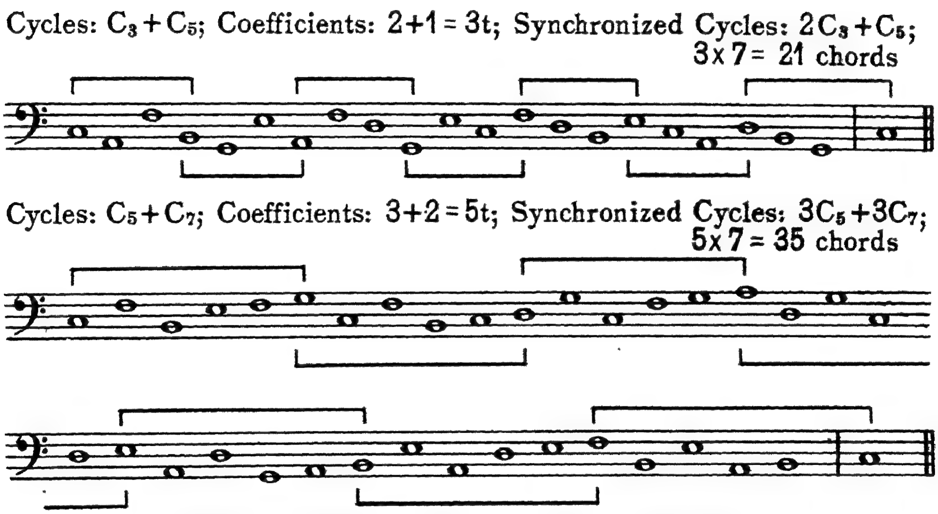


Figure 5. Binomial cycles, binomial coefficients.

*See Book I.

**The distributive power-groups are discussed by Schillinger in Chapter 12 of Book I. Examples of the different series of growth and acceleration are presented in Chapter 14 of the same book. Among the useful series of acceleration for musical purposes are the following:

1) natural harmonic series: 1, 2, 3, 4, 5, 6, 7, 8, 9, etc.;

2) arithmetical progressions: 1, 3, 5, 7, 9, etc.;

3) geometrical progression: 1, 2, 4, 8, 16, etc.;

4) power series: 2, 4, 8, 16, 32, etc.;

5) summation series: 1, 2, 3, 5, 8, 13, 21, etc.;

6) arithmetical progressions with variable differences: $1+1$, $2+2$, $4+3$, $7+4$, $11+5$, $16+6$, $22+7$, etc.

7) prime number series: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc. (Ed.)

Binomial Cycles, Coefficient-Groups with the number of terms divisible by 2

Cycles: $C_7 + C_3$; Coefficients: $r_{4 \div 3} = 3+1+2+2+1+3 = 12t$

Synchronized Cycles: $3C_7 + C_3 + 2C_7 + 2C_3 + C_7 + 3C_3$; $12 \times 7 = 84$ chords

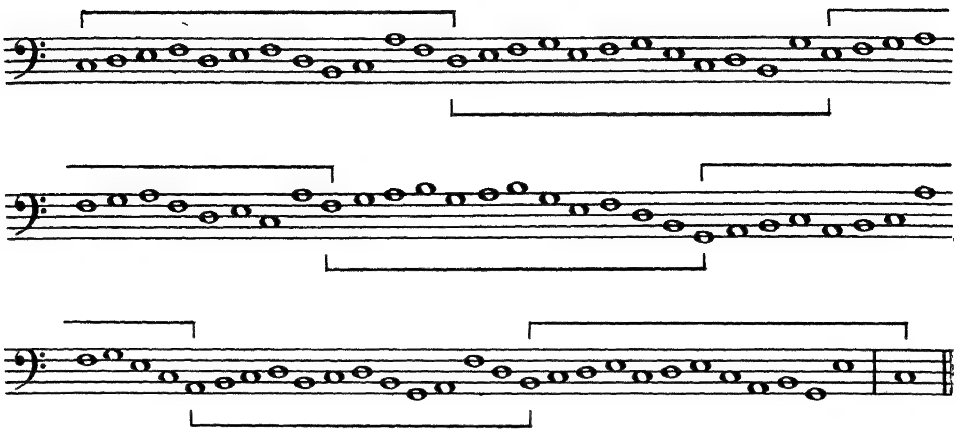


Figure 6. Binomial cycles, coefficient-groups with number of terms divisible by 2.

Binomial Cycles, Coefficient-Groups producing interference with the cycles (not divisible by 2)

Cycles: $C_5 + C_3$ Coefficients: $3+1+2 = 6t$

Synchronized Cycles: $3C_5 + C_3 + 2C_5 + 3C_3 + C_5 + 2C_3$

Synchronized coefficients: $6t \times 2 = 12t$; $12 \times 7 = 84$ chords

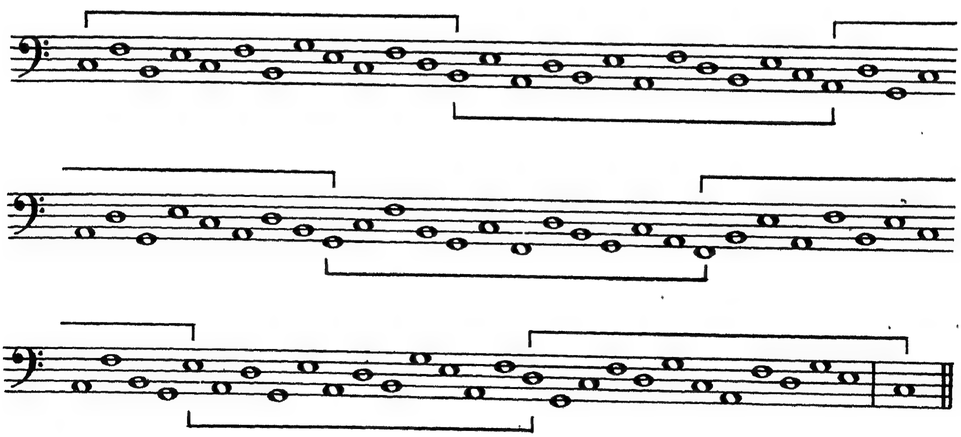


Figure 7. Binomial cycles, coefficient-groups producing interference with the cycles.

Trinomial Cycles, Trinomial Coefficients

Cycles: $C_3 + C_5 + C_7$ Coefficients: $4+1+3=8t$

Synchronized Cycles: $4C_3 + C_5 + 3C_7$; $8 \times 7 = 56$ chords

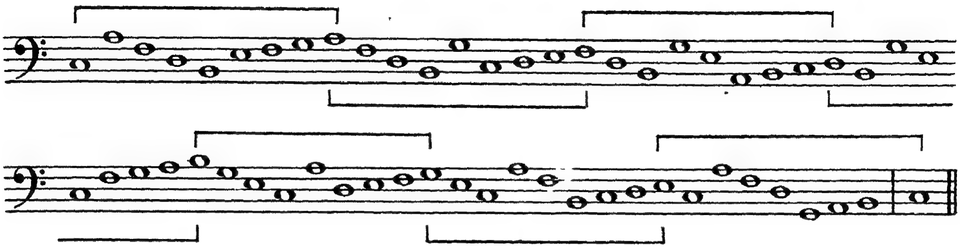


Figure 8. Trinomial cycles, trinomial coefficients.

Trinomial Cycles, Coefficient-Groups with the number of terms divisible by 3

Cycles: $C_7 + C_3 + C_5$; Coefficients: $r_{5 \div 2} = 2+2+1+1+2+2 = 10t$

Synchronized Cycles: $2C_7 + 2C_3 + C_5 + C_7 + 2C_3 + 2C_5$; $10 \times 7 = 70$ chords

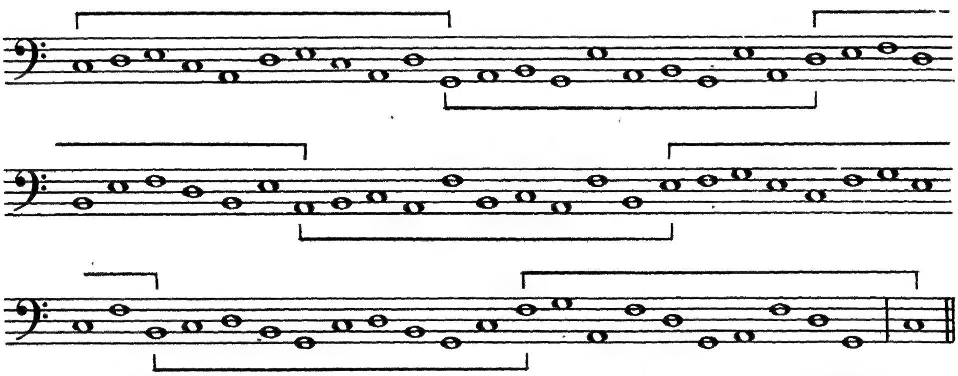


Figure 9. Trinomial cycles, coefficient-groups with number of terms divisible by 3.

Trinomial Cycles, Coefficient-Groups producing interference with the cycles (not divisible by 3)

Cycles: $C_7 + C_5 + C_3$; Coefficients: $3+1=4t$

Synchronized Cycles: $3C_7 + C_5 + 3C_3 + C_7 + 3C_5 + C_3$

Synchronized Coefficients: $4t \times 3 = 12t$; $12 \times 7 = 84$ chords

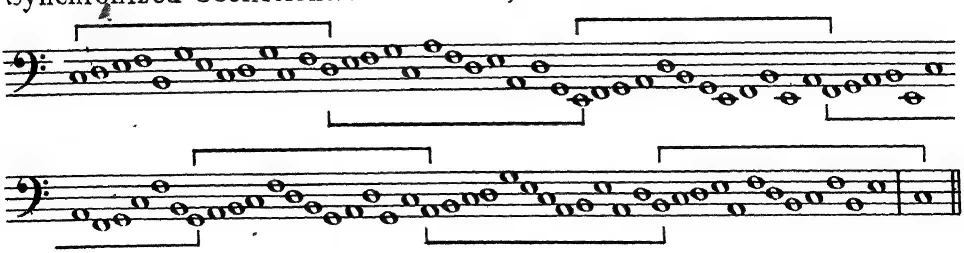


Figure 10. Trinomial cycles, coefficient-groups not divisible by 3.

The *style* of harmonic progressions depends entirely on the form of cycles employed. No composer confines himself to one definite cycle, yet it is the predominance of a certain cycle over others that makes his music immediately recognizable to the listener. In one case it may be that the beginning of a progression is expressed through the cadences of a certain cycle; in another case it may be a prominent coefficient group that makes such music sound distinctly different from other music.

The style of harmonic progressions can be defined as: a definite form of *selective cycles.* Both the combination of cycles (their sequence) and the coefficient group determining their recurrence are the factors of a *style* of harmonic progressions.

B. Historical Development of Cycle Styles

There is much that needs to be said about the historical development of the cycles, for there are already some wrong notions established in this field.

Though the common belief is that progression from the tonic to the dominant and back to the tonic (ending cadence in C_5) is the foundation of diatonic harmony, historical evidence, as well as mathematical analysis, prove the contrary.

During the course of centuries of European musical history, parallel to the development of counterpoint, there was an awakening of harmonic consciousness. The latter can be traced, in its current forms, back to the 15th century A.D. At that time *harmony* meant *concord*—an agreeable, consonant, stabilized sonority of several voices simultaneously sustained.

Concordant progressions could be achieved through consonant chords moving in consonant relations. Obviously such progressions require common tones; these can be expressed as C_3. As *tonality*—i.e., an organized progression of tonal cycles—was at that time in a state of fermentation, it is natural to expect that the cycle of the third would appear in both positive (C_3) and negative (C_{-3}) forms.

The following are a few illustrations taken from the music of the 15th and 16th centuries.

Opening of "Ave Regina Coelorum"— Leonel Power, c. 1460

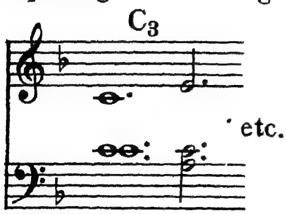
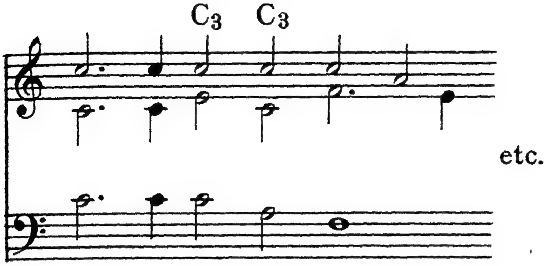


Figure 11. Cycle of the third (continued).

"Benedicta Tu" MS. Pepysian 1236, Madrigal Collection, Cambridge, c. 1460



"Deutsches Lied"_ Adam von Fulda (1470)

Giulio Caccini (1550-1618)

Figure 11. Cycle of the third (concluded).

The cycle of the seventh (C_7), on the other hand, has a purely *contrapuntal* derivation. When the two leading tones (the upper and the lower) move in a cadence into their respective tonics (like b→c and d→c) by means of contrary motion in two voices, we obtain the ending cadence of C_7. Further development of the third part was undoubtedly necessitated by the desire for fuller sonorities. This introduced an extra tone (f in a chord of b) with which the remaining tones form S(6), i.e., a third-sixth-chord or a sixth-chord, the first inversion of the root-chord: S(5).

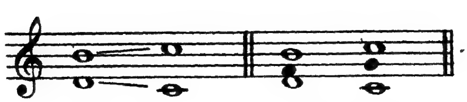


Figure 12. Cycle of seventh (C_7).

It is only natural to expect the predominance of the C_7 in contrapuntal music. Cadences—such as in *Figure 12*—are most standardized in 13th and 14th century European music; see Guillaume de Machault's (1300-1377) *Mass for the Coronation of Charles V.**

The appearance of the cycle of the fifth occurred at a later date, by which time C_3 and C_7 were already in use. I offer the following hypothesis of the origin of C_5. The positive form might have occurred as a pedal-point development where, by sustaining the tonic and changing the remaining two tones to their leading tones, the sequence would represent C_5. Another interpretation of the origin of C_5 is the one on which the present system of harmony is based, i.e., *omission of intermediate links in a series.* (This principle ties up musical harmony with the harmonic structure of crystals as used in crystallographic analysis.)

Figure 13. Cycle of the fifth (C_5).

The origin of the negative form of the cycle of the fifth (C_{-5}) is due to the desire to acquire a concord supporting a leading tone. Let b be a leading tone in the scale of c. The most concordant combination of tones in pre-Bach times, i.e., in the *mean temperament* tuning system,** which harmonized the tone b was the G-chord (g, b, d). But, in the movement from G-chord to C-chord, the form of the cycle is positive. In reality both forms, the positive and the negative, are the beginning and the ending cadences. Compare *Figure 14* with *Figure 13*.

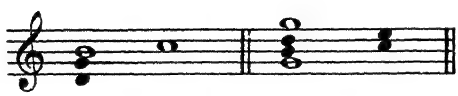


Figure 14. Cycle of the fifth (C_5). (See pp. 363 and 386).

1. Richard Wagner

The development of harmonic progressions in the European music of the last three centuries can be easily traced back to its sources. The style of every composer is *hybrid*, yet the quantitative predominance of certain ingredients (like the cycles appearing with the different coefficients of recurrence) produces individual characteristics.

In the following exposition, I will *confine* the concept of "style" to harmonic progressions in the diatonic system.

Richard Wagner was the greatest representative of C_3 in the 19th century. This statement is backed by actual statistical analysis of tonal cycles in his works, as compared to those of his contemporaries and predecessors. C_5 was the universal vogue of the whole century preceding Wagner. In fact, it is not even

*Recording issued by Gramophone Shop of New York City.

**Officially recognized in Europe before the advent of *equal temperament* tuning. (Ed.)

necessary to analyze all the works of Wagner; the most characteristic progressions may be found at the *beginning* of his preludes to music-dramas and also in the various cadences.

The beginnings of the major works of any composer are important for the reason that they cannot be casual: the beginning is the "calling card" of a composer. The importance of cadences as determinants of harmonic styles was stressed by our contemporary Alfredo Casella in a paper *Evolution of Harmony from the Authentic Cadence.*

Wagner, being German and being an intentionally Germanic composer, undoubtedly had done some research into earlier German music, for he intended to deal with the subjects of German mythology in which he was well versed. Fifteenth century German music discloses such an abundance of C_3 that it is only natural to expect there would be strong influence by such an authentic source of Germanic music on Wagner's creations. In his time, Wagner's harmonic progressions sounded revolutionary because many things had been forgotten in four hundred years, and the archaic acquired a flavor of the modernistic. So far as the development of diatonic progressions in Wagner's music is concerned, it appears to the unbiased analyst that the whole mission of Wagner's life was to develop a consistent combined cadence in C_3.

Starting with an early work like *Tannhauser*, we find that the very beginning of the overture is typical in this respect.

Figure 15. Opening of Tannhäuser.

Later on, we find more extended progressions of C_3, as in the aria of Wolfram von Eschenbach (the scene of the Minnesingers' contest):

Figure 16. Aria of Wolfram von Eschenbach.

Lohengrin abounds even more in C_3 than *Tannhäuser*. In the "Farewell to the Swan", as in many other passages in the same opera, we find the characteristic back-and-forth fluctuation: $C_3 + C_{-3}$.

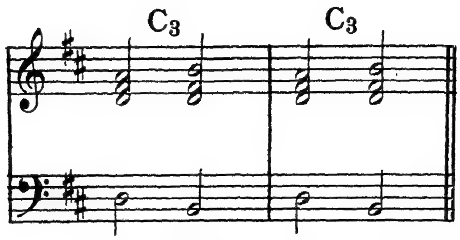


Figure 17. "Farewell to the Swan".

In forming his cadences, Wagner sometimes paid tribute to the dominating "dominant" of Beethoven (C_5). This produced combined hybrid cadences, which are characteristic of *Lohengrin*. The first part of such a cadence is the beginning cadence in C_3 while the second part is the ending cadence in C_5: I — IV — V — I.

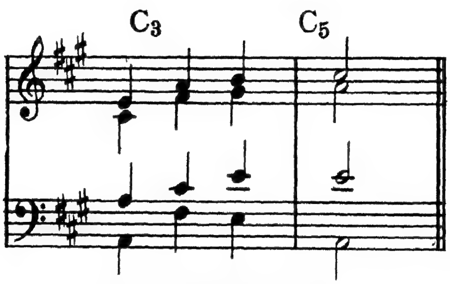


Figure 18. Lohengrin.

Though he dealt with types of progression other than diatonic in the course of his career, Wagner came back to diatonic purity in its most complete and consistent form in his last work, *Parsifal*. The beginning of the prelude to Act I reveals that the composer came to a realization of the combined cadence of C_3: I — VI — III;

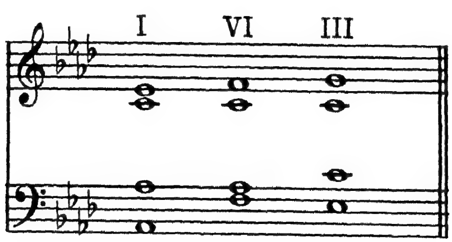


Figure 19. Beginning of Parsifal.

The more extensive sequences of C_3 are: I — VI — IV — II;

Figure 20. Parsifal *motif.*

The complete combined cadence appears in the "Procession of the Knights of the Grail": I — VI — III — I.

Figure 21. Procession of The Knights of The Grail.

2. Hegemony of C_5, 1750-1850

The second half of the 18th century and the first half of the 19th century are the period of the hegemony of the dominant and C_5 in all its aspects in general. The latter are: continuous progressions of C_5; starting, ending and combined cadences (I — IV — I; I — V — I; I — IV — V — I). The main bodies of music possessing these characteristics are the Italian opera and the Viennese School.

To the first belong Monteverdi, Scarlatti, Pergolesi, Rossini, and Verdi. The second is represented by Dittersdorf, Haydn, Mozart, Beethoven, and Schubert. Today this style has disintegrated into the least imaginative creations in the field of popular music. Nevertheless, it is the stronghold of harmony in educational music institutions.

Here are a few illustrations of C_5 style in the early sonatas for piano by Ludwig van Beethoven: Sonata Op. 7, Largo; Sonata Op. 13, Adagio Cantabile.

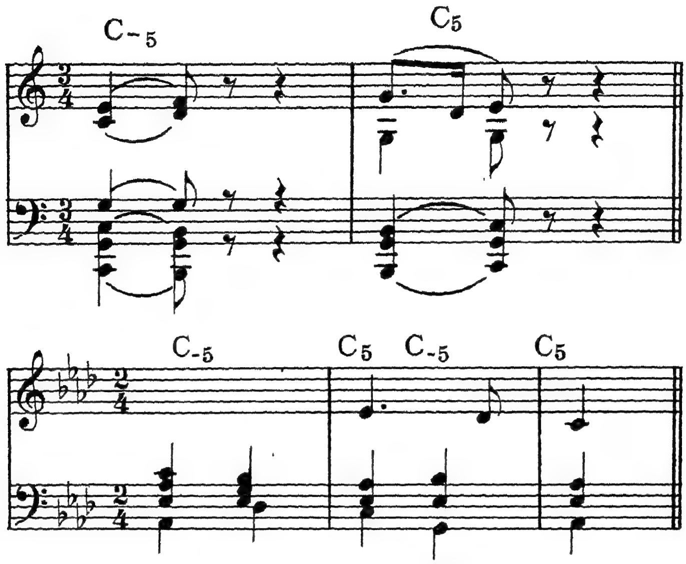


Figure 22. C_5 style in early Beethoven sonatas.

Any number of illustrations can be found in Mozart's and Beethoven's symphonies, particularly in the conclusive parts of the last movements.

3. C_7 in Bach

Assuming that the historical origin of the cycle of the seventh can be traced back to contrapuntal cadences, it would be only logical to expect to find evidence of C_7 in the works of the great contrapuntalists. I choose for the illustration of C_7, as characteristic starting progressions, some of the well-known Preludes to Fugues taken from the First Volume of the *Well-Tempered Clavichord* by Johann Sebastian Bach: Prelude I; Prelude III; Prelude V.

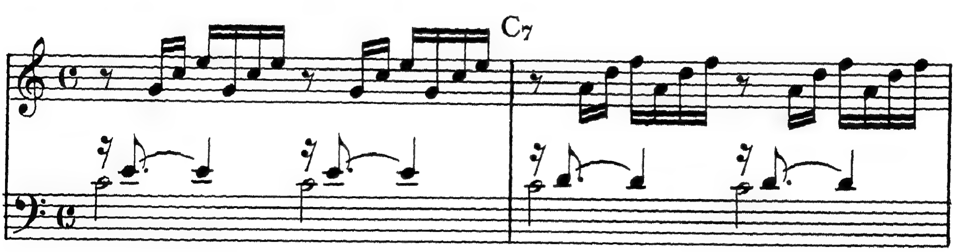


Figure 23. C_7 in Bach (continued).

Figure 23. C_7 in Bach (*concluded*).

Bach's famous *Chaconne in D-minor* for violin discloses the same characteristics: the first chord is d, and the second chord is e—which makes C_7.

A consistent and ripe style of diatonic progression corresponds to a consistent use of one form, either positive or negative, and *not* to an indiscriminate mixture of both. Many theorists confuse the hybrid of positive and negative forms with *modal progressions*, which these theorists have never defined clearly. In reality, *modal progressions* are in no respect different from *tonal progressions* except for scale structure. Both types (tonal and modal) can be either positive, or negative, or hybrid. Modes can be obtained by the direct change of key signatures, as set forth in my theory of pitch-scales (transposition to one axis).*

Here is an example, which is typical of Moussorgsky from the opera *Boris Godounov*.

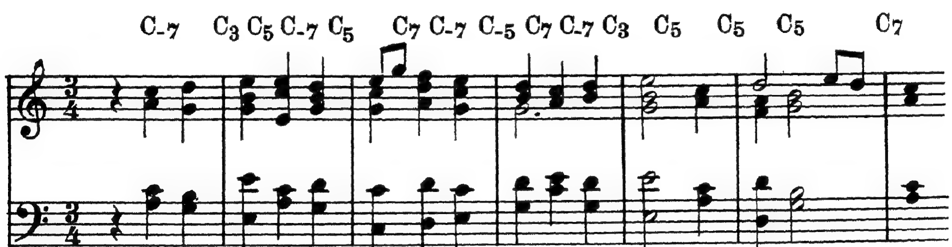


Figure 24. Hybrid of positive and negative forms

In the above example, the mode (scale) is Cd_5, the fifth derivative scale of the natural major in the key of C, known as the Aeolian mode; the progression of tonal cycles is a hybrid of positive and negative forms.

*See Book II.

C. Transformations of S(5).

In traditional courses in harmony the problems of progressions and voice-leading are treated as inseparable. Each pair of chords is described as a sequence and as a form of voice-leading. Thus each case becomes an individual case where the movement of voices is described in terms of melodic intervals—like: "a fifth down", "a second up", "a leap in soprano", "a sustained tone in alto", etc. No person of normal mentality can ever memorize all the rules and exceptions offered in such courses. In addition to this unsatisfactory form of presentation of the subject of harmony, one finds out very soon that the abundance of rules covers very limited material, mostly the harmony of the second-rate 18th century European composers.

The main defect of existing theories of harmony is in the use of the *descriptive method*. Each case is analyzed apart from all other cases and without yielding any general underlying principles. But the mathematical treatment of this subject discloses the *general properties* of the positions and movements of the voices in terms of *transformations of the chordal functions*.

Any chord, no matter of what structure, is from a mathematical standpoint an *assemblage of pitch units*, or a *group of conjugated functions* (elements). These functions are the different pitch-units distributed in each group, assemblage, or chord, according to the different number of voices (parts) and the intervals between the latter.

In groups with three functions, known as three-part structures (S = 3p), the functions are a, b and c. These functions behave through *general forms of transformation* and not through any musical specifications.

As in this branch we are dealing with so-called four-part harmony, we have to define the meaning of this expression more precisely.

When an S(5) constitutes a chord-structure, the functions of the chord are: *the root, the third and the fifth* or 1, 3 and 5. In their general form they correspond to a, b and c, i.e., a = 1, b = 3, and c = 5. The bass of such harmony is a constant root-tone, i.e., const. 1 or const. a.

Thus the transformation of functions affects all parts except the bass. Here, therefore, we are dealing with groups consisting of three functions.

Such groups have two fundamental transformations: (1) clockwise ($\circlearrowright$) and (2) counterclockwise ($\circlearrowleft$)

The clockwise transformation is:

$$\begin{tikzcd}[column sep=small] & a \arrow[dr] & \\ c \arrow[ur] & & b \arrow[ll] \end{tikzcd}$$

The counterclockwise transformation is:

$$\begin{tikzcd}[column sep=small] & a \arrow[dl] & \\ c \arrow[rr] & & b \arrow[ul] \end{tikzcd}$$

*

Each of these transformations has two meanings: the first meaning is to be read—

a is followed by b
b " " " c
c " " " a,
for the $\circlearrowright$; and

a is followed by c
c " " " b
b " " " a
for the $\circlearrowleft$

discloses the mechanism of the *position of a chord.*

The second meaning is to be read—

a transforms into b
b " " c
c " " a,
for the $\circlearrowright$; and

a transforms into c
c " " b
b " " a,
for the $\circlearrowleft$.

These constitute *the forms of voice-leading.*

1. Positions

The different positions of S(5) = 1, 3, 5 can be obtained by constructing the chordal functions downward from each phase of the transformations.

$$\begin{array}{l|ccc} & a & b & c \\ & b & c & a \\ \downarrow & c & a & b \\ & & \circlearrowright & \end{array} \quad \text{and} \quad \begin{array}{ccc|l} a & c & b & \\ c & b & a & \\ b & a & c & \downarrow \\ & \circlearrowleft & & \end{array}$$

Substituting 1, 3, 5 for a, b, c, we obtain

$$\begin{array}{l|ccc} & 1 & 3 & 5 \\ & 3 & 5 & 1 \\ \downarrow & 5 & 1 & 3 \\ & & \circlearrowright & \end{array} \quad \text{and} \quad \begin{array}{ccc|l} 1 & 5 & 3 & \\ 5 & 3 & 1 & \\ 3 & 1 & 5 & \downarrow \\ & \circlearrowleft & & \end{array}$$

The clockwise positions are commonly known as "open", and the counter-clockwise as "closed."

Here are the positions for S(5) = 4 + 3 = c − e − g. The bass is added to double the root.

Positions

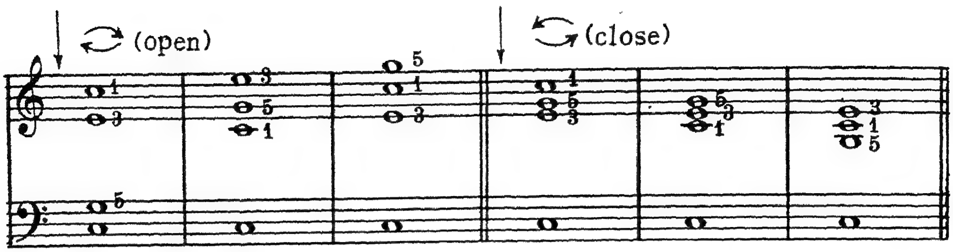


Figure 25. Open and close positions of S5.

D. Voice-Leading

The movement of the individual voices follows the groups of transformation in this form: a̲ of the first chord transforms into b̲ of the following chord; b̲ of the first chord transforms into c̲ of the following chord; c̲ of the first chord transforms into a̲ of the following chord. The above three forms constitute *clockwise* voice-leading.

For *counterclockwise* voice-leading, the reading must follow this order: a̲ of the first chord transforms into c̲ of the following chord; c̲ of the first chord transforms into b̲ of the following chord; b̲ of the first chord transforms into a̲ of the following chord.

a → b		a → c
b → c	and	c → b
c → a		b → a

Applying the above transformations to 1, 3, 5 of the S(5), we obtain:

1 → 3		1 → 5
3 → 5	and	5 → 3
5 → 1		3 → 1

Clockwise form: the root of the first chord becomes the third of the next chord; the third of the first chord becomes the fifth of the next chord; the fifth of the first chord becomes the root of the next chord.

Counterclockwise form: the root of the first chord becomes the fifth of the next chord; the fifth of the first chord becomes the third of the next chord; the third of the first chord becomes the root of the next chord.

Both forms apply to all tonal cycles. Let us take C_3 in the natural major, for example. The first chord is C = c − e − g and the next chord is A = a − − c − e.

Clockwise form gives the following reading:

c → c
e → e
g → a

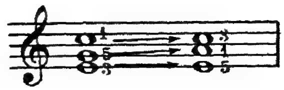


Figure 26. Clockwise transformation of C_3.

Counterclockwise form gives the following reading:

c → e
g → c
e → a

Figure 27. Counterclockwise transformation of C_3.

Let us take C_5 in the same scale. The chords are: C = c − e − g and F = f − a − c.

c → a
e → c
g → f

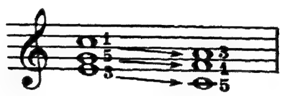


Figure 28. Clockwise transformation of C_5.

c → c
g → a
e → f

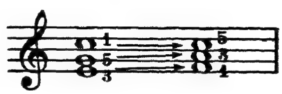
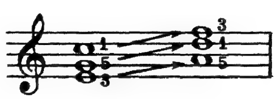


Figure 29. Counterclockwise transformation of C_5.

Let us take C_7 in the same scale. The chords are: C = c − e − g and D = d − f − a.

c → f
e → a
g → d



or

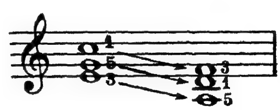


Figure 30. Clockwise transformation of C_7.

Both forms of ⇄ are acceptable in this case, as the intervals in both directions are nearly equidistant.

c → a
g → f
e → d

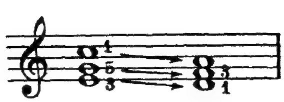


Figure 31. Counterclockwise transformation of C_7.

Each tonal cycle permits a continuous progression through one form of transformation. In the following table const. 1 in the bass is added. The commas indicate an octave variation introduced when the extension of range becomes impractical.

In C_7 both directions are combined, offering the most practical form for the range.

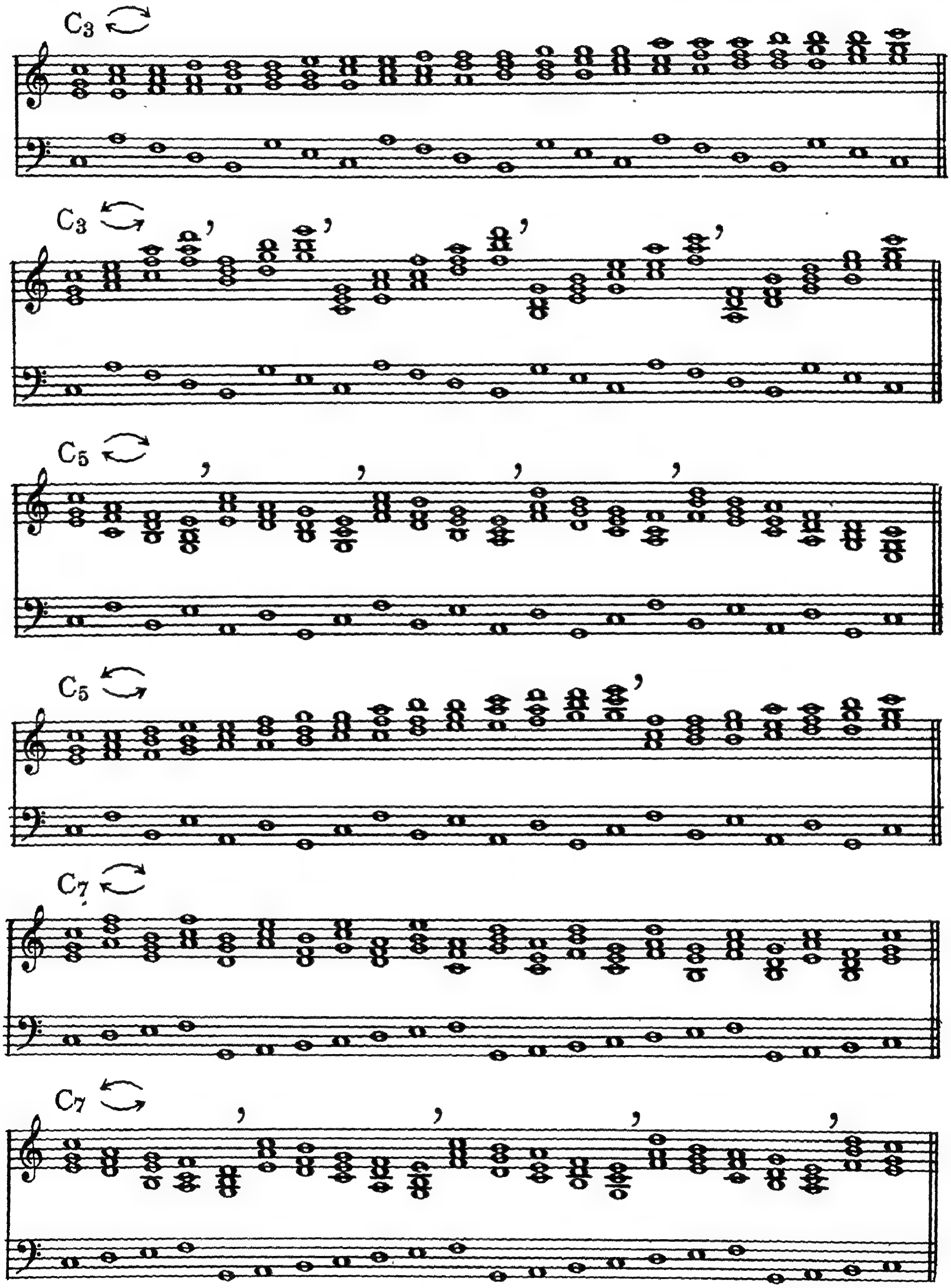


Figure 32. Clockwise and counterclockwise transformations of C_3, C_5, C_7.

Both clockwise and counterclockwise transformations are applicable to *all* positions for the starting chord. When the first chord is in the ↻ (open) position, the entire progression remains automatically in such a position. When the first chord is in ↺ (close) position, the entire progression remains in that position. This constancy of position (open or close) is not affected either by the constancy of the tonal cycles or by the lack of such constancy.

The transition from close to open position and vice-versa can be accomplished through the use of the following formula:

Constant b transformation

	Const. 3
a → c	1 → 5
b → b	3 → 3
c → a	5 → 1

It is best to have 3 in the upper voice for such purposes, as in some positions voices will otherwise cross. Function 3 from close to open position moves upward to function 3 of the following chord. Reverse the procedure from open to close.

Figure 33. Transitions from open to close positions and vice-versa.

Continuous application of const. 3 transformation produces a consistent variation of the ↻ and the ↺ positions, regardless of the sequence of tonal cycles.

The following table offers continuous progressions through const. cycles and const. 3 transformation.

Constant 3 Transformations

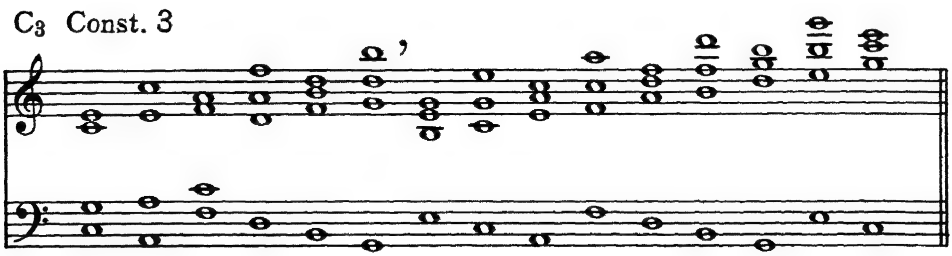
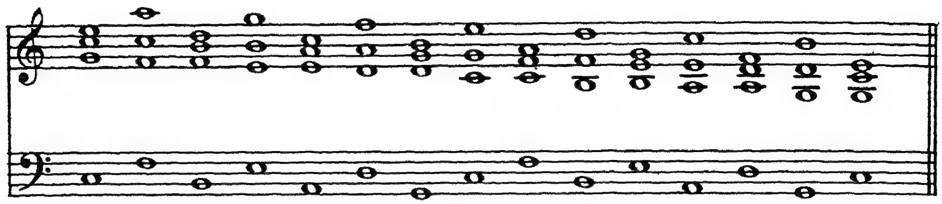


Figure 34. Continuous progressions through constant cycles and constant 3 transformations (continued).

C_5 Const. 3



C_7 Const. 3

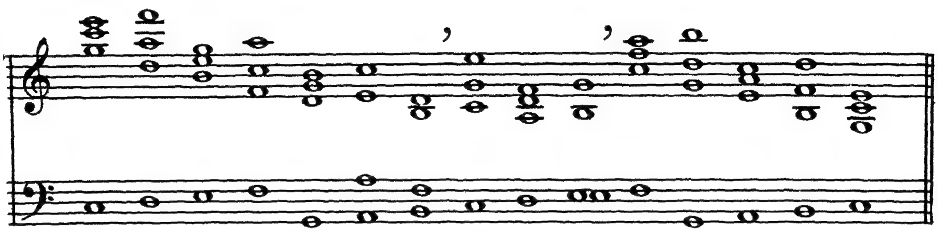


Figure 34. Continuous progressions through constant cycles and constant 3 transformations (concluded).

E. How Cycles and Transformations are Related

There are four forms of relationship between cycles and transformations with regard to the variability of both:

(1) const.-cycle, const.-transformation;
(2) const.-cycle, variable transformation;
(3) variable cycle, const.-transformation;
(4) variable cycle, variable transformation.

The forms of transformation produce their own periodic groups which may be superimposed on the groups of cycles.

Monomial forms of transformations (const. transformations):

(1) ↻, (2) ↺, (3) const. 3.

Binomial forms of transformations:

(1) ↻ + ↺, (2) ↺ + ↻

Here const. 3 is excluded because of the crossing of inner voices.

When coefficients of recurrence are applied to the forms of transformations, *selective transformation-groups* are produced.

For example: 2 ↻ + ↺; 3 ↺ + 2 ↻; 2 ↻ + ↺ + ↻ + 2 ↺; 4 ↻ + ↺ + + 3 ↻ + 2 ↺ + 2 ↻ + 3 ↺ + ↻ + 4 ↺; ↺ + 2 ↻ + 3 ↺ + 5 ↻ + 8 ↺; 4 ↻ + 2 ↺ + 2 ↻ + ↺.

Although the groups of tonal cycles, as well as the forms of transformations, may be chosen freely with the writing of each subsequent chord, nevertheless rhythmic planning of both cycles and transformations guarantees a greater regularity and, therefore, greater unity of style.

Here are examples of variable transformations applied to constant tonal cycles.

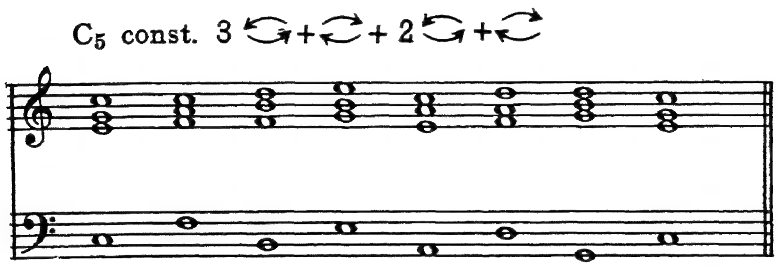


Figure 35. Variable transformations of constant tonal cycles.

Examples of variable transformations applied to variable tonal cycles.

Figure 36. Variable transformations of variable tonal cycles (continued).

$2C_7 + C_3 + 3C_5$; 4↶ + 2↷ + 2↶ + ↷

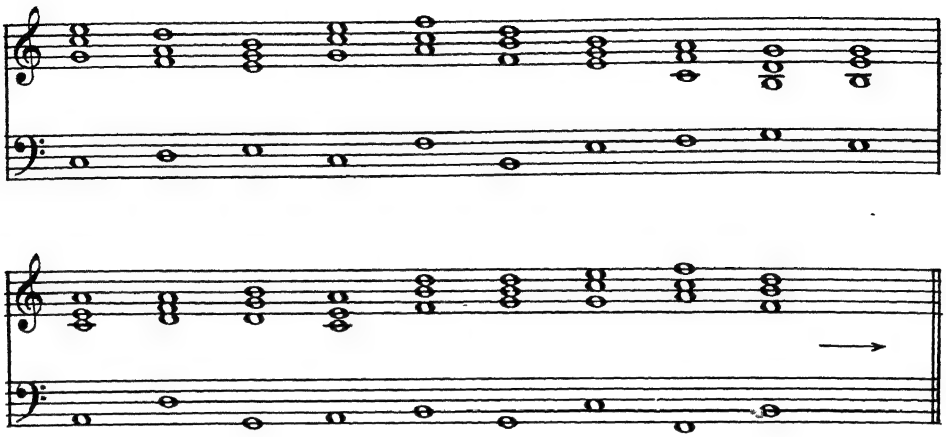


Figure 36. Variable transformations of variable tonal cycles (concluded).

All forms of harmonic continuity, because of their properties of redistribution, modal variability and convertibility, are subject to the following modifications:

(1) Placement of the voice representing constant function, and originally appearing in the bass, in any other voice, i.e., tenor, alto or soprano. There are four forms of such distribution:

s	s	s	S
a	a	A	a
t	T	t	t
B	b	b	b

Capital letters represent the voice functioning as const. 1.

Original

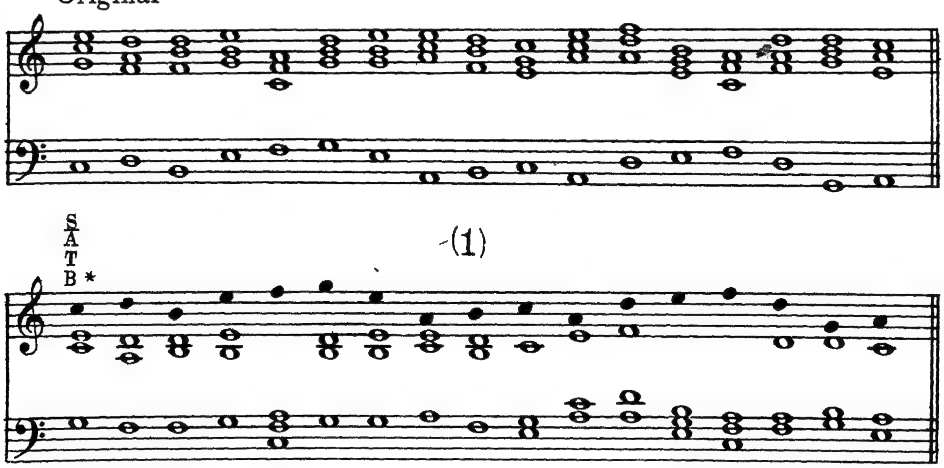


Figure 37. Varying position of constant voice.

*In Schillinger's original M.S., the constant voice is indicated by notes in red; here, however, they are indicated by quarter-notes, to avoid the complication of printing a second color. (Ed.)

(2) General redistribution (vertical permutations) of all voices according to the 24 variations of 4 elements.

T
S
B
A

(2)

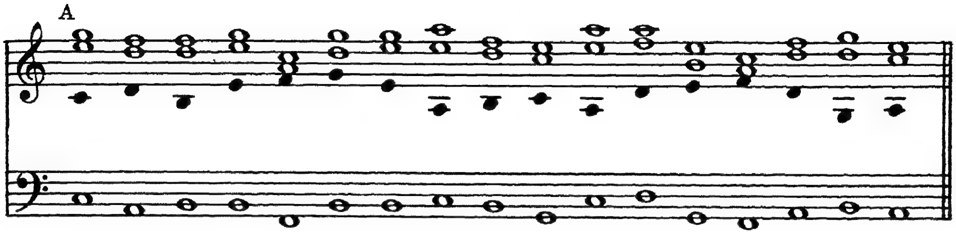


Figure 38. General redistribution of all voices.

(3) Geometrical inversions: ⓐ, ⓑ, ⓒ and ⓓ for any or all forms of distribution of the four voices.

Original ⓓ

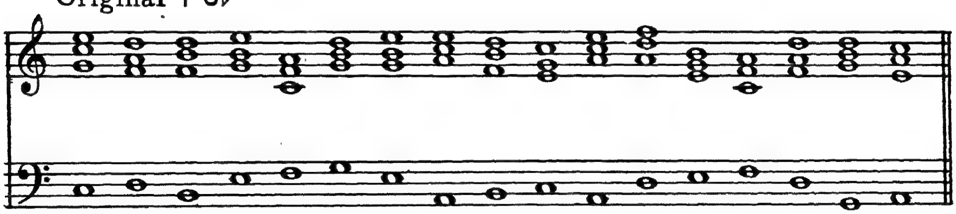
(3)

Figure 39. Geometrical inversion, position ⓓ.

(4) Modal variation by means of *modal transposition*, i.e., direct change of key signature, without relocating the notes on the staff.

(4)

Original + 3♭



Original + (b♭ + f♯) = G mel. minor: d_3

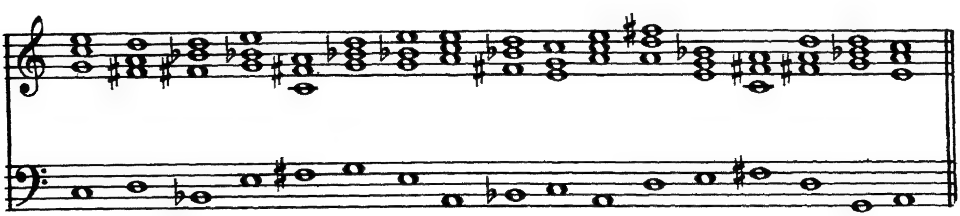


Figure 40. Modal variation by modal transposition.

E. The Negative Form

As previously indicated, the negative form of harmony can be obtained by direct reading of the positive form in position ⓑ.

Here, for the sake of clarity, I offer some technical details which explain the theoretical side of the negative form.

According to the definition given of the harmony scale in the negative form, we obtain the latter by means of further expansions of HS. In the positive form we use: HS_{E_0} (= C_3), HS_{E_1} (= C_5) and HS_{E_2} (= C_7).

By further expanding HS, we acquire the cycles of the negative form: HS_{E_3} (= C_{-7}), HS_{E_4} (= C_{-5}), HS_{E_5} (= C_{-3}).

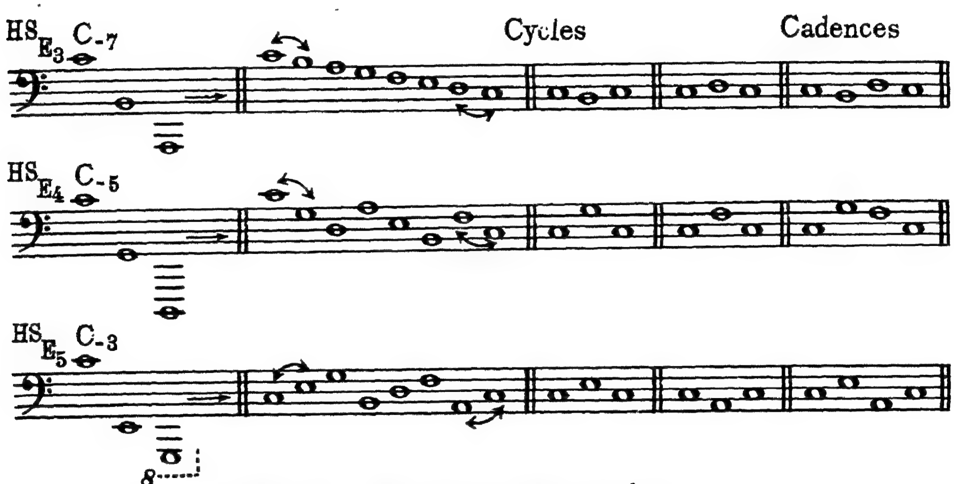


Figure 41. Cycles of the negative form.

As in this negative form, the chord-structures are built downward from a given pitch unit, such a pitch unit becomes the root-tone of the negative structure: *the negative root* (− 1). All chord-structures of the negative form, according to the previous definition, derive from HSⓑ. Thus, in order to construct a *negative S (5)*, it is necessary to take the next pitch-unit downward, which becomes *the negative third* (− 3), and the next unit downward from the latter, which becomes *the negative fifth* (− 5).

For example, if we start from c̲ as a − 1, we obtain a negative S (5) where a is − 3 and f is − 5.

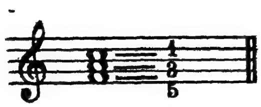


Figure 42. Natural C-major.

Positions of chords, as they were expressed through transformation, remain identical in the negative form, providing that they are constructed upward. In such a case, the addition of a const. 1 in the bass must be, strictly speaking, transferred to the soprano.

Here is how a negative CS (5) would appear in its four-part settings.

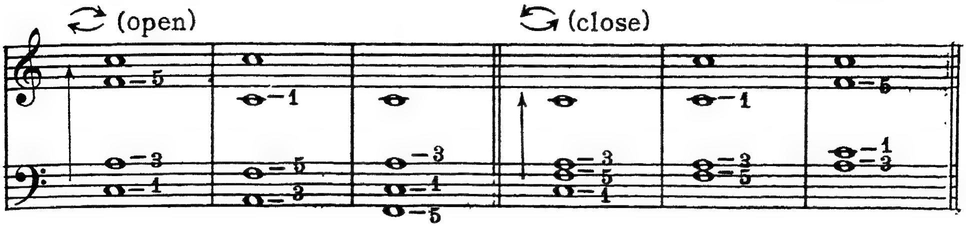


Figure 43. Four-part setting of negative CS (5).

Under such conditions, if the chord is constructed downward, the reversal of ⟳ and ⟲ reading takes place.

Transformations as applied to voice-leading possess the same reversibility: if everything is read downward, the ⟳ and the ⟲ transformations correspond to the positive form, while in the upward reading the ⟳ becomes the ⟲, and vice-versa.

Let us connect two chords in the negative cycle of the third: CS (5) + C_3 + + ES (5).

CS (5) = −1, −3, −5 = c − a − f.
ES (5) = −1, −3, −5 = e − c − a.

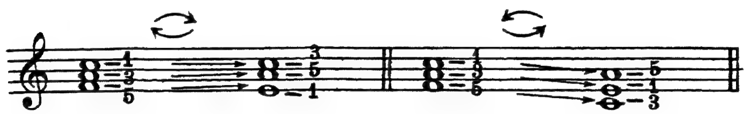


Figure 44. CS (5) + C_3 + ES (5).

It is easy to see that in the upward reading, chord C corresponds to F while chord E corresponds to A. Transposing this upward reading to C, we find that this progression is C → E. This proves the reversibility of tonal cycles and the correctness of reading the positive form of progressions in position ⓑ when the negative form is desired.

The mixture of positive and negative forms in continuity does not change the situation, but merely reverses the characteristics of voice-leading with regard to positive and negative forms. For example, C_3 in ⟳ in the positive system produces two sustained common tones. In order to obtain an analogous pattern of voice-leading in C_{-3}, it is necessary to reverse the transformation, i.e., to use the ⟲ form in this case.

CHAPTER 3

THE SYMMETRIC SYSTEM OF HARMONY

DIATONIC harmony can be best defined as *a system in which chord-structures as well as chord-progressions derive from a given scale.* The structural constitution of pitch assemblages, known as chords, as well as the actual intonation of the sequences of root-tones, known as tonal cycles, are entirely conditioned by the structural constitution of the scale, which is *the source of intonation.*

Symmetric harmony is *a system of pre-selected chord-structures and pre-selected chord progressions,* one independent of the other. In the symmetric system of harmony, *scale* is the result; scale is the *consequence of chords in motion.* The selection of intonation for structures is independent of the selection of intonation for the progressions.

A. Structures of S(5)

In this part of my treatment of harmony only such three-part structures will be used as satisfy our definition of the *special* theory of harmony. The ingredients of chord-structures here are limited to 3 and 4 semitones. Under such limitations only *four forms of S*(5) are possible. It should be remembered, however, that the number of all possible three-part structures amounts to 55, which is the general number of three-unit scales from one axis.

Table of S(5)

$S_1(5) = 4 + 3$, known as a major triad;
$S_2(5) = 3 + 4$, known as a minor triad;
$S_3(5) = 4 + 4$, known as an augmented triad;
$S_4(5) = 3 + 3$, known as a diminished triad.

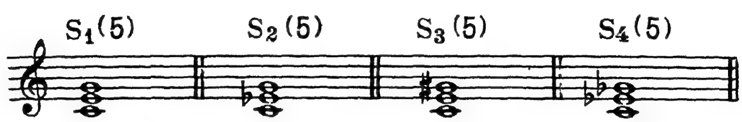


Figure 45. Table of S (5) structures.

Inasmuch as S(5) will be the only structure treated at present, we shall simplify the above expressions to the following form:

$$S_1;\ S_2;\ S_3;\ S_4.$$

Regardless of what the chord-progression may be, the structural constitution of chords appearing in such a progression may be either constant or variable. Constant structures will be considered as monomial progressions of structures, while the variable structures will be considered as binomial, trinomial and polynomial structural groups.

1. Monomial Forms of S(5)

$S_1 + \ldots.$
$S_2 + \ldots.$
$S_3 + \ldots.$
$S_4 + \ldots.$
Total: 4 forms

2. Binomial forms of S(5)

$S_1 + S_2$	$S_2 + S_3$	$S_3 + S_4$
$S_1 + S_3$	$S_2 + S_4$	
$S_1 + S_4$		

6 combinations, 2 permutations each.
Total: 12 forms

3. Trinomial forms of S(5)

$S_1 + S_1 + S_2$	$S_2 + S_2 + S_3$	$S_3 + S_3 + S_4$
$S_1 + S_1 + S_3$	$S_2 + S_2 + S_4$	
$S_1 + S_1 + S_4$		
$S_1 + S_2 + S_2$	$S_2 + S_3 + S_3$	$S_3 + S_4 + S_4$
$S_1 + S_3 + S_3$	$S_2 + S_4 + S_4$	
$S_1 + S_4 + S_4$		

12 combinations, 3 permutations each.
Total: 36 forms

$S_1 + S_2 + S_3$	$S_2 + S_3 + S_4$
$S_1 + S_2 + S_4$	
$S_1 + S_3 + S_4$	

4 combinations, 6 permutations each.
Total: 24 forms.

The total of all trinomials: $36 + 24 = 60$.

4. Quadrinomial forms of S(5)

$S_1 + S_1 + S_1 + S_2$	$S_2 + S_2 + S_2 + S_3$	$S_3 + S_3 + S_3 + S_4$
$S_1 + S_1 + S_1 + S_3$	$S_2 + S_2 + S_2 + S_4$	
$S_1 + S_1 + S_1 + S_4$		
$S_1 + S_2 + S_2 + S_2$	$S_2 + S_3 + S_3 + S_3$	$S_3 + S_4 + S_4 + S_4$
$S_1 + S_3 + S_3 + S_3$	$S_2 + S_4 + S_4 + S_4$	
$S_1 + S_4 + S_4 + S_4$		

12 combinations, 4 permutations each.
Total: 48 forms.

$S_1 + S_1 + S_2 + S_2$	$S_2 + S_2 + S_3 + S_3$	$S_3 + S_3 + S_4 + S_4$
$S_1 + S_1 + S_3 + S_3$	$S_2 + S_2 + S_4 + S_4$	
$S_1 + S_1 + S_4 + S_4$		

6 combinations, 6 permutations each.
Total: 36 forms

$S_1 + S_1 + S_2 + S_3$ $\quad$ $S_2 + S_2 + S_3 + S_4$
$S_1 + S_1 + S_2 + S_4$
$S_1 + S_1 + S_3 + S_4$

$S_1 + S_2 + S_2 + S_3$ $\quad$ $S_2 + S_3 + S_3 + S_4$
$S_1 + S_2 + S_2 + S_4$
$S_1 + S_3 + S_3 + S_4$

$S_1 + S_2 + S_3 + S_3$ $\quad$ $S_2 + S_3 + S_4 + S_4$
$S_1 + S_2 + S_4 + S_4$
$S_1 + S_3 + S_4 + S_4$

12 combinations, 12 permutations each.
Total: 144 forms.

$S_1 + S_2 + S_3 + S_4$

1 combination, 24 permutations.
Total: 24 froms.

The total of all quadrinomials: $48 + 36 + 144 + 24 = 252$.

In addition to all these fundamental forms of the groups of S(5), which represent a *neutral harmonic continuity of structures*, there are groups with coefficients of recurrence, which represent a *selective harmonic continuity of structures*. The latter are subject to individual selection.

Any rhythmic groups* may be used as coefficients of recurrence.

Examples

(1) $2S_1 + S_3$

(2) $3S_3 + S_2$

(3) $3S_1 + 2S_3 + S_2$

(4) $2S_2 + S_1 + S_2 + 2S_1$

(5) $2S_1 + S_2 + S_3 + 2S_4$

(6) $3S_1 + S_2 + 2S_1 + 2S_2 + S_1 + 3S_2$

(7) $3S_1 + S_3 + 2S_2 + 2S_1 + S_3 + 3S_2$

(8) $4S_3 + 2S_2 + 2S_3 + S_2$

(9) $2S_1 + S_2 + S_1 + S_2 + S_1 + S_2 + 2S_1 + 2S_2 + S_1 + S_2 + S_1 + S_2 +$
$+ S_1 + 2S_2$

(10) $4S_1 + 2S_2 + 2S_4 + 2S_1 + S_2 + S_4 + 2S_1 + S_2 + S_4$

(11) $S_1 + 2S_2 + 3S_4 + 5S_3$

(12) $S_2 + 3S_1 + 4S_2 + 7S_1$

*In this brief sentence, Schillinger reminds the reader that all of his basic rhythmic procedures, set forth in great detail in the earlier section of rhythm, may be applied as a means of determining the pattern of coefficients of structures. (Ed.)

B. SYMMETRIC PROGRESSIONS. SYMMETRIC ZERO CYCLE (C_0)

A group of chords with a common root-tone but with variable positions and variable structures produces a symmetric zero cycle (C_0).

Such a group may be an independent form of harmonic continuity as well as a portion of other symmetric forms of harmonic continuity.

Coefficients of recurrence in the groups of structures, when used in a continuity of C_0, acquire the following meaning: a structure with a coefficient greater than one changes its positions until the next structure appears. The change of structure requires the preservation of the position of the chord.

This can be expressed as *a form of interdependence of structures and their positions in the C_0*:

S const. ——————position var.
S var. ——————position const.

For instance, in a case of $3S_1 + S_3 + 2S_2 = S_1 + S_1 + S_1 + S_3 + S_2 + S_2$, the constant and variable positions appear as follows:

var.	var.	const.	const.	var.
$S_1 +$	$S_1 +$	$S_1 + S_3$	$+$	$S_2 + S_2$

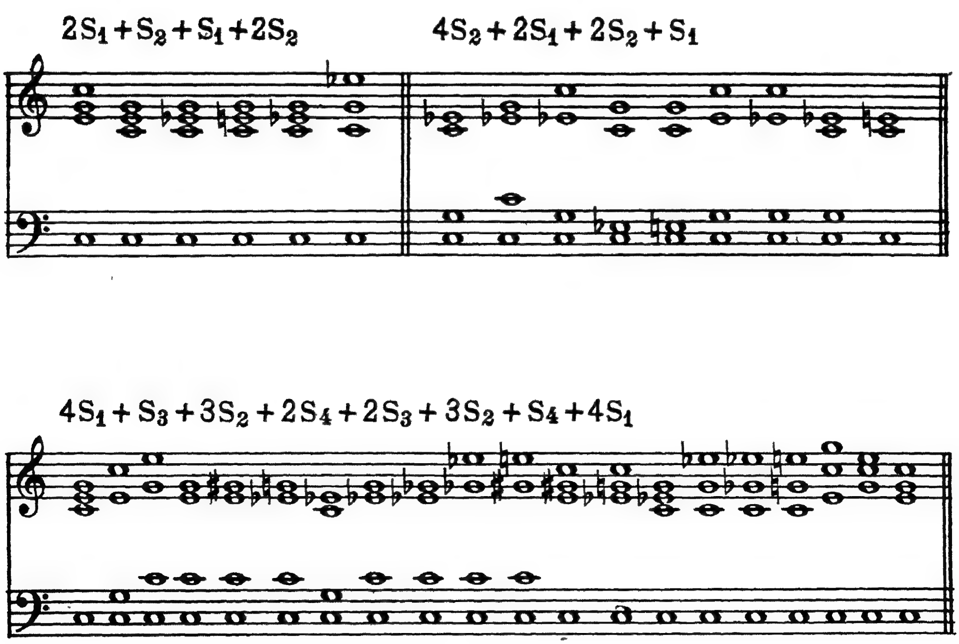


Figure 46. Harmonic continuity in C_0 (continued).

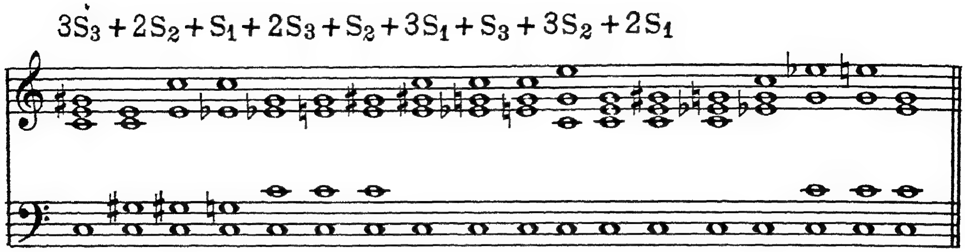


Figure 46. Harmonic continuity in C_0 (concluded).

CHAPTER 4

DIATONIC-SYMMETRIC SYSTEM OF HARMONY
(Type II)

THE diatonic-symmetric system of harmony must satisfy two requirements:

(1) all root-tones of the diatonic-symmetric system must belong to one scale of the First Group;

(2) all chord structures must be pre-selected; they are not affected by the intonation of scale formed by the root-tones.

In this system of harmony, structural groups must be superimposed upon the progressions of the root-tones belonging to one scale. This form of harmony has advantages over the diatonic system itself, to which I refer as Type I. Like the diatonic system, the diatonic-symmetric system produces a united tonality, which is due to the structural unity of the scale. Unlike the diatonic system, the diatonic-symmetric system is not bound to use the structures which are considered defective in the equal temperament [like $S_4(5)$, for example], as the individual structures and the structural groups are a matter of free choice.

Unlike the diatonic system, the diatonic-symmetric system has a greater variety of intonations, as the pre-selected structures unavoidably introduce new accidentals (alterations), which implies a modulatory character without destroying the unity of the tonality.

Examples of Harmony Type II.

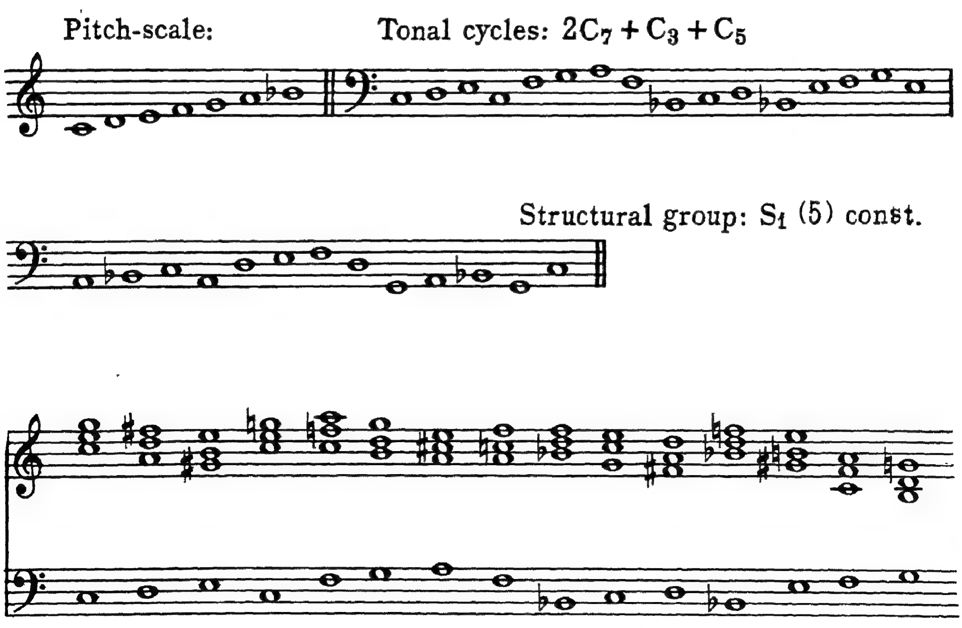
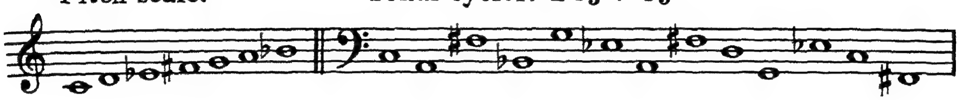


Figure 47: Diatonic-symmetric system (continued).

Figure 47. Diatonic-symmetric system (concluded).

Pitch-scale: Tonal cycles: $2C_3 + C_5$



Structural group: $S_1 + S_3 + 2S_2$

Figure 48. Diatonic-symmetric system (continued).

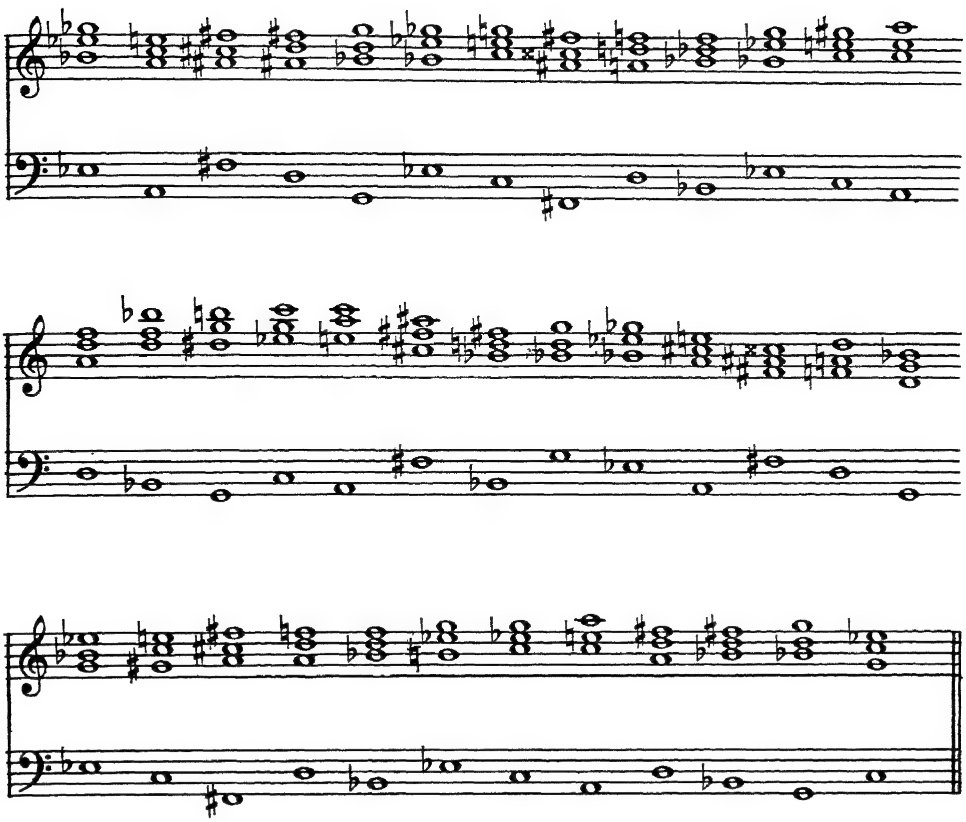


Figure 48. Diatonic-symmetric system (concluded).

CHAPTER 5

THE SYMMETRIC SYSTEM OF HARMONY

(*Type III*)

THE symmetric system of harmony of the third type must satisfy the following requirements:

(1) the root-tones and their progressions are the roots of two (i.e., $\sqrt{2}$, $\sqrt[3]{2}$, $\sqrt[4]{2}$, $\sqrt[6]{2}$, $\sqrt[12]{2}$), that is, the points of symmetry of an octave.

(2) chord structures are pre-selected.

As a consequence of motion through symmetric roots, each voice of harmony produces one of the pitch-scales of the third group.

Symmetric C_0 represents one tonic;

$\sqrt{2}$ represents two tonics;
$\sqrt[3]{2}$ represents three tonics;
$\sqrt[4]{2}$ represents four tonics;
$\sqrt[6]{2}$ represents six tonics;
$\sqrt[12]{2}$ represents twelve tonics.

The correspondences of the tonal cycles and the symmetric roots are as follows:

One tonic: C——————C
C_0

Two tonics: C——F♯——C
C_5 C_{-5}

Three tonics: C——A♭——E——C
C_3 C_3 C_3
C——E——A♭——C
C_{-3} C_{-3} C_{-3}

Four tonics: C——A——F♯——E♭——C
C_3 C_3 C_3 C_3
C——E♭——F♯——A——C
C_{-3} C_{-3} C_{-3} C_{-3}

Six tonics: C——D——E——F♯——A♭——B♭——C
C_7 C_7 C_7 C_7 C_7 C_7
C——B♭——A♭——F♯——E——D——C
C_{-7} C_{-7} C_{-7} C_{-7} C_{-7} C_{-7}

Twelve tonics: C——D♭——D♮——E♭——E♮ . . .
C_7 C_7 C_7 C_7
C——B——B♭——A——A♭
C_{-7} C_{-7} C_{-7} C_{-7}

Transformations with regard to positions and voice-leading remain the same as in the diatonic system. In case of doubt, cancel *all* the accidentals and test the leading of voices that way.

A. Two Tonics

Two tonics break an octave into two uniform intervals. The second tonic (T_2) being the $\sqrt{2}$ produces the center of an octave. This property makes the *two-tonic system reversible.* All points of intonation in the ↻ as well as in the ↺ transformation are identical, that is, both clockwise and counterclockwise voice-leading produce the same pattern of motion. This is true only in the case of two tonics.

Two tonics form a *continuous system,* i.e., the recurring tonic does not appear in its original position. Two tonics produce a *triple recurrence-cycle* before the original position falls on the first tonic (T_1) for the ↻ and the ↺. Const. 3 produces a closed system.

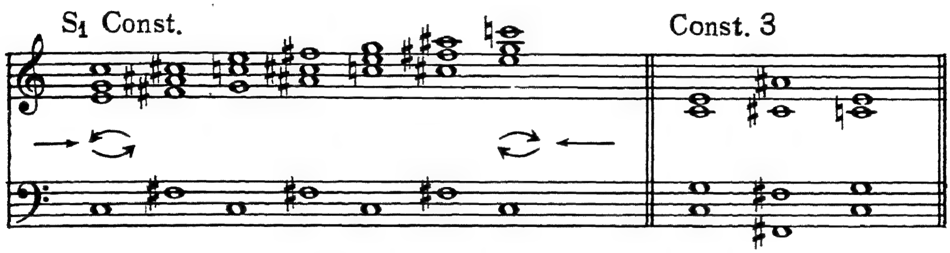


Figure 49. S_1 const. and const.3.

The upper voice of harmony produces the following scale: c – d♭ – e – f♯ – g – a♯ – (c) = (1+3) + 2 + (1+3) + 2. All other voices of the above progression produce the same scale starting from its different phases.

It is easy to see that this scale belongs to the third group and is constructed on two tonics.

By selecting other structures and structural groups of S(5), one can get other scales of the third group. For example, the use of S_2 const. produces the following scale: c – d♭ – e♭ – f♯ – g – a – (c) = (1+2) + 3 + (1+2) + 3.

Structural groups may be used in two ways:

(1) S changes with each tonic;
(2) the groups of S produce C_0 on each tonic.

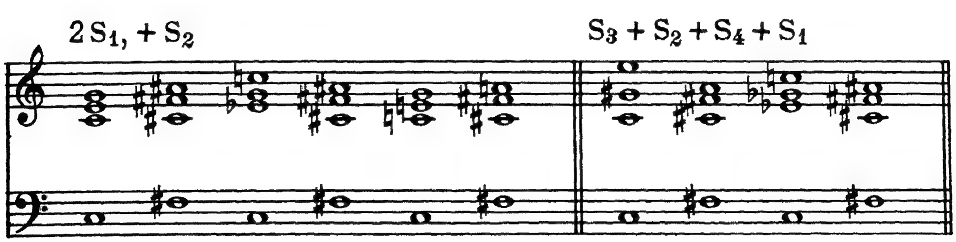


Figure 50. S changes with each tonic.

Figure 51. S produces C_0 on each tonic.

Combinations of the preceding two methods in the structural selection of each tonic of any one symmetric system are applicable to all symmetric sytems.

Figure 52. Structural selection of each tonic.

Longer progressions may be obtained through the use of longer structural groups, such as rhythmic resultants, power-groups, series of growth, etc.

In some cases, the number of terms in the structural group produces interference against the number of tonics in the symmetric system.

Example:

$$T_1,\quad T_2;\quad 2S_1+S_2+S_1+S_2+S_1+S_2+2S_1.$$

$(S_1T_1+S_1T_2+S_2T_1+S_1T_2+S_2T_1+S_1T_2+S_2T_1+S_1T_2+S_1T_1)+(S_1T_2+$
$+S_1T_1+S_2T_2+S_1T_1+S_2T_2+S_1T_1+S_2T_2+S_1T_1+S_1T_2).$

B. Three Tonics

Three tonics produce a closed system for ↻ and ↺, and a continuous system (two recurrence-cycles) for const. 3.

Figure 53. Three tonics.

C. Four Tonics

Four tonics produce a continuous system (three recurrence-cycles) for ↻ and ↺, and a closed system for const. 3.

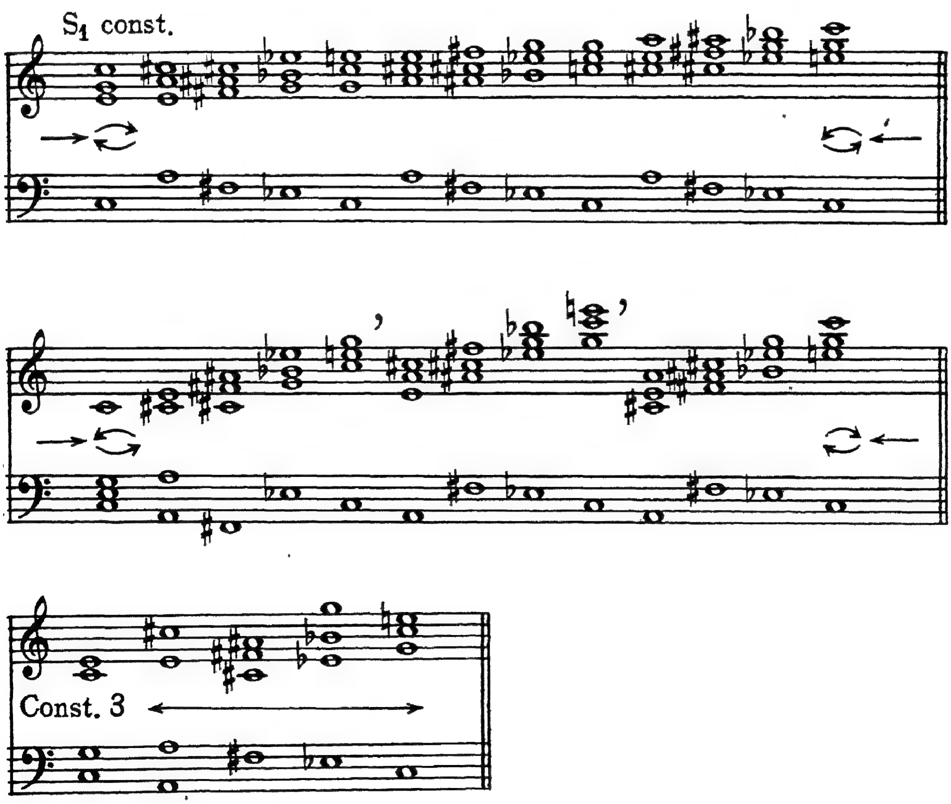


Figure 54. Four tonics.

D. Six Tonics

Six tonics produce a closed system for ↻ and ↺, as well as for the const. 3.

S$_1$ const.

Figure 55. Six tonics.

E. Twelve Tonics

Twelve tonics produce a closed system for ↻ and ↺, as well as for the const. 3.

S$_1$ const.

Figure 56. Twelve tonics.

CHAPTER 6

VARIABLE DOUBLINGS IN HARMONY

HARMONY, in many cases conceived as an accompaniment, may be given a self-sufficient character by means of *variable doublings*. This device gives to chord progressions a *greater versatility of sonority and voice-leading* than the one usually observed.

Variable doublings comprise the three functions of S(5). Thus, the root, the third or the fifth can be doubled. The notation to be used is: S(5)①, S(5)③ and S(5)⑤.

When the root-tone remains in the bass, S(5)① is the only case of doubling where all three functions (1, 3, 5) appear in the upper three parts.

The following represents a comparative table of functions in the three upper parts under various forms of doubling.

S(5)① = 1, 3, 5
S(5)③ = 3, 3, 5
S(5)⑤ = 3, 5, 5

In cases S(5)③ and S(5)⑤, only three positions are possible for each case. Black notes represent variants where unison is substituted for an octave.

Positions

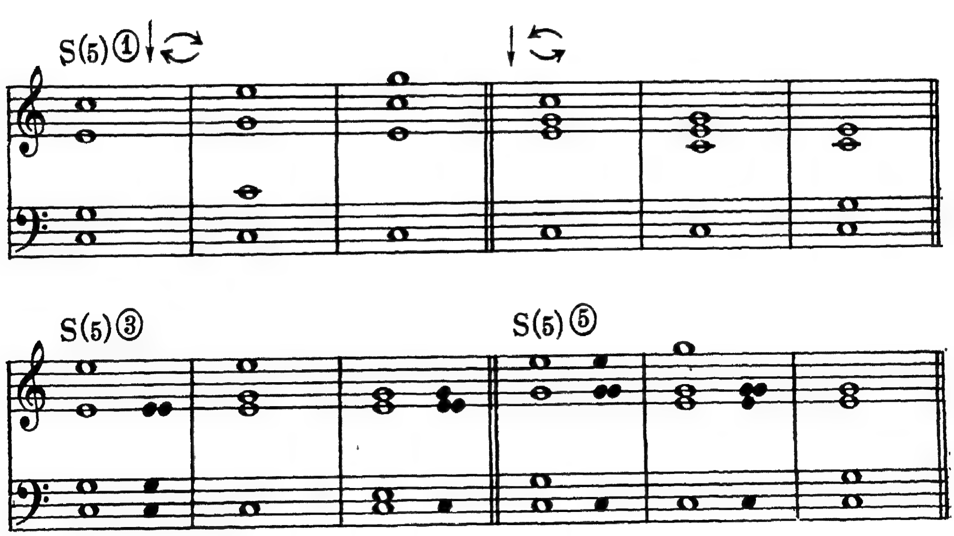


Figure 57. Various forms of doubling in 3 upper parts.

Transformations

S(5)① ⟷ S(5)③

5 ↔ 5	5 ↔ 3	5 ↔ 3
3 ↔ 3	3 ↔ 5	3 ↔ 3
1 ↔ 3	1 ↔ 3	1 ↔ 5

C_3 const. C_5 const. C_7 const.

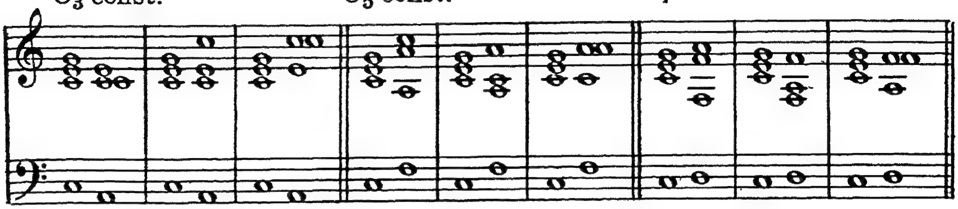


Figure 58. Transformation S(5)① ⟷ S(5)③.

S(5)③ ⟷ S(5)③

5 ↔ 3
3 ↔ 5
3 ↔ 3

C_3 const. C_5 const. C_7 const.

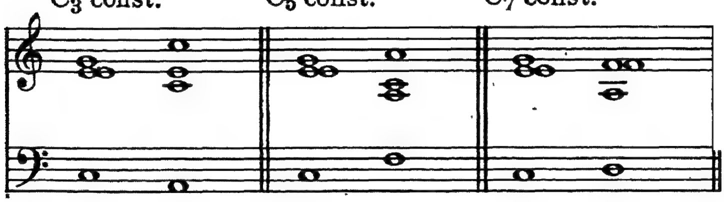


Figure 59. Transformation of S(5)③ ⟷ S(5)③.

S(5)① ⟷ S(5)⑤

5 ↔ 5	5 ↔ 3	5 ↔ 5
3 ↔ 5	3 ↔ 5	3 ↔ 3
1 ↔ 3	1 ↔ 5	1 ↔ 5

C_3 const. C_5 const. C_7 const.

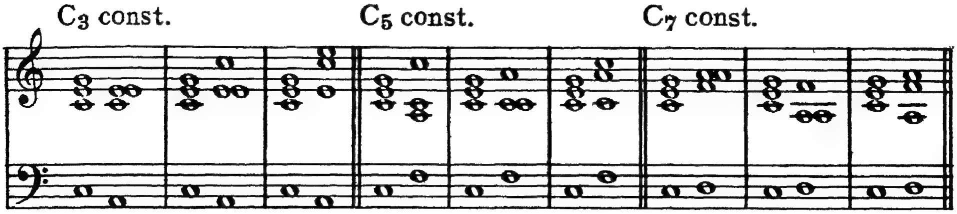


Figure 60. Transformation of S(5)① ⟷ S(5)⑤.

$$S(5)^{\textcircled{5}} \longleftrightarrow S(5)^{\textcircled{5}}$$

$$\begin{array}{c} 5 \leftrightarrow 3 \\ 5 \leftrightarrow 5 \\ 3 \leftrightarrow 5 \end{array}$$

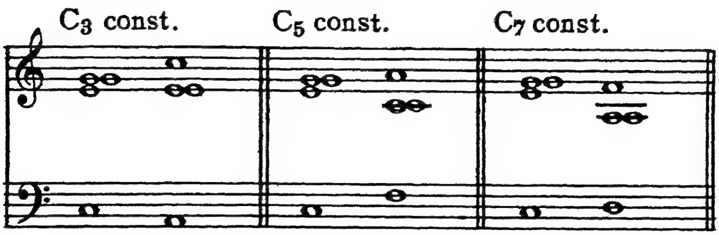


Figure 61. Transformation of $S(5)^{\textcircled{5}} \longleftrightarrow S(5)^{\textcircled{5}}$.

$$S(5)^{\textcircled{3}} \longleftrightarrow S(5)^{\textcircled{5}}$$

$$\begin{array}{c} 5 \leftrightarrow 5 \\ 3 \leftrightarrow 5 \\ 3 \leftrightarrow 3 \end{array}$$

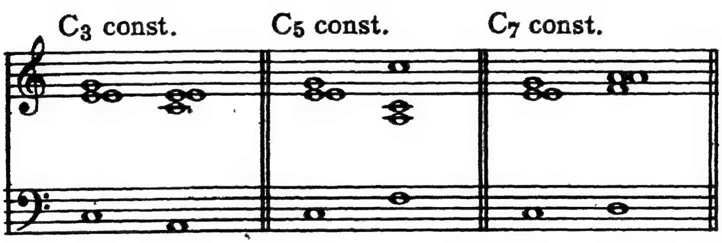


Figure 62. Transformation of $S(5)^{\textcircled{3}} \longleftrightarrow S(5)^{\textcircled{5}}$.

In reading these tables, consider *identical directions of the arrows* for the sequence of structures and for the corresponding transformations.

Note that there always are three transformations when $S(5)^{\textcircled{1}}$ participates and only one when it does not.

Musical tables in the above figures are devised from an initial chord in the same position. Similar tables can be constructed from all positions as well as in reverse sequence; also in the cycles of the negative form.

Variable doublings are subject to distributive arrangement and can be superimposed on any desirable cycle-group.

Example: $2C_3 + C_5 + C_7;\ S(5)^{1} + 2S(5)^{3} + S(5)^{5}$.

$H^{\rightarrow} = S(5)^{1} + C_3 + S(5)^{3} + C_3 + S(5)^{3} + C_5 + S(5)^{5} + C_7 + $
$+ S(5)^{1}$.

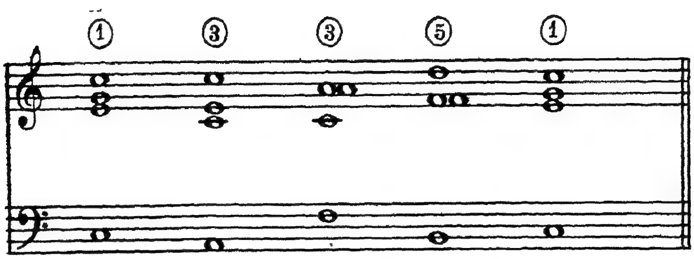


Figure 63. Variable doublings superimposed on a cycle-group.

Example: $2C_5 + C_3 + C_5 + 2C_7; S(5)^{3} + S(5)^{1} + S(5)^{3} + S(5)^{5}$.

$H^{\rightarrow} = S(5)^{3} + C_5 + S(5)^{1} + C_5 + S(5)^{3} + C_3 + S(5)^{5} + C_5 +$
$+ S(5)^{3} + C_7 + S(5)^{1} + C_7 + S(5)^{3} + C_5 + S(5)^{5} + C_5 + S(5)^{3} +$
$+ C_3 + S(5)^{1} + C_5 + S(5)^{3} + C_7 + S(5)^{5} + C_7 + S(5)^{3}$.

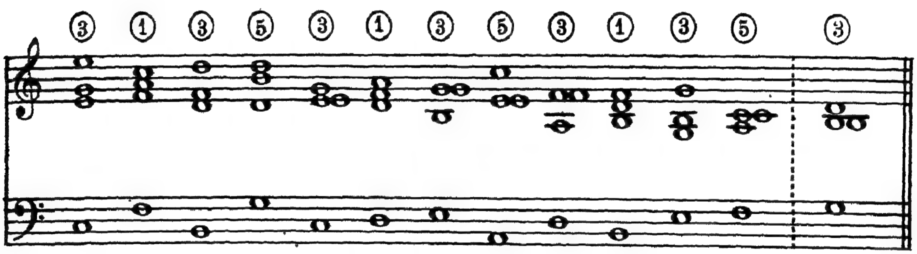


Figure 64. Variable doublings superimposed on a cycle-group.

Variable doublings are applicable to all types of harmonic progressions, thus including types II and III.

Type II (diatonic-symmetric).
$H^{\rightarrow}$ as in the preceding example.
$S^{\rightarrow} = 2S_2 + S_3 + S_1$

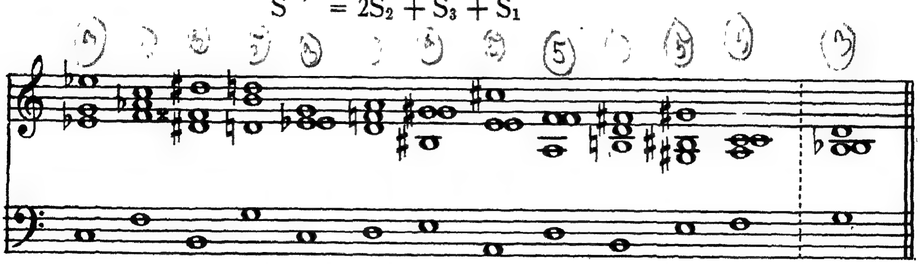


Figure 65. Variable doublings are applicabe to type II.

Type III (symmetric).

$$H^{\rightarrow}(6T) = T_1S_1⑤ + T_2S_2③ + T_3S_1① + T_4S_3③ + T_5S_4⑤ + T_6S_1③ + T_1S_2⑤.$$

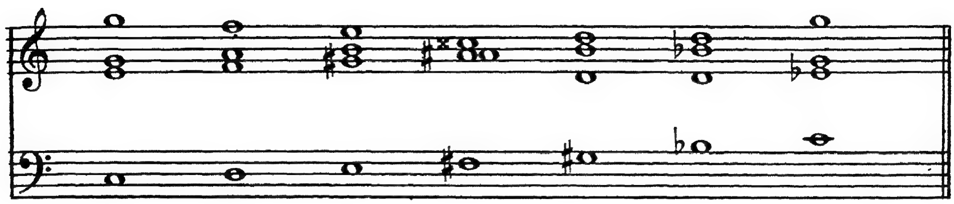


Figure 66. Variable doublings are applicable to type III.

CHAPTER 7

INVERSIONS OF THE S(5) CHORD

THE usual technique of inversions, strictly speaking, is unnecessary to a composer. The reason is that through vertical permutation of the positions of parts in any harmonic continuity of S(5), the inversions appear automatically when inner or upper parts become the bass parts. This technique was fully described in Book Three, *Variations of Music by Means of Geometrical Projection*, in the section on continuity of geometrical inversions.*

For an *analyst* or a *teacher*, however, a thorough systematization of the classical technique of inversions is a necessity, for there is no other branch of harmony where the confusion is greater and the information less reliable.

The first inversion of S(5) is known as a "sixth-chord" or a "third-sixth-chord" and is expressed in this notation by the symbol S(6). The only condition under which S(5) becomes an S(6) is when the *third* (3) appears in the bass. The positions of the upper voices are not affected by such a change, but the forms of doublings are affected. Which doublings are appropriate in each case, will be discussed later.

Assuming that any S(6) may be either S(6)①, or S(6)③, or S(6)⑤, we obtain the following Table of Positions:

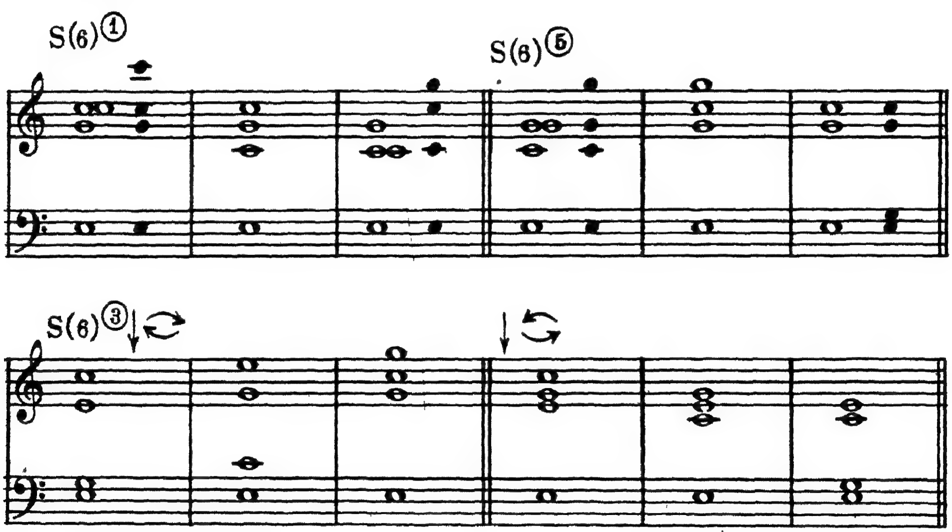


Figure 67. Positions of S(6).

It is easy to memorize the above table, as S(6)① and S(6)⑤ positions are systematized through the following characteristics: (1) the doubled function appears above the remaining functions; (2) the doubled function surrounds the remaining function; (3) the doubled function appears below the remaining function.

*See Book III, Chapter 1, pp. 200-203.

S(6)③ is identical with S(5) positions, except that the bass has constant 3.

Harmonic progressions ($H^{\rightarrow}$) consisting of S(5) and S(6) are based on the following combinations by two:

1. S(5) → S(5); 2. S(5) → S(6); 3. S(6) → S(5); 4. S(6) → S(6).

As the first case is covered by the previous technique, we are concerned, for the present, with the last three cases.

All the following transformations, being applied to voice-leading, are reversible, as in the case of variable doublings of S(5). Tonal cycles are always measured through *root-tones*.

S(5) ←——		——→ S(6)①
5 ↔ 5	5 ↔ 1	5 ↔ 1
3 ↔ 1	3 ↔ 5	3 ↔ 1
1 ↔ 1	1 ↔ 1	1 ↔ 5

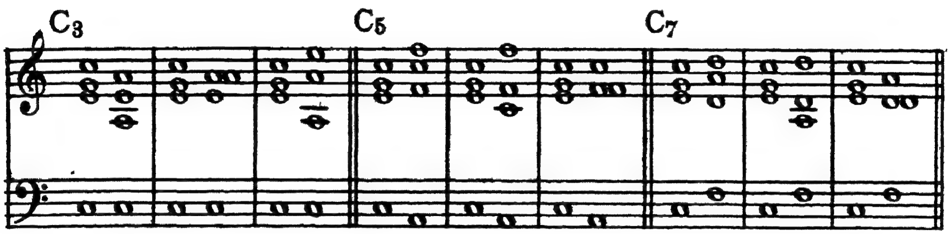


Figure 68. Transformations of S(5) ←——→ S(6)①.

S(5) ←——		——→ S(6)⑤
5 ↔ 1	5 ↔ 5	5 ↔ 5
3 ↔ 5	3 ↔ 1	3 ↔ 5
1 ↔ 5	1 ↔ 5	1 ↔ 1

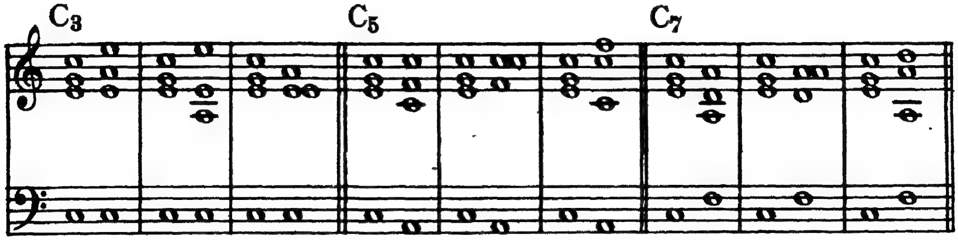


Figure 69. Transformation of S(5) ←——→ S(6)⑤.

S(5) ⟷	S(6)③
5 ↔ 3	5 ↔ 1
3 ↔ 1	3 ↔ 3
1 ↔ 5	1 ↔ 5
→	Const. 3
	Const. 3 ←

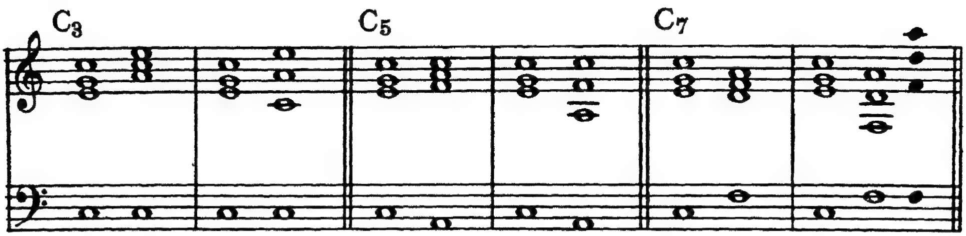


Figure 70. Transformation of S(5) ⟷ S(6)③.

S(6)① ⟷ S(6)①

5 ↔ 1
1 ↔ 5
1 ↔ 1

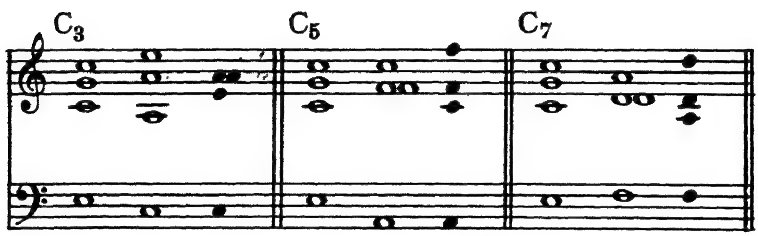


Figure 71. Transformation of S(6)① ⟷ S(6)①.

S(6)① ⟷ S(6)⑤

5 ↔ 5
1 ↔ 1
1 ↔ 5

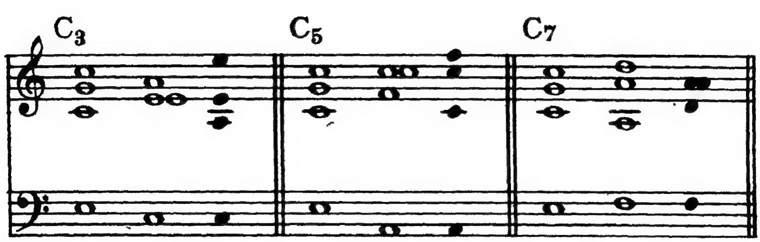


Figure 72. Transformation of S(6)① ⟷ S(6)⑤.

S(6)⑤ ⟷ S(6)⑤

5 ↔ 1
5 ↔ 5
1 ↔ 5

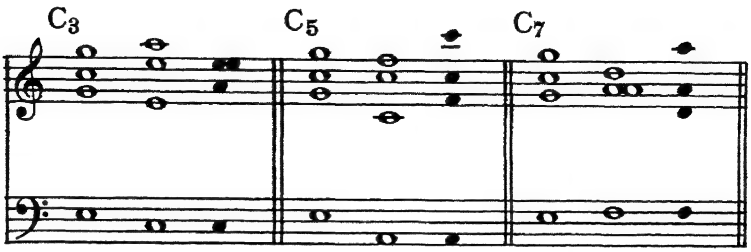


Figure 73. Transformation of S(6)⑤ ⟷ S(6)⑤.

S(6)① ⟷ S(6)③

5 ↔ 5	5 ↔ 1	5 ↔ 3
1 ↔ 3	1 ↔ 5	1 ↔ 1
1 ↔ 1	1 ↔ 3	1 ↔ 5

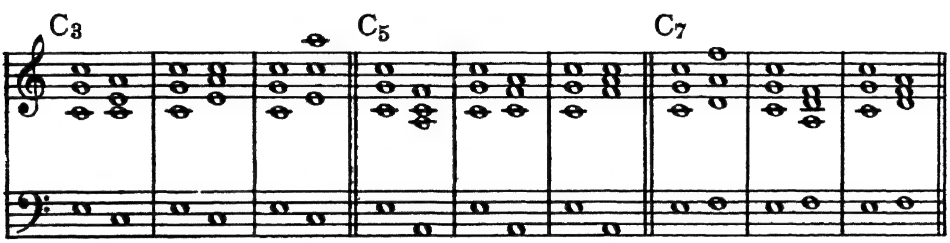


Figure 74. Transformation of S(6)① ⟷ S(6)③.

S(6)③ ⟷ S(6)③

5 ↔ 1	5 ↔ 3
3 ↔ 5	3 ↔ 1
1 ↔ 3	1 ↔ 5

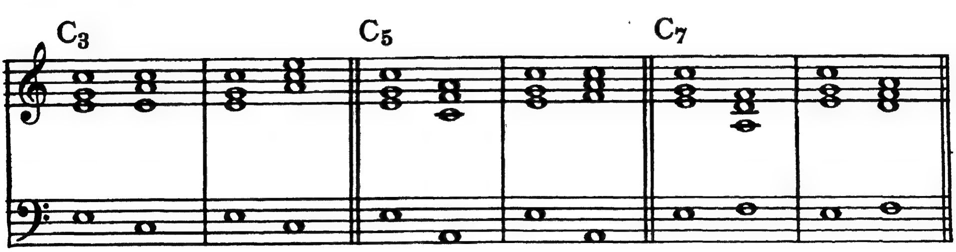


Figure 75. Transformation of S(6)③ ⟷ S(6)③

S(6)⑤ ←	→	S(6)③
$5 \leftrightarrow 5$	$5 \leftrightarrow 1$	$5 \leftrightarrow 3$
$5 \leftrightarrow 3$	$5 \leftrightarrow 5$	$5 \leftrightarrow 1$
$1 \leftrightarrow 1$	$1 \leftrightarrow 3$	$1 \leftrightarrow 5$

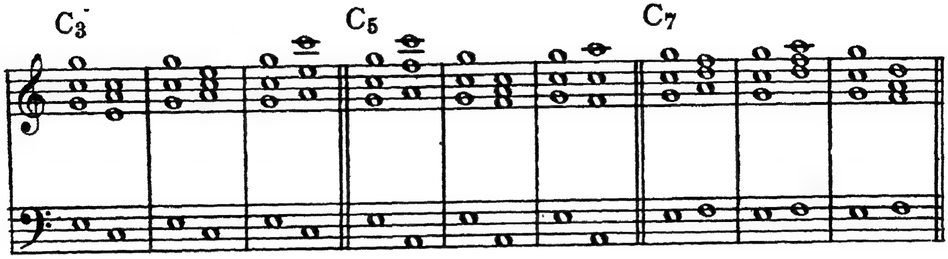


Figure 76. Transformation of S(6)⑤ ⟷ S(6)③.

Any variants which conform to identical transformations (like the black notes in some of the preceding tables) are as acceptable as those in the tables.

A. Doublings of S(6)

Musical habits are formed comparatively rapidly. Once they assume the form of natural reactions, they influence us more than the purely acoustical factors. This is particularly true in the case of doublings of S(6). The mere fact that identical doublings in the different musical contexts affect us in a different way, shows that our auditory reactions in music are not natural but conditioned.

The principle offered here are based on a comparative study of the respective forms of music.

There are two technical factors affecting the doubling in an S(6):

(1) the structure of the chord;

(2) the degree of the scale (on which the chord is constructed).

These two influences are ever-present, regardless of the type to which the respective harmonic continuity belongs.

While in harmonic progressions of type II and III, the structure of the chord is the most influential factor—in the diatonic progressions (type I) it is exactly the reverse. The influence of a constant pitch-scale is so overwhelming that each chord becomes associated with its definite position in the scale. Thus, one chord begins to sound to us as a dominant and another as a tonic, a mediant or a leading tone. This hierarchy of the various chords calls for the different forms of doubling, particularly when the respective chords appear in the different inversions.

The following is most practical for use in diatonic progressions.

Strong Factor			Weak Factor		
The degree of the scale	Regular Doubling	Irreg. Doubling	The structure of the chord	Regular Doubling	Irreg. Doubling
I, IV, V, VI	1, 5	3	$S_1(6)$	1, 5	3
II, III, VII	3	1, 5	$S_2(6)$	3	1, 5
			$S_3(6)$	3	——
			$S_4(6)$	1, 3, 5	——

Figure 77. Table for doubling in diatonic progressions.

Regular doublings are *statistically predominant.* Irregular doublings, in most cases, are the result of *melodic* tendencies.

In reading the above table, give preference to the strong factor, except in the case of $S_3(6)$ and $S_4(6)$. It is customary to believe that an $S_1(6)$ must have doubled root or fifth. But in reality it seldom happens when such a chord belongs to II, III or VII. Naturally, all our habits with regard to doublings are formed on the more customary major and minor scales. The above table will work perfectly when applied to such scales. There will be no discrepancy when $S_3(6)$ and $S_4(6)$ are compared with the data on the left side of the table, as such structures do not occur on the main degrees of the usual scales.

In using less familiar scales, however, one or another type of doubling will not make as much difference. Yet in such cases the structure may become a more influential factor, though the sequence is diatonic.

In types II and III the most practical forms of doublings are:

Structure	Regular Doubling	Irregular Doubling
$S_1(6)$	1, 5	3
$S_2(6)$	1, 5	3
$S_3(6)$	3	—
$S_4(6)$	1, 3, 5	—

Figure 78. Forms of doubling for types II and III.

B. Continuity of S(5) and S(6)

The comparative characteristic of S(5) is its stability, due to the presence of the root-tone in the bass. The absence of the root-tone in the bass of S(6) deprives this structure of such stability.

Composition of continuity consisting of S(5) and S(6) results in an interplay of stable and unstable units or groups. The following fundamental forms of continuity utilizing the above-mentioned structures are possible:

(1) S(5) const. ——— stable

(2) S(6) const. ——— unstable

(3) [S(5) + S(6)] + . . . alternate

(4) 2S(5) + S(6) + S(5) + 2S(6)

3S(5) + S(6) + 2S(5) + 2S(6) + S(5) + 3S(6)

4S(5) + S(6) + 3S(5) + 2S(6) + 2S(5) + 3S(6) + S(5) + 4S(6)

⟶ increasing instability

increasing stability ⟵

(5) 4S(5) + 2S(6) + 2S(5) + S(6)

⟶ proportionately decreasing ratios

proportionately increasing ratios ⟵

(6) S(5) + 2S(6) + 3S(5) + 5S(6) + 8S(5) + 13S(6)

⟶ progressive over-balancing of unstable elements

S(6) + 2S(5) + 3S(6) + 5S(5) + 8S(6) + 13S(5)

⟶ progressive over-balancing of stable elements

Many other forms of the distribution of S(5) and S(6) may be devised on the basis of my theory of rhythm.*

Diatonic

S(6) Const.; $2C_7 + 2C_5 + C_3 + C_5$

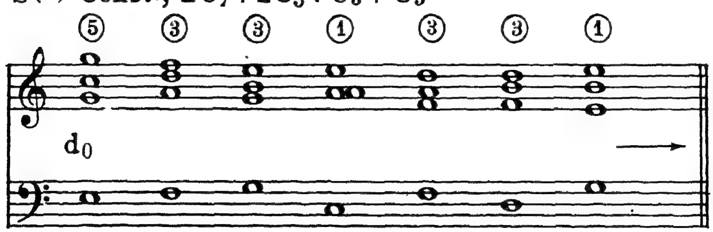
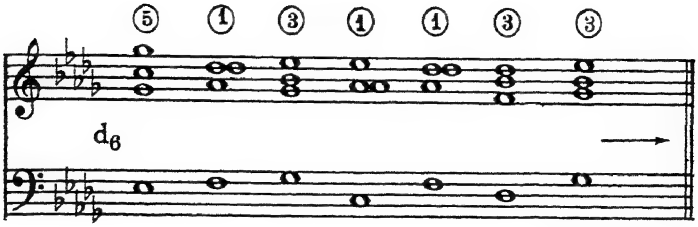


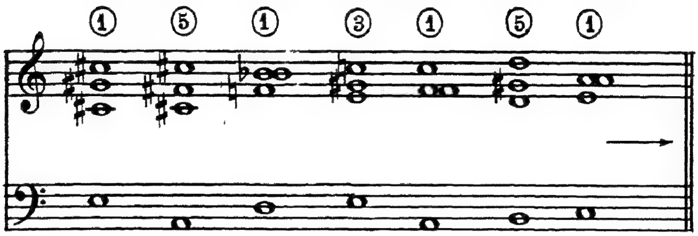
Figure 79. Progressions in diatonic, diatonic-symmetric and symmetric (continued).

*See Book I.



Diatonic-Symmetric

$2S_2(6)+S_1(6)+S_3(6)+S_1(6)+S_4(6)$; $2C_5+C_7+C_5+2C_7$



Symmetric

$S_3(6)+S_2(6)+S_4(6)+2S_1(6)$; Six tonics

Figure 79. Progressions in diatonic, diatonic-symmetric and symmetric. (concluded).

Diatonic

$3S(6)+S(5)+2S(6)+2S(5)+S(6)+3S(5)$; $2C_5+C_7$

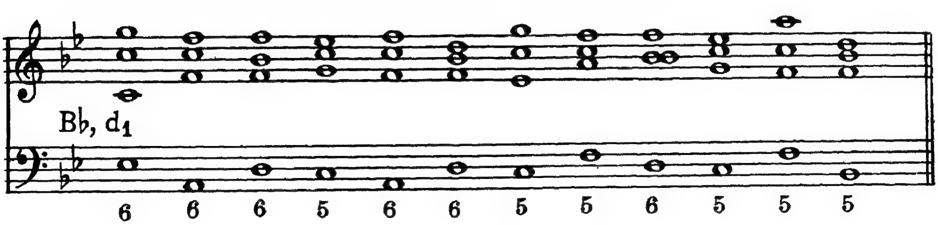
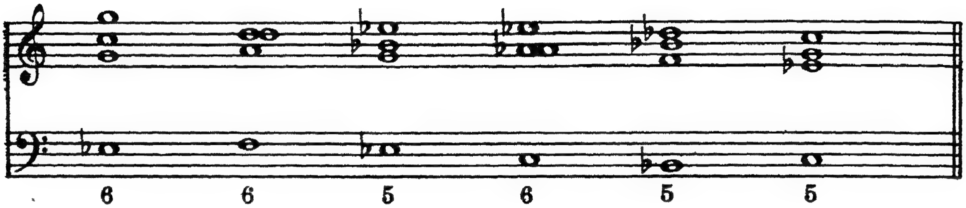


Figure 80. Progressions in diatonic, diatonic-symmetric and symmetric (continued).

Diatonic-Symmetric

$2S_2(6)+S_1(5)+S_1(6)+2S_2(5)$; $2C_7+C_5$; Scale of roots: Aeolian



Symmetric

$\{[S_1(5)+S_2(6)]\,T_1+[S_4(6)+S_1(5)]\,T_2\}+\dots$ Four tonics

Figure 80. Progressions in diatonic, diatonic-symmetric and symmetric. (*concluded*).

CHAPTER 8

GROUPS WITH PASSING CHORDS

A. Passing Sixth-chords

A GROUP with a passing S(6) is a *pre-set combination* of three chords: namely, S(5) + S(6) + S(5). *Every passing chord occupies the center of its group, appears on a weak beat* and has a *doubled bass.* The complete expression for a group (G) with passing sixth-chord is:

$$G_6 = S(5) + S(6)^{\textcircled{3}} + S(5).$$

This formula is not reversible in actual intonation. The relationship between the extreme chords of G_6 is C–5. This relationship remains constant in all cases of classical music.

We shall extend this principle to all cycles. Under such conditions G_6 retains the following characteristics:

(1) The transformation between the extreme chords of the group is *always clockwise* for both the positive and the negative cycles.
(2) The bass progression is: $1 \rightarrow 3 \rightarrow 1$, which necessitates the first condition.

In the classical form of G_6, the bass moves by the thirds. Thus, 3 in the bass under S(6) is a third above its preceding position under the first S(5), and a third below its following position under the last S(5).

In order to obtain G_6, it is necessary to connect S(5) with the next S(5) through C–5 and add the intermediate third of the first chord in the bass, without moving the remaining voices.

$$G = S(5) + S(6)^{\textcircled{3}} + S(5)$$

C–5

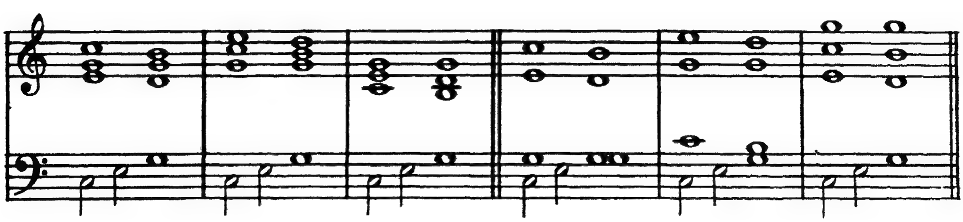


Figure 81. Passing sixth chords.

There are three melodic forms for the bass movement.

Figure 82. Melodic forms for bass.

Combinations of these three forms in sequence produce a very flexible bass part and, being repeated with one G_6, make expressive cadences of a Mozartian flavor.

Figure 83. Combination of figures 81 and 82.

B. CONTINUITY OF G_6.

Continuity of such groups can be obtained by connecting them through the tonal cycles.

Connecting by C_5 closes the sequence, while C_3 and C_7 produce a progression of $7G_6$.

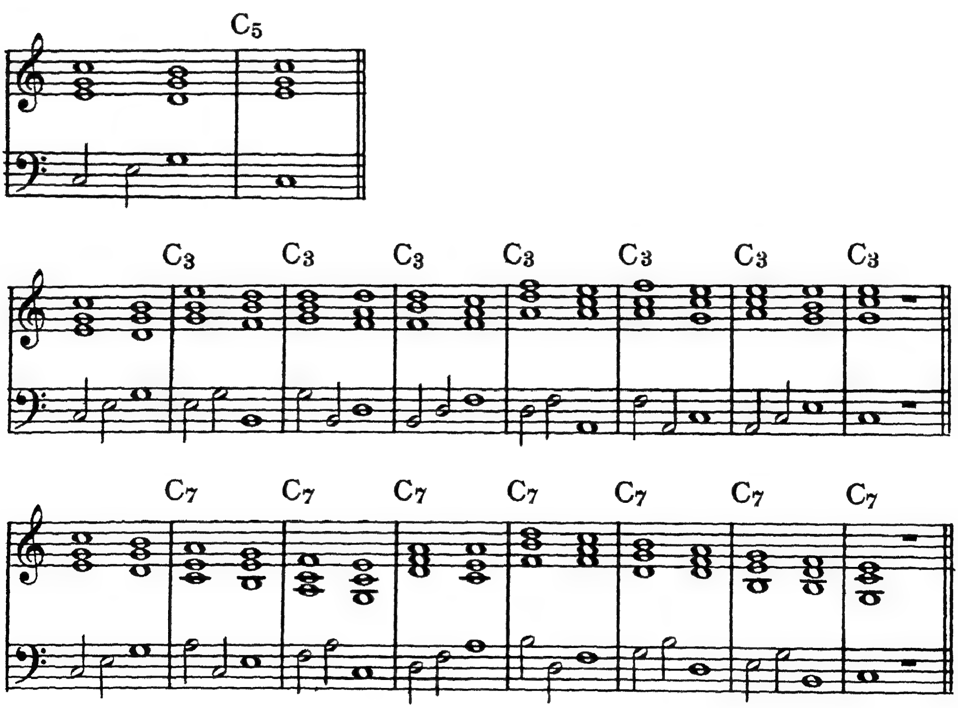


Figure 84. Continuity of G_6

Further versatility of G_6 progressions can be achieved by varying the cycles between the groups. Any time a decisive cadence is desirable, C_5 must be introduced, as this cycle closes the progression.

$$H^{\rightarrow} = G_6 + C_7 + G_6 + C_7 + G_6 + C_3 + G_6 + C_7 + G_6 + \\ + C_3 + G_6 + C_3 + G_6 + C_5$$

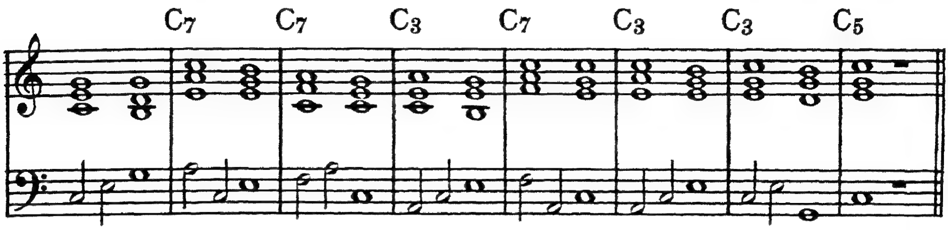


Figure 85. Varying cycles between G_6.

C. Generalization of G_6

In addition to the classical form of G_6, other forms can be developed through the use of other than C–5 cycles within the group. Of course, each cycle produces its own characteristic bass pattern.

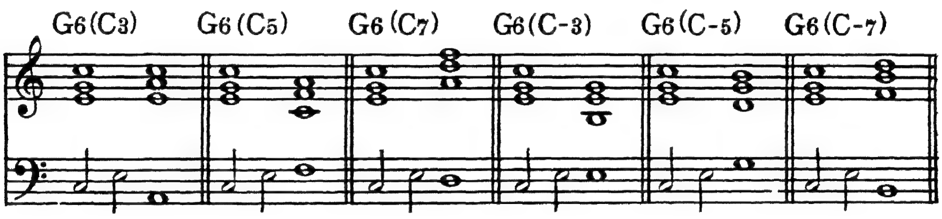


Figure 86. Various forms of G_6.

The respective variations of the bass pattern will be as follows:

Figure 87. Variations of bass pattern for figure 86.

D. Continuity of the Generalized G_6

Such a continuity can be developed through the selective progressions of the various forms of G_6 combined with the various cycle connections between the groups.

Example:

$$H^{\rightarrow} = G_6(C\text{-}5) + C_7 + G_6(C_3) + C_5 + G(C\text{-}7) + C_3 + \\ + G(C\text{-}5) + C_5 + G(C_7) + C_5$$

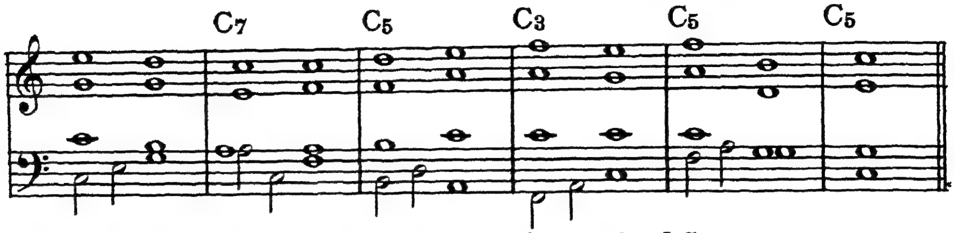


Figure 88. Continuity of generalized G_6.

E. Generalization of the Passing Third

It follows from the technique of groups with a passing sixth-chord that the first two chords, i.e., S(5) and S(6)③, belong to C_0, and that the position of the three upper parts does not change until the last chord of the group appears. This last chord, S(5), can be in any relation but C_0 with the preceding chord.

If we think of the appearance of the third in the bass during S(6)③ merely as a passing third, it is easy to see that this entire technique can be generalized. The passing 3 can be used after any S(5), providing the transformation between the latter and the following S(5) is *clockwise* for all the cycles. Such a device can be applied to any progression of S(5) with the root-tones in the bass.

Example:

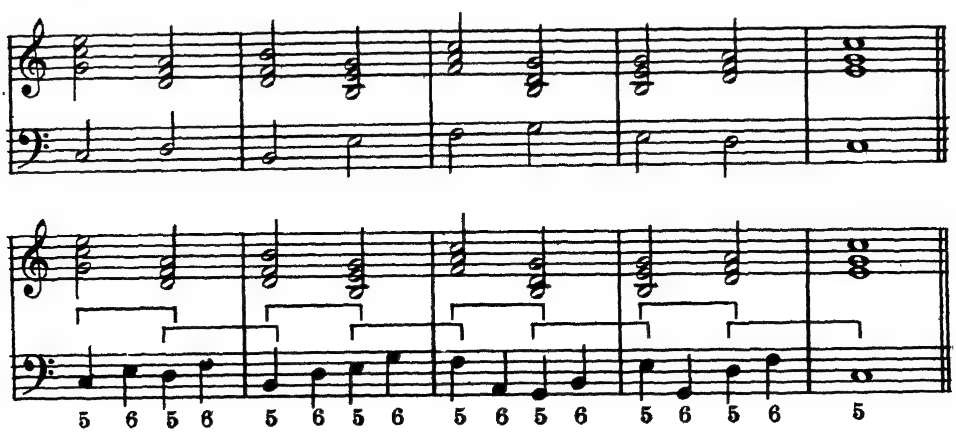


Figure 89. Passing third illustrated.

The effect of such harmonic continuity is one of overlapping groups of G_6, as marked in the preceding figure.

F. Applications of G_6 to Diatonic-Symmetric (Type II) and Symmetric (Type III) Progressions

The use of structures of S(5) and S(6)③ in the groups with a passing sixth-chord must satisfy the following requirement: *the adjacent S(5) and S(6)③ of one group must have identical structures.*

This requirement does not affect the form of the last S(5) of a group; neither does it influence the selection of the forms of S(5) in the adjacent groups.

As each G_6 consists of three places, two of which are identical, the number of structural combinations for the individual groups equals $4^2 = 16$.

$S_1 + S_1$	$S_2 + S_1$	$S_3 + S_1$	$S_4 + S_1$
$S_1 + S_2$	$S_2 + S_2$	$S_3 + S_2$	$S_4 + S_2$
$S_1 + S_3$	$S_2 + S_3$	$S_3 + S_3$	$S_4 + S_3$
$S_1 + S_4$	$S_2 + S_4$	$S_3 + S_4$	$S_4 + S_4$

Thus, we obtain 16 forms of G_6 with the following distribution of structural combinations:

$$G_6 = S_1(5) + S_1(6)③ + S_1(5)$$
$$G_6 = S_1(5) + S_1(6)③ + S_2(5)$$
$$G_6 = S_1(5) + S_1(6)③ + S_3(5)$$
$$G_6 = S_1(5) + S_1(6)③ + S_4(5)$$

$$G_6 = S_2(5) + S_2(6)③ + S_1(5)$$
$$G_6 = S_2(5) + S_2(6)③ + S_2(5)$$
$$G_6 = S_2(5) + S_2(6)③ + S_3(5)$$
$$G_6 = S_2(5) + S_2(6)③ + S_4(5)$$

$$G_6 = S_3(5) + S_3(6)③ + S_1(5)$$
$$G_6 = S_3(5) + S_3(6)③ + S_2(5)$$
$$G_6 = S_3(5) + S_3(6)③ + S_3(5)$$
$$G_6 = S_3(5) + S_3(6)③ + S_4(5)$$

$$G_6 = S_4(5) + S_4(6)③ + S_1(5)$$
$$G_6 = S_4(5) + S_4(6)③ + S_2(5)$$
$$G_6 = S_4(5) + S_4(6)③ + S_3(5)$$
$$G_6 = S_4(5) + S_4(6)③ + S_4(5)$$

As the melodic interval in the bass, while moving from the root (1) in $S(5)$ to the third (3) in $S(6)$③, is identical for the forms S_1 and S_3, as well as S_2 and S_4, the total quantity of intonations in the bass part for one type of G_6 is $\frac{4}{2} = 2$.

$$S_1 + S_1$$
$$S_1 + S_2$$
$$S_2 + S_1$$
$$S_2 + S_2$$

As each intonation has 3 melodic forms and there are two different intonations, the total number of intonations combined with melodic forms in the bass part is 2x 3 = 6.

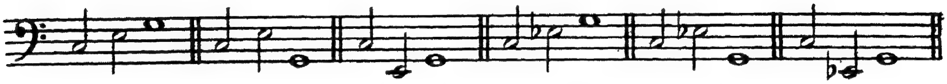


Figure 90. Six melodic forms in the bass part.

1. Progressions of Type II.

Example:

Forms of S: $S_2(5) + S_2(6)$③ $+ S_1(5)$
$H^{\rightarrow} = G_6(C\text{–}5) + C_3 + G_6(C\text{–}5) + C_7 + G_6(C\text{–}5) + C_5$.

Figure 91. Progression of type II.

Example:

Forms of S: $[S_1(5) + S_1(6)$③ $+ S_2(5)] + [S_3(5) + S_3(6)$③ $+ S_2(5)]$
$H^{\rightarrow}$ = as in the preceding example

Figure 92. Progression of type II.

Example:

Forms of S: $S_2(5) + S_2(6)^{③} + S_2(5)$
$H^{\rightarrow}$ = as in Figure 88.

Figure 93. Progression of type II.

Generalization of the passing third is applicable to this type of harmonic progression as well. The following is an application of the structural group $2S_1 + S_2 + 2S_1 + S_2 + 2S_1$ to Figure 89.

Figure 94. Generalization of passing third in type II.

2. Progressions of Type III.

Applications of G_6 to symmetrical systems of tonics disclose many unexplored possibilities, among which the two-tonic system deserves particular attention. As intervals forming the two tonics are equidistant, the passing tones of $S(6)^{③}$, which in turn may also be equidistant from T_1 and T_2, produce, in the bass movement, diminished seventh-chords in symmetric harmonization—a device heretofore unknown.

The justification for the use of G_6 in the symmetrical systems of tonics is based on the following deductions from the original classical form, i.e., G_6(C–5).

(Diatonic) (Symmetric)

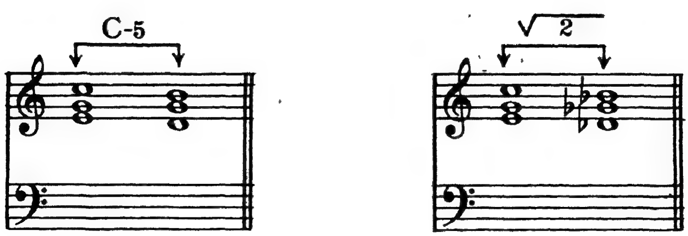


Figure 95. Justification for use of G_6 in symmetric systems of tonics.

The above-mentioned equidistancy of the two tonics permits retention of $H^{\rightarrow} = 3G_6$ until the cycle closes. Selecting S_1 for the entire G_6, we obtain:

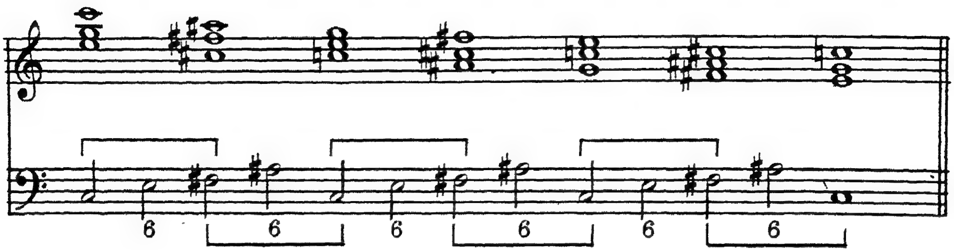


Figure 96. Progression of type III.

The overlapping of groups, indicated by the brackets in the above Figure, is an invariant of the symmetrical systems. Thus the passing third can be considered a general device for progressions of type III.

The number of bass patterns for the cycle of the *two tonics* equals: $2^2 = 4$.

The number of intonations in each cycle of the two tonics equals: $2^2 = 4$. The latter is due to the use of the different forms of S(5). The interval between 1 and 3 equals 4, and is identical for $S_1(5)$ and $S_3(5)$. The interval between 1 and 3 equals 3, and is identical for $S_2(5)$ and $S_4(5)$. Thus, by distributing the different structures through two tonics, we obtain the following combinations:

$S_1(T_1) + S_1(T_2)$	
$S_1(T_1) + S_3(T_2)$	identical intonations
$S_3(T_1) + S_1(T_2)$	in the bass part
$S_3(T_1) + S_3(T_2)$	
$S_2(T_1) + S_2(T_2)$	
$S_2(T_1) + S_4(T_2)$	identical intonations
$S_4(T_1) + S_2(T_2)$	in the bass part
$S_4(T_1) + S_4(T_2)$	
$S_1(T_1) + S_2(T_2)$	
$S_1(T_1) + S_4(T_2)$	identical intonations
$S_3(T_1) + S_2(T_2)$	in the bass part
$S_3(T_1) + S_4(T_2)$	
$S_2(T_1) + S_1(T_2)$	
$S_2(T_1) + S_3(T_2)$	identical intonations
$S_4(T_1) + S_1(T_1)$	in the bass part
$S_4(T_1) + S_3(T_2)$	

The following is a table of intonations and melodic forms in the bass part on two tonics. Total: $4^2 = 16$.

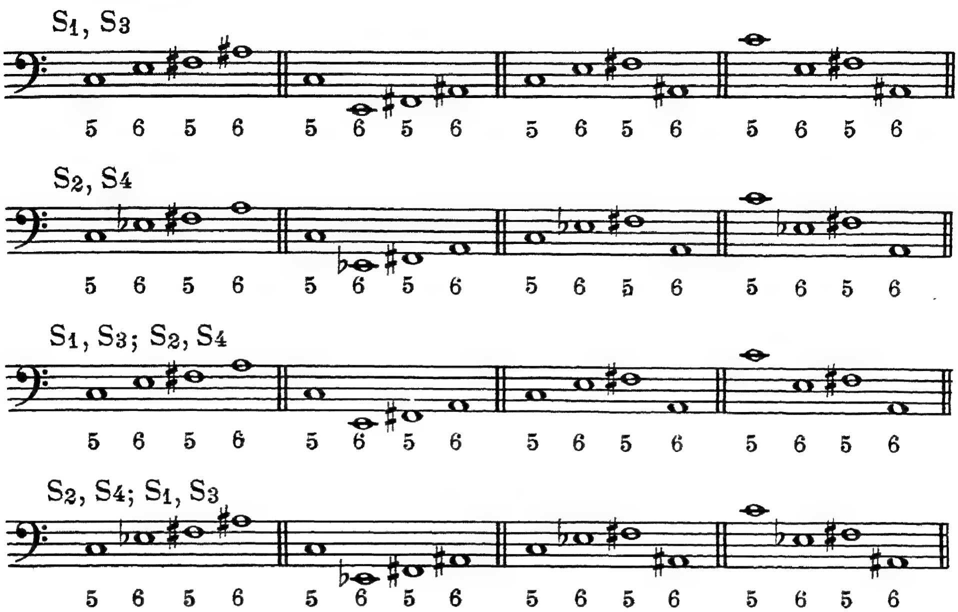


Figure 97. Intonations and melodic forms in bass part on two tonics.

The above combinations can be incorporated into a versatile continuity of G_6 on two tonics.

Example:

Figure 98. Continuity of G_6 on two tonics.

Application of G_6 to *three tonics* produces 8 melodic forms in the bass part: $2^3 = 8$.

$$T_1a_2 + T_2a_2 + T_3a_2$$
$$T_1b_2 + T_2a_2 + T_3a_2$$
$$T_1a_2 + T_2b_2 + T_3a_2$$
$$T_1a_2 + T_2a_2 + T_3b_2$$
$$T_1b_2 + T_2b_2 + T_3a_2$$
$$T_1b_2 + T_2a_2 + T_3b_2$$
$$T_1a_2 + T_2b_2 + T_3b_2$$
$$T_1b_2 + T_2b_2 + T_3b_2$$

Figure 99. Application of G_6 to three tonics.

The number of distributions of the different S through three tonics is $4^3 = 64$, while the number of non-identical intonations is $2^3 = 8$.

Non-identical intonations:

$S_1(T_1) + S_1(T_2) + S_1(T_3)$	$S_2(T_1) + S_2(T_2) + S_1(T_3)$
$S_1(T_1) + S_1(T_2) + S_2(T_3)$	$S_2(T_1) + S_1(T_2) + S_2(T_3)$
$S_1(T_1) + S_2(T_2) + S_1(T_3)$	$S_1(T_1) + S_2(T_2) + S_2(T_3)$
$S_2(T_1) + S_1(T_2) + S_1(T_3)$	$S_2(T_1) + S_2(T_2) + S_2(T_3)$

The total number of different intonations and melodic forms in the bass part is $8^2 = 64$.

Figure 100. Examples of continuity of G_6 on three tonics.

Application of G_6 to *four tonics* produces $2^4 = 16$ melodic forms in the bass part. The number of distributions of the four forms of S through four tonics produces $4^4 = 256$ intonations. The number of intonations in the bass part is limited to $2^4 = 16$. Thus the total number of intonations and melodic forms in the bass part is $16^2 = 256$.

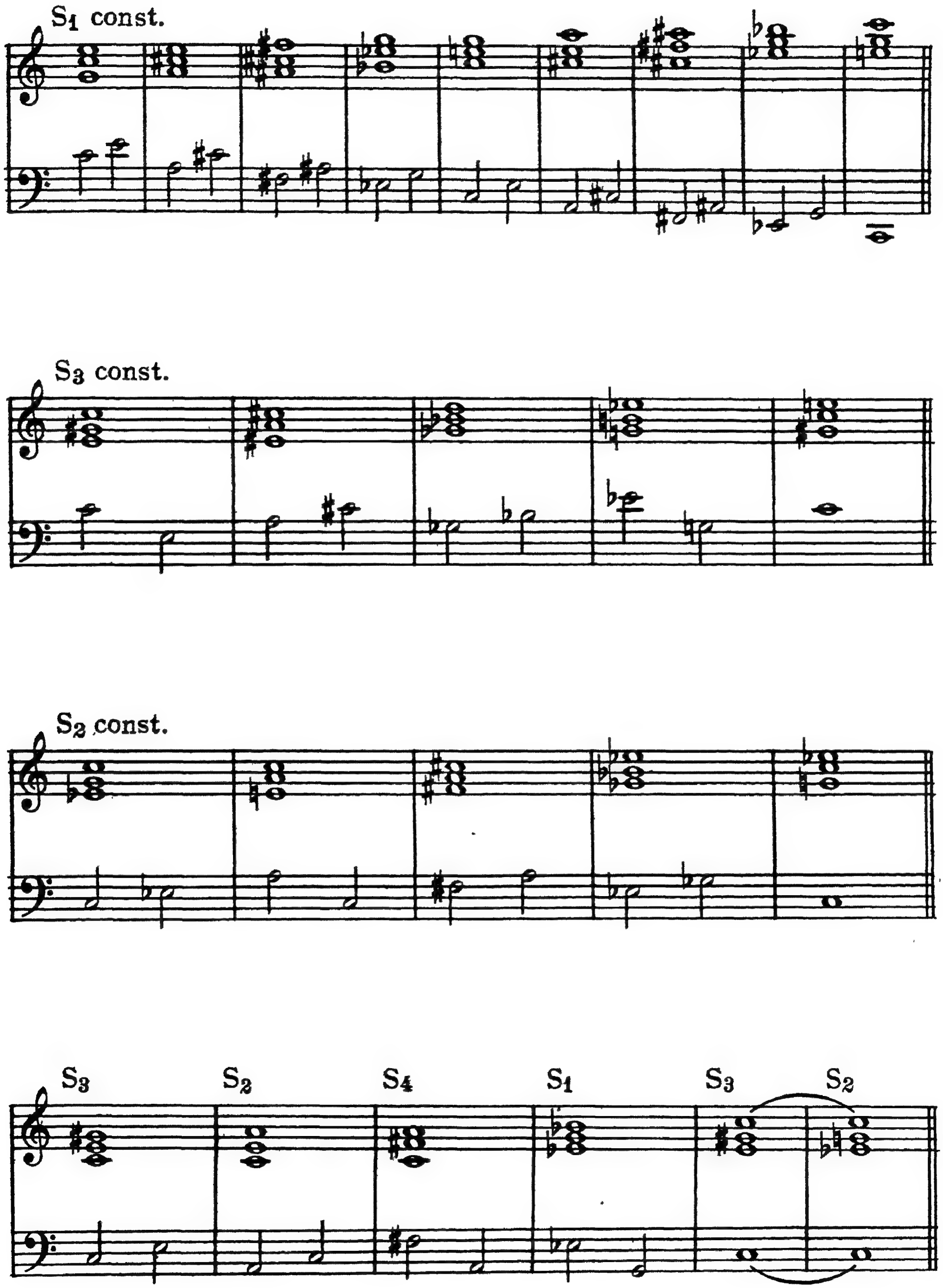


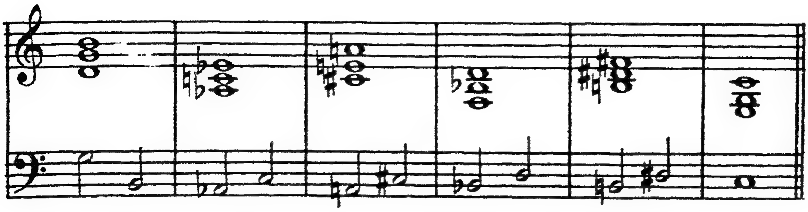
Figure 101. Continuity of G_6 on four tonics.

Application of G_6 to *six tonics* produces $2^6 = 64$ melodic forms in the bass part. The number of distributions of the four forms of S through four tonics produces $4^6 = 4096$ intonations. The number of intonations in the bass part is $2^6 = 64$. The total number of intonations and melodic forms in the bass part is $64^2 = 4096$.

Figure 102. Examples of continuity of G_6 on six tonics.

Application of G_6 to *twelve tonics* produces $2^{12} = 4096$ melodic forms in the bass part. The number of distributions of the four forms of S through four tonics produces $4^{12} = 16{,}777{,}216$. The number of intonations in the bass part is $2^{12} = 4096$. The total number of intonations and melodic forms in the bass part is $4096^2 = 16{,}777{,}216$.

Figure 103. Continuity of G_6 on twelve tonics (continued).



S_1 const.

Figure 103. Continuity of G_6 on twelve tonics (concluded).

G. Passing Fourth-Sixth Chords: $S\binom{6}{4}$

The second inversion of S(5) is a *fourth-sixth chord*: $S\binom{6}{4}$. This name derives from the old *basso continuo* or *generalbass*, where intervals were measured from the bass.

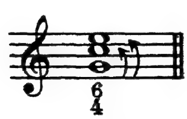


Figure 104. Passing $S\binom{6}{4}$.

$S\binom{6}{4}$ has a fifth (5) in the bass while the three upper parts have the six usual arrangements.

The use of $S\binom{6}{4}$ in classical music is a very peculiar one. This chord appears only in definite pre-set combinations. One of them is the *group with a passing fourth-sixth chord*: G^6_4.

As in the case of G_6, the passing chord itself appears on a weak beat, being surrounded by the two other chords, and has a doubled fifth: $S^6_4$⑤. The two other chords of G^6_4 are: S(5) and S(6). The latter can have two forms of doubling (regardless of the chord-structure): S(6)① and S(6)⑤.

The group with a passing fourth-sixth chord, contrary to G_6, is *reversible*.

$$G_4^6 = S(5) + S(\tfrac{6}{4}) + S(6).$$

This property being combined with the choice of two possible doublings produces four variants.

$$G_4^6\uparrow① = S(5) + S(\tfrac{6}{4}) + S(6)①$$
$$G_4^6\downarrow① = S(6)① + S(\tfrac{6}{4}) + S(5)$$
$$G_4^6\uparrow⑤ = S(5) + S(\tfrac{6}{4}) + S(6)⑤$$
$$G_4^6\downarrow⑤ = S(6)⑤ + S(\tfrac{6}{4}) + S(5)$$

The arrows in the above formulae specify the directions of the bass pattern which is always scalewise, and therefore can be either ascending or descending.

The bass pattern is developed on *three adjacent pitch-units*, which correspond to the three chords of G_4^6.

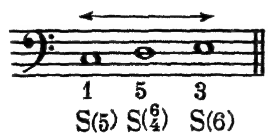


Figure 105. Bass pattern.

Arabic numerals represent the respective chordal functions.

Transformations between S(5) and $S(\tfrac{6}{4})$ in the G_4^6: *as the bass moves from 1 to 5*, when read in upward motion, *the three upper voices must move clockwise* in order to get the transformation of 1 into 3.

Figure 106. Transformations between S (5) and S $(\tfrac{6}{4})$.

The transition from $S(\tfrac{6}{4})$ into S(6)① or S(6)⑤ follows the forms of transformations, where two identical functions participate, as in the cases of S(5) ↔ S(6)① and S(5) ↔ S(6)⑤.

However, classical technique adopted definite routines concerning this transition:

(1) *one part* must carry out a melodic form *reciprocal to the bass* (i.e., position ⓑ of the bass melody);

(2) it is *this reciprocal part* that deviates from its path in order to *supply the doubling of the fifth* in an S(6)⑤.

Under such conditions G_4^6 acquires the following appearances:

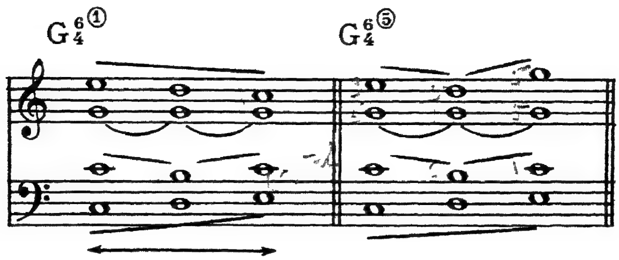


Figure 107. $G\frac{6}{4}$ transformations.

In the sequence of operations the following items should be considered in the order indicated:

(1) bass
(2) part reciprocating the bass
(3) common tone
(4) part supplying the third for $S(\frac{6}{4})$

The relations between the chords of G_4^6 are as follows:

$$C_0$$

$$S(5) + C\text{–}5 + S(\tfrac{6}{4}) + C_5 + S(6)$$

$$C_0$$

$$S(6) + C\text{–}5 + S(\tfrac{6}{4}) + C_5 + S(5)$$

Each group can be carried out in 6 positions which depend on the starting position.

The following is the table of all four forms of G_4^6 in one position.

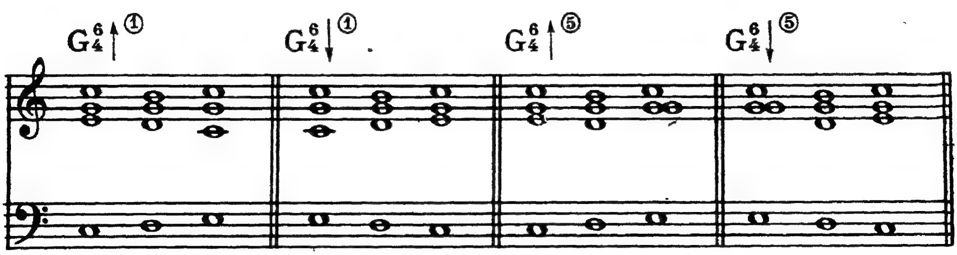


Figure 108. Forms of G_4^6 in one position.

The different forms of G_4^6 can be connected by means of tonal cycles and their coefficients of recurrence can be specified.

It is desirable to make the following tables:

(1) $G^{6}_{4}\uparrow 1$ const.; C_3 const., C_5 const., C_7 const.
(2) $G^{6}_{4}\downarrow 1$ const.; C_3 const., C_5 const., C_7 const.
(3) $G^{6}_{4}\uparrow 5$ const.; C_3 const., C_5 const., C_7 const.
(4) $G^{6}_{4}\downarrow 5$ const.; C_3 const., C_5 const., C_7 const.
(5) $G^{6}_{4}\uparrow 1$ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$
(6) $G^{6}_{4}\downarrow 1$ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$
(7) $G^{6}_{4}\uparrow 5$ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$
(8) $G^{6}_{4}\downarrow 5$ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$
(9) $G^{6}_{4}\uparrow 1 + G^{6}_{4}\downarrow 5 + G^{6}_{4}\uparrow 5 + G^{6}_{4}\downarrow 1$; C_3 const.
(10) " " " " ; C_5 const.
(11) " " " " ; C_7 const.
(12) " " " " ; $C^{\rightarrow} = C_3 + C_5 + C_7$

$C^{\rightarrow}$ is the symbol of a group of cycles (cycle continuity).

Continuity of G^{6}_{4}, when connected through a constant tonal cycle, consists of seven cycles: $C^{\rightarrow} = 7C$.

Example:

$G^{6}_{4}\uparrow 1$ const. $C^{\rightarrow} = C_3$ const.

Figure 109. Continuity of G^{6}_{4}.

Continuity of G^{6}_{4} of different forms and connection through different cycle-groups can be applied in its present form to diatonic progressions.

G^{6}_{4} in symmetric progressions of types II and III requires *identical structures for the two extreme chords of one group.* This requirement does not affect the middle chord of the group, i.e., $S(^{6}_{4})$, nor does it influence the selection of structures for the following groups.

Examples of continuity with G^6_4
in progressions of types I and II.

$$H^{\rightarrow} = 2G^6_4\uparrow + G^6_4\downarrow + G^6_4\uparrow + 2G^6_4\downarrow;\ C^{\rightarrow} = C_5 + 2C_7 + 2C_3 + C_5.$$

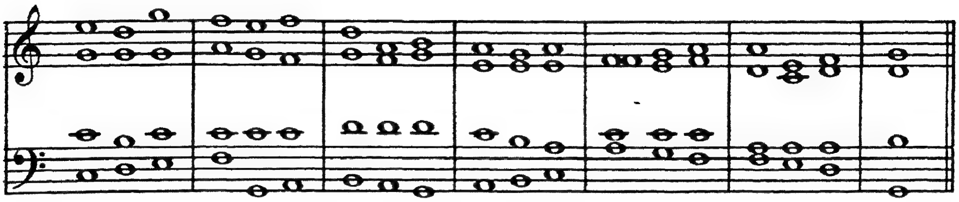


Figure 110. Continuity with G^6_4 in progressions of type I.

$H^{\rightarrow}$ and $C^{\rightarrow}$ as in the preceding example.
$S^{\rightarrow} = 2(S_1 + S_2) + (S_3 + S_2)$.

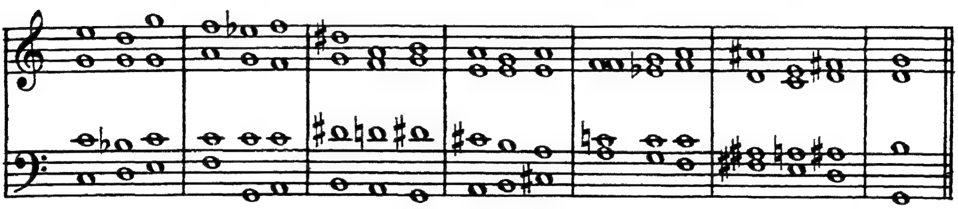


Figure 111. Continuity with G^6_4 in progressions of type II.

Application of G^6_4 to symmetric systems requires the following sequence of tonics:

$$GH^{\rightarrow} = (T_1 + T_2 + T_1) + (T_2 + T_3 + T_2) + (T_3 + T_4 + T_3) + \ldots$$

For example, the three-tonic system must be distributed as follows:

$$GH^{\rightarrow} = (T_1 + T_2 + T_1) + (T_2 + T_3 + T_2) + (T_3 + T_1 + T_3).$$

The number of tonics in the respective system specifies the cycle. Each group may begin with either S(5) or S(6).

Each group acquires the following distribution of inversions:

$$G^6_4 = T_1S(5) + T_2S(^6_4) + T_1S(6)\ .$$

Under such conditions, each tonic appears in all the three inversions.

Table of G_4^6 applied to all symmetric systems

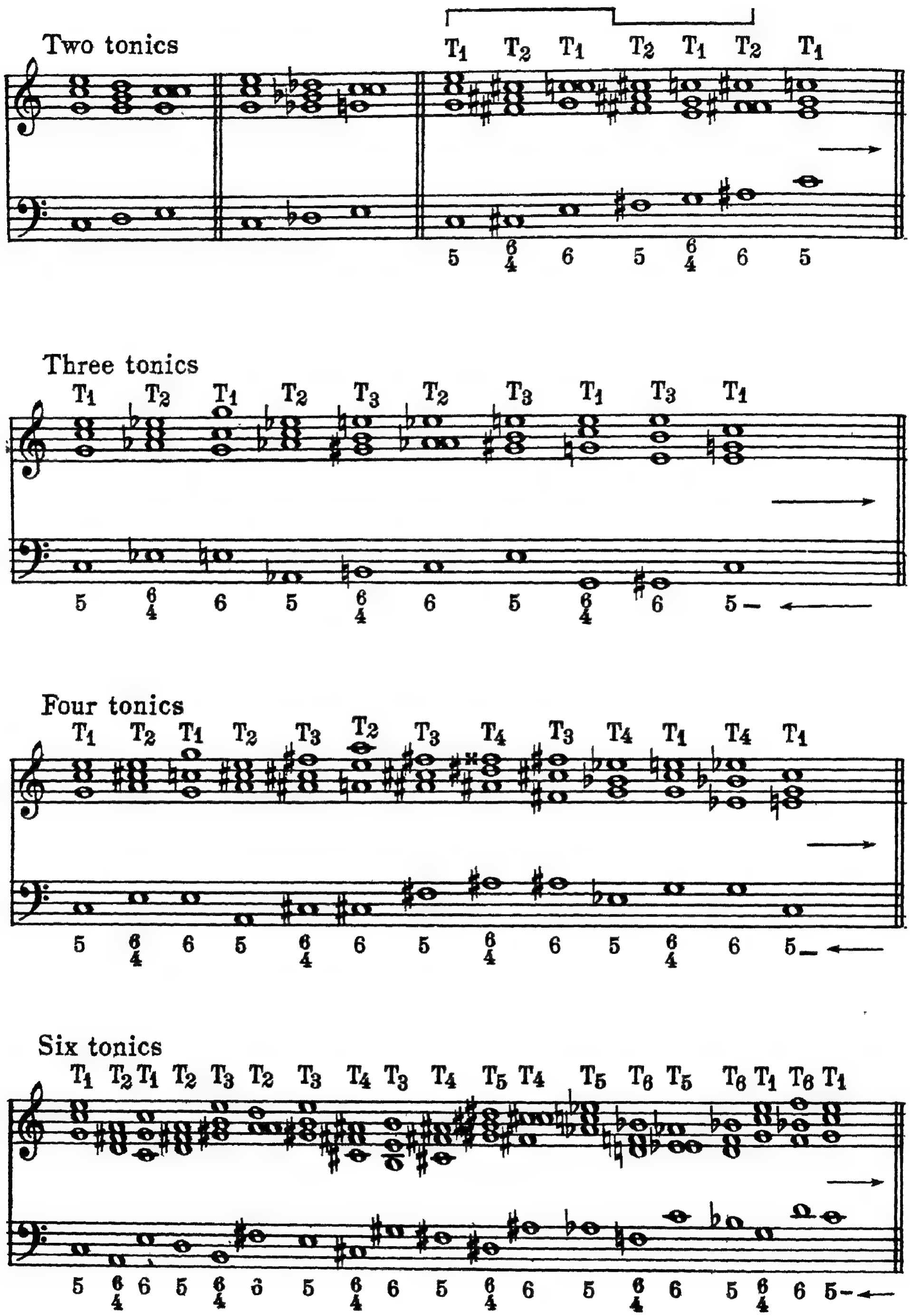
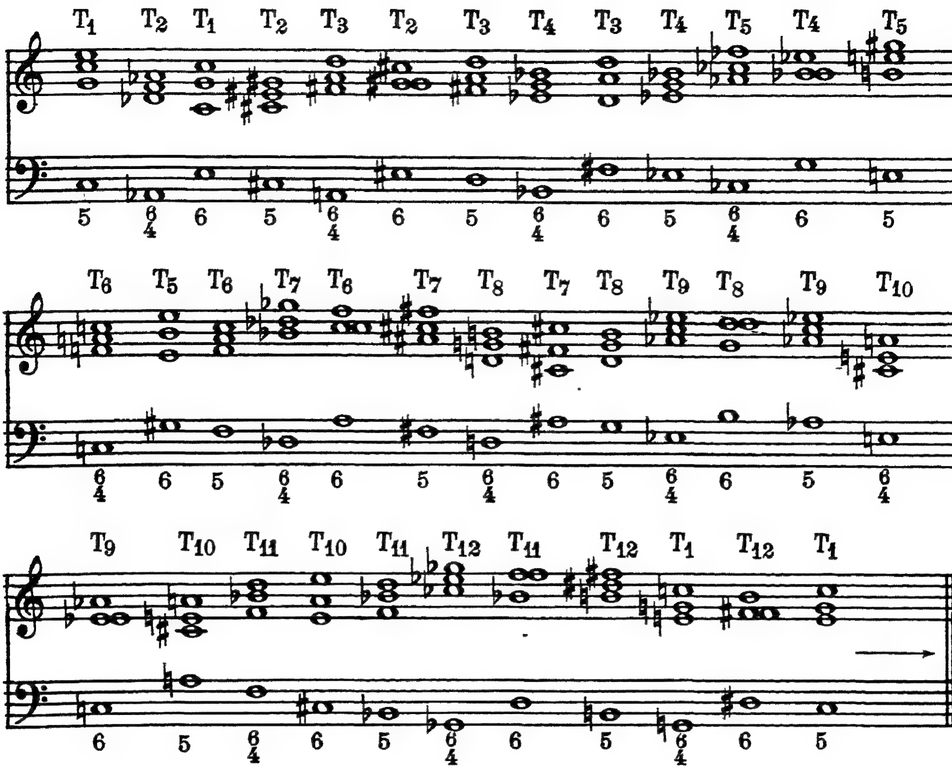
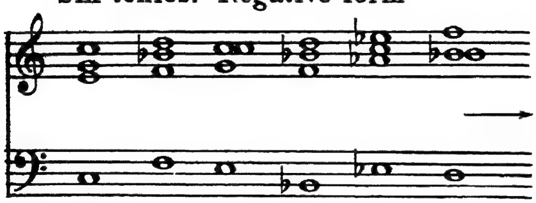


Figure 112. G_4^6 applied to symmetric systems (continued).

Twelve tonics



Six tonics: Negative form



Twelve tonics: Negative form

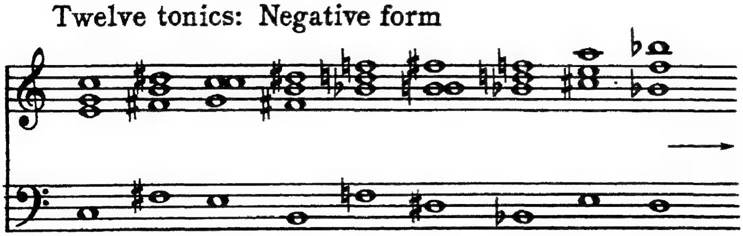


Figure 112. G^6_4 applied to symmetric systems (concluded).

Other negative forms are not as practical: inversions weaken tonality.

Example of variation of structures and directions.

Four tonics.

$$GH^{\rightarrow} = [S_1(5) + S_2(\tfrac{6}{4}) + S_1(6)] + [S_2(6) + S_1(\tfrac{6}{4}) + S_2(5)] + \\ + [S_3(6) + S_4(\tfrac{6}{4}) + S_3(5)] + [S_2(5) + S_3(\tfrac{6}{4}) + S_2(6)]$$

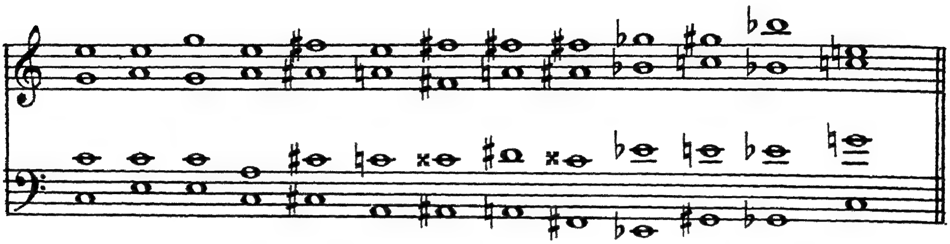


Figure 113. Variation of structures and directions.

H. Cycles and Groups Mixed

Tonal cycles can be introduced into the continuity of groups, and groups can be introduced into the continuity of cycles.

It is convenient to plan the mixed form of cycle-group continuity by bars (T).

Bars of cycles and bars of groups can be assigned to have different coefficients of recurrence.

When planning such a continuity in advance, it is important to remember that there is always a cycle-connection *between the bars.*

Examples:

$$H^{\rightarrow} = 2TC + TG + TC + 2TG = (C_5 + C_3) + C_7 + (C_3 + C_7) + \\ + C_5 + G_6 + C_7 + (C_3 + C_3) + C_5 + G^6_4|① + C_7 + G^6_4|⑤ + C_3.$$

Type I

Figure 114. Cycles and groups mixed (continued).

Type II

Figure 114. Cycles and groups mixed (concluded).

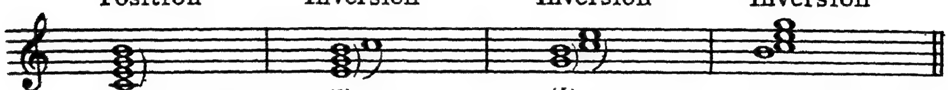
CHAPTER 9

THE SEVENTH CHORD

THE seventh chord, in the diatonic as in other systems, has the following positions:

A. Diatonic System

Fundamental Position	The First Inversion	The Second Inversion	The Third Inversion



S(7) Seventh Chord	S $\binom{6}{5}$ Fifth-Sixth Chord	S $\binom{4}{3}$ Third-Fourth Chord	S (2) Second Chord

Figure 115. Inversions of the seventh chord.

A seventh-chord, including all of its inversions, has 24 positions altogether. The classical system of harmony is based on the *postulate of resolving seventh: the seventh moves one step down.*

Figure 116. Resolving the seventh

This postulate provides a means for the continuous progression of S(7); in addition, it is the basis of the entire system of diatonic continuity (cycles).

One movement is required to produce C_3: the movement of the seventh alone. This results in a clockwise transformation.

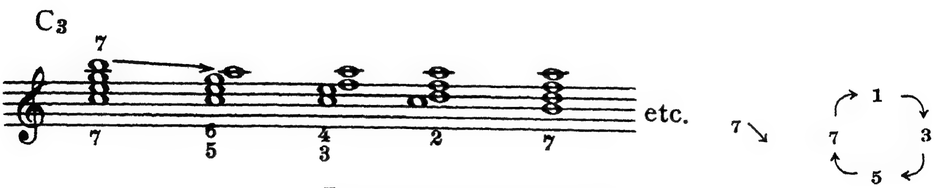


Figure 117. Producing C_3

Two movements are required to produce C_5: the movement of both the seventh and of the fifth, each moving one step down. This results in a crosswise transformation.

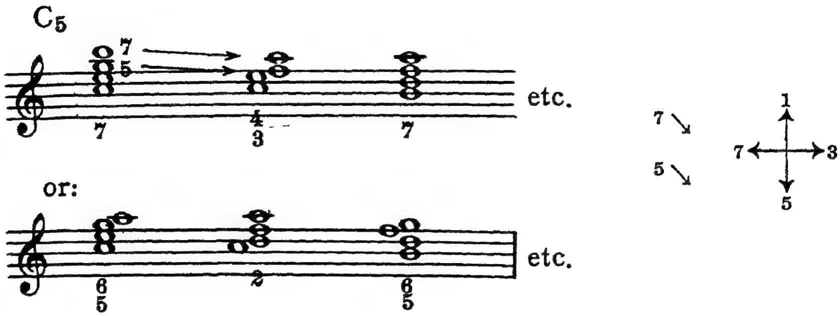


Figure 118. Producing C_5.

Three movements are required to produce C_7: the movement of the seventh, of the fifth, and of the third, each moving one step down. This results in a counter-clockwise transformation.

Skipping two chords in C_3, we obtain:

Figure 119. Producing C_7.

This type of music may be found among contrapuntalists of the 17th and 18th centuries. Palestrina, Bach and Händel obtained similar results by means of suspensions.

Assigning a system of cycles, we can produce a continuity of S(7). The starting chord may be taken in any position.

Example: $C_5 + C_3 + C_5 + C_7 + C_3 + C_3 + C_5$

Figure 120. Continuity of S(7).

This continuity—being entirely satisfactory harmonically—may prove in some cases to be unsatisfactory melodically because of the continuous downward movement of all voices. When it is desirable to do so, this characteristic may be eliminated by means of two devices:

(1) exchange of the common tones
(2) octave inversion of the common tones

The same continuity of cycles assumes the following form:

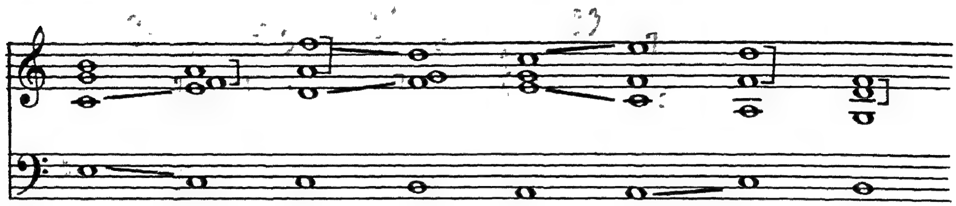


Figure 121. The same continuity with a more satisfactory melody.

Since C_7 does not provide common tones, the use of the above devices in C_7 is precluded.

As continuity of the second type offers better melodic forms for all voices, it may be desirable to pre-set certain melodic forms in advance. For example, it is possible to obtain, by means of continuous C_5, the following form of descent through two parallel axes (b) or (d)—as in the music of Frederic Chopin.

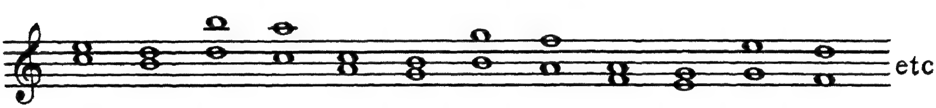


Figure 122. Continuous C_5 through two parallel axes.

This may be harmonized as follows:

Figure 123. Harmonizing the continuum of figure 122.

Diatonic C_0 becomes a necessity in order to avoid that excess of saturation typical of the continuity of S(7) with variable cycles.

The principle of moving continuously through C_0 is based on the exchange and inversion of common tones.

The exchange and inversion of adjacent functions brings the utmost satisfaction. Nevertheless, it is not desirable to use the two extreme functions for such a purpose since they produce a certain amount of harshness.

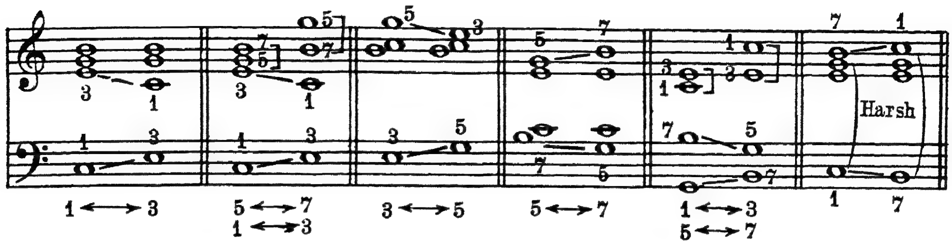


Figure 124. Inversion of adjacent functions.

An example of continuity of C_0:

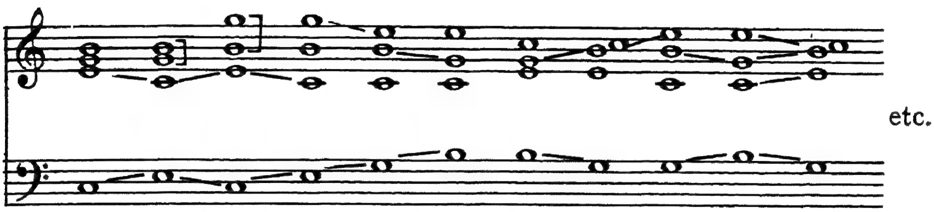


Figure 125. Continuity of the C_0.

The final form of continuity of S(7) consists of mixtures of all cycles (including C_0) based on a rhythmic composition of the coefficients of recurrence.

Example: $2C_5 + C_0 + 2C_3 + C_0 + 2C_7 + C_0$

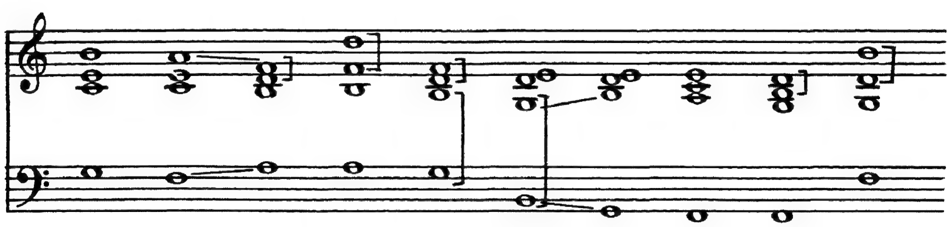


Figure 126. Final form, continuity of S(7).

B. The Resolution of S(7).

Resolution of an S(7) into an S(5) in all positions and inversions may be defined as a *transition from four functions to three functions.*

S(5) in four-part harmony and with a normal doubling (doubled root) consists of:

1, 1, 3, 5

And S(7) consists of:

1, 3, 5, 7

Thus, when a transition occurs, the root takes the place of the seventh. Therefore the resolution is provided through the *motion of S(7) → S(7) and the substitution of one for the seven,* i.e., the function which would otherwise have become a seventh in the continuity of seventh-chords now becomes a root-tone in order to achieve a resolution.

$7 \rightarrow 1$
$5 \rightarrow 7 \rightarrow 1$
$3 \rightarrow 5$
$1 \rightarrow 3$

Note: Do not move S(7) → S(5) in C_0.

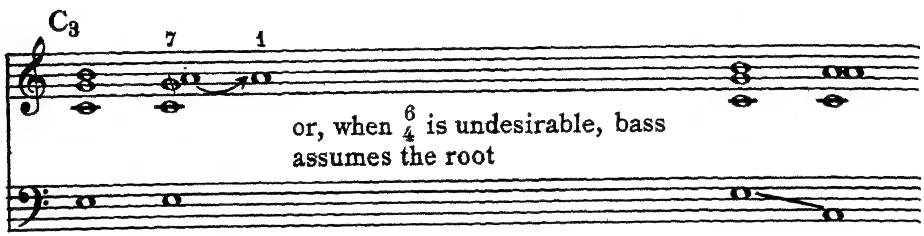


Figure 127. Resolutions in diatonic cycles.

This case provides an explanation of why a tonic triad acquires a tripled root and loses its fifth:

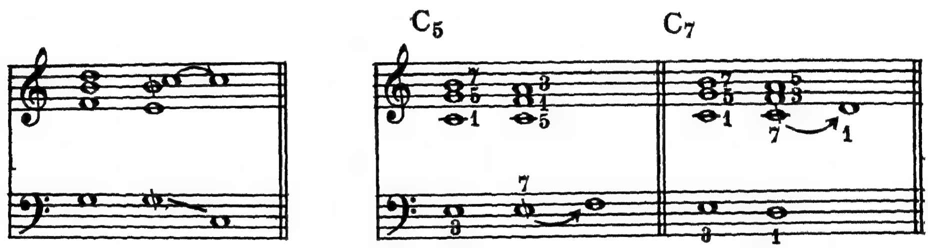


Figure 128. Tonic triad acquires a tripled root.

1. Preparation of S(7)

There are three methods of preparing an S(7), i.e., of transition from S(5) to S(7):

(1) suspending
(2) descending
(3) ascending

The first method is the only one producing the *positive* (C_3, C_5, C_7) cycles.

Methods (2) and (3) are the outcome of the *intrusion of melodic factors* into harmony. These are obviously in conflict with the nature of harmony (like those groups with passing chords we have already studied) as they produce

negative cycles, and these in turn contradict the postulate of the resolving seventh universally observed in classical music.

The technique of preparing the seventh consists of assigning a certain consonant function (1, 3, 5) to become a dissonant function (7) and of either sustaining the assigned function of the S(5) over the bar line, or moving it one step downward or upward.

The last two forms of a seventh conventionally occur on a *weak* beat.

Here are different positions, inversions and cycles of the S(5) → S(7) transition.

(1) Suspending:

1 | 7 — C_7 3 | 7 — C_5 5 | 7 — C_3

(2) Descending:

| 1 ↘ 7 | C_0 | 3 ↘ 7 | C_{-3} | 5 ↘ 7 | C_{-5}

(3) Ascending:

| 1 ↗ 7 | C_{-3} | 3 ↗ 7 | C_{-5} | 5 ↗ 7 | C_{-7}

Figure 129. Different positions of transition of S(5) → S(7)

(1) Suspending

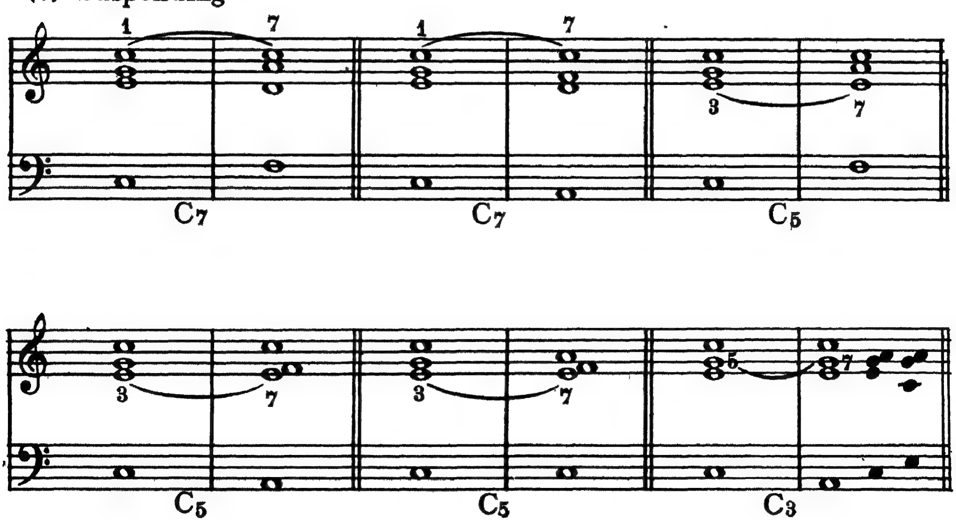


Figure 130. Preparation of S(7) (continued).

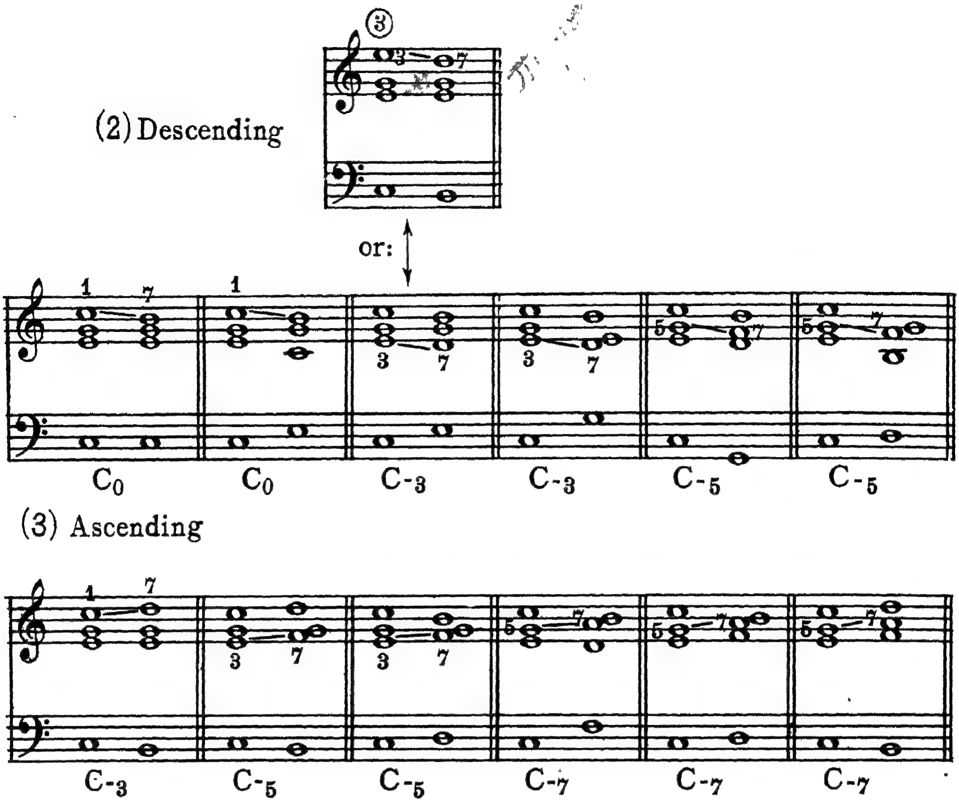


Figure 130. Preparation of S(7)
(concluded).

A mixture of zero, positive and negative cycles, provides the final form of continuity based on S(5) and S(7).

For more efficient planning of such continuity, use bar lines for the layout. The preparation of S(7) may be either positive or negative; the resolution is always positive.

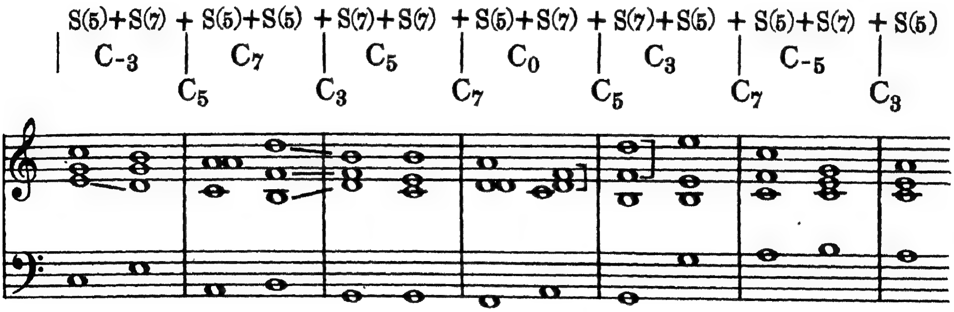


Figure 131. Preparation of S(7)

C. With Negative Cycles.

The *negative* system of tonal cycles may be used as an independent system. The negative system is in reality a geometrical inversion of the positive system. Every principle, rule or regulation of the positive system thus becomes its own converse in the negative.

Chord structures become $E_{1\textcircled{b}}$ of the original scale. Chord progressions are based on $E_{1\textcircled{a}}$ which forms the C_{-3}. Clockwise transformations become counterclockwise and vice versa.

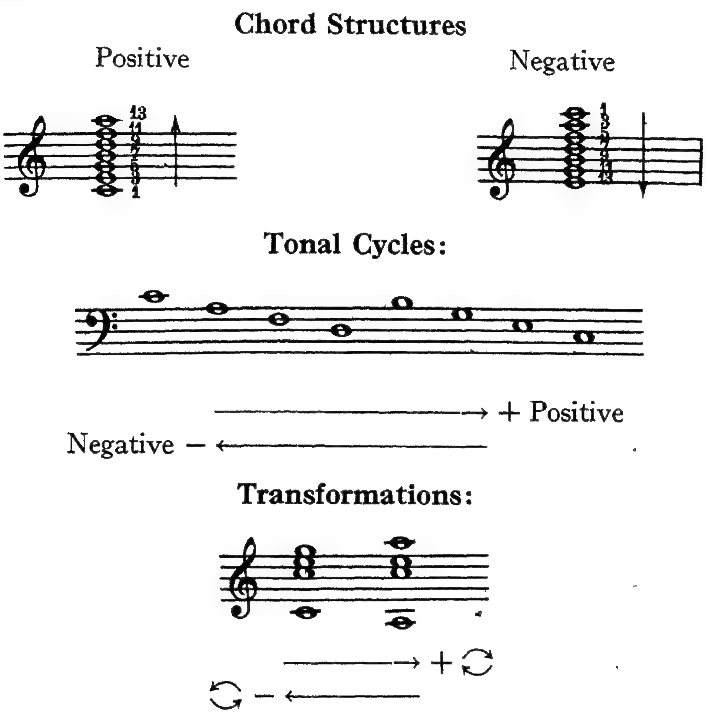


Figure 132. Negative Cycles.

The postulate of resolving seventh for the negative system must be read. *the negative seventh moves one step up.* The C_{-5} requires the negative seventh and negative fifth to move one step up. The C_{-7} requires all tones except the root to move up. This system may be of great advantage in building climaxes.*

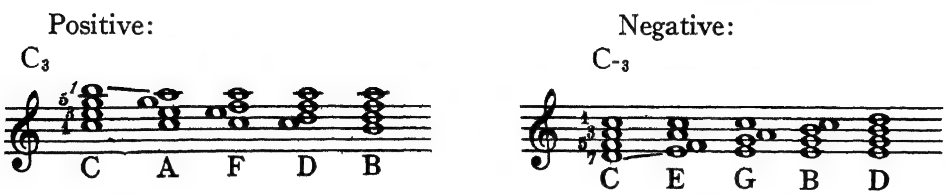


Figure 133. Positive (C_3) and negative (C_{-3}).

*Passing observations of this character, though made casually by Schillinger, should be noted carefully by the student. They offer valuable ideas and techniques which may be successfully exploited in composition and arranging. (Ed.)

The root-tone of the negative system is the seventh of the positive, and *vice-versa.*

It is easv to see how the other cycles would operate.

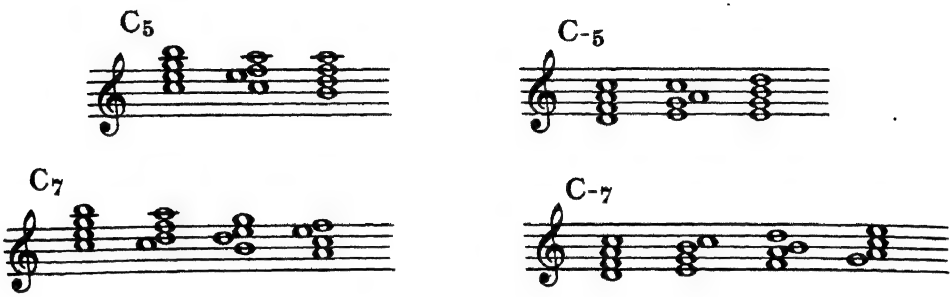


Figure 134. Positive (C_5, C_7) and negative (C_{-5}, C_{-7}).

If one wishes to read the negative system as if it were positive, the technique must be changed as follows:

The C_{-3} requires the ascending of 1
The C_{-5} " " " " 1 and 3
The C_{-7} " " " " 1, 3 and 5

1. Special Applications of S(7)

S(7) finds its application in G^6_4, either as the first or the last chord of the group.

The following forms are possible:

$$S(5) + S(^6_4) + S(^6_5) \qquad S(7) + S(^6_4) + S(6)$$
$$\longleftrightarrow \qquad\qquad \longleftrightarrow$$
$$S(5) + S(^6_4) + S(^4_3) \qquad S(7) + S(^6_4) + S(^4_3)$$
$$\longleftrightarrow \qquad\qquad \longleftrightarrow$$
$$S(5) + S(^6_4) + S(2) \qquad S(7) + S(^6_4) + S(2)$$
$$\longleftrightarrow \qquad\qquad \longleftrightarrow$$

Figure 135. Forms of S(7).

The cycle between the extreme chords of G^6_4 may be either C_0, or C_3, or C_5.

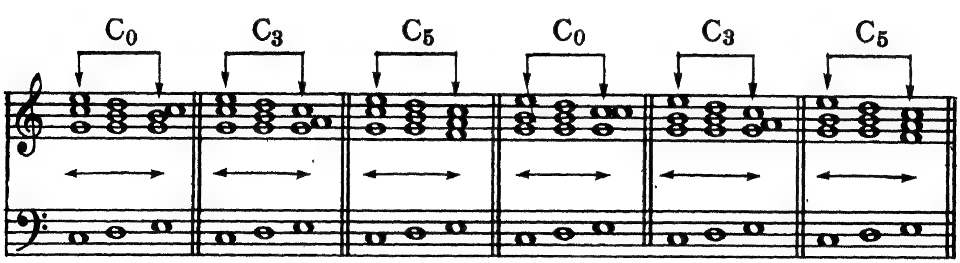


Figure 136. Cycle between exteme chords of G^6_4.

Besides G^6_4 there is a special group in which $S(\frac{4}{3})$ is used as a passing chord. There are two forms of this group.

$$\text{(A)}\quad G^4_{3(5)} = \underrightarrow{S(5)^{3} + S(\tfrac{4}{3}) + S(7) \text{ or } S(5)}$$

$$\text{(B)}\quad G^4_{3(6)} = \underrightarrow{S(6)^{5} + S(\tfrac{4}{3}) + S(7) \text{ or } S(5)}$$

Figure 137. $S(\frac{4}{3})$ used as a passing chord.

These two forms may be used in one direction only. All positions are available.

The rule of voice-leading is: the bass and one of the voices of doubling move stepwise down; common tones are sustained.

The cycle between the extreme chords in the first form is C_3; in the second form it is C_0.

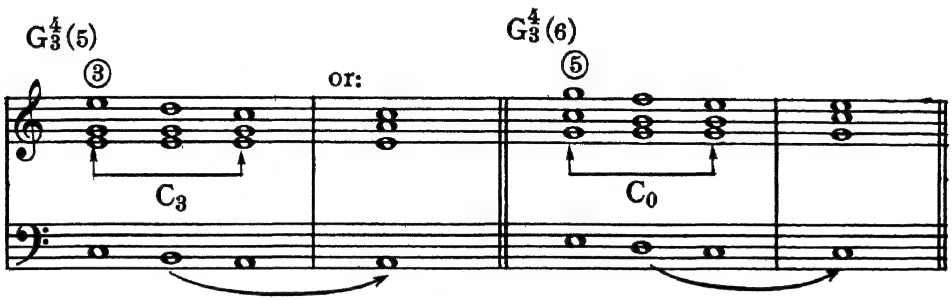


Figure 138. Voice-leading exemplified.

2. Cadences

The following applications of S(7) are commonly known:

(1) IV_7 $I^6_4 — V_7$ I_5
(2) IV^6_5 " " "
(3) II^6_5 " " "
(4) II^4_3 " " "

In addition to this, the following forms may be offered:

(5) Any of the previous forms $I^6_4 — III^6_5$ I_5
(6) " $I^6_4 — VII^6_5$ I_5

Besides these, there are two ecclesiastic forms:

(1) $I_5 — IV^{5}_{(II^6_5)}$ I_5
(2) $I_5 — IV^{5}_{(VII^4_3)}$ I_5

Figure 139. Applications of S(7).

D. S(7) IN THE SYMMETRIC ZERO CYCLE (C_0).

Symmetric C_0 exhibits extraordinary versatility with S(7): seven structures of the latter have been in use.

If the forms of S(7) had been evolved scientifically, they would have been obtained in the following order:

Taking c – e – g – b♭ (4 + 3 + 3) as the most common form and producing variations thereof, we obtain two other forms:

c – e♭ – g – b♭ (3 + 4 + 3)
and c – e♭ – g♭ – b♭ (3 + 3 + 4)

Taking another form, c – e – g – b (4 + 3 + 4), we obtain two other forms:

c – e – g♯ – b (4 + 4 + 3)
and c – e♭ – g – b (3 + 4 + 4)

These two groups of three are distinctly different; but inasmuch as music has made use of them for some time, our ears accept mixing all of them in one harmonic continuity.

Besides these six forms there is a c – e♭ – g♭ – b♭♭ (3 + 3 + 3 + 3); and there might have been c – e – g♯ – b♯ (4 + 4 + 4 + 4), if it were not for the fact that c – b♯ is an enharmonic octave.

A continuity on symmetric C_0 of all seven structures offers 5,040 permutations. Thus a c-chord alone can move (without changing its position and without coefficients of recurrence being applied) for 5,040 × 7 = 35,280 chords.

The method of selecting the best of the available progressions must be based on the following principle: **the best progressions on symmetric C_0 are due to identity of steps or to contrary motion.**

(1) Identity of Steps:

[all semitones]

(2) Contrary motion:

Figure 140. Best progressions on symmetric C_0.

The principle of variation of chord-structures and their positions remains the same as in S(5):

Structure		*Position*
Constant	————————	Variable
Variable	————————	Constant

S(7) in the following table has a dual system of indications: letter symbols and adjectives.* The adjectives are chosen so that they do not pertain to degrees of a particular scale but to structure alone. Thus, so common an adjective as "dominant" must be abandoned.

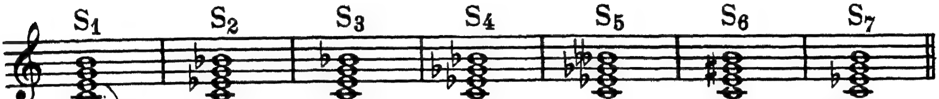


Figure 141. S(7) Table of structures

1. An example of Continuity in C_0:

Structures: $S_3 + S_7 + S_4 + S_1$
Coefficients $(r5 \div 4)$: $4S_3 + S_7 + 3S_4 + 2S_1 + 2S_3 + 3S_7 + S_4 + 4S_1$

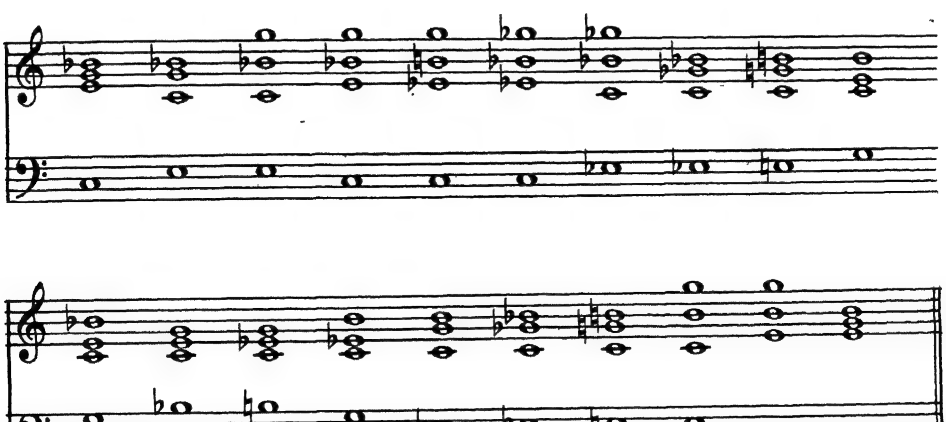


Figure 142. Continuity in C_0

As in S(5), any combination of the forms of S(7) by 2, 3, 4, 5, 6 and 7 may be used.

*By *adjective*, Schillinger means the descriptive word used to indicate the *shape* of an S(7). (Ed.)

2. S(7) in Type III (Symmetric).

As in previous cases, in dealing with symmetrical tonics, we may apply C_0 either to any of the tonics or with a continuous change of chord structures, a change occurring with each tonic.

When structures of S(5) and S(7) have to be specified in one continuity, they must have full indications:

$S_1(5)$; $S_2(5)$; $S_3(5)$ $S_4(5)$ and
$S_1(7)$; $S_2(7)$; $S_3(7)$; $S_4(7)$; $S_5(7)$; $S_6(7)$; $S_7(7)$

3. Two Tonics ($\sqrt{2}$)

As the $\sqrt{2}$ forms the center of the octave, the progression $1 \rightarrow \sqrt{2}$ (C→F♯) is positive and $\sqrt{2} \rightarrow 2$ (F♯→C) is negative.

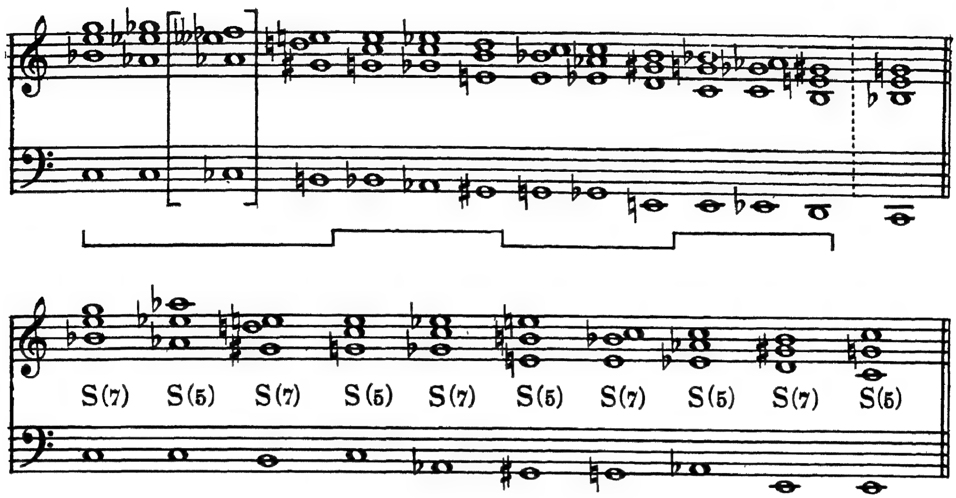
The system of Two Tonics which was continuous on S(5) becomes closed on S(7). Transformations correspond to C_5.

Example of continuity

Figure 143. Continuity on two tonics.

4. Three Tonics ($\sqrt[3]{2}$)

Continuous system: moves four times; transformations correspond to C_3. To obtain S(7) after an S(5), use the position which would correspond to a continuous progression of S(7).



An example of continuity:

Figure 144. Three tonics $\sqrt[3]{2}$

5. Four Tonics ($\sqrt[4]{2}$)

A closed system: transformations correspond to C_3; S(7) after S(5) as in the three-tonic system.

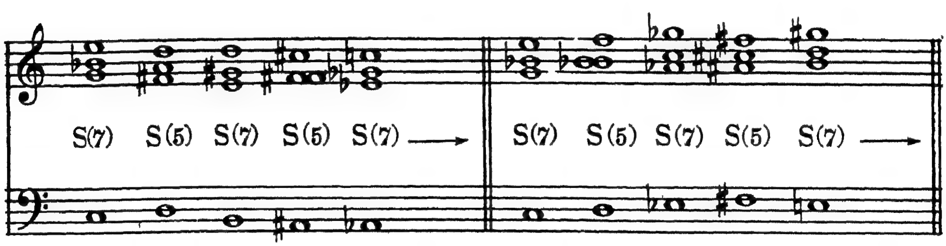
Figure 145. Four tonics (continued).

Example of continuity:

Figure 145. Four tonics (concluded).

6. Six Tonics ($\sqrt[6]{2}$)

A continuous system: moves two times; transformations correspond to C_7; S(7) after S(5) as in previous cases; both positive and negative progressions are fully satisfactory; to obtain the negative progressions, read the positive ones backwards.

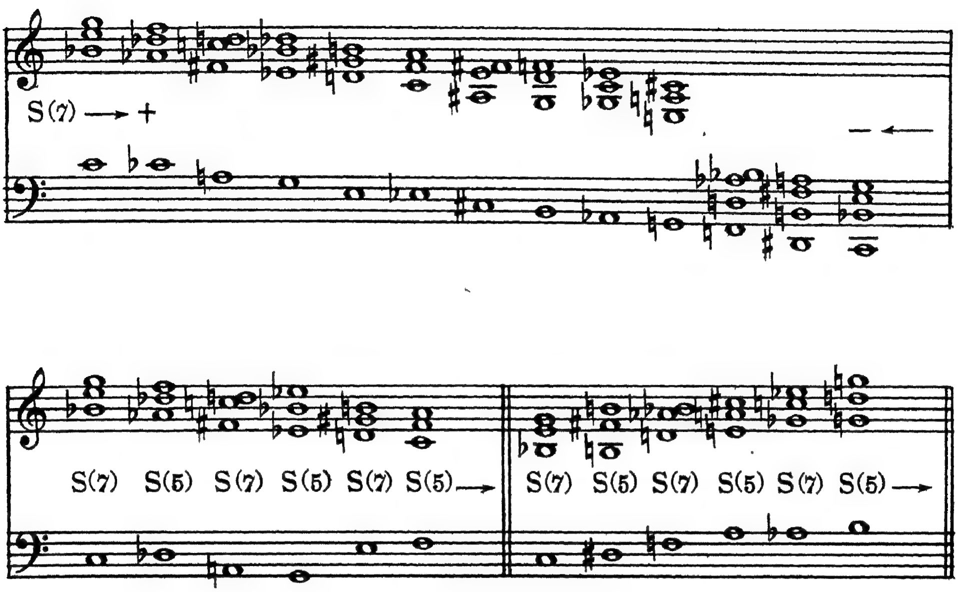


Example of continuity

Figure 146. Six tonics.

7. Twelve Tonics ($\sqrt[12]{2}$)

A closed system: all specifications and applications as in the six-tonic system.



Example of continuity

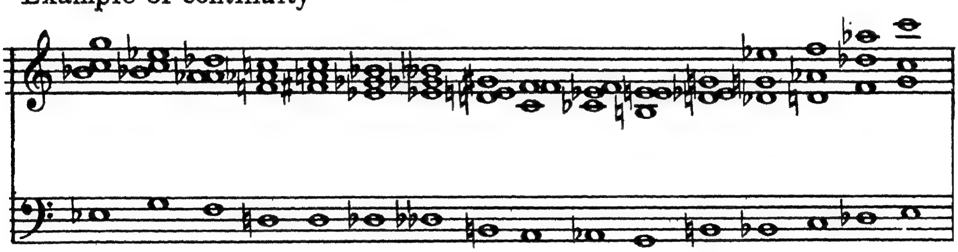


Figure 147. Twelve tonics.

E. Hybrid 5-Part Harmony

The technique of continuous S(7) makes it possible to evolve a hybrid five-part harmony, in which the bass is a constant root tone and the four upper functions assume variable forms of S(7) with respect to the bass.

By placing an S(7) on either the root, or the third, or the fifth, or the seventh of the bass root, we obtain all forms of S in five-part harmony. An S(5) has to be represented with the addition of 13th (the so-called "added sixth").

Forms of Chords in Hybrid Five-Part (4 + 1) *Harmony*

The 4 Upper Parts	5 3 1	7 5 3	9 7 5	11 9 7	13 11 9
S	13	1	3	5	7
The Bass	1	1	1	1	1
The forms of Tension	S(5)	S(7)	S(9)	S(11)	S(13)

Figure 148. Chord forms in hybrid five-part harmony.

It is possible to move either forms or any of the combinations of forms continuously in any rhythmic form of continuity.

Note that the tonal cycles do *not* correspond in the upper four parts to the tonal cycles in the bass when the forms of tension are variable. For example f — a — c — e may be 3 — 5 — 7 — 9 in a DS(9) or 7 — 9 — 11 — 13 in a GS(13). In such a case, a progression C_5 for the bass with S(9) → S(13) produces C_0 for the upper four parts.

The principle of exchange and octave-inversion of the common tones holds true.

Three forms of harmonic continuity will be used in the following illustrations (these forms of continuity are applicable in four-part harmony as well). When chord structures of greater tension are desired, and also when compensation for the diatonic system's deficiencies is required, it is often desirable to use pre-selected *forms* of chord-structures which nevertheless *move diatonically*. Such a system has a bass belonging to one definite diatonic scale, while the chord structures acquire various accidentals in order to produce a definite sonority. In the general classification of harmonic progressions, this latter type is known as *diatonic-symmetric*.

1. Three Types of Harmonic Progressions

I. Diatonic
II. Diatonic-Symmetric
III. Symmetric

The following examples will be worked out in all three types of harmonic continuity. Constant and variable forms of tension will be offered.

In order to select a desirable form or forms of structure for the different forms of tension, it is advisable to select a scale first, as such a scale offers the manifold of forms of tension. For example, if the scale selected is c — d — e — f♯ — g — a — b♭; S(5) = c — e — g — a; S(7) = c — e — g — b♭; S(9) = c — e — g — b♭ — d; S(11) = c — g — b♭ — d — f♯; S(13) = c — b♭ — d — f♯ — a.

Though the same scale would be *ideal* for the progression, it is not impossible and not undesirable to use some other scale for the chord-progressions.

2. Tables and Examples

(a) *Continuity of S(5)* [*monomials*]

Scale: c — d — e — f♯ — g — a — b♭

Type I

Type II

Type III

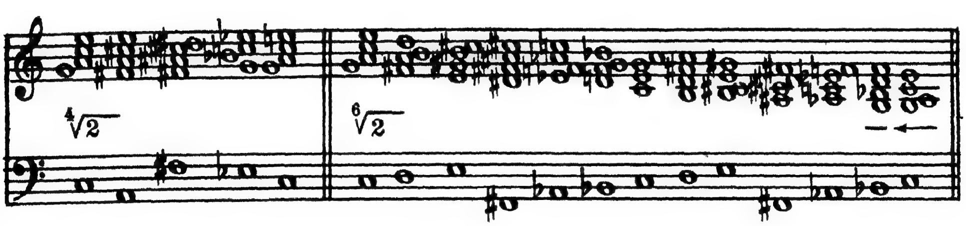


Figure 149. Continuity of S(5) monomials.

(b) *Continuity of S(7)* [*monomials*]

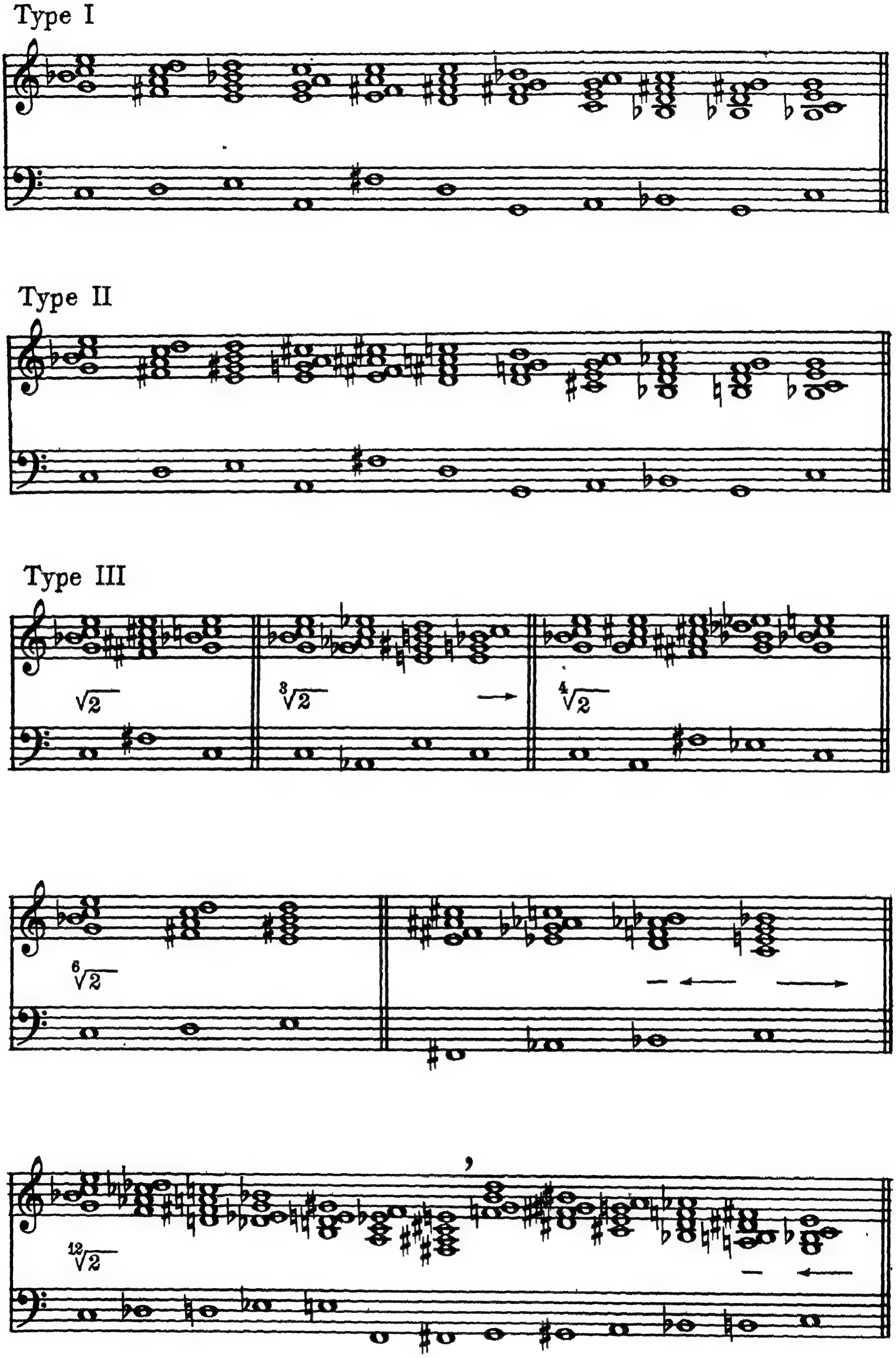


Figure 150. Continuity of S(7) monomials.

(c) *Continuity of S(9)* [*monomials*)

Type I

Type II

Type III

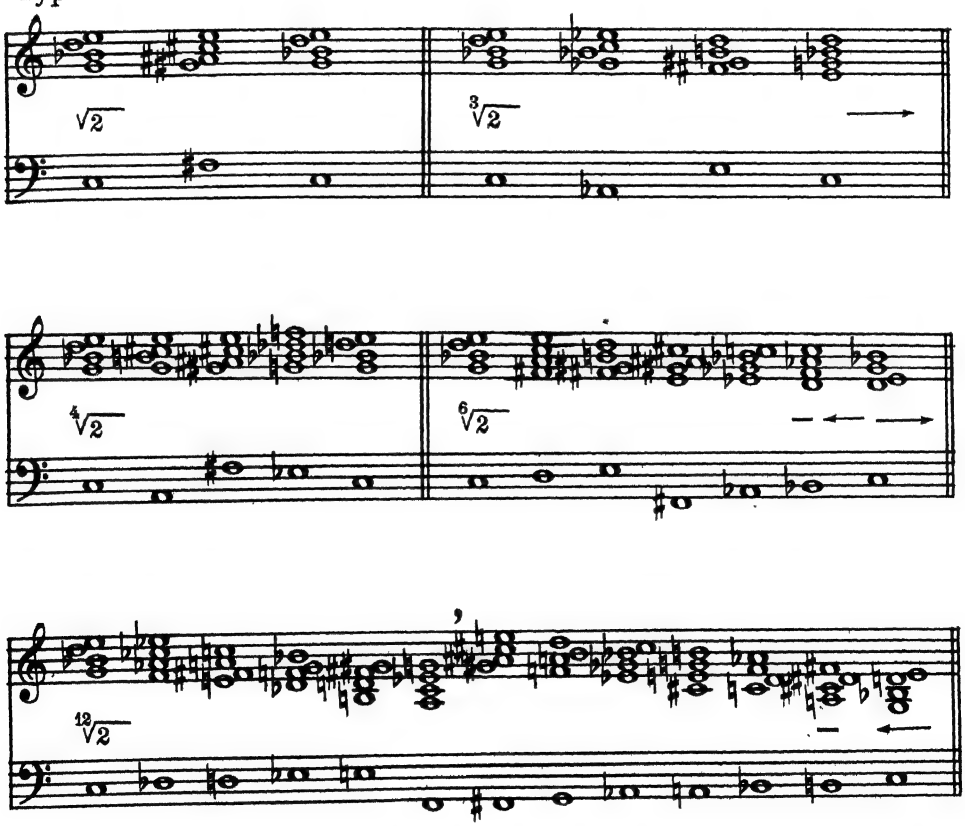


Figure 151. Continuity of S(9) monomials.

(d) *Continuity of S(11)* [*monomials*]

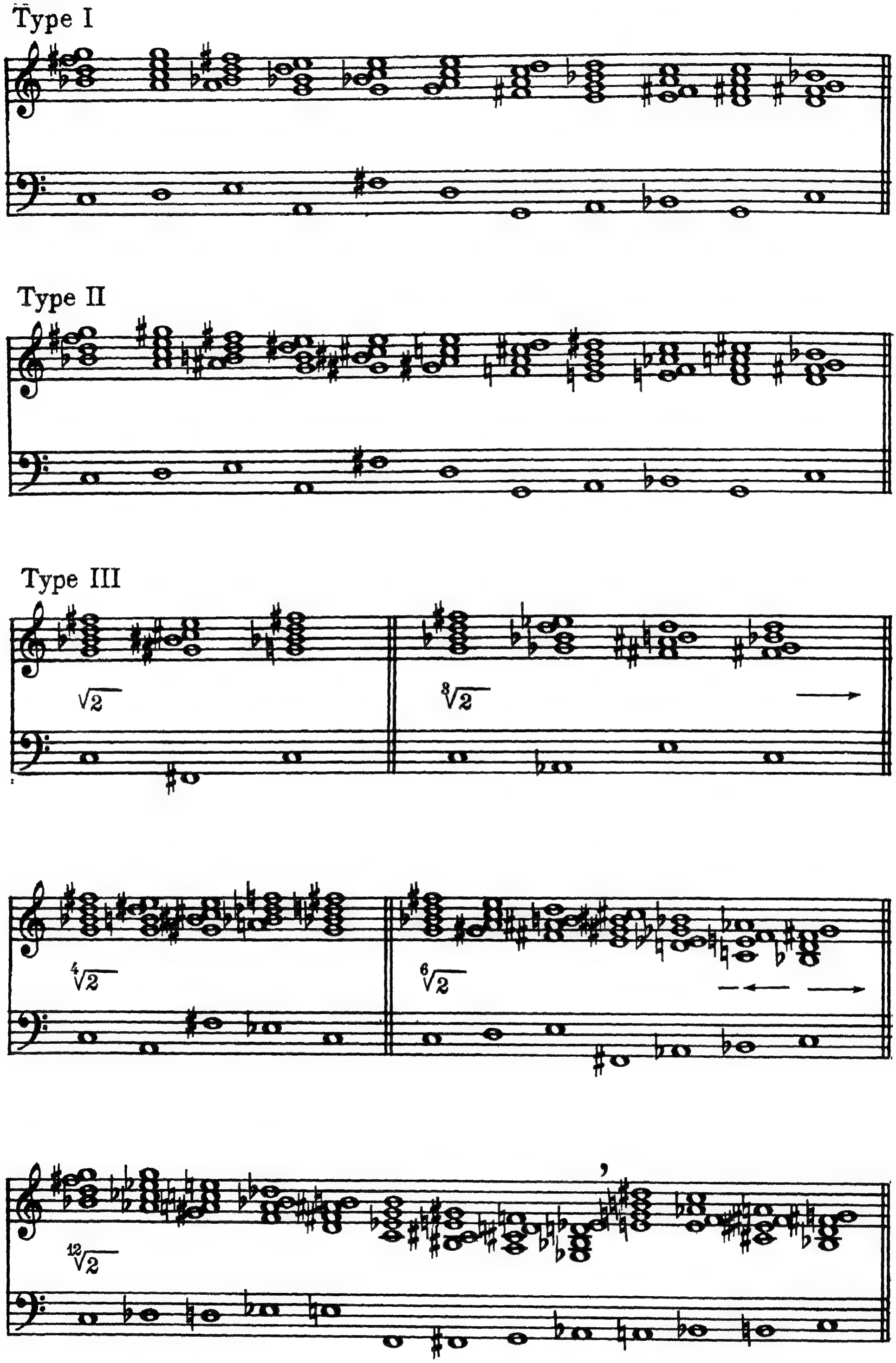


Figure 152. Continuity of S (11) monomials.

(e) *Continuity of S(13)* [*monomials*)

Type I

Type II

Type III

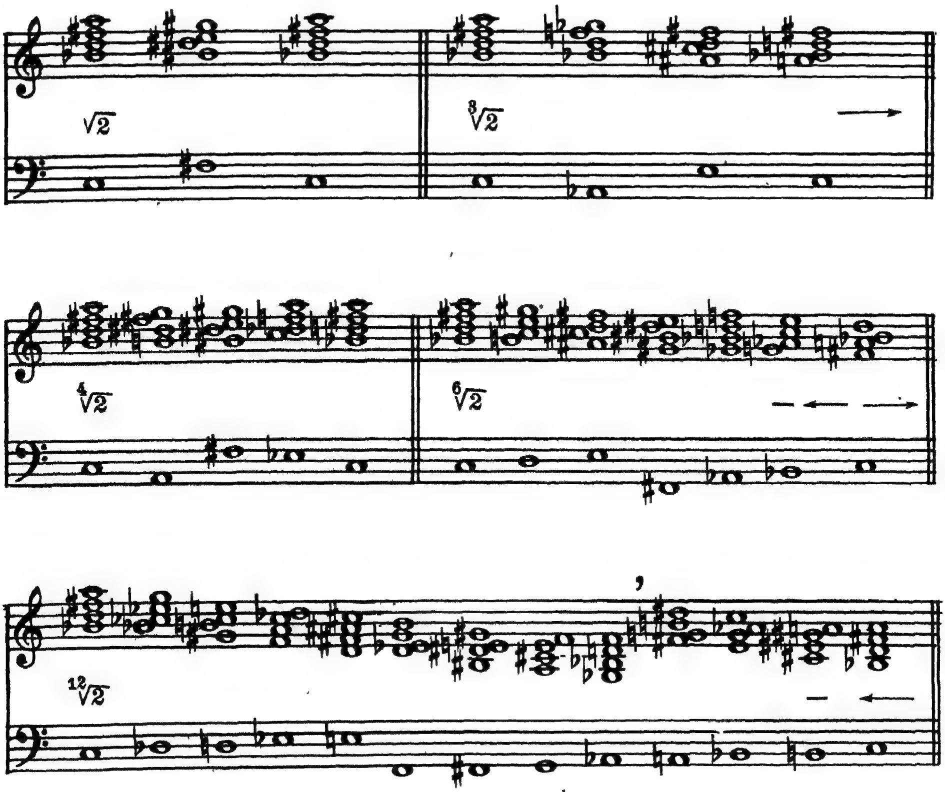


Figure 153. Continuity of S (13) monomials.

Combinations by two (binomials), three (trinomials), four (quadrinomials) and five (quintinomials) may be devised in a similar way.

3. Table of Combinations

The Arabic numerals in the following tables represent the chord structures (S):

a. *Combinations by 2*

5 + 7	7 + 9	9 + 11	11 + 13
5 + 9	7 + 11	9 + 13	
5 + 11	7 + 13		
5 + 13			

10 combinations, 2 permutations each
Total: $10 \times 2 = 20$

b. *Combinations by 3*

5 + 7 + 9	7 + 9 + 11	9 + 11 + 13
5 + 7 + 11	7 + 9 + 13	
5 + 7 + 13	7 + 11 + 13	
5 + 9 + 11		
5 + 9 + 13		
5 + 11 + 13		

10 combinations, 6 permutations each
Total: $10 \times 6 = 60$

c. *Combinations by 4*

5 + 7 + 9 + 11	7 + 9 + 11 + 13
5 + 7 + 9 + 13	
5 + 7 + 11 + 13	
5 + 9 + 11 + 13	

5 combinations, 24 permutations each
Total: $5 \times 24 = 120$

d. *Combinations by 5*

5 + 7 + 9 + 11 + 13

1 combination, 120 permutations
Total: $1 \times 120 = 120$

All other cases of trinomial, quadrinomial, quintinomial and larger combinations are treated as coefficients of recurrence.

Example: $S^{\rightarrow} = 2S(5) + S(7) + 2S(9) = S(5) + S(5) + S(7) + S(9) +$ $+ (S(9)$, i.e., a quintinomial with two identical pairs.

Coefficients of recurrence may be applied to the composition of continuity consisting of the forms of variable tension.

Examples:

Type I. 2S(5) + S(9) + S(13) + 2S(7)

Type II

Type III $\sqrt[3]{2}$

Figure 154. Coefficients of recurrence applied to variable tension continuity.

CHAPTER 10

THE NINTH CHORD

A. S(9) in the Diatonic System

NINTH-CHORDS in four-part harmony are used *with the root-tone in the bass only*, thus operating as a hybrid four-part harmony—like S(5) with the doubled root. The three upper parts are 3, 7 and 9. The 7 and the 9 are subject to resolution through stepwise downward motion.

If one function resolves at a time, it is always the higher one (the ninth). A resolution of one function at a time produces C_0. Other cycles derive from the simultaneous resolutions of two functions (the ninth and the seventh). No consecutive S(9)'s are possible through this particular type of system for S(9) alternates with S(7) and S(5).

The reason for first resolving the 9th rather than the 7th in C_0 is that the latter procedure would result in a chord-structure alien to the usual seven-unit diatonic scales; the intervals in the three upper voices are fourths.

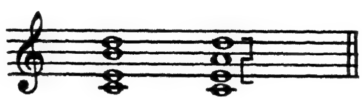


Figure 155. Resolving the ninth.

1. Positions of S(9)

As the bass remains constant, the three upper voices are subject to six permutations resulting in corresponding distributions.

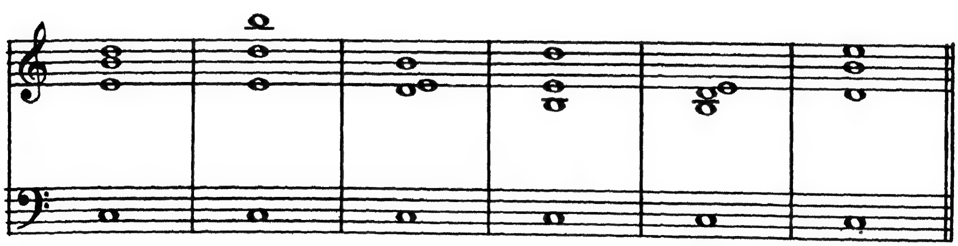


Figure 156. Table of positions of S(9).

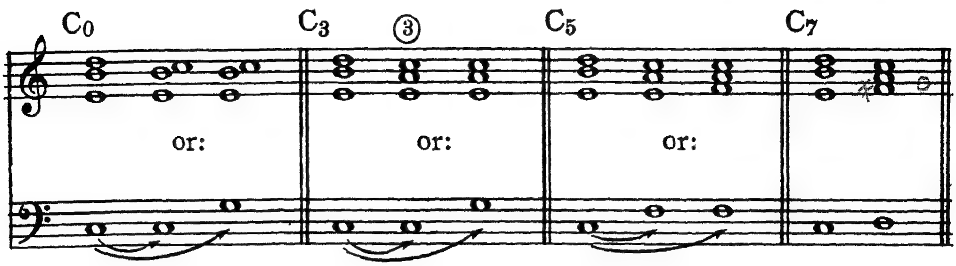


Figure 157. Resolutions of S(9).

The resolutions (except in C_0) produce positive cycles only. In C_3 they are characteristic of Mozart, Clementi and others of the same period. C_5 (the second resolution) is the most commonly known, especially with b♭ in the first chord (making a "dominant chord" of F-major).

C_7 is characteristic of Bach and contrapuntalists who developed such progressions from the idea of two pairs of voices moving in thirds in contrary motion. Read the last measure with b♭ and f♯ and add S(5) g-minor. All these cases of resolution were known to the classics through *melodic manipulations* (i.e., as a part of their contrapuntal heritage) and not through the idea of those independent structures we call S(9).

Preparation of S(9) bears a great similarity to the preparation of S(7). There is even an absolute correspondence in the cycles with respect to technical procedures.

The same three methods constitute the technique of preparation (suspending, descending, ascending).

2. Table of Preparations

(1) *Suspending*:

3 — 9	5 — 9	7 — 9
1 — 7	3 — 7	5 — 7
C_7	C_5	C_3

(2) *Descending*:

3 ↘ 9	5 ↘ 9	7 ↘ 9
1 ↘ 7	3 ↘ 7	5 ↘ 7
C_0	C_{-3}	C_{-5}

(3) *Ascending*:

3 ↗ 9	5 ↗ 9	7 ↗ 9
1 ↗ 7	3 ↗ 7	5 ↗ 7
C_{-3}	C_{-5}	C_{-7}

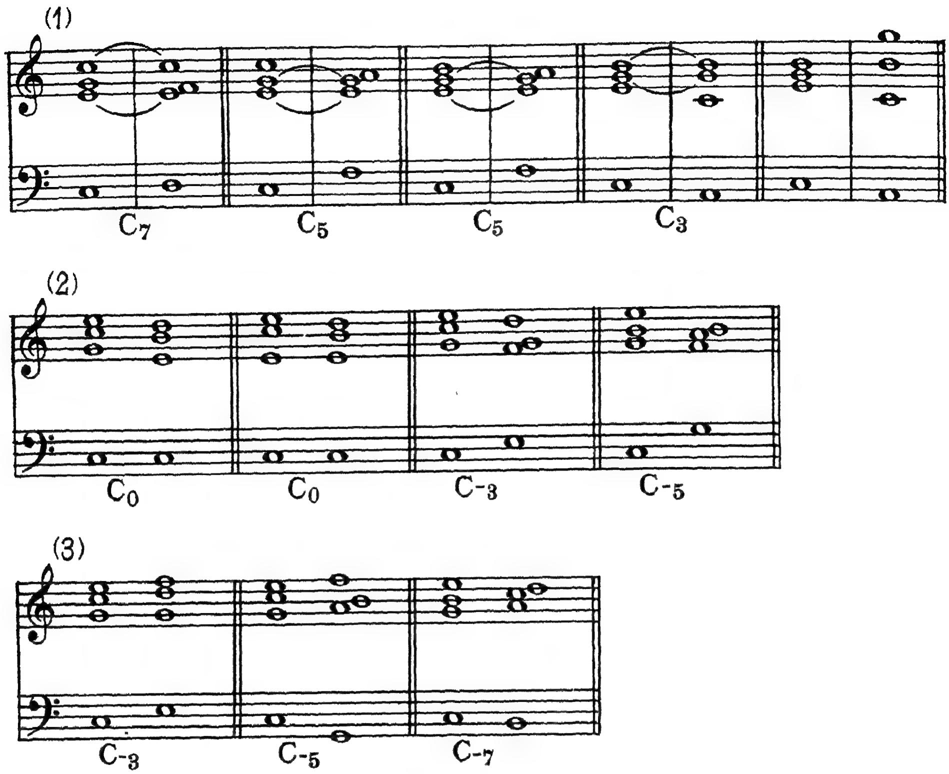


Figure 158. Preparations of S(9).

It follows from the above chart that some of the preparations of S(9) require an S(5); some require S(7); and some allow both. It is practical to have S(5) or S(7) preparing S(9) with the root in the bass.

The first form of preparation was known to the classics as a double suspension.

Figure 159. Double suspension.

A similar cadence was used in major.

Here is another example of a characteristic classical cadence:

Figure 160. Characteristic classical cadence.

*Figure 161. Example of continuity containing S(9).**

*At this point in the original manuscript, Schillinger writes: "If the student is to grasp all the implications of the foregoing material, he should work out the following instructions as home-work:

(1) Make complete tables of preparations and resolutions from all positions.

(2) Write diatonic continuity containing S(9).

(3) Make some modal transpositions of the examples thus obtained.

(4) Write continuity containing S(9) in the second type (diatonic-symmetric) of harmony. Select chord-structures from the examples of hybrid five-part harmony."

(Ed.)

B. S(9) in the Symmetric System

The classical (preparation-resolution) technique just described—and commonly used in the diatonic system—is also applicable to the symmetric system. Symmetric roots correspond to the respect cycles: C_5, to $\sqrt{2}$; C_3, to $\sqrt[3]{2}$ and $\sqrt[4]{2}$; C_7, to $\sqrt[6]{2}$ and $\sqrt[12]{2}$. With this in view, a continuity consisting of S(5), S(7) and S(9), and operated through classical technique, may be offered.

Symmetric C_0 is quite fruitless when S(9) alone is used, for the upper three functions (3, 7, 9) produce an incomplete seventh-chord, the permutations of which ($3 \leftrightarrow 7$, $3 \leftrightarrow 9$) sound awkward. There is one exception: $7 \leftrightarrow 9$.

As S(9) in hybrid four-part harmony is an incomplete structure—5 is omitted—the adjectives descriptive of chord structure may be applied only with a certain allowance for the 5th.

There are two distinctly different families of S(9), not to be mixed except when in C_0:

(1) *The minor seventh family.*
(2) *The major seventh family.*

The **minor 7th family** includes the following structures:

Figure 162. Minor 7th family.

To these the following adjectives may be applied in their respective order:

$7\flat S_1$ — large.
$7\flat S_2$ — diminished.
$7\flat S_3$ — minor.
$7\flat S_4$ — small.

The **major 7th family** includes the following structures:

Figure 163. Major 7th family.

The respective adjectives are:

$7\natural S_1$ — major.
$7\natural S_2$ — augmented I.
$7\natural S_3$ — augmented II.

These are the only possible forms.

It seems that all combinations of the two families, except those producing consecutive sevenths ($7\flat S_4 \leftrightarrow 7\natural S_1$; $7\natural S_2 \leftrightarrow 7\natural S_3$; $7\flat S_3 \leftrightarrow 7\natural S_2$; $7\flat S_1 \leftrightarrow 7\flat S_4$), are satisfactory when in C_0. On the different roots, the forms of S(9) must belong to one family.

Figure 164. Example of C_0 continuity.

Full indication for S(9) when used in combinations with S(5) and S(7):

$7\flat S_1(9)$; $7\flat S_2(9)$; $7\flat S_3(9)$; $7\flat S_4(9)$
$7\natural S_1(9)$; $7\natural S_2(9)$; $7\natural S_3(9)$

Two tonics ($\sqrt{2}$). The technique corresponds to C_5.

Figure 165. Two tonics ($\sqrt{2}$).

To resolve the last chord of the preceding table, use position ⓑ of the resolution technique.

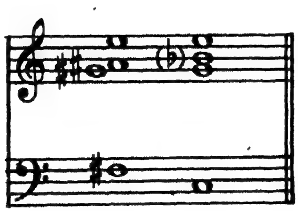


Figure 166. Resolution of last chord of figure 165.

Figure 167. Example of continuity.

Three tonics ($\sqrt[3]{2}$). The technique corresponds to C_3.

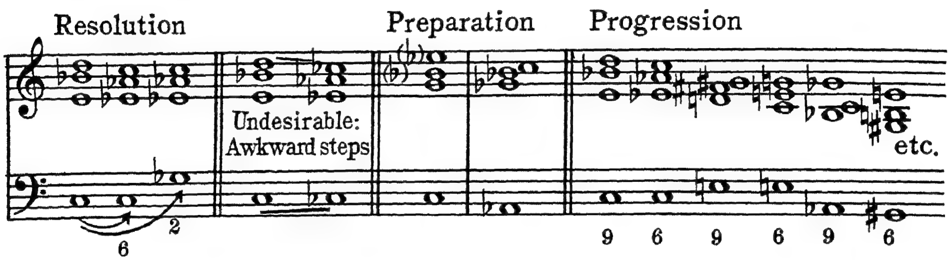


Figure 168. Three tonics.

In order to acquire a complete understanding of voice-leading in the preceding table of progressions (9 — 6 — 9 — 6 etc.), one should construct mentally an S(7) instead of an S(6). Then the first two chords will appear in the following positions:

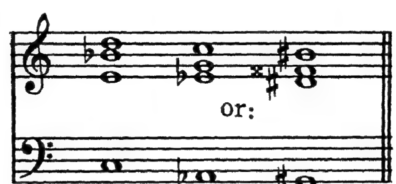


Figure 169. Positions of S(9) to S(6).

It is clear now that d♯ and f𝄪 are the necessary 7 and 9 of the following chord.

Figure 170. Example of continuity (continued).

Figure 170. Example of continuity (concluded).

Four tonics ($\sqrt[4]{2}$). The technique corresponds to C_3.

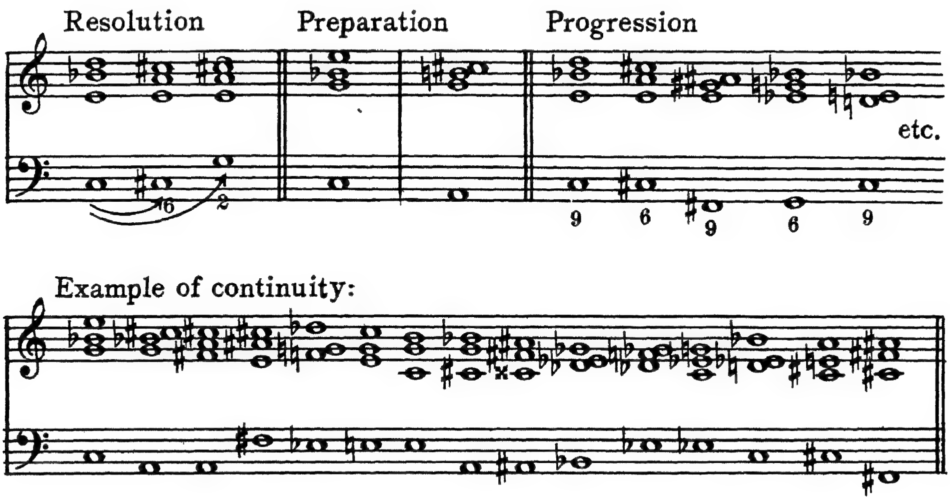


Figure 171. Four tonics.

Six tonics ($\sqrt[6]{2}$). The technique corresponds to C_7.

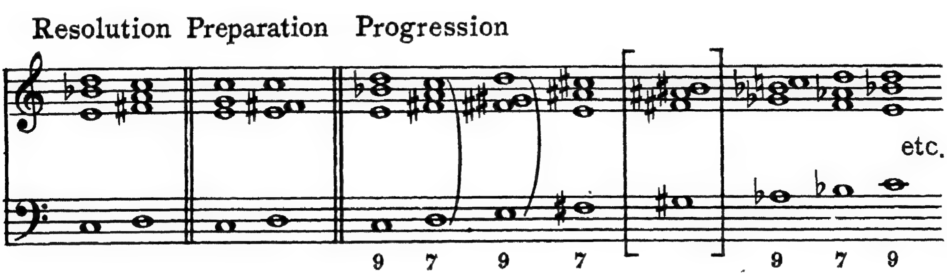


Figure 172. Six tonics $\sqrt[6]{2}$.

The above consecutive sevenths are unavoidable with *this* technique.

The position of every S(9) is based on the assumption that the preceding chord was S(5) and not S(7).

Figure 173. Continuity: $S(9) + S(7) + S(5)$.

The negative system which may be obtained by reading the above tables in position ⓑ is not as desirable with these media as the positive. The same concerns the following $\sqrt[12]{2}$. More plastic devices (general forms of transformations) will be offered later.

Twelve tonics ($\sqrt[12]{2}$). The technique corresponds to C_7.

Figure 174. Twelve tonics $\sqrt[12]{2}$

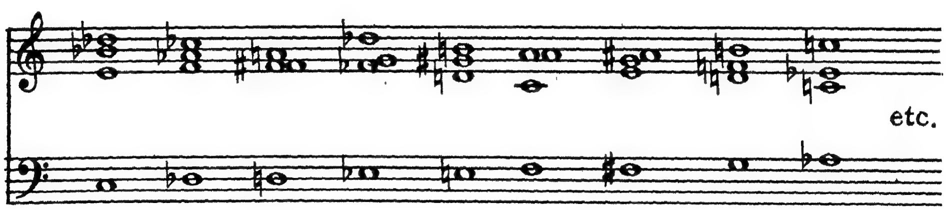


Figure 175. Continuity: $S(9) + S(7) + S(5)$.*

*In the original manuscript, Schillinger suggests that the implications of this material he studied through the following: "Exercises in the different symmetric systems containing $S(5)$, $S(7)$ and $S(9)$ with application of different structures and the C_0 between the roots." (Ed.)

CHAPTER 11

THE ELEVENTH CHORD

A. S(11) IN THE DIATONIC SYSTEM

IN FOUR-PART harmony, eleventh chords [S(11)] are used with the root-tone *in the bass only,* thus forming a hybrid four-part harmony [like that formed by S(5) with the doubled root]. The three upper parts consist of 7, 9, 11. An S(11) has an advantage over S(9) in that the upper functions form a complete S(5). All three upper functions are subject to resolution through stepwise downward motion. Resolutions of fewer than the three upper functions produce C_0.

No consecutive S(11)'s are possible in *this* particular system. They alternate with the other structures.

For reasons explained in the previous chapter, the C_0 resolutions must follow in the direction of the decreasing functions: if only one is resolved, 11 must be resolved first; then, 9; then, 7. When two functions resolve simultaneously, they are 11 and 9. An S(11) allows a continuous chain of resolutions'

S(11) ↘
S(9) 9 ↘
S(7) 7 ↘
S(6)③

An eleventh-chord through resolution of the eleventh becomes a ninth-chord; a ninth-chord through resolution of the ninth becomes an incomplete seventh-chord (without a fifth), or a complete S($\frac{4}{3}$) as in the corresponding resolutions of S(9); an incomplete seventh-chord through resolution of the seventh becomes a sixth-chord with doubled third.

1. Positions of S(11)

As the bass remains constant, the three upper voices are subject to six permutations. Seventh, ninth and eleventh form a triad corresponding to a root, a third and a fifth while the bass corresponds to the pitch-unit one degree higher than the root of the triad.

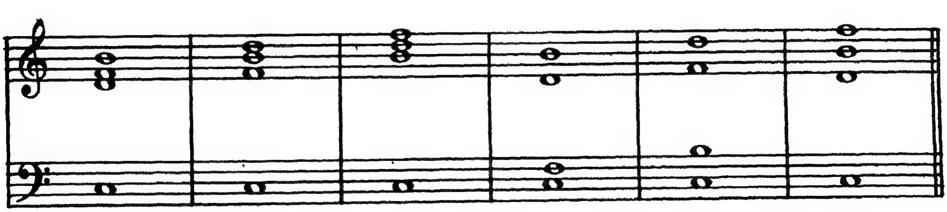


Figure 176. Positions of S(11).

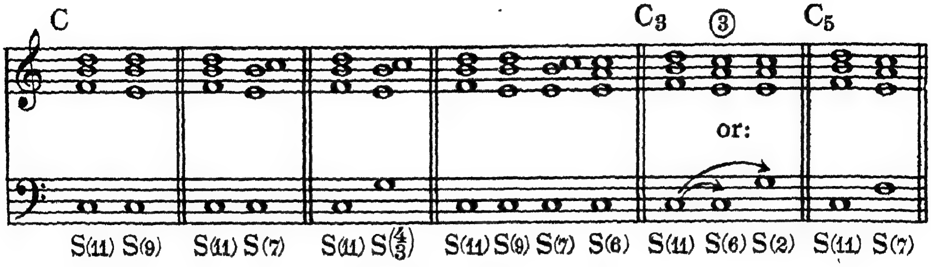


Figure 177. Resolutions of S(11).

As it follows from the above table, when S(11) resolves into S(9) in C_0, S(9) has its proper structural constitution (i.e., 1, 3, 7, 9).The C_7 resolution does not appear on this table for the reason that the structural constitution of S(9) into which S(11) would resolve is 1, 5, 7, 9, and this does not sound satisfactory, according to *our* musical habits.

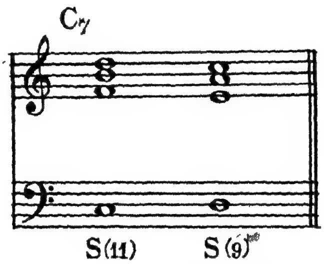


Figure 178. S(11) ⟶ S(9).

The above resolutions correspond to the classical resolutions of the triple suspensions

B. Preparation of S(11)

Preparation of S(11) in the positive cycles has a cyclic correspondence to the preparation of S(7) and S(9) through suspensions. Nevertheless, the manner of reasoning is somewhat different in this case.

As S(11) has an appearance of an S(5) with a bass corresponding to the pitch-unit one degree higher than the root of the triad, the most logical assumption is: take S(5)①, move its bass one step up and this will produce an S(11) with a proper structural constitution. In such a case, the relation of the three stationary upper functions is C_0. The tones being common tones, may be inverted or exchanged.

The first case gives a clue to the preparation of other cycles (positive and negative as well). The method of preparation implies merely the more gradual transformation (↻ or ↺) of the three upper functions.

To prepare S(11) after an S(5) in C_0, move all upper functions down scalewise and leave the bass stationary (which is the converse of the first proposition).

Preparations of S(11)

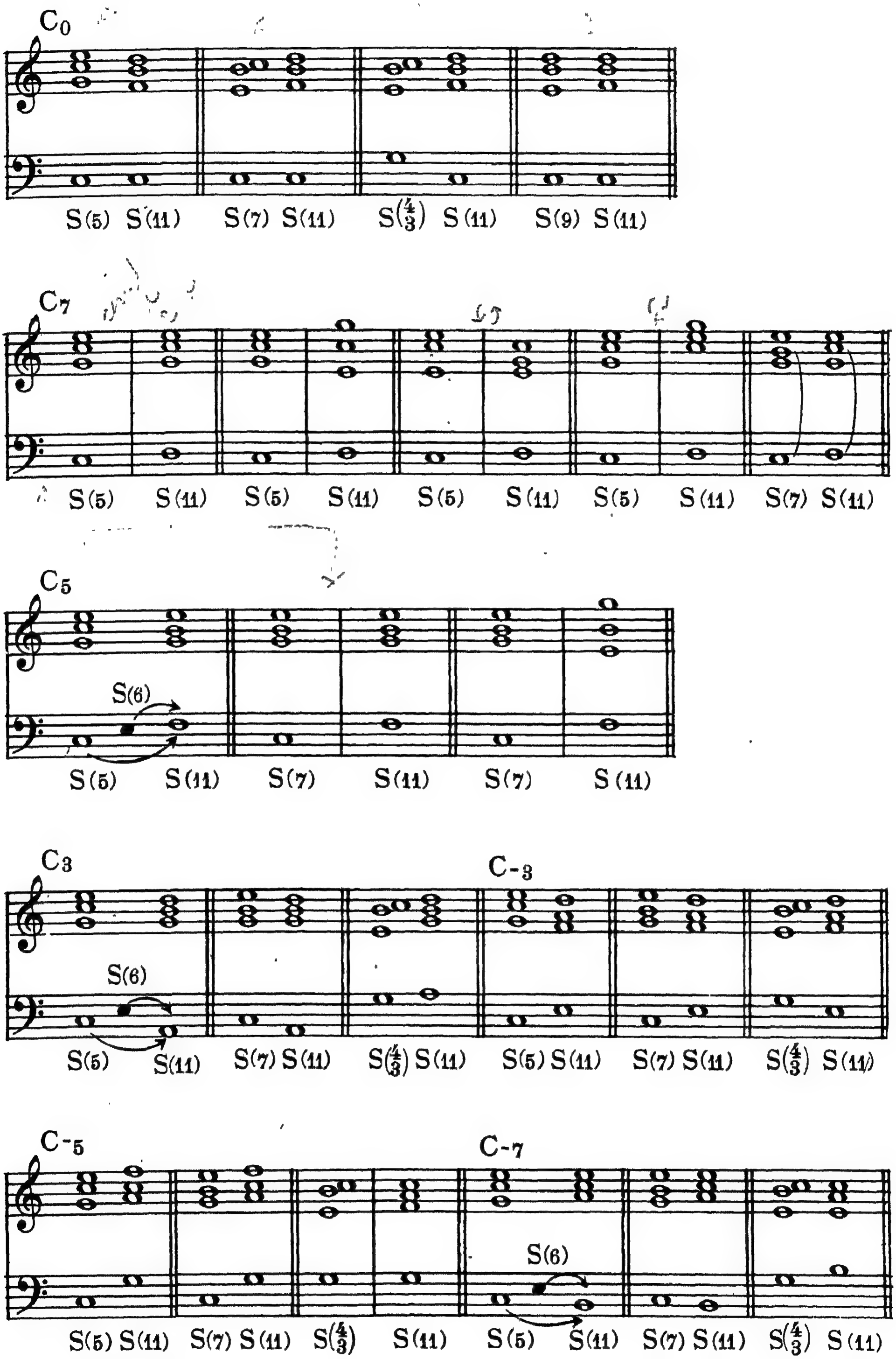


Figure 179. Preparations of S(11).

When all tones are held in common in the three upper parts, it is advisable to use the over-the-bar suspension method. (See page 462.)

When some of the upper parts move and some remain stationary, either the within-the-bar or the over-the-bar preparation may be used.

Characteristic progressions and cadences, in which all forms of tension [from S(5) to S(11)] are applied, would be:

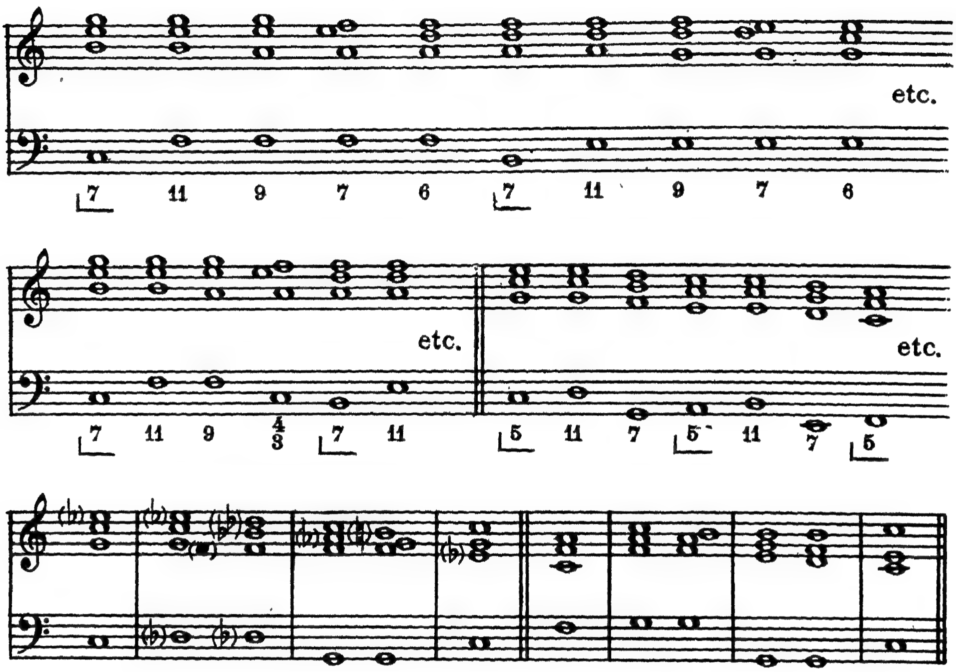


Figure 180. Characteristic progressions and cadences employing S(5) to S(11).

Example of Continuity Containing S(11)

Figure 181. Continuity containing S(11).

C. S(11) in the Symmetric System

The above technique of diatonic progressions containing S(11) is applicable to the symmetric system as well. The cyclic correspondence previously used remains the same. Thus, preparations of S(11) are possible in all systems of the symmetric roots, whereas resolutions can be performed only when the acting cycle is C_3 ($\sqrt[3]{2}$ and $\sqrt[4]{2}$) and C_5 ($\sqrt{2}$). There is no difficulty with any preparation of S(11) after a resolution, as the latter always consists of 1, 3, 5 and therefore may be connected with the following chord through the usual transformations.

Unlike S(9), S(11) produces a highly satisfactory C_0, due to the presence of all functions without gaps in the three upper parts.

As with the ninth-chords, there are two distinctly different families of S(11) which are not to be mixed, except when in C_0. The distinction becomes even greater than before and the mixing becomes still more "dangerous."

The structural constitution of S(11) permits the classification of such structures as S(5) with regard to their upper functions.

The Minor Seventh Family

The Major Seventh Family

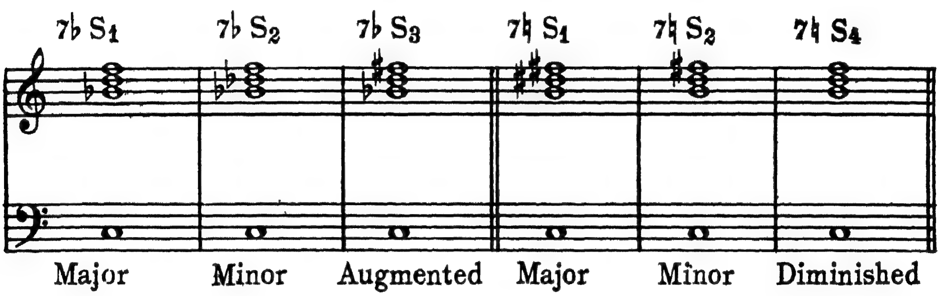


Figure 182. Forms of S (11).

There are two less common forms. The diminished in the first group and the augmented in the second group.

Figure 183. 7♭ S_4 (11); 7♮ S_3 (11).

The selection of better progressions in C_0 for the continuity of S(11) must be analogous to the selection of forms for S(5). If desired, consecutive sevenths may be avoided by permutations.

Figure 184. Example of C_0 continuity.

Full indications for S(11) when used in combination with other structures:

$7\flat S_1(11)$; $7\flat S_2(11)$; $7\flat S_3(11)$
$7\natural S_1(11)$; $7\natural S_2(11)$; $7\natural S_3(11)$

Two tonics ($\sqrt{2}$). The technique corresponds to C_5; clockwise or counterclockwise transformations for continuous S(11).

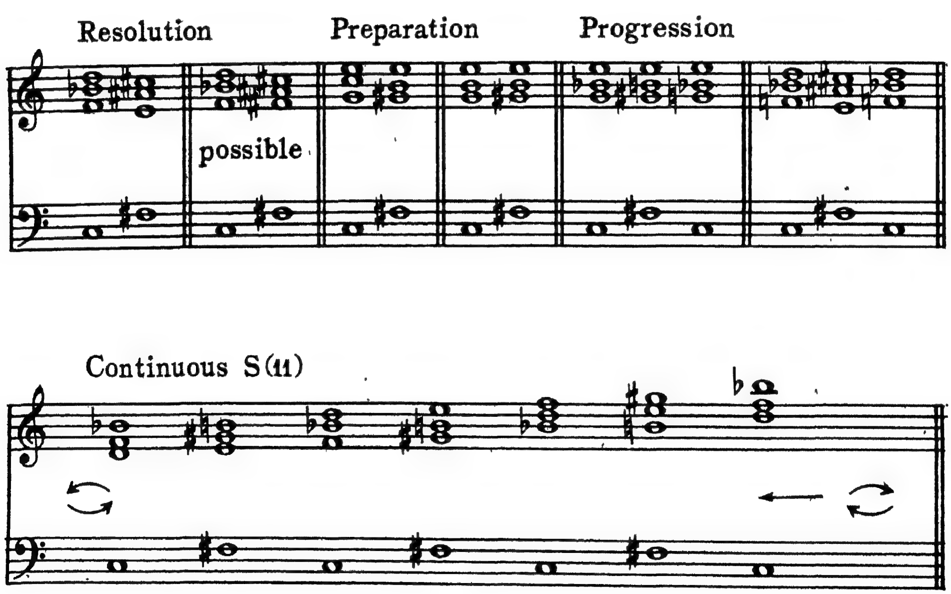


Figure 185. Two tonics $\sqrt{2}$.

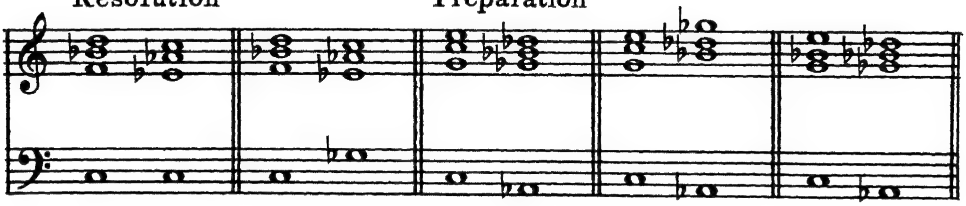
You may consider the upper three parts either as 7, 9, 11 in ↻ and ↺ transformations or as 1, 3, 5 with a displaced bass.

Example of Continuity

Figure 185. Two tonics (concluded)

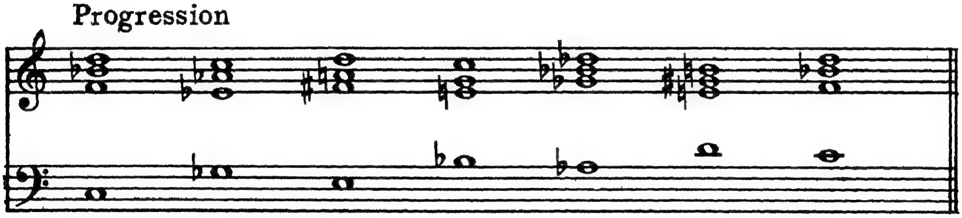
Three tonics ($\sqrt[3]{2}$). The technique corresponds to C_3 or to the ↻ and ↺ transformations.

Resolution Preparation



Progression

Progression



Continuous S (11)

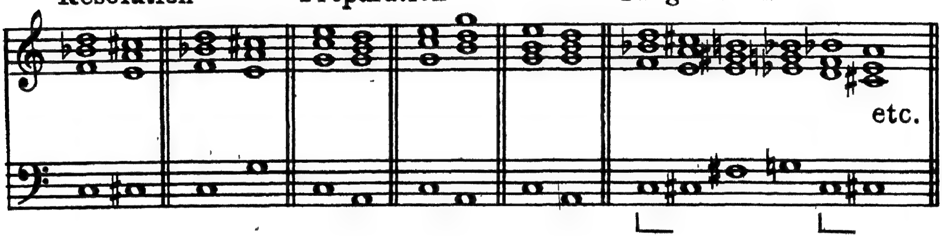
Figure 186. Three tonics $\sqrt[3]{2}$.

Example of Continuity

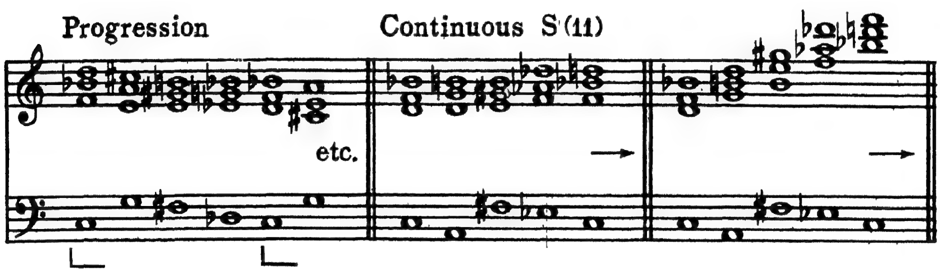
Figure 186. Three tonics (concluded).

Four tonics ($\sqrt[4]{2}$). The technique corresponds to C_3 or to the ↻ and ↺ transformations.

Resolution Preparation Progression



Progression Continuous S (11)



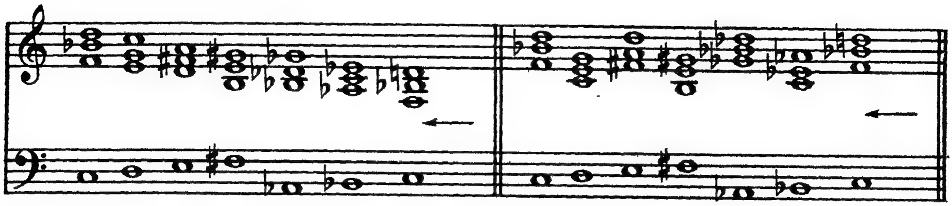
Example of Continuity

Figure 187. Four tonics $\sqrt[4]{2}$.

With the complexity of the harmony above, the consecutive ninths (if they are both major and move on a whole tone) are perfectly admissible.

Six tonics ($\sqrt[6]{2}$). Use ↻ and ↺ transformations only.

Continuous S(11)



Example of Continuity

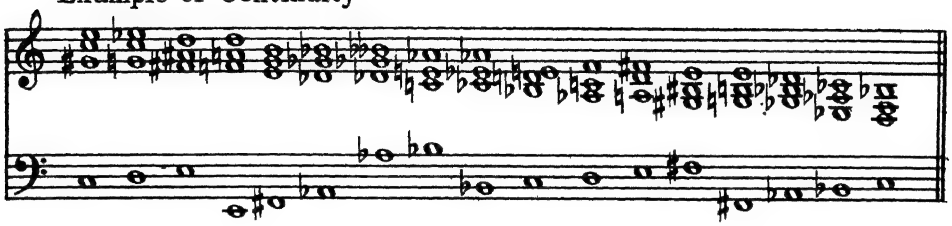
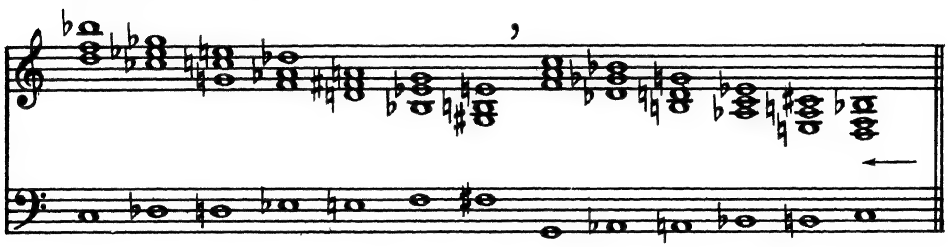
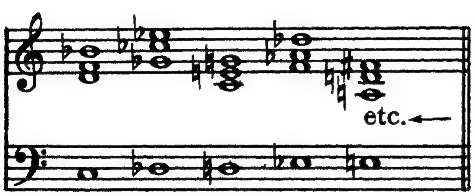


Figure 188. Six tonics.

Twelve tonics ($\sqrt[12]{2}$). Use ↻ and ↺ transformations only.

Continuous S(11)





Example of Continuity

Figure 189. Twelve tonics $\sqrt[12]{2}$*

*In the original manuscript, Schillinger suggests that the following work be done: "As with S(9), utilize various structures, forms and progressions on S(11). The transformation technique is applicable to diatonic and diatonic-symmetric progressions as well." (Ed.)

D. In Hybrid Four-Part Harmony

The general technique of transformations for groups with three functions may now be adopted for a generalization of the forms of voice-leading in hybrid four-part harmony. The three upper parts perform the transformations corresponding to the groups with three functions, and the bass remains constant.

The following technique is applicable to any type of harmonic progression: diatonic, diatonic-symmetric, or symmetric. The specifications for the following forms of S are chosen *with respect to their sonority*. Those marked with an asterisk in the following tables are less commonly used than the unmarked ones. The charts of transformations for the latter are worked out; the reader may easily substitute them for those marked with the asterisk.

Forms of Hybrid Four-Part (3 + 1) Harmony

The Three upper parts.	5 3 1	5 3 13	7 5 3	7 3 1	9 7 3	9 7 1	11 9 7	13 9 7	13 11 7
The bass.	1	1	1	1	1	1	1	1	1
Forms of tension.	S(5)	* S(5)	S(7)	* S(7)	S(9)	* S(9)	S(11)	S(13)	* S(13)

Figure 190. Forms of hybrid four-part harmony.

When the numerals expressing the functions in a group are identical with the numerals of the succeeding group, certain forms of transformation—such as constant abc—may be eliminated because of their complete parallelism. When the numerals in the two allied groups are partly identical, some of the forms (constant a, constant b, constant c) give either favorable or unfavorable partial parallelisms. The partial parallelisms are favorable when the parallel motion forms desirable intervals with the bass. They are unfavorable when the motion causes a consecutive motion of the seventh or ninth with the bass (consecutive seventh, consecutive ninth).

Inasmuch as the actual *quality* of voice-leading depends on the structures of the two allied chords, the student will be able—upon completion of all these charts in musical notation—to make his own preferential selection.

When the numerals in the two allied groups are either partly or totally different, often the *constant abc* transformation becomes the most favorable form of voice-leading. There is a natural compensation at work in this case. ***Homogeneous*** structures are compensated by ***heterogeneous*** transformations—and heterogeneous structures are compensated by homogeneous transformations. For example, if the allied groups are both S(5), the constant abc transformation

would be unconventional: $1 \rightarrow 1$, $3 \rightarrow 3$, $5 \rightarrow 5$, which gives consecutive octaves and fifths. On the contrary, when the functions have different numerals, the smoothest voice-leading results from this particular transformation.

When two allied groups have different or partly different numerals for their functions, the first group becomes the *original* group and the succeeding group beomes the *prime* group. When a transformation between two such groups is performed, the prime group in turn becomes the original group for the next transformation.

The Original Group			The Prime Group		
	a			a^1	
c		b	c^1		b^1

For example, by connecting S(5) + S(9) + S(13) we obtain the following numerals in their corresponding order:

S(5)			S(9)			S(13)		
	1			3			7	
5		3	9		7	13		9

When the functions of S(5) are connected to the functions of S(9), the first group is the original group; the second is the prime group. When the functions of S(9) are connected to S(13), the functions of S(9) form the original group, and the functions of S(13) form the prime group.

Here is a complete table of transformations.

Forms of Transformations in the Homogeneous Groups

↻	↺	Const. a	Const. b	Const. c	Const. abc
a c b	a c b	a c b	a c b	a c b	a c b
a → b b → c c → a 5	a → c c → b b → a 6	a → a b → c c → b 2	a → c b → b c → a 3	a → b b → a c → c 4	a → a 1 b → b 2 c → c 3 1

Figure 191. Transformations in the homogeneous groups.

Forms of Transformations in the Heterogeneous Groups

The Original Group.	The Prime Group.
a	a^1
c b	c^1 b^1

$\circlearrowright$	$\circlearrowleft$	Const. a	Const. b	Const. c	Const. abc
$a \to b^1$ $b \to c^1$ $c \to a^1$	$a \to c^1$ $c \to b^1$ $b \to a^1$	$a \to a^1$ $b \to c^1$ $c \to b^1$	$a \to c^1$ $b \to b^1$ $c \to a^1$	$a \to b^1$ $b \to a^1$ $c \to c^1$	$a \to a^1$ $b \to b^1$ $c \to c^1$

Figure 192. Transformations in the heterogeneous groups.

Here are all the combinations for the two allied groups, applied to all forms of tension.

Binomial Combinations of the Original and the Prime Groups

$S(5) \leftrightarrow S(7)$	$S(7) \leftrightarrow S(9)$	$S(9) \leftrightarrow S(11)$	$S(11) \leftrightarrow S(13)$
$S(5) \leftrightarrow S(9)$	$S(7) \leftrightarrow S(11)$	$S(9) \leftrightarrow S(13)$	
$S(5) \leftrightarrow S(11)$	$S(7) \leftrightarrow S(13)$		
$S(5) \leftrightarrow S(13)$			

10 Combinations, 2 permutations each.
Total number of cases: $10 \times 2 = 20$.

Figure 193. Binomial combinations.

The following pages contain tables of transformations for the 20 binomials consisting of one original and one prime group. Each S tension is represented in this table by one structure only. The sequence of the forms of transformations in this table remains the same for all cases: (1) $\circlearrowright$; (2) $\circlearrowleft$; (3) Const. a; (4) Const. b; (5) Const. c; (6) Const. abc.

1. **Table of transformations for the twenty binomials.**

		S(5) ⟶ S(7)			
$1 \rightarrow 5$ $3 \rightarrow 7$ $5 \rightarrow 3$	$1 \rightarrow 7$ $3 \rightarrow 3$ $5 \rightarrow 5$ b	$1 \rightarrow 3$ a $3 \rightarrow 7$ c $5 \rightarrow 5$ b	$1 \rightarrow 7$ c $3 \rightarrow 5$ b $5 \rightarrow 3$ a	$1 \rightarrow 5$ b $3 \rightarrow 3$ a $5 \rightarrow 7$ c	$1 \rightarrow 3$ $3 \rightarrow 5$ $5 \rightarrow 7$
2	4	5	1	3	6

		S(7) ⟶ S(5)			
$3 \rightarrow 3$ $5 \rightarrow 5$ $7 \rightarrow 1$	$3 \rightarrow 5$ $5 \rightarrow 1$ $7 \rightarrow 3$	$3 \rightarrow 1$ $5 \rightarrow 5$ $7 \rightarrow 3$	$3 \rightarrow 5$ $5 \rightarrow 3$ $7 \rightarrow 1$	$3 \rightarrow 3$ $5 \rightarrow 1$ $7 \rightarrow 5$	$3 \rightarrow 1$ $5 \rightarrow 3$ $7 \rightarrow 5$
2	4	5	1	3	6

		S(5) ⟶ S(9)			
$1 \rightarrow 7$ $3 \rightarrow 9$ $5 \rightarrow 3$	$1 \rightarrow 9$ $3 \rightarrow 3$ $5 \rightarrow 7$	$1 \rightarrow 3$ $3 \rightarrow 9$ $5 \rightarrow 7$	$1 \rightarrow 9$ $3 \rightarrow 7$ $5 \rightarrow 3$	$1 \rightarrow 7$ $3 \rightarrow 3$ $5 \rightarrow 9$	$1 \rightarrow 3$ $3 \rightarrow 7$ $5 \rightarrow 9$

		S(9) ⟶ S(5)			
$3 \rightarrow 3$ $7 \rightarrow 5$ $9 \rightarrow 1$	$3 \rightarrow 5$ $7 \rightarrow 1$ $9 \rightarrow 3$	$3 \rightarrow 1$ $7 \rightarrow 5$ $9 \rightarrow 3$	$3 \rightarrow 5$ $7 \rightarrow 3$ $9 \rightarrow 1$	$3 \rightarrow 3$ $7 \rightarrow 1$ $9 \rightarrow 5$	$3 \rightarrow 1$ $7 \rightarrow 3$ $9 \rightarrow 5$

Figure 194. Transformations of binomial combinations.

		S(5) ——→ S(11)			
1 → 9 3 → 11 5 → 7	1 → 11 3 → 7 5 → 9	1 → 7 3 → 11 5 → 9	1 → 11 3 → 9 5 → 7	1 → 9 3 → 7 5 → 11	1 → 7 3 — 9 5 → 11

		S(11) ——→ S(5)			
7 → 3 9 → 5 11 → 1	7 → 5 9 → 1 11 → 3	7 → 1 9 → 5 11 → 3	7 → 5 9 → 3 11 → 1	7 → 3 9 → 1 11 → 5	7 → 1 9 → 3 11 → 5

		S(5) ——→ S(13)			
1 → 9 3 → 13 5 → 7	1 → 13 3 → 7 5 → 9	1 → 7 3 → 13 5 → 9	1 → 13 3 → 9 5 → 7	1 → 9 3 → 7 5 → 13	1 → 7 3 → 9 5 → 13

		S(13) ——→ S(5)			
7 → 3 9 → 5 13 → 1	7 → 5 9 → 1 13 → 3	7 → 1 9 → 5 13 → 3	7 → 5 9 → 3 13 → 1	7 → 3 9 → 1 13 → 5	7 → 1 9 → 3 13 → 5

Figure 195. Transformations of binomial combinations.

S(7) ⟶ S(9)					
3 → 7 5 → 9 7 → 3	3 → 9 5 → 3 7 → 7	3 → 3 5 → 9 7 → 7	3 → 9 5 → 7 7 → 3	3 → 7 5 → 3 7 → 9	3 → 3 5 → 7 7 → 9

S(9) ⟶ S(7)					
3 → 5 7 → 7 9 → 3	3 → 7 7 → 3 9 → 5	3 → 3 7 → 7 9 → 5	3 → 7 7 → 5 9 → 3	3 → 5 7 → 3 9 → 7	3 → 3 7 → 5 9 → 7

S(7) ⟶ S(11)					
3 → 9 5 → 11 7 → 7	3 → 11 5 → 7 7 → 9	3 → 7 5 → 11 7 → 9	3 → 11 5 → 9 7 → 7	3 → 9 5 → 7 7 → 11	3 → 7 5 → 9 7 → 11

S(11) ⟶ S(7)					
7 → 5 9 → 7 11 → 3	7 → 7 9 → 3 11 → 5	7 → 3 9 → 7 11 → 5	7 → 7 9 → 5 11 → 3	7 → 5 9 → 3 11 → 7	7 → 3 9 → 5 11 → 7

Figure 196. Transformations of binomial combinations.

		S(7) ⟶ S(13)			
3 → 9 5 → 13 7 → 7	3 → 13 5 → 7 7 → 9	3 → 7 5 → 13 7 → 9	3 → 13 5 → 9 7 → 7	3 → 9 5 → 7 7 → 13	3 → 7 5 → 9 7 → 13

		S(13) ⟶ S(7)			
7 → 5 9 → 7 13 → 3	7 → 7 9 → 3 13 → 5	7 → 3 9 → 7 13 → 5	7 → 7 9 → 5 13 → 3	7 → 5 9 → 3 13 → 7	7 → 3 9 → 5 13 → 7

		S(9) ⟶ S(11)			
3 → 9 7 → 11 9 → 7	3 → 11 7 → 7 9 → 9	3 → 7 7 → 11 9 → 9	3 → 11 7 → 9 9 → 7	3 → 9 7 → 7 9 → 11	3 → 7 7 → 9 9 → 11

		S(11) ⟶ S(9)			
7 → 7 9 → 9 11 → 3	7 → 9 9 → 3 11 → 7	7 → 3 9 → 9 11 → 7	7 → 9 9 → 7 11 → 3	7 → 7 9 → 3 11 → 9	7 → 3 9 → 7 11 → 9

Figure 197. Transformations of binomial combinations.

*

		S(9) ——→ S(13)			
3 → 9 7 → 13 9 → 7	3 → 13 7 → 7 9 → 9	3 → 7 7 → 13 9 → 9	3 → 13 7 → 9 9 → 7	3 → 9 7 → 7 9 → 13	3 → 7 7 → 9 9 → 13

		S(13) ——→ S(9)			
7 → 7 9 → 9 13 → 3	7 → 9 9 → 3 13 → 7	7 → 3 9 → 9 13 → 7	7 → 9 9 → 7 13 → 3	7 → 7 9 → 3 13 → 9	7 → 3 9 → 7 13 → 9

		S(11) ——→ S(13)			
7 → 9 9 → 13 11 → 7	7 → 13 9 → 7 11 → 9	7 → 7 9 → 13 11 → 9	7 → 13 9 → 9 11 → 7	7 → 9 9 → 7 11 → 13	7 → 7 9 → 9 11 → 13

		S(13) ——→ S(11)			
7 → 9 9 → 11 13 → 7	7 → 11 9 → 7 13 → 9	7 → 7 9 → 11 13 → 9	7 → 11 9 → 9 13 → 7	7 → 9 9 → 7 13 → 11	7 → 7 9 → 9 13 → 11

Figure 198. Transformations of binomial combinations.

S(5) ⟶ S(7)

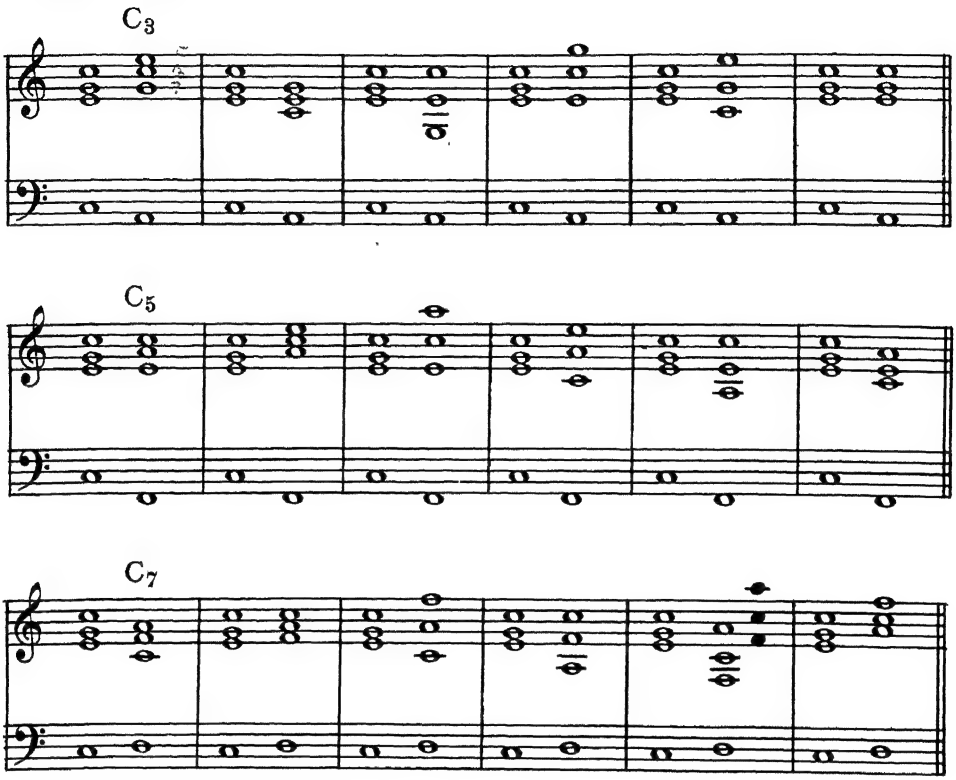


Figure 199. S(5) ⟶ S(7).

S(7) ⟶ S(5)

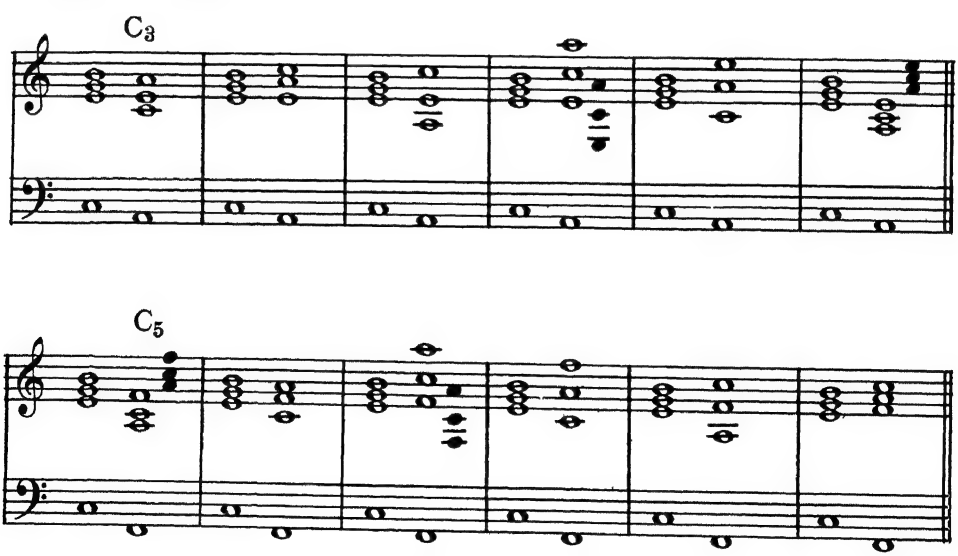


Figure 200. S(7) → S(5) (continued).

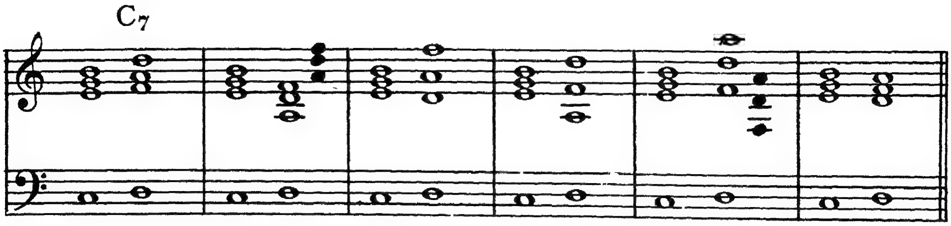


Figure 200. $S(7) \rightarrow S(5)$ *(concluded).*

$S(5) \rightarrow S(9)$

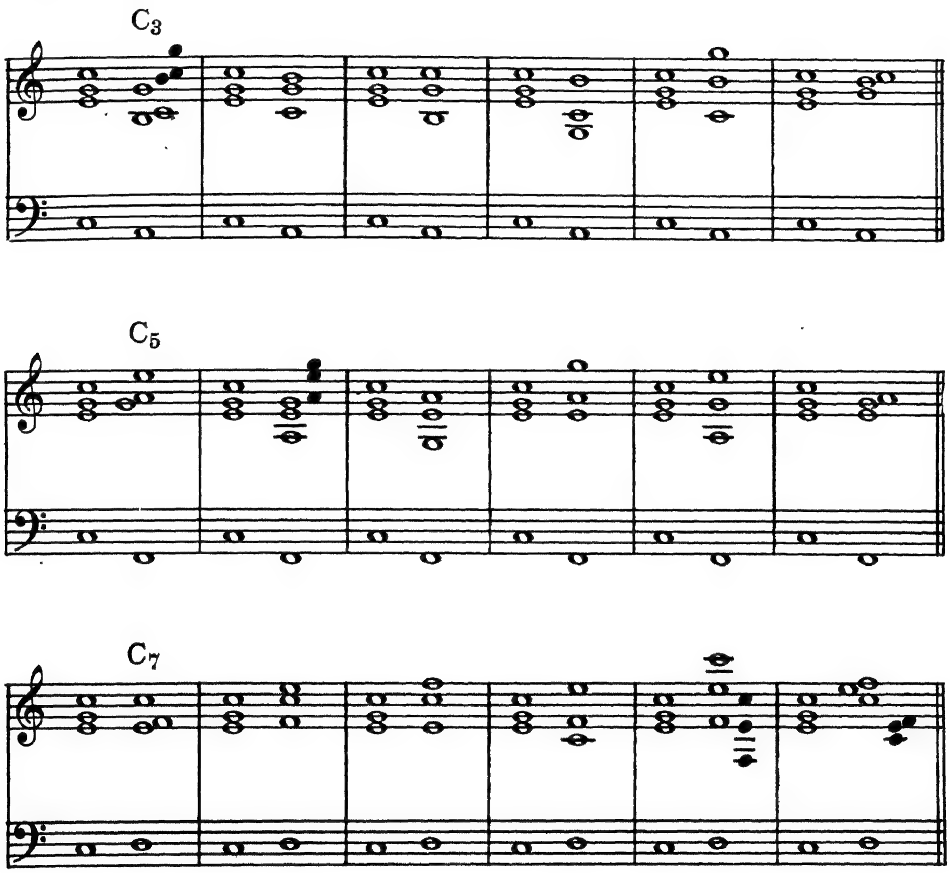


Figure 201. $S(5) \rightarrow S(9)$*.

*In the original manuscript, Schillinger suggests that the student make additional tables for: $S(5) \rightarrow S(5)$; $S(7) \rightarrow S(7)$; $S(9) \rightarrow S(9)$; $S(11) \rightarrow S(11)$; $S(13) \rightarrow S(13)$. (Ed.)

It is easy to work out all cases in musical notation by applying each case to all three tonal cycles.

As in previous cases, continuity may be composed in all three types of harmony (diatonic, diatonic-symmetric and symmetric). Structures of different tension may be selected for the composition of continuity. Different individual styles depend upon the coefficients of recurrence applied to structures of differing tensions.

The first of the following two examples of continuity is produced through structures of constant form and tension [S(13)]; the second illustrates a continuity of variable forms and variable tensions distributed through $r_{3\div2}$.

Continuity of Groups with Identical Functions

S(13) → Type II. Scale: b♭-harm., d_1.

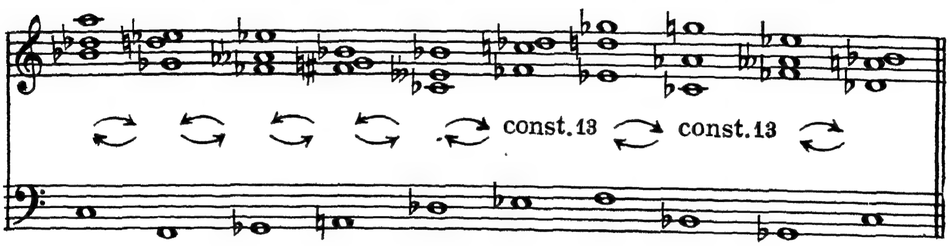


Figure 202. Structures of constant form and tension S (13).

Continuity of Groups with Different Functions

2S(9) + S(7) + S(11) + S(13) + 2S(11); Type III. $\sqrt[6]{2}$

Figure 203. Variable forms and tensions through $r_{3\div2}$.

CHAPTER 12

GENERALIZATION OF SYMMETRIC PROGRESSIONS

THE forms of symmetric progressions heretofore used in this portion of my discussion of harmony were based on a *monomial* symmetry of the uniform intervals of an octave.

But in order to obtain other mixtures (binomials, trinomials and polynomials) of the original forms of symmetry within an octave, it is necessary to establish a general nomenclature for all intervals of an octave. As all intervals are special cases of the twelve-fold symmetry, any diatonic form may be considered a special case of symmetry as well.

The system of enumeration of intervals may follow the upward or downward direction from any established axis point. As both directions include *all* intervals (which means both positive and negative tonal cycles), the matter of preference must be determined by the quantitative predominance of the type of intervals generally used. It seems that the descending system is the more practical, for smaller numbers can then be used to express the positive steps on three and four tonics; the negative, on six and twelve tonics.

In the following exposition, the descending system will be used exclusively. This need not prevent one from using the ascending system.

Scales of Intervals within one Octave Range:

Descending System:	*Ascending System:*
c → c = 0	c → c = 0
c → b = 1	c → d♭ = 1
c → b♭ = 2	c → d = 2
c → a = 3	c → e♭ = 3
c → a♭ = 4	c → e = 4
c → g = 5	c → f = 5
c → f♯ = 6	c → f♯ = 6
c → f = 7	c → g = 7
c → e = 8	c → a♭ = 8
c → e♭ = 9	c → a = 9
c → d = 10	c → b♭ = 10
c → d♭ = 11	c → b = 11
c → c_1 = 12	c → c^1 = 12

Monomials

Two Tonics:	$6 + 6$
Three Tonics:	$4 + 4 + 4$ or $8 + 8 + 8$
Four Tonics:	$3 + 3 + 3 + 3$ or $9 + 9 + 9 + 9$
Six Tonics:	$2 + 2 + 2 + 2 + 2 + 2$ or $10 + 10 + 10 + 10 + 10 + 10$
Twelve Tonics:	$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$
or	$11+11+11+11+11+11+11+11+11+11+11+11$

Figure 204. Intervals and tonics within one octave.

So approached, each constant system of tonics becomes a form of monomial periodicity of a certain pitch-interval, expressible in the form of a constant number-value, which in turn expresses the quantity of semitones from the preceding pitch-unit.

In the framework of this system, the problem of mixing various tonics (or any interval-steps in general) becomes reduced to the process of composing binomials, trinomials or any more extended groups (such as rhythmic resultants, their modifications through permutations and powers, series of growth), i.e., to the rhythmic distribution of steps.

The vitality of such groups, i.e., the periodicity of their recurrence until the completion of their cycle, depends upon the divisibility-properties of the sums of their interval-quantities. The total sum of all number-values expressing the intervals becomes a divisor of 12, or any multiple thereof. This signifies the motion of a certain group through an octave (or octaves).

For example, a binomial $3 + 2$ has 12 recurrences until it completes its cycle, as $3 + 2 = 5$, and the smallest multiple of 12, divisible by 5 is 60. This is true of all prime numbers when used as divisors.

C – A – G – E – D – B – A – F♯ – E – C♯ – B

B – G♯ – F♯ – D♯ – C♯ – A♯ – G♯ – F – E♭

E♭ – C – B♭ – G – F – D – C

Figure 205. Binomial 3 + 2.

This property makes mixtures of three and four tonics very desirable when a long harmonic span is necessary without a variety of steps.

The process of division serves as a testing tool of the vitality of compound symmetric groups.

Two tonics close after two cycles, as $6 + 6 = 12$, or $\frac{12}{6} = 2$;

$r4{\div}3$ closes after one cycle, as $3 + 1 + 2 + 2 + 1 + 3 = 12$, and $\frac{12}{12} = 1$;

$r5{\div}4$ closes after three cycles, as $4 + 1 + 3 + 2 + 2 + 3 + 1 + 4 = 20$, and $\frac{60}{20} = 3$.

Greater variety without deviating from a given style may be achieved by means of permutations of the members of a group. For example, a group with a short span may be revitalized through permutations:

(3+1+2) + (3+2+1) + (2+3+1) + (1+3+2) + (1+2+3) + (2+1+3)

or: C – A – G♯ – F♯ – E♭ – D♭ – C – B♭ – G – F♯ – F♮ – D – C

C – B♮ – A – F♯ – E♮ – E♭ – C

Figure 206. Permutating a group with a short span.

The selection of number values is left to the composer's discretion; if he wants to obtain the tonic-dominant character of classical music, the only thing he needs is an excess of the value 5.

Anyone equipped with this method can dodge extremities of style by a cautious selection of the coefficients of recurrence. For instance, in order to produce that style of progressions which lies somewhere between Wagner and Ravel, it is necessary to have the 5, the 3, and the 10 in a certain proportion—such as: 2_{3+5+10}, i.e.,

C – A – F♯ – C♯ – D♯ – C – A – E – F♯, etc.

Naturally, selection of the *tensions* and of the *forms* of structures in definite proportions is as important as selection of the forms of progressions when a certain definite style must be produced.

On the other hand, this method offers a fascinating pastime, as one can produce chord progressions from any number combinations. Thus, a telephone directory becomes a source of inspiration.

Columbus 5 – 7573

5 + 7 + 5 + 7 + 3 is equivalent to
C – G – C – G – C – A.

This progression closes after 4 cycles:

C – G – C – G – C – A – E – A – E – A – F♯ –

F♯ – C♯ – F♯ – C♯ – F♯ – D♯ – A♯ – D♯ – A♯ – D♯ – C

Figure 207. Chord progressions from a telephone number.

When zeros occur in a number-combination, they represent zero-steps, i.e., zero cycles (C_0). Then the form of tension, the structure, or the position of a chord has to be changed.

Example of Continuity:

Figure 208. Progression: $r_{5 \div 3}$.

A. Generalized Symmetric Progressions as Applied to Modulation Problems

The *rhythm of chord progressions* expressed in number-values may serve the purpose of transition from one key to another. This procedure can be approached in two ways: (1) as a problem of connecting the tonic chords of the preceding and the following key; or (2) as a problem of connecting any chord of the preceding key to any chord of the following key. The last case requires movement through diatonic cycles in both the preceding and the succeeding keys.

The technique of performing such modulations, based on the rhythm of symmetric progressions, consists of two steps:

(1) detection of the number-value expressing the interval between the two chords, where such connection must be established;

(2) composition of a rhythmic group from the numeral expressing the interval between the above-mentioned chords. For example, if one wants to perform a modulation by means of symmetric progressions from the chord C (which may or may not be in the key of C) to the chord E♭ (which may or may not be in the key of E♭), the first procedure to perform is to compose rhythm from the interval 9. The techniques set forth in the *Theory of Rhythm** offer many ways of composing such groups: composition of binomials, trinomials or larger groups from the original number, or any permutation thereof.

The number of terms in a group will define the number of chords for the modulatory transition. Breaking up number 9 into binomials, we obtain: 8 + 1, 7 + 2, 6 + 3, 5 + 4, and their reciprocals. When a binomial is used in this sense, the two chords are connected through one intermediate chord. For example, taking 5 + 4 we acquire: C — G — E♭. If more chords are desired, any other rhythmic group may be devised from number 9. For example, 4 + 1 + 4, which will give C — A♭ — G — E♭, i.e., two intermediate chords.

*See Book I. (Ed.)

When a number-value expressing the interval between the two chords to be connected through modulation is a *small* number, it is necessary to add the invariant 12. This places the same pitch-unit (or the root of the chord) in a different octave without changing its intonation. For example, if a modulation from a chord of C to the chord of B♭ is required, such an addition becomes very desirable.

$$
\begin{aligned}
C &\rightarrow B\flat = 2\\
B\flat &\rightarrow B_1\flat = 12\\
12 + 2 &= 14
\end{aligned}
$$

Some rhythms derived from the value 14:

$$7 + 7 = C - F - B\flat$$
$$5 + 2 + 2 + 5 = C - G - F - E\flat - B\flat$$

In cases such as this, rhythmic resultants may be used as well, providing the necessary changes are made.

$r_{4 \div 3} = 3 + 1 + 2 + 2 + 1 + 3$

Readjustment:

$3 + 1 + 2 + 2 + 1 + 3 + 2 = C - A - A\flat - F\sharp - F\flat - E\flat - C - B\flat$

Or:

$r_{5 \div 3} = 3 + 2 + 1 + 3 + 1 + 2 + 3$

Readjustment:

$3 + 2 + 1 + 2 + 1 + 2 + 3 = C - A - G - F\sharp - E - E\flat - D\flat - B\flat$

All these procedures guarantee the appearance of the desirable B♭ point.

When a modulation of still greater extension is required, the invariant of addition becomes 24 or 36—or even a higher multiple of 12—from which rhythmic groups may be composed.

Many persons engaged in the work of "arranging" find this type of transition more effective than the modulations ordinarily used. Naturally, selection of structures of different tension and form may be made according to the requirements of the general style of harmony used in a particular arrangement. The best modulations will result from use of that symmetry which may be detected in any given piece of music.

Even when the tonic-dominant progression is characteristic of harmonic continuity, this method may be used with success—it simply requires the composition of a rhythmic group in which the original value is 5. In this seemingly limited case there is still a choice of steps: 4 + 1; 3 + 2; 2 + 3; 1 + 4.

Examples of Modulations Through Symmetric Groups

(1) Key of C to Key of E♭; i = 9

Symmetric Group: 1 + 3 + 1 + 3 + 1 (r_3 of $\frac{9}{9}$ series)

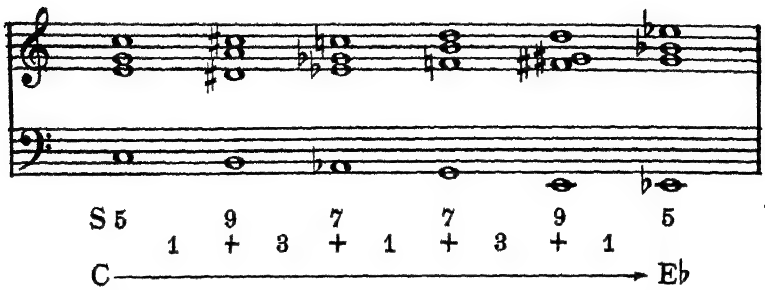


Figure 209. Modulation through symmetric group: C → E♭.

(2) Key of C to Key of E♭

Chords to be Connected: D——B♭; i = 4; 4 + 12 = 16

Symmetric Group: $\underline{r_{4 \div 3}}$ = 3 + 1 + 2 + 1 + 1 + 1 + 1 + 2 + 1 + 3

Figure 210. Modulation through symmetric group: D → B♭.

CHAPTER 13

THE CHROMATIC SYSTEM OF HARMONY

THE basis of the chromatic is: *transformation of diatonic chordal functions* into *chromatic* chordal functions and back into *diatonic*. Chromatic continuity evolved on this basis emphasizes various phenomena of harmony which do *not* confine themselves to diatonic or symmetric systems. What are usually known as "modulations" are simply a special case of the whole chromatic system. Chord progressions usually called "alien chord progressions" find their exhaustive explanation in the chromatic system.

Wagner was the first composer to manipulate intuitively this type of harmonic continuity. Not having any basic theoretical principle for handling such progressions, Wagner often wrote them in an enharmonically confusing way. (Note, also, that J. S. Bach made an unsuccessful attempt to move in chromatic systems; see *The Well-Tempered Clavichord*, Vol. I, Fugue 6, measure 16). It is necessary, for analytical purposes, to rewrite such music in its proper notation, i.e., chromatically rather than enharmonically. A more consistent *notation* of chromatic continuity may be found among such *followers* of Wagnerian harmony as Borodin and Rimsky-Korsakov.

The chromatic system of harmonic continuity is based on progressions of *chromatic groups*. Every chromatic group consists of *three* chords which express the three stages of the following mechanical process: balance—tension—release. These three moments correspond to the diatonic-chromatic-diatonic transformation.

A chromatic group may consist of one or more simultaneous *operations*. Such operations are alterations of diatonic tones into chromatic tones, by raising or lowering them. The initial diatonic tone of a chromatic group and the next alteration have the same *name*, but the ensuing release results in a pitch-unit of a new name.

The two general forms of chromatic operations are these:

$$\begin{array}{lllll} & & & & y \\ & & & \nearrow & \\ & & x\sharp & & \\ & \nearrow & & & \\ (1)\ x & & & & \end{array}$$

$$\begin{array}{lllll} (2)\ x & & & & \\ & \searrow & & & \\ & & x\flat & & \\ & & & \searrow & \\ & & & & y \end{array}$$

In their application to musical names these general forms may become, for instance, g – g♯ – a or g – g♭ – f. Such steps are always semitones. At each such moment of release in a chromatic group a new chordal function (and, in some cases, the same) becomes the starting point of the next chromatic group; thus the whole evolves into an infinite chromatic continuity.

Such a continuity has the following appearance:

$$\begin{array}{lllll} \underline{d - ch - d} & & & & \\ & \underline{d - ch - d} & & & , \text{etc.} \\ & & \underline{d - ch - d} & & \end{array}$$

Chromatic continuity in such a form offers a very practical measure distribution, for it permits one to place two chords in each measure. Such a distribution places the release of tension on the downbeat and this sounds satisfactory to our ears, probably because we have acquired the habit of hearing them in such a distribution.

As in diatonic progressions it is the commonness of tones or the resolution of chordal functions—or as in symmetric progressions, it is the symmetric roots—which become the controls of motion, so in chromatic progressions such stimuli are the chromatic alterations of the diatonic tones.

In addition to the form of continuity of chromatic groups presented in the preceding diagram, two other forms are possible. The latter do not necessarily require the technique of the chromatic system. The first of these additional forms of continuity produces an overlapping over one term:

$$\begin{array}{llll} (1) & \underline{d - ch - d} & & \\ & & \underline{d - ch - d} & \\ & & & \underline{d - ch - d} \end{array}$$

Note that the second part produces the first term of a chromatic group, while the first one produces the second term.

$$\begin{array}{ll} (2) & \overline{d - ch - d} \\ & \underline{d - ch - d} \end{array}$$

Note that two or more parts of harmony coincide in their transformation in time, although the form of transformation may be different in each part.

Any chord acquiring a chromatic alteration becomes more "intense" than the same chord without such alteration. If the middle term of a chromatic group has to be intensified, the following forms of tension may constitute a chromatic group:

S(5)	S(7)	S(5)
S(5)	S(7)	S(7)
S(7)	S(7)	S(5)
S(7)	S(7)	S(7)

The only combination which is undesirable—it produces an effect of weakness—is that in which the middle term is S(5).

Operations in a given chromatic group correspond to a group of chordal functions which may be assigned to any form of alterations. For technical reasons 4-part harmony is here limited to S(5) and S(7) forms with their inversions; so all transformations of functions in the chromatic group deal with the four lower functions, 9, 11 and 13 being excluded.

Numerical Table of Transformations for the Chromatic Groups.*

1–1–1	3–3–3	5–5–5	7–7–7
1–1–3 1–3–1 3–1–1	3–3–1 3–1–3 1–3–3	5–5–1 5–1–5 1–5–5	7–7–1 7–1–7 1–7–7
1–1–5 1–5–1 5–1–1	3–3–5 3–5–3 5–3–3	5–5–3 5–3–5 3–5–5	7–7–3 7–3–7 3–7–7
1–1–7 1–7–1 7–1–1	3–3–7 3–7–3 7–3–3	5–5–7 5–7–5 7–5–5	7–7–5 7–5–7 5–7–7
1–3–5 1–5–3 5–1–3 3–1–5 3–5–1 5–3–1	1–3–7 1–7–3 7–1–3 3–1–7 3–7–1 7–3–1	1–5–7 1–7–5 7–1–5 5–1–7 5–7–1 7–5–1	3–5–7 3–7–5 7–3–5 5–3–7 5–7–3 7–5–3

Figure 211. Transformations for chromatic groups.

Some of these combinations must be excluded because of the adherence of the seventh to the classical system of voice-leading, the descending resolution.

The preceding table offers 16 different versions for each starting function (1, 3, 5, 7). In addition to this, any middle chord of a chromatic group may assume one of the seven forms of S(7); any of the last chords of a chromatic group may have either one of four forms of S(5) or one of seven forms of S(7).

*The table is to be read this way: "1 – 3 – 5" means, for example, that a tone—let us say, C of the major triad, C-E-G—selected for chromatic alteration will be the 1 (root) of the first of the three chords in the chromatic groups; it will be, when altered (say, C#), the 3 of the second chord; and it will be, when the alteration is completed (say, D), the 5 of the third chord. (Ed.)

Thus, each starting point offers either 28 or 49 forms. The total number of starting points for one function equals 16. These quantities must be multiplied by 16 in order to show the total number of cases.

$$28 \times 16 = 448$$
$$49 \times 16 = 784$$

This applies to one initial function only, and, as any group may start with any of the four functions, the total quantity is $4(784 + 448) = 4{,}928$. A number of these cases eventually exclude themselves because of the above-mentioned limitation imposed by the tradition of voice-leading.

Actual realization of chromatic groups must be accomplished on what we may call the two fundamental bases: the major and the minor. This concept of *harmonic basis* refers to any *three adjacent chordal functions*, such as:

5	7	9	11	13
3	5	7	9	11
1	3	5	7	9

Owing to practical limitations this section of my discussion of harmony will deal with the first $\left(\begin{smallmatrix}5\\3\\1\end{smallmatrix}\right)$ basis only.* The terms major and minor correspond to the structural constitution in the usual sense: major $= 4 + 3$, and minor $= 3 + 4$. All fundamental chromatic operations are derived from these two bases.

Major Basis	Minor Basis
1♯	1♭
3♭	3♯
5♯	5♭

These six forms of chromatic operations (3 on each basis) are used independently. Chromatic operations available from the major basis are: raising of the root-tone; lowering of the third; raising of the fifth. Note that they are the opposite of those of the minor basis.

*But observe that in dealing with an S(9), S(11), or S(13) in cases in which any three tones form a triad, the chromatic operation may be performed on these three tones *as if* they were a single triad rather than part of a larger structure. (Ed.)

Examples of Chromatic Groups: One Operation

Table of Transformations.

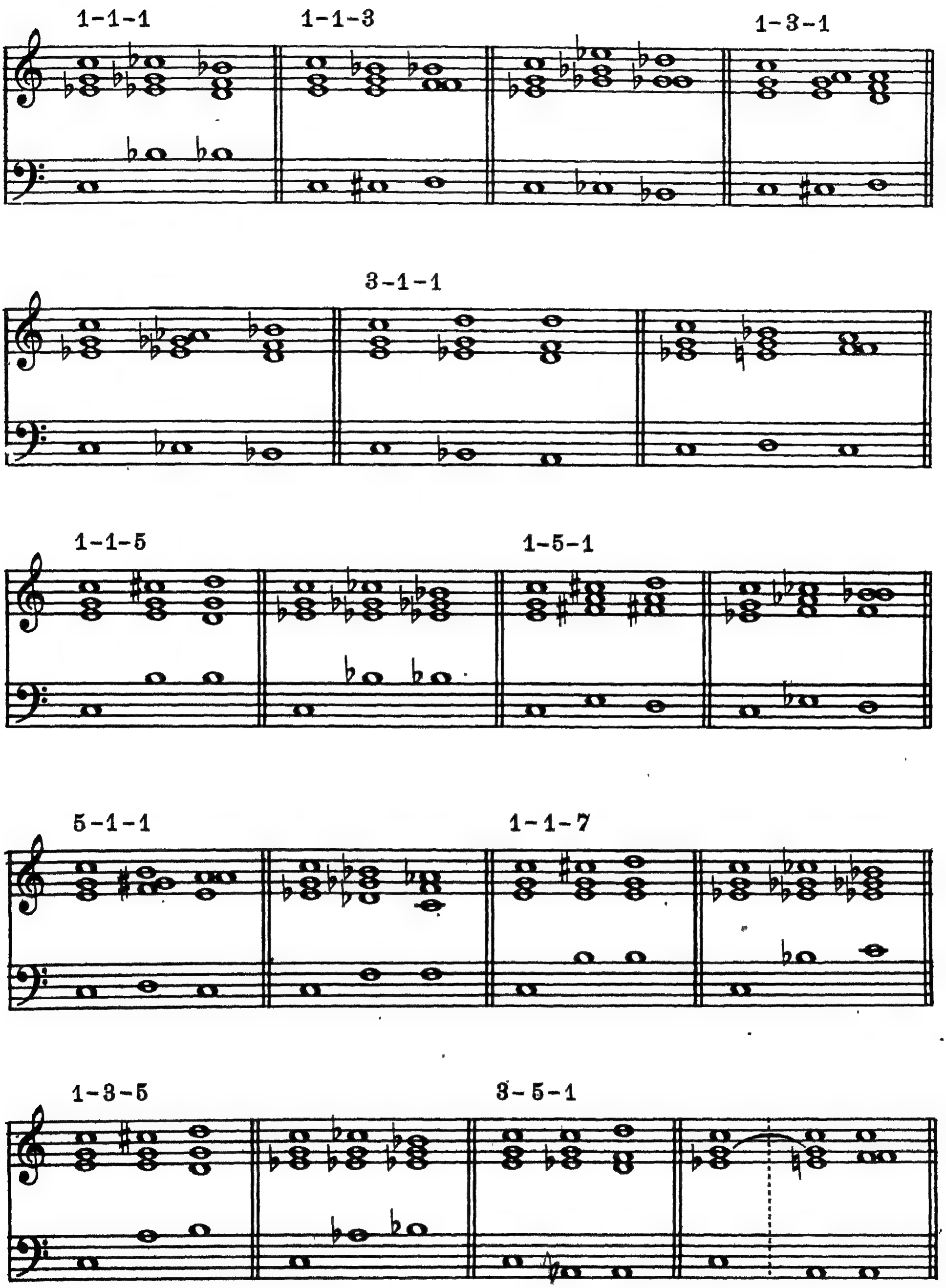


Figure 212. One operation transformations (continued).

Figure 212. One operation transformations (concluded).

The reader will wish to try to find the remaining cases through the table of transformations of the chordal functions, remembering that the classical system of voice-leading (so far as special harmony is concerned) must be carried out through the chromatic continuity: a seventh either descends or remains (as in traditional cadences); it may even go up one semitone because of the chord structure, yet it *must retain its original name*—as in d — d# in the last case..

Through selection of different chromatic groups (which may be used, of course, with coefficients of recurrence) a chromatic continuity may be composed.

From the explanation offered thus far, it will be obvious that every last chord of the preceding group (and therefore the first chord of the following group) must be of major or minor basis. Operations from other bases will be explained in the following lesson.

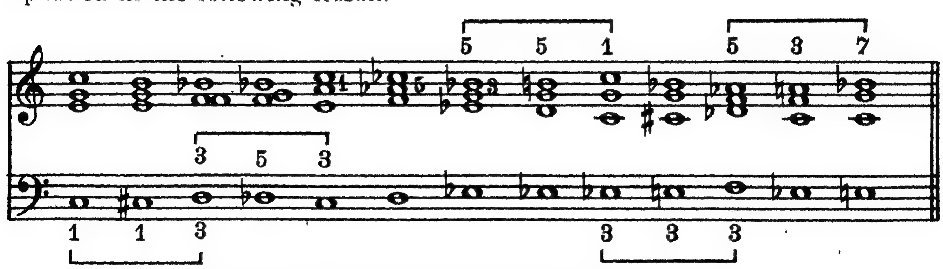


Figure 213. Example of chromatic continuity.

A. Operations from $S_3(5)$ and $S_4(5)$ Bases

As the 3 or $S_3(5)$ is identical with the 3 of $S_1(5)$, the fundamental operations correspond to those for $S_1(5)$. They are:

(1) raising of 1
(2) lowering of 3

Function 5 does not participate in the fundamental operations as it is already altered; and, as the form of the middle chord is pre-selected, the fifth requires rectification* in many cases, although it retains its name. All forms of doublings are acceptable.

As the 3 of $S_4(5)$ is identical with the 3 of $S_2(5)$, the fundamental operations correspond to those for $S_2(5)$. They are:

(1) lowering of 1
(2) raising of 3

The fifth does not participate in the fundamental operations but may be rectified.

Figure 214. Operations from an augmented S_3 (5) basis.

*The meaning of *rectification* in this context is explained on page 503. (Ed.)

Figure 215. Operations from a diminished S_4 (5) basis.

Figure 216. Chromatic continuity including all bases.

B. Chromatic Alteration of the Seventh

Because of the classical tendency toward a downward resolution of the seventh, chromatic alterations in this case conventionally follow the same direction. This lowering of the seventh (both major and minor) can be carried out from all forms of S(7). If the seventh is minor, it is more practical to have it as sharp or natural, since lowering of the flat produces a double-flat. Do *not* operate from a diminished seventh.

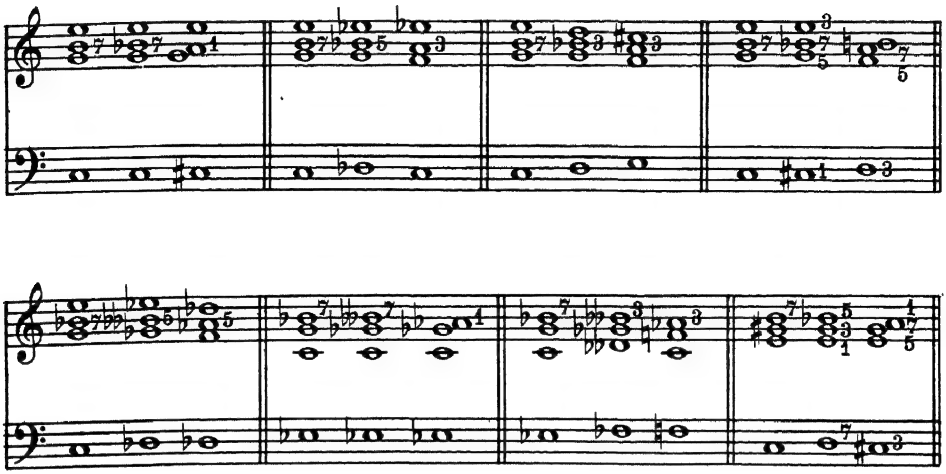


Figure 217: Examples of operations from the seventh.

All the single operations may now be incorporated into a final example of a form of chromatic continuity:

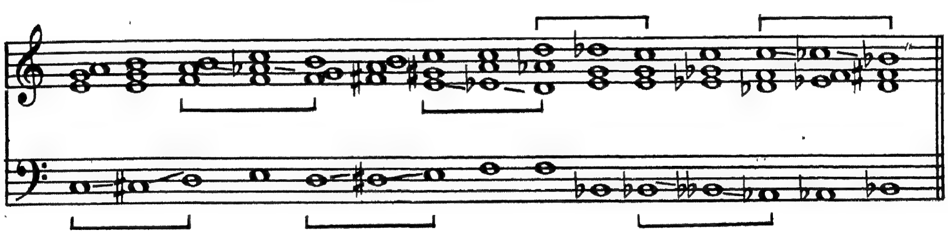


Figure 218. Operations from 1, 3, 5 and 7, all bases.

C. Parallel Double Chromatics

Parallel double chromatics occur when *fundamental operations are performed from an opposite base.* In such a case the *rectification* of the third is required.

If, for example, we decide to lower the 1 of the $S_1(5)$ basis, it becomes necessary to adjust 3 to its proper basis, i.e., in this case to lower it.

We shall consider the alterations of 1 and 5 as *fundamental*; the correction of 3, as *complementary*.

The following table represents all operations.

Parallel Double Chromatics.

$S_1(5)$ basis		$S_2(5)$ basis	
Fundamental	1♭	Fundamental	1♯
Complementary	3♭	Complementary	3♯
Fundamental	5♭	Fundamental	5♯
Complementary	3♭	Complementary	3♯

Figure 219. Parallel double chromatics.

The fundamental chromatics represent the middle term of a complete chromatic group, whereas the complementary chromatics do not necessarily perform the conclusive movement designated by their alterations. Thus, the scheme of chromatic groups for the parallel double chromatics is generalized as follows:

(1) $x \nearrow x\# \nearrow y$ (fundamental)
$\quad\; x^1 \nearrow x^1\#$ (complementary)

(2) $x \searrow x\flat \searrow y$ (fundamental)
$\quad\; x^1 \searrow x^1\flat$ (complementary)

For example, if c – c♭ – b♭ is a fundamental operation, the complementary chromatic is e – e♭. The complementary chromatic e♭ does not necessarily move into d. It may remain, or it may even move upward—depending on the chordal function assigned to it.

The same is true of the ascending chromatics. If c – c♯ – d is the fundamental operation, the complementary chromatic is e♭ – e. The complementary chromatic e does not necessarily move to f; it may remain, or even move downward, depending on the chordal function assigned to it.

The assignment of chordal functions must be performed for the two simultaneous operations: fundamental and complementary. It is practical to designate the ascending alterations as: $\genfrac{}{}{0pt}{}{3}{1}$ or $\genfrac{}{}{0pt}{}{5}{3}$, and the descending—as: $\genfrac{}{}{0pt}{}{7}{5}$ or $\genfrac{}{}{0pt}{}{5}{3}$.

This protects the resulting harmonic continuity from a wrong direction and sometimes from an excess of accidentals, particularly in reference to the middle term of a chromatic group.

Figure 220. Double parallel chromatics.

By assigning the opposite bases, we can obtain double parallel chromatics at any desirable point in the chromatic continuity.

Figure 221. Continuity of double parallel chromatics.

Double parallel chromatics are the quintessence of chromatic style in harmony. It is these chromatics that created the unmistakable character of Wagnerian and post-Wagnerian music. While an analysis of the music of Borodin, Rimsky-Korsakov, Franck or Delius does not present any difficulties to an analyst familiar with my theory, the music of Wagner often requires transcribing into *chromatic* rather than enharmonic notation. One of the progressions typical of Wagner's later period, for example, (we find much of it in his *Parsifal*) is:

Figure 222. Typical Wagner.

But when transcribed into chromatic notation, it has the following appearance:

Figure 223. Previous figure transcribed into chromatic notation.

This corresponds to the $S_1(5)$ basis: $\begin{matrix} 3\searrow \\ 1\searrow \end{matrix}$

There are many instances in which double parallel chromatics are evolved on the basis of passing chromatic tones; they are abundant in the music of Rimsky-Korsakov, Borodin and lately have become very common in American popular and show songs (*Cuban Love Song, The Man I Love*, for example.) The historical source of passing chromatic tones, however—the technique of which I shall discuss later—is Chopin rather than Wagner or the post-Wagnerians.

D. Triple and Quadruple Parallel Chromatics

Triple parallel chromatics occur when the 1 is raised in $S_4(5)$ basis. This, being the fundamental operation, requires the correction of the third (3♯) and of the fifth (5♯). The triple alterations become:

$$\begin{matrix} 5 \\ 3 \\ 1 \end{matrix} \quad \text{or} \quad \begin{matrix} 7 \\ 5 \\ 3 \end{matrix}$$

Figure 224. Triple parallel chromatics.

Quadruple parallel chromatics occur when the 1 is raised in $S_6(7)$ basis [diminished seventh-chord]. This requires the alteration of all remaining functions, i.e., 3♯, 5♯ and 7♯. This is the only interpretation satisfying those cases of chromatic parallel motion of the diminished seventh-chords—such as that found in Beethoven's Piano Sonata No. 7, the *largo* movement, (measure 20 from the end and the following five bars in relation to the adjacent harmonic context). Such a continuous chain of quadruple parallelisms takes place when the same operation is performed several times in succession.

As the chromatic system is limited to four functions (1, 3, 5, 7), quadruple parallel chromatics remain with their original assignments (while being altered).

Figure 225. Quadruple parallel chromatics.

By combining all forms of chromatic operations, i.e., single, double, triple and quadruple, we obtain an example of the final form of mixed chromatic continuity. See Figure 226 on the following page.

Figure 226. Continuity of mixed chromatic operations.

E. Enharmonic Treatment of the Chromatic System

By *reversing* the original directions of chromatic operations, we more than double the original resources of the chromatic system.

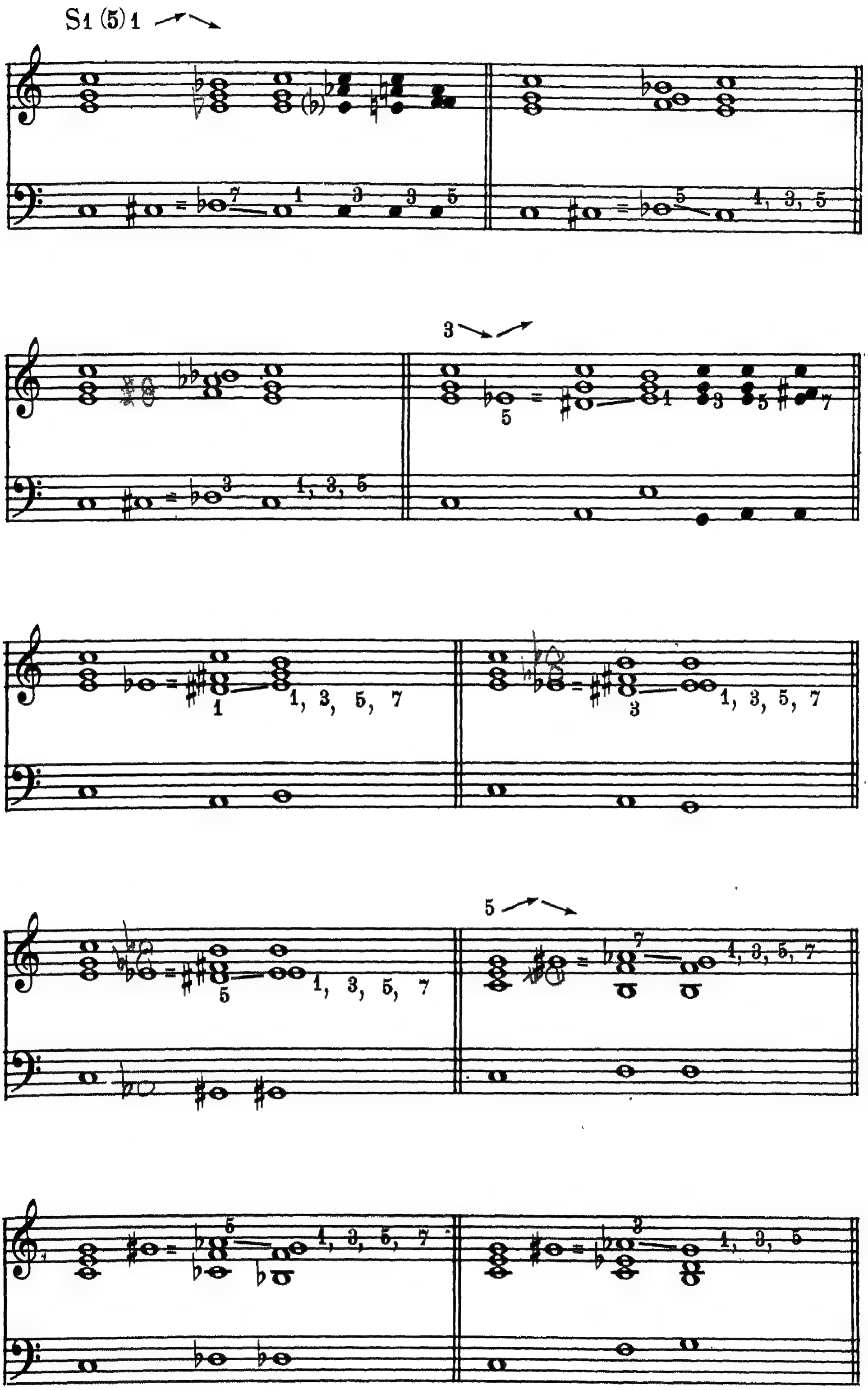
Enharmonic treatment of chromatic groups consists of the substitution of raising for lowering, and *vice versa*. This changes the original direction of a group and brings the second, or "tension," chord to a new point of release in the third chord.

The following formula expresses all conditions necessary for the enharmonic treatment.

$$(1)\quad x \nearrow x\sharp = y\flat \searrow z\ (1, 3, 5, 7)$$

$$(2)\quad x \searrow x\flat = y\sharp \nearrow z\ (1, 3, 5, 7)$$

Progressions of this kind are characteristic of the post-Wagnerian composers —see Borodin's opera *Prince Igor*, Rimsky-Korsakov's opera *Coq d'Or* and Moussorgsky's *Khovanschina*.


S1 (5) 1
3
5
1, 3, 5, 7
1, 3, 5

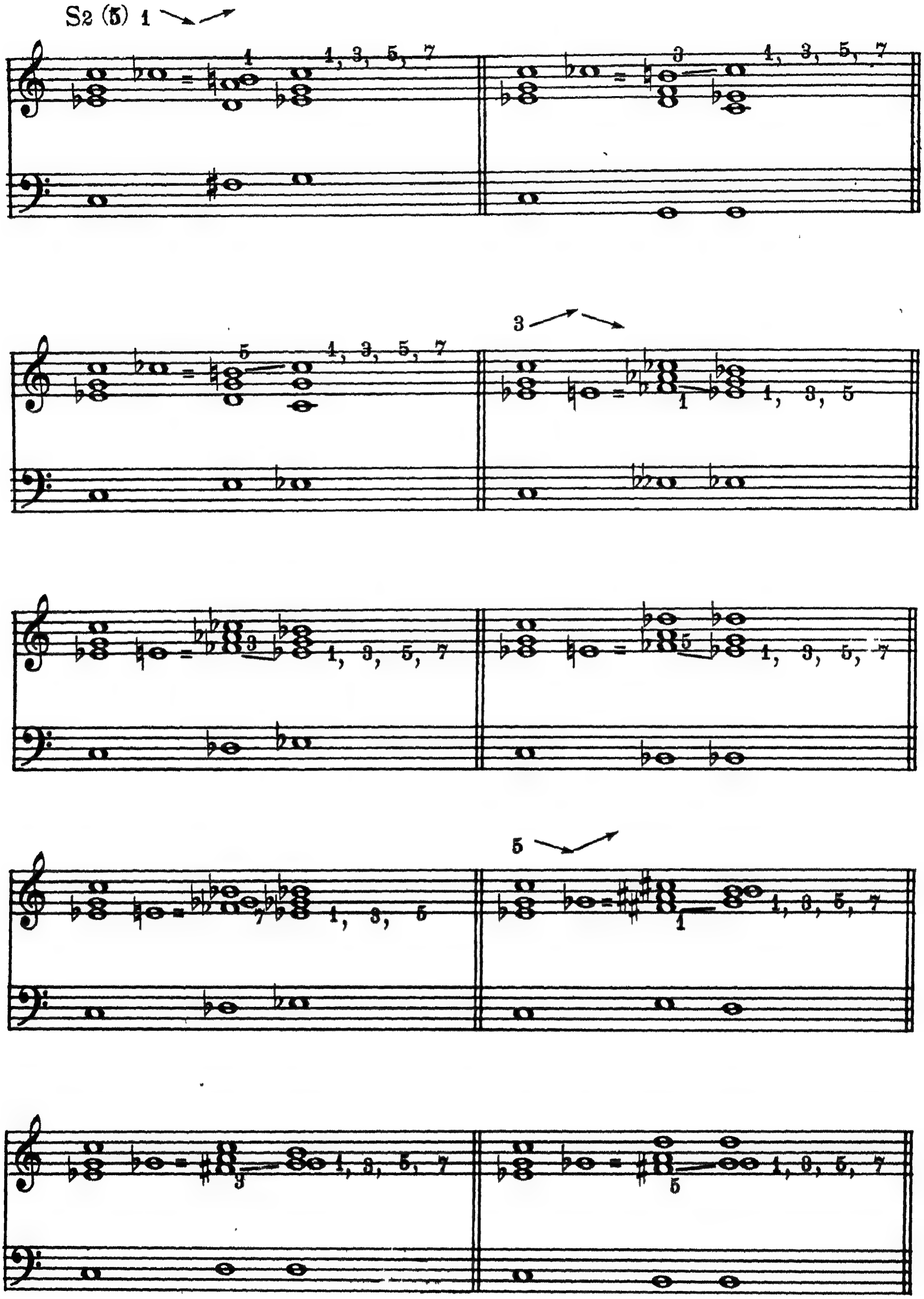


Figure 227. Enharmonic treatment of chromatic system (concluded).

In using double or triple chromatics, all or some of the altered functions can be enharmonized.

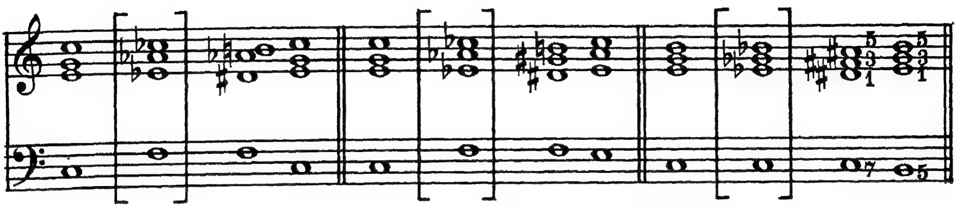


Figure 228. Enharmonic treatment of double and triple chromatics.

F. Overlapping Chromatic Groups

The use of overlapping chromatic groups produces a highly saturated form of chromatic continuity. Alterations in the two overlapping groups may be either both ascending, or both descending—or one of the groups can be ascending while the other is descending. The choice of ascending and descending groups *depends on the possibilities presented by the preceding groups during the moment of alteration.*

The general form of overlapping chromatic groups is:

$$\begin{array}{lllll} d - ch - d & & \\ & d - ch - d \end{array}$$

This scheme, being applied to ascending and descending alterations, offers 4 variants.

(1) $x \nearrow x\sharp \nearrow y$

$\quad\quad x \nearrow x\sharp \nearrow y$

(2) $x \searrow x\flat \searrow y$

$\quad\quad x \nearrow x\sharp \nearrow y$

(3) $x \nearrow x\sharp \nearrow y$

$x \searrow x\flat \searrow y$

(4) $x \searrow x\flat \searrow y$

$x \searrow x\flat \searrow y$

Thus parallel as well as contrary forms are possible.

Each of the mutually overlapping groups has a *single* chromatic operation.

Figure 229. Examples of overlapping chromatic groups.

The sequence in which such groups can be constructed is as follows: (For purposes of illustration, we use figure 229 just given. The procedure in other cases is similar).

Write the first chord first:

1.

Figure 230. Procedure for constructing overlapping chromatics.

The next step is to make operations in one voice; in this example, 1♯ was chosen in the bass:

2.

Figure 231. Step 2.

The next step is to construct the middle chord of this group; 1♯ was assumed to remain 1, which yielded the C♯ seventh-chord:

3.

Figure 232. Step 3.

The next step is *to estimate the possibilities of other voices with regard to chromatic alterations.*

The b → b♭ step permits us to construct a chord which necessitates the inclusion of d and b♭. Another possibility might have been to produce g → g♯, which would also permit the use of d in the bass, as in the second example of the figure just given. The third possibility might have been the step e → e♯, in the alto voice, which also permits the use of d. Even such steps as e → e♭ or g → g♭ would be possible, although the latter would require an augmented S(7), i.e. (reading upward) d — g♭ — e♭ — b♭.

Figure 233. Continuity of overlapping chromatic groups.

G. Coinciding Chromatic Groups

The technique of evolving *coinciding chromatic groups* is quite different from all the chromatic techniques previously described. It is more similar to the technique of *passing chromatic tones*, which we shall discuss later.

Coinciding chromatic groups are evolved as a form of contrary motion in those *two voices which are a doubling of one function of the chord with which the group begins.*

The general form of a coinciding chromatic group is:

$$\begin{array}{c} d - ch - d \\ d - ch - d \end{array}$$

The contrary directions of the chromatic operations may be either outward or inward:

$$(1)\quad \begin{array}{lllll} & & & & y \\ & & & \nearrow & \\ & & x\sharp & & \\ & \nearrow & & & \\ x & & & & \\ x & & & & \\ & \searrow & & & \\ & & x\flat & & \\ & & & \searrow & \\ & & & & y \end{array}$$

$$(2)\quad \begin{array}{lllll} x & & & & \\ & \searrow & & & \\ & & x\flat & & \\ & & & \searrow & \\ & & & & y \\ & & & & y \\ & & & \nearrow & \\ & & x\sharp & & \\ & \nearrow & & & \\ x & & & & \end{array}$$

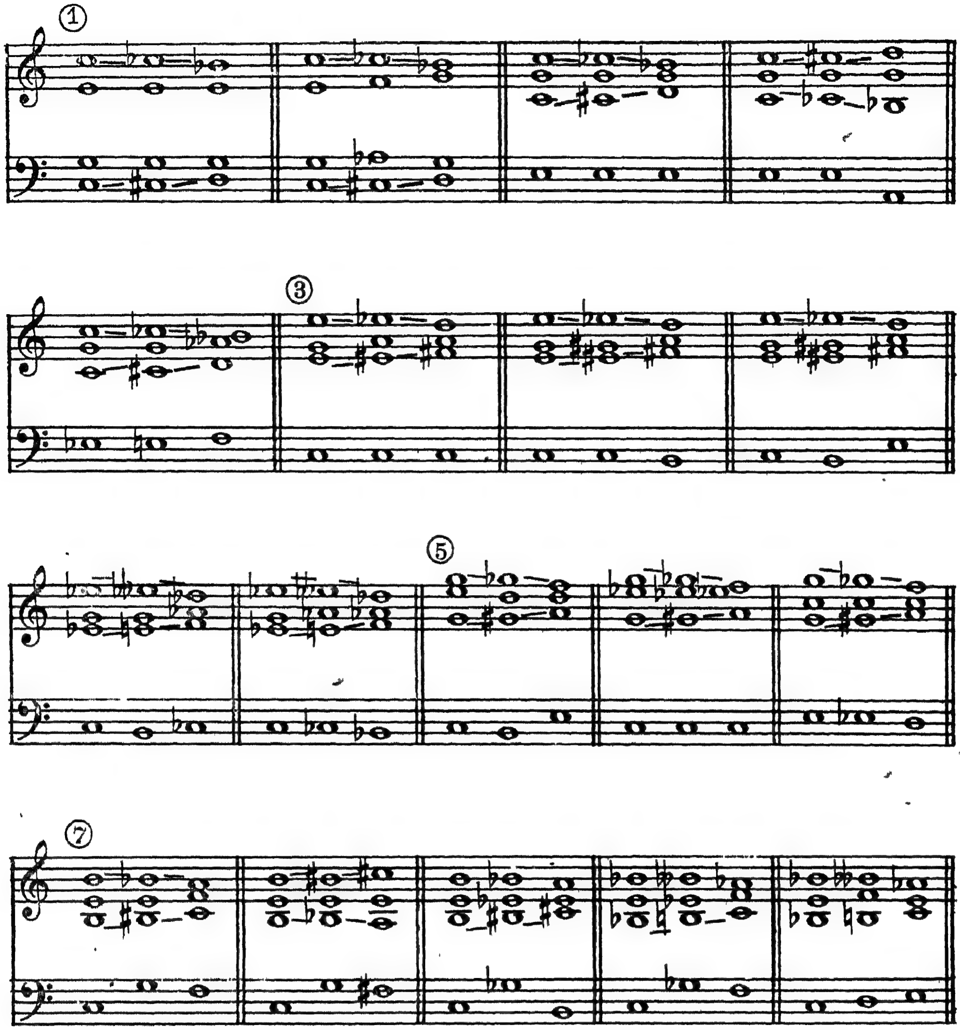
Assignment of the two remaining functions in the middle chord of a coinciding group can be performed by considering them enharmonically instead of by sonority.

For instance, in a group

$$\begin{array}{lllll} c & & & & \\ & \searrow & & & \\ & & c\flat & & \\ & & & \searrow & \\ & & & & b\flat \\ & & & & d \\ & & & \nearrow & \\ & & c\sharp & & \\ & \nearrow & & & \\ c & & & & \end{array}$$

the $\genfrac{}{}{0pt}{}{c\flat}{c\sharp}$ interval can be read enharmonically, i.e., as $\genfrac{}{}{0pt}{}{b}{c\sharp}$ in which case it becomes $\genfrac{}{}{0pt}{}{7}{1}$ or $\genfrac{}{}{0pt}{}{9}{3}$, etc. It is easy then to find the two remaining functions, like 3 and 5. Thus we can construct a chord $c\sharp - e - g - b$.

As coinciding chromatics result from doubling, it is very important to have full control of the variable doubling technique. The doubling of 1, 3, 5 and also 7 (major or minor) must be used intentionally in all forms and inversions of S(5) and S(7)—the latter, naturally, to obtain the doubled 7.



(Notation of chromatic operations as in all other forms of chromatic groups).

Figure 234. Coinciding chromatic groups.

It is important to remember in executing the coinciding chromatic groups that the first procedure is to establish the chromatic operations.

1.

Figure 235. Step 1 in constructing a coinciding chromatic group.

—and the second procedure is to add the two missing functions.

2.

Figure 236. Step 2.

After doing this, the final step is to assign the functions in the last chord of the group.

3.

Figure 237. Step 3.

All coinciding groups are reversible. In moving from an octave inward by semitones, the *last term of the group produces a minor sixth.* In moving outward from unison or octave, the *last term of the group produces a major third.*

It is important to take these considerations into account while evolving a continuity of coinciding chromatic groups. Any such group can start from any two voices producing (vertically) a unison, an octave, a major third or a minor sixth.

The following are all movements and directions with respect to c.

(1) c ↗ c♯ ↗ d
c ↘ c♭ ↘ b♭

(2) e ↘ e♭ ↘ d
c ↗ c♯ ↗ d

(3) c ↘ c♭ ↘ b♭

d
c ↗ c♯ ↗ d

b♭
(4) a♭ ↗ a♮ ↗ b♭
c ↘ c♭ ↘ b♭

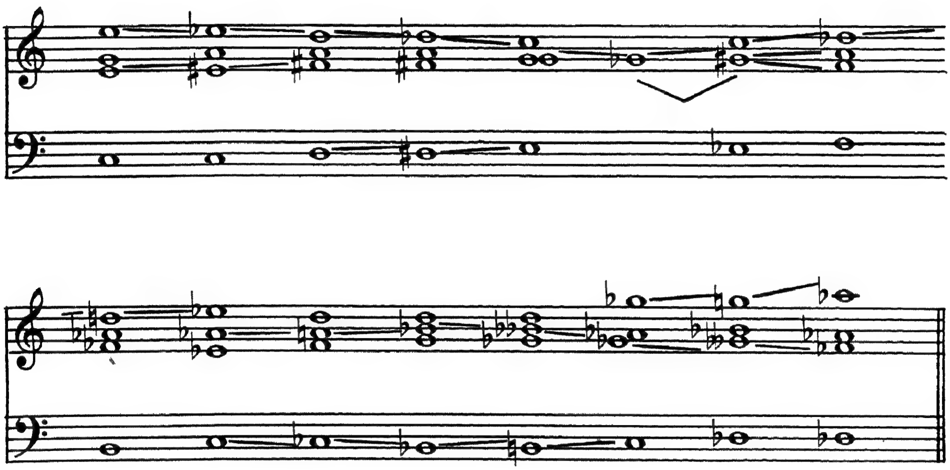


Figure 238. Continuity of coinciding chromatic groups.

All techniques of chromatic harmony may now be utilized in the mixed forms of chromatic continuity.

CHAPTER 14

MODULATIONS IN THE CHROMATIC SYSTEM

MODULATION—that is, key-to-key (or, more generally, scale-to-scale) transition—may be accomplished by means of chromatic alterations—in addition to the technique previously offered, i.e., modulation by means of Generalized Symmetric Progressions. The theory of modulation is, in other words, a special case of the entire chromatic system of harmony. Viewed in such a way, the present explanation *absorbs all possible cases of modulation* and offers a unified picture of all forms pertaining to key changes.

As our musical system operates with *seven names*, i.e., c, d, e, f, g, a, b, every possibility within the seven-unit scales must be a combination thereof. The actual intonations resulting from such combinations of the seven names are due to various combinations of naturals and accidentals, sharps and flats.

This permits a form of reasoning whereby *each musical name may become the root-tone of a chord linking the preceding to the following key*. As all *names* are common to all keys and all chords within the keys (so far as special harmony is concerned), every chord may be presumed to be a common chord between any preceding and following key.

For example, a transition from the key of C to the key of G may be accomplished by means of seven modulations:

	Preceding Key	*Common Chord*	*Following Key*
(1)	C	C	G
(2)	C	D	G
(3)	C	E	G
(4)	C	F	G
(5)	C	G	G
(6)	C	A	G
(7)	C	B	G

Figure 239. Modulations.

The fifth case is the least practical one; it anticipates the following key and makes the appearance of the latter too obvious.

Combining all forms of modulations from all keys to all keys and assuming that every chord may be a common chord, we obtain the following table of musical names in naturals:

C - D - E - F - G - A - B

D - E - F - G - A - B - C

E - F - G - A - B - C - D

F - G - A - B - C - D - E

G - A - B - C - D - E - F

A - B - C - D - E - F - G

B - C - D - E - F - G - A

Figure 240. Table of musical names.

From the above table it follows that a common chord may be established *on any degree of the preceding key*—which, in turn, corresponds to a *certain degree of the following key.*

Applying this principle to the figure just given, we obtain the following key-chord correspondence:

Key	*Chord*	*Key*
C	C	G
	I = IV	
C	D	G
	II = V	
C	E	G
	III = VI	
C	F	G
	IV = VII	
C	G	G
	V = I	
C	A	G
	VI = II	
C	B	G
	VII = III	

Figure 241. Key-chord correspondences.

The rest of the necessary procedure in key-to-key modulation is simply the *chromatic readjustment of accidentals.* The following key demands that the tones be adapted to its *real key signature.* Thus *all names* which are not the same in pitch in both the preceding and the following signatures must be altered. When all names of the common chord are common tones (identical pitches), modulation becomes *diatonic*, i.e., the intonation of the preceding and following keys, in this particular chord, coincides. Thus, *diatonic modulation is a special case of chromatic modulation.*

The technique includes:

(1) the preceding key has to be developed through diatonic cycles;

(2) the particular common name chord has to be selected;

(3) the corresponding chromatic alterations have to be made;

(4) the correspondence of the degrees has to be established;

(5) after the common chord is repeated with the accidentals of the following key (preferably in the form of a seventh chord), the continuation—and, possibly, completion—has to be performed through the diatonic cycles of the following key..

When a full completion is needed, a cadence may be added. The common chord thus has the significance of the middle chord of a chromatic group. In case the modulation becomes diatonic, there is no repetition of the common chord and there is no need to have the latter in the form of a seventh chord.

Figure 242. Examples of modulation.

There are some cases—mostly those in which the preceding and the following keys are one semitone apart, or in which the common chord is one semitone away from the following key—in which alterations cause *consecutive sevenths* or an awkward hidden seventh. If such steps are to be avoided, it is necessary to have the common chord first as S(5), then as S(7). To avoid the hidden seventh, double the fifth in S(5), i.e., use S(5)⑤ for the first common chord.

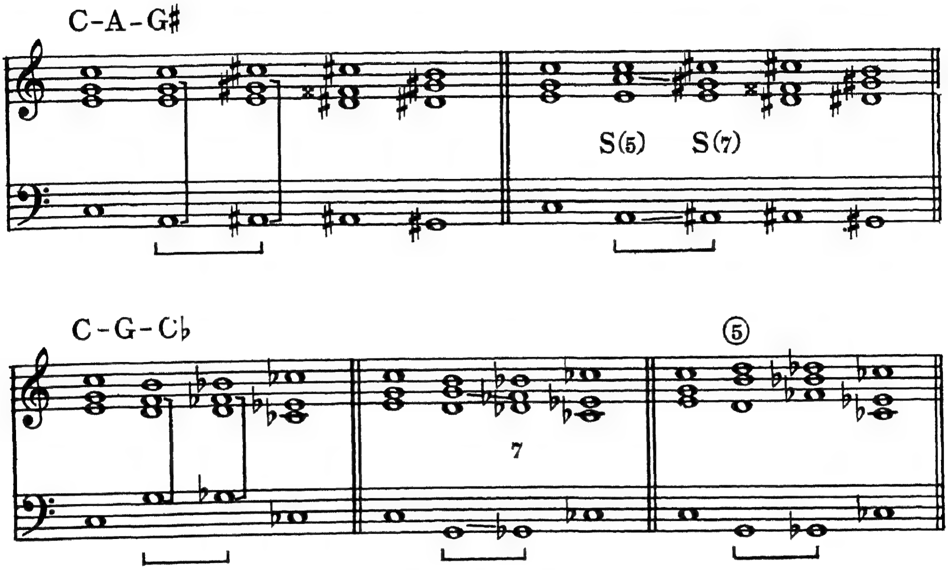


Figure 243. To avoid consecutive or "hidden" sevenths.

This theory of modulation is applicable to all seven-unit scales in which each musical name appears once and in which none of the seven intonations (pitches) coincide. There are 36 such fundamental scale structures, and each of the 36 has six derivative scales (modes) by pitch permutations, producing a total of 36 × 7 = 252 scales.

The ears of our audiences—and often those of composers themselves—are accustomed to modulations dealing with *natural major*, *harmonic major*, *harmonic minor* and *melodic minor*. All other seven-unit scales, even the modes of these scales, sound new and strange. Therefore a free use of 252 scales offers a new and virgin field to a composer who wants to achieve originality without departing from the established trends of musical reasoning.

Examples of original key-scale relations in modulation.

(1) Key of Cd_0: c — d♭ — e♭ — f — g — a — b; modulation to the key of Ed_3: a — b — c♯ — d♯ — e — f — g; common chord: F.

(2) Key of Cd_1: d — e — f♯ — g♯ — a — b — c; modulation to the key of B♭d_5: g — a — b♭ — c — d — e — f♯; common chord: E.

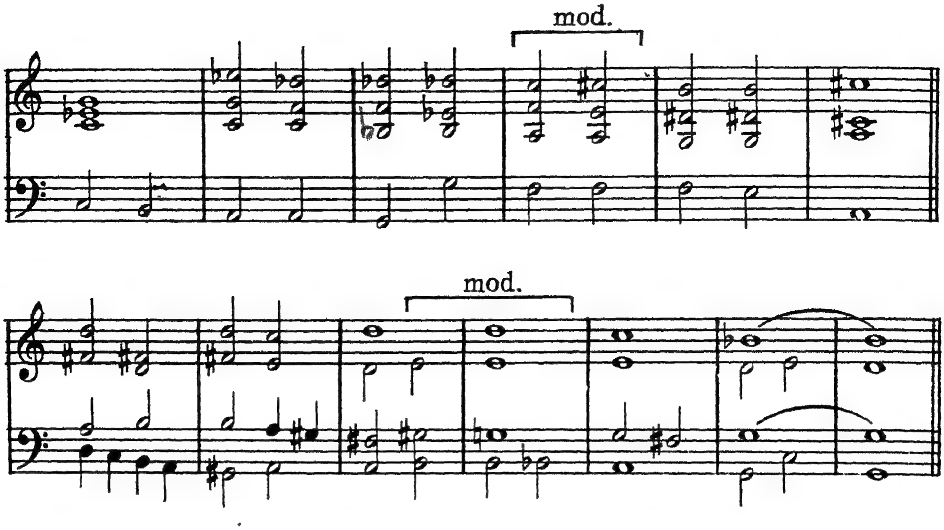


Figure 244. Original key-scale relations in modulation.

The readjustment of accidentals of the following key may be performed gradually (one by one) when an instantaneous change would produce an effect of too abrupt a character, as is usually the case when there is considerable difference between the real signature of the preceding and the real signature of the following key (see the modulation in the preceding example). We shall call such cases *extended modulations.*

Key I: Cd_0 Nat. major;
Key II: G♯d_0 Nat. major;
common chord: A.

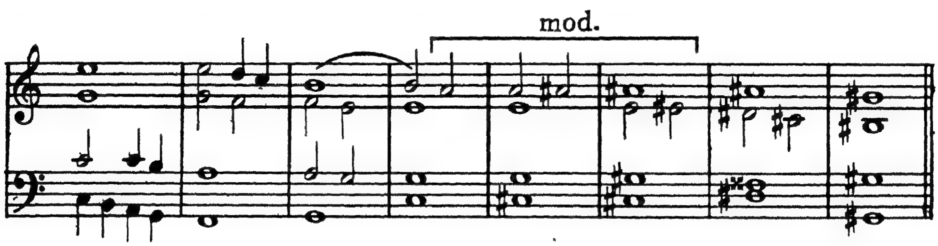


Figure 245. Example of an extended modulation.

The reader is now in a position to work out a systematic tabulation of all modulations, if necessary. Here is offered one example of a table comprising all the modulations between two *keys* and two *scales.* More such tables—between other pairs of keys and scales—may be worked out in a similar way.

Example of all modulations between two keys and two scales.

Key I: Cd_0 Nat. major;
Key II: $E\flat d_0$ Nat. major; common chords: (1) C; (2) D; (3) E; (4) F; (5) G; (6) A; (7) B.

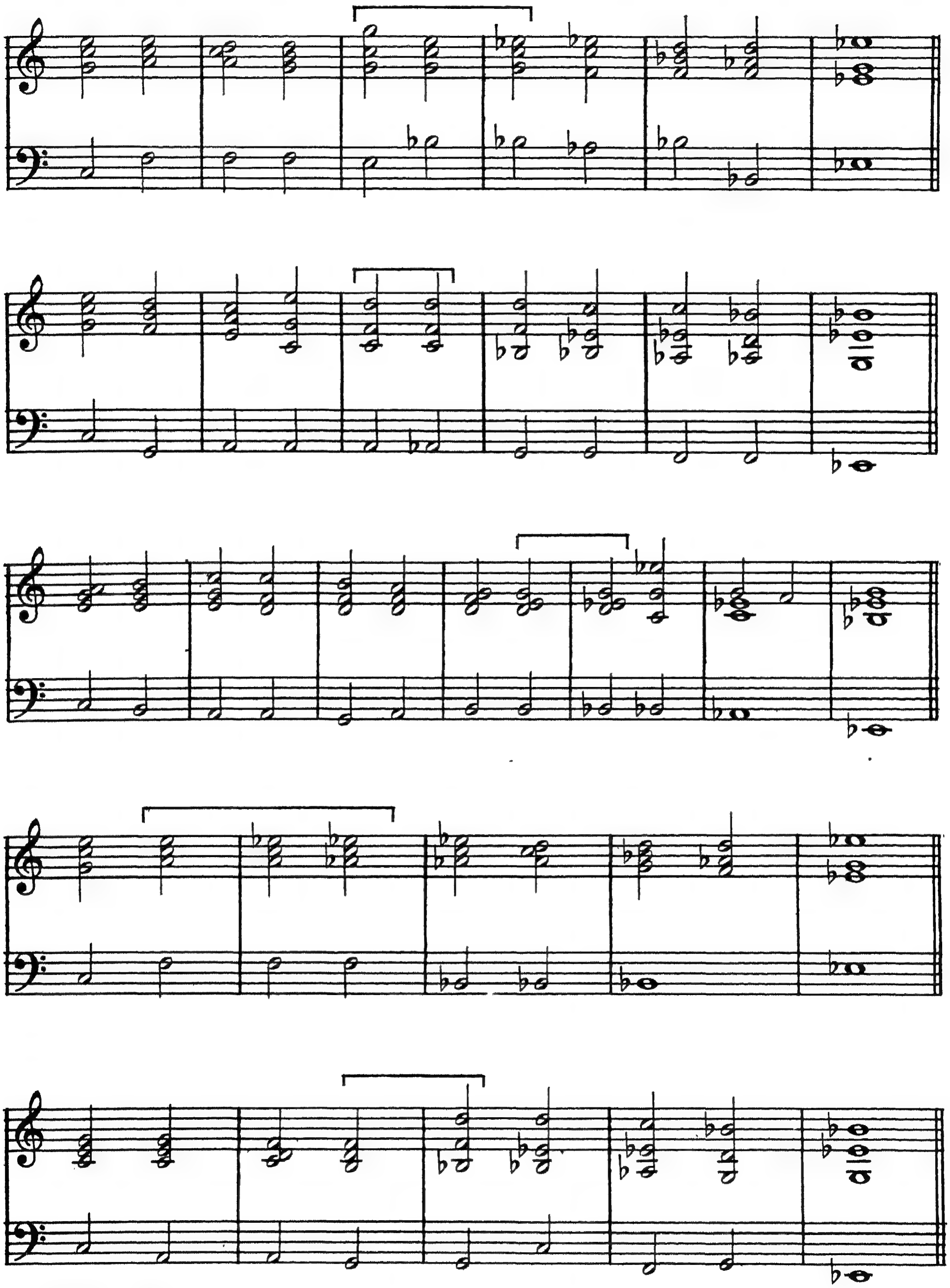


Figure 246. All modulations between two keys and two scales (continued).

Figure 246. All modulations between two keys and two scales (concluded).

A. Indirect Modulation

It happens that the typical, trivial academic modulations into remote key signatures by means of intermediate keys are known to lack musical interest. These academic modulations sound very unimaginative, indeed, especially after one has listened to a symphony by Schubert or by Mendelssohn—not to mention the modulations of such leading composers who came after Schubert and Mendelssohn, as Wagner, Brahms, or Franck.

But if academic modulations are analytically dissected and if the cause of their triviality is found, then, perhaps, the same analysis will lead us to an explanation of the modulatory secrets of Schubert, Mendelssohn, Wagner and others. Such was the reasoning which led me to the discovery of the *theory of indirect modulations.*

The fundamental solution of this problem (i.e., the selection of intermediate keys to link the initial and the terminal key) lies in establishing a *scale of key-signatures.*

Let us take the established key signatures, i.e., those which are real for natural major. Let us next assume the starting point to be "zero accidentals" (i.e., key of C), which will become the axis of symmetry for the reciprocal position of the opposite accidentals. For example, 3 sharps above the axis are equidistant with 3 flats below the axis.

Under such conditions the scale of key-signatures, which we shall here limit to seven accidentals, will assume the following appearance:

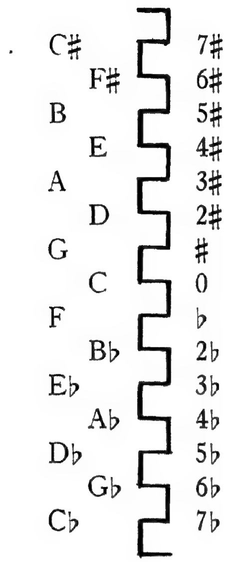


Figure 247. The Scale of Key Signatures for the Natural Major.

The academic method of planning intermediate keys for remote modulations exhibits a definite tendency (even though it is usually expressed through a whole system of complicated rules) which tendency can be formulated as: the *accumulation of sharps* when the preceding key has fewer sharps than the following key, and the *accumulation of flats* when the preceding key has fewer flats than the following key. When such a tendency is carried into practice directly, the outcome of such planning can be graphically represented as: *scalewise motion through the scale of key signatures.* The latter would be one general rule working in all such cases and would exclude all other rules.

Let us, for illustration, apply this rule to the planning of a typical academic modulation. Major and minor keys are frequently alternated—and we shall follow this precedent: let Db major be the preceding and E major be the following key; by drawing a scalewise graph between the limits of the preceding and the following key we obtain the key-sequence shown in Figure 248 on the following page.

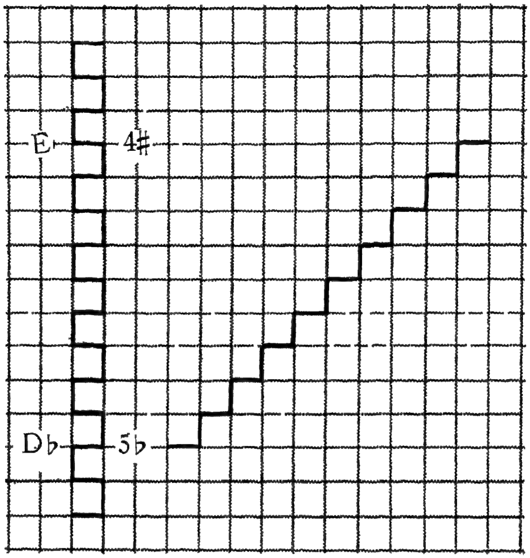


Figure 248. Scalewise graph D♭ major to E major.

The graph above can be read as follows:

D♭ + f + E♭ + g + F + a + G + b + A + E

The capital letters here are used to represent major keys; the small letters, to represent minor keys.

In some cases the academic procedure provides a direct transition through minor sub-dominants of the major and major dominants of the minor keys. In applying this method to the above case we would obtain a shorter scheme:

D♭ − f − E♭ − g − F − a − E.

—where a-minor is the subdominant of the following key, E-major; in such a case the graph would assume the following appearance:

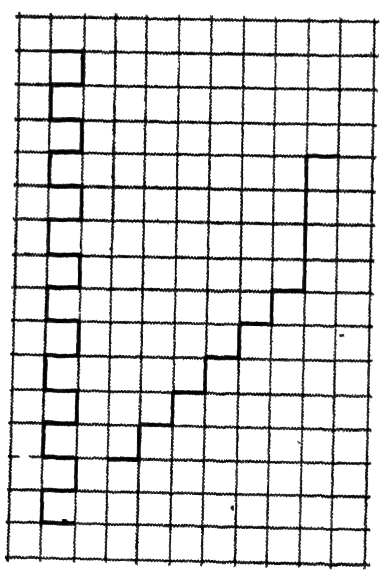


Figure 249. Shortening the modulation.

One could extend this principle still further to make wider gaps on the scale of key signatures; but this would not help, for the trajectory still remains predominantly scalewise—and such a form never produces anything of interest.

Thus we arrive at the conclusion that, by producing more dramatic forms of trajectories on the scale of key signatures, we can obtain more expressive modulations.

The fact is that *any trajectory which is not scalewise produces modulations with musical interest.* Various forms of resistance, binary and ternary axes—as set forth in my earlier discussion of the theory of melody—constitute such material.

Here are a few examples of the planning of such modulations to remote keys.

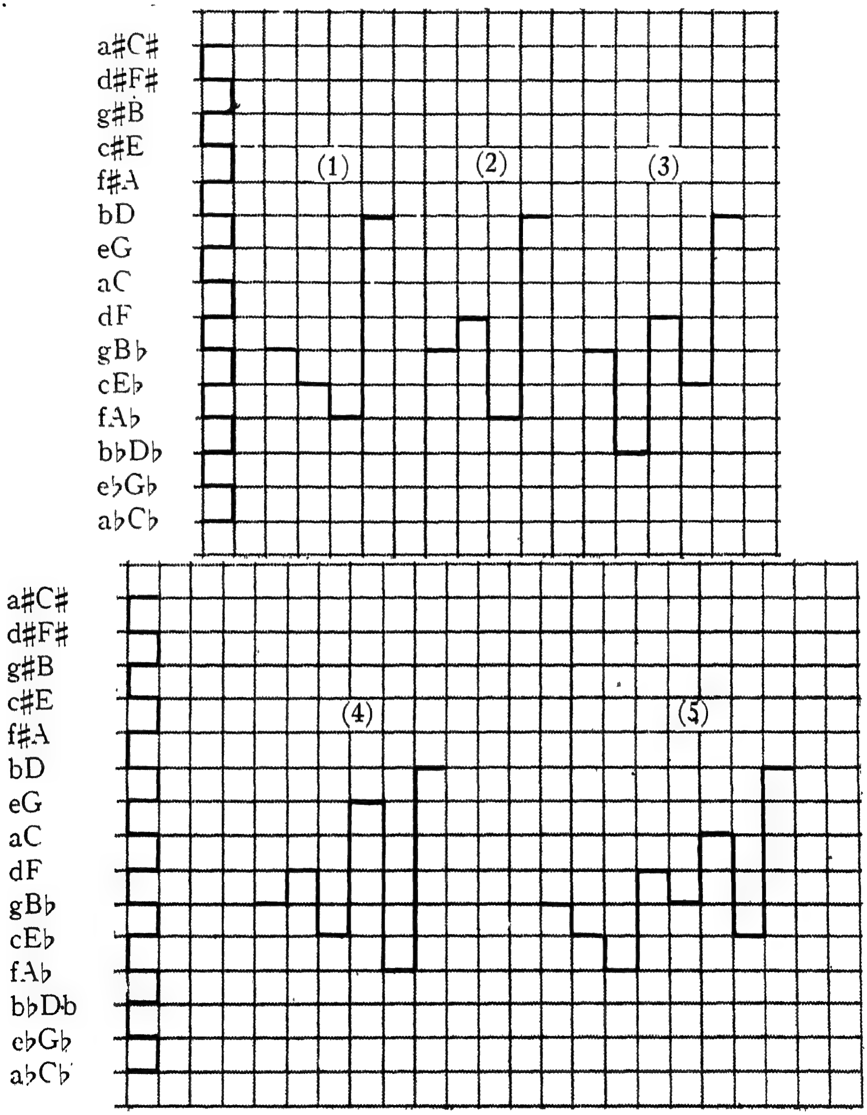


Figure 250. Modulations which are not scalewise.

Deciphering the graph, we get:

(1) B♭ + c + A♭ + D; B♭ + c + A♭ + b;
g + E♭ + f + D; g + E♭ + f + b.

(2) B♭ + d + A♭ + D; B♭ + d + A♭ + b;
g + F + A♭ + D; g + F + A♭ + b.

(3) B♭ + G♭ + d + E♭ + D; B♭ + G♭ + d + E♭ + b;
g + b♭ + F + c + D; g + b♭ + F + c + b.

(4) B♭ + d + E♭ + e + A♭ + D; B♭ + d + E♭ + e + A♭ + b;
g + F + c + G + f + D; g + F + c + G + f + b.

(5) B♭ + c + A♭ + d + g + C + c + D;
B♭ + c + A♭ + d + g + C + E♭ + b;
g + E♭ + f + d + B♭ + a + c + D;
g + E♭ + f + d + B♭ + a + E♭ + b.

Figure 251. The previous figure deciphered.

Indirect modulations can be plotted as *key sequences with specific time arrangement.* The amount of time allowed to each intermediate key is a matter of *rhythmic* distribution. The latter can be expressed either in chord-units (H) or in time group units (T)—which is, practically, the same thing when the number of H in each T is constant (i.e., when the number of chords in each bar is the same).

Let us take as an example the first modulation in group (4) of Figure 250.

B♭ + d + E♭ + e + A♭ + D.

As the durations of the first and the last key are specified *a priori*, it is necessary to plan the duration of intermediate keys only. The intermediate keys should present some definite equivalent of time with regard to the preceding and the following keys, as well as to the duration of the entire modulatory group.

Let us assume that music developing in the first and the last key is based on 8T as a structural unit. Then if we want to allow 8T for the entire modulation group, we can easily distribute the four intermediate keys. The simplest solution in this case would be: $\frac{8}{4} = 2$, i.e., 2T per key. Now time-key representation takes the following appearance:

B♭8T + d2T + E♭2T + e2T + A♭2T + D8T.

The above scheme can be plotted as shown in Figure 252 on the following page.

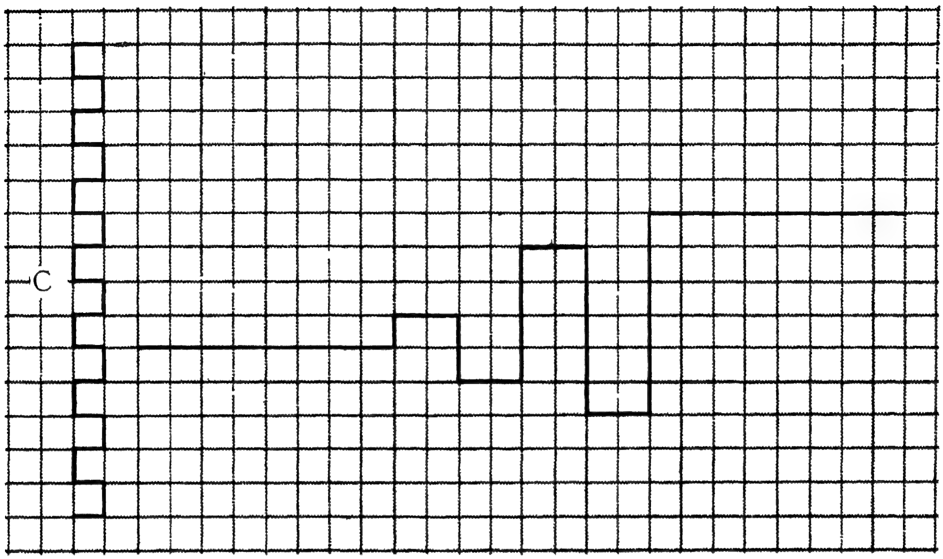


Figure 252. First modulation of figure 250 plotted to 8T structure.

It is easy to see that the same scheme can be represented through the quantity of H. For example, assuming that there are three chords per T, and substituting 3H for each T, we obtain:

$$B\flat 24H + d6H + E\flat 6H + e6H + A\flat 6H + D24H.$$

Any non-uniform distribution of intermediate keys must conform to the rhythmic series to which the *factorial continuity* of the theme belongs.

For example, let us assume that the above described 8T-groups belong to $\frac{8}{8}$ series; we want to find a proper form of distribution for the four intermediate keys, and we want to express it in T-units—and this amounts to the construction of a quadrinomial in $\frac{8}{8}$ series. Of the three trinomials of this series, i.e. $3+3+2$, $3+2+3$ and $2+3+3$, we may choose any one. Let us take the first trinomial and evolve a *binomial split-unit group* out of the first term: $3 = 2+1$. Then the quadrinomial acquires the following form: $2+1+3+2$.

Applying this quadrinomial to the modulation group under discussion, we obtain:

$$B\flat 8T + d2T + E\flat T + e3T + A\flat 2T + D8T.$$

In applying such key-time schemes, it is well to carry them out as closely as possible, although there is no need for absolute mathematical precision. Only the *total duration* of the entire modulation group must be carried out *exactly*.

Example of indirect modulation with key-time planning:

(B♭8T) + d2T + E♭T + e3T + A♭2T + (D8T)

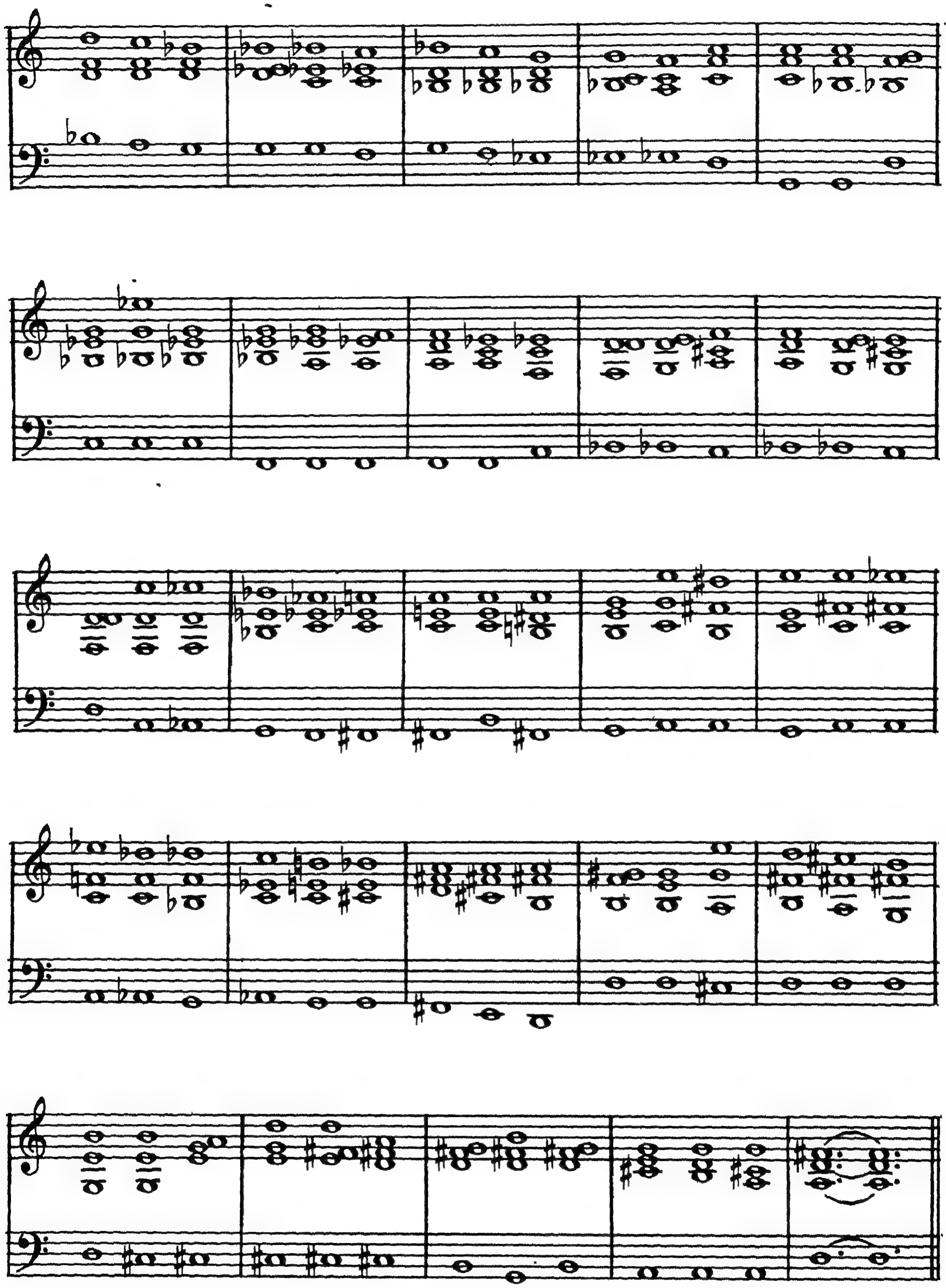


Figure 253 Indirect modulation with key-time planning.

CHAPTER 15

THE PASSING SEVENTH GENERALIZED

As we have seen before, the preparation of the seventh in C_0 requires a descending step from the root-tone. In C_3 the seventh while resolving, becomes a new root-tone. This fact permits us to develop a *continuity of the passing seventh* when C_3 is constant. All transformations are applicable. All chords must be S(5).

Figure 254. Passing seventh.

By reading the above figure backwards, we obtain an ascending scale in the bass.

The cycle in such a case becomes negative (C_{-3}) and the transformation is ⟲.

Figure 255. Previous figure read backwards.

Examples of the other forms of transformation:

Figure 256. Other forms of transformation.

In each case the role of the bass can be transferred to soprano.

Figure 257. Transferring role of bass to soprano.

A great flexibility of the melodic form can be achieved by a leap of a seventh upward for the positive cycle, and a leap of a seventh downward for the negative cycle.

Figure 258. Adding flexibility of melodic form.

Melodic forms and control of the leaps can be accomplished by pre-set forms of distribution of the scalewise steps. For example, a coefficient group can determine the number of pitch-units moving in succession until the leap occurs.

Figure 259. Pre-set form of distribution of scalewise steps (continued).

Figure 259. Pre-set form of distribution of scalewise steps (concluded).

A variety of melodic forms may also be obtained by mixing C_3 and C_{-3}.

$$4C_{-3} + 3C_3$$

Figure 260. Mixing C_3 and C_{-3}

All forms of the *generalized passing seventh* are applicable to modal transposition, as well as to progressions in harmony of types II and III. The latter must be confined to $\sqrt[3]{2}$ and $\sqrt[4]{2}$, (three and four tonics), as only these two systems correspond to the C_3 and C_{-3}.

(A) Phrygian (d_2)

(B) Persian

Figure 261. Modal transposition.

In type II all structures must be specified as S(7) and S(9) in such a way as to conform to the seventh of one family.

Example: large + 2 minor + small.

Figure 262. In type II, all structures must be S(7) and S(9).

The use of but one consistent S for the whole progression results in a consistent scale in the moving voice for each half of the entire cycle.

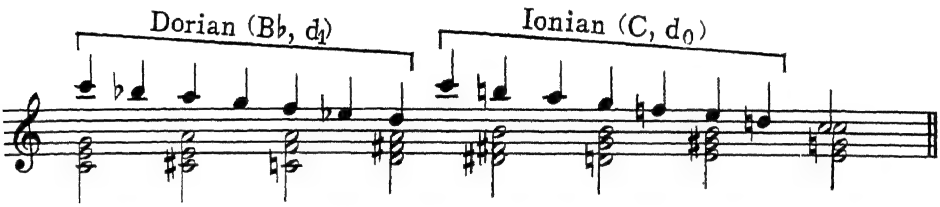


Figure 263. S(7) large constant.

A. Generalized Passing Seventh in Progressions of Type III

The fundamental material for this technique is the progressions based on three and four tonics.

In the case of three tonics the interval between the roots equals 4 semitones. This makes it possible to use *three forms of the seventh*: the major, the minor and the diminished.

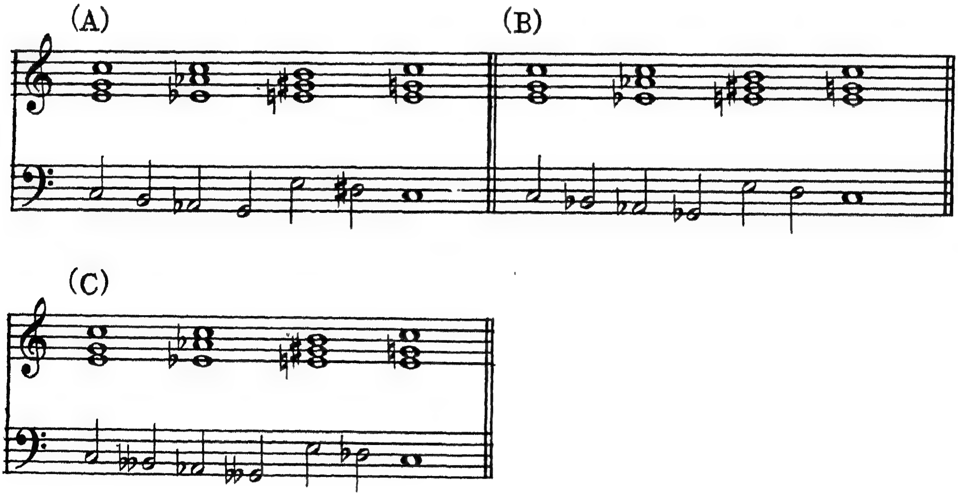


Figure 264. Passing 7th in $\sqrt[3]{2}$.

In this field, by means of symmetric chord progressions and the passing seventh device (which need not always "pass" in the conventional sense), we obtain pitch-scales of the *third group*. The number of tonics in the scales corresponds to the number of tonics in the chord progression. As in previous cases, the bass part can be placed *above* the remaining three voices and, as before, it is subject also to *octave variation* (leap into the adjacent octave).

As the 1, 3 and 5 of the three voices may be $S_1(5)$, $S_2(5)$, $S_3(5)$ or $S_4(5)$, it is possible to obtain automatically some of the *new structures* of S(7), which belong to the category usually known as "altered chords."

In four tonics the interval between the roots equals 3 semitones. This gives us a choice of a *major* and a *minor seventh*.

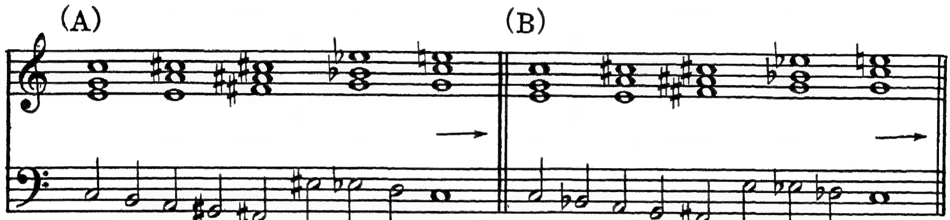


Figure 265. Passing 7th in $\sqrt[4]{2}$.

The above two cases produce the Arabian scale called "String of Pearls" (*Zer ef Kend*) in its two versions. The ascending forms can be obtained by reversing the cycles:

C – E – A♭ – C for the three tonics
and C – E♭ – F♯ – A for the four tonics.

By *mixing the positive and the negative forms*, we can acquire a more diversified melodic structure.

Figure 266. Mixing positive and negative forms.

This *mixture of structures and of the sevenths* introduces still greater variety into symmetric progressions and results in *mixed scales.*

Figure 267. Mixture of structures and of sevenths.

Further development of this field may be obtained through the assumption that *1 followed by 7 in* C_0 can be used in any symmetric system other than C_3.

Figure 268. Passing seventh in $\sqrt{2}$, $\sqrt[6]{2}$, $\sqrt[12]{2}$.

Under such conditions, i.e., without the necessity of resolving the seventh by scalewise downward motion, it is possible to apply the technique of the passing seventh to *generalized symmetric progressions.*

Figure 269. Applying the passing seventh to generalized symmetric progressions.

B. Generalization of Passing Chromatic Tones

As there are three forms of the seventh available in the system of three tonics, we may now incorporate all three into a continuous progression.

Figure 270. Three forms of the seventh in one progression.

Applying the same to the negative form of three tonics, we obtain:

Figure 271. Negative form of three tonics.

Likewise, the system of four tonics offers two forms of the seventh—we shall now use them in succession:

Figure 272. Four tonics offer two forms of the seventh.

The same approach is applicable to the negative form of four tonics:

Figure 273. Negative form of four tonics.

All forms of the passing seventh between the roots of symmetric systems produce *continuous chromatic passages* connecting the roots.

The number of pitch-units of the chromatic scale to be harmonized by one chord equals *12 divided by the number of symmetric roots.* Thus, the system of two tonics requires $\frac{12}{2} = 6$, i.e., six units of the chromatic scale to be harmonized by one chord.

Figure 274. Two tonics.

The system of six tonics requires $\frac{12}{6} = 2$, i.e., two units of the chromatic scale to be harmonized by one chord.

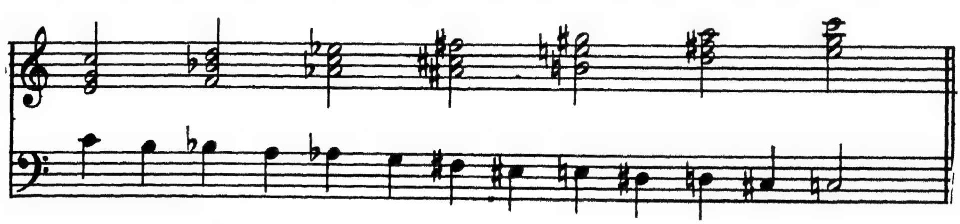


Figure 275. Six Tonics (continued).

Figure 275. Six Tonics (concluded).

In the system of twelve tonics, each pitch-unit of the chromatic scale must be harmonized by one chord, as $\frac{12}{12} = 1$.

In this way we establish an interrelation between the complete form of symmetry of our tuning system ($\sqrt[12]{2}$) and the sub-systems of this symmetry ($\sqrt{2}$, $\sqrt[3]{2}$, $\sqrt[4]{2}$ and $\sqrt[6]{2}$).

A mathematical representation of the forms of symmetric harmonization of the chromatic scale would be:

Form of symmetry	Number of chromatic units per chord
1	12
$\sqrt{2}$	6
$\sqrt[3]{2}$	4
$\sqrt[4]{2}$	3
$\sqrt[6]{2}$	2
$\sqrt[12]{2}$	1

Figure 276. Symmetric harmonization of chromatic scale.

We see that a *variable quantity of units of the chromatic scale may be harmonized by means of generalized symmetric progressions.*

Example: $6 + 1 + 4 + 2 + 3$

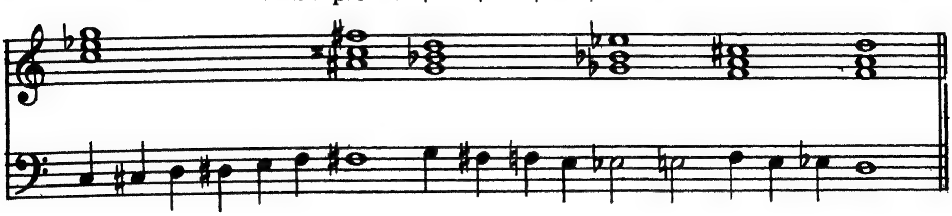


Figure 277. Harmonizing by means of generalized symmetric progressions.

Leaps are applicable to chromatic passages in the same way as they have been applied to the passing seventh. Such leaps can be performed from any point; the structures may be varied.

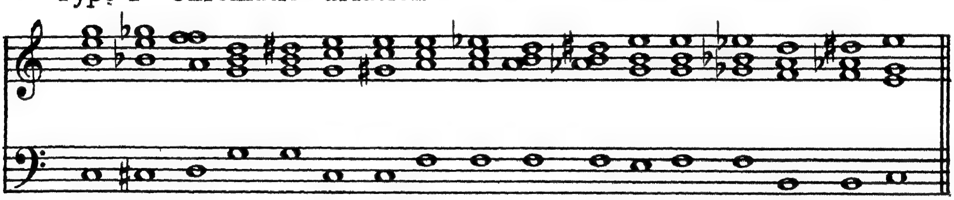
Figure 278. Leaps are applicable to chromatic passages.

When passing chromatics fill the intervals between the symmetric roots, they result in chromatic passages *within* symmetric progressions.

In addition to this technique, there is a possibility of *filling the interval of a whole tone in any type of harmonic progressions, after the voice-leading* related to the given type of progression *is completed.*

Type I= Original

Type I= Chromatic variation



Type II= Original

Figure 279. Passing chromatic tones (continued).

Type II = Chromatic variation

Type III = Original

Type III = Chromatic variation

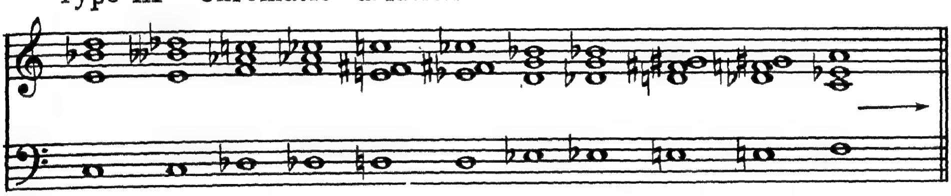


Figure 279. Passing chromatic tones (concluded).

Although a chromatic harmonic continuity (of any form) offers but limited possibilities for the insertion of passing chromatics, such a procedure can nevertheless be accomplished if and when necessary.

Chromatic = Original

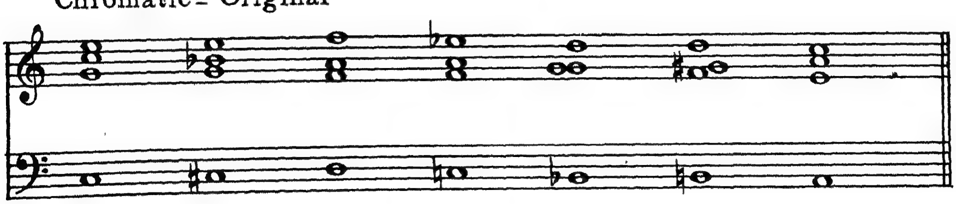


Figure 280. Passing chromatics inserted into chromatic harmonic continuity (continued).

Chromatic = Chromatic variation

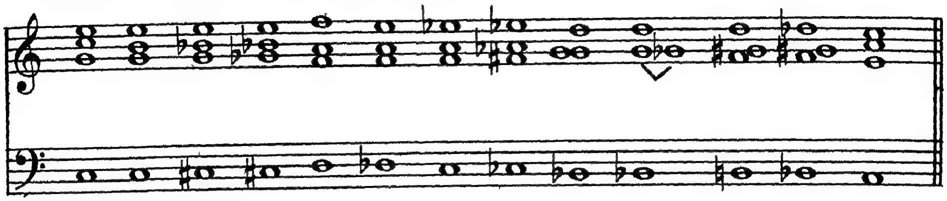


Figure 280. Passing chromatics may be inserted into chromatic harmonic continuity. (concluded)

C. Altered Chords

Pitch assemblages produced by one or more simultaneous passing chromatics are usually known under the name of "*altered chords.*"

Whereas the usual academic scope of information regarding "altered chords" is very limited, it becomes virtually *inexhaustible*—and the forms of altered chords become all but *limitless*—when evolved through the technique of passing chromatic tones.

Some of the altered chords, although *different* in their written forms, *correspond* in their *intonation* to forms already studied in this special theory of harmony. These structures, familiar in their intonation, frequently necessitate entirely new progressions (as compared with the familiar types of progressions)

For example, a chord—

Figure 281.

may be known in the key of harmonic G♯—minor (II) in the following notation:

Figure 282. Same chord in G♯ minor (II).

Yet in the first notation it moves into A-minor $(I)_A$—

Figure 283. Moves into A minor (I) in first notation.

—whereas in the second notation it could move through any cycle in the key of G♯ – minor.

Some other altered chords *do not correspond to any of the structures previously classified,* as they contain an interval 2—and all structures previously classified contain 3 and 4 only.

For example:

Figure 284. Other altered chords.

—where $\frac{\text{f}}{\text{d}\sharp}$ is an interval of two semitones.

In order to obtain a progression where such altered chords occur, it is necessary to start with a chord produced by passing chromatics and *alternate continuously the altered and the regular chords.*

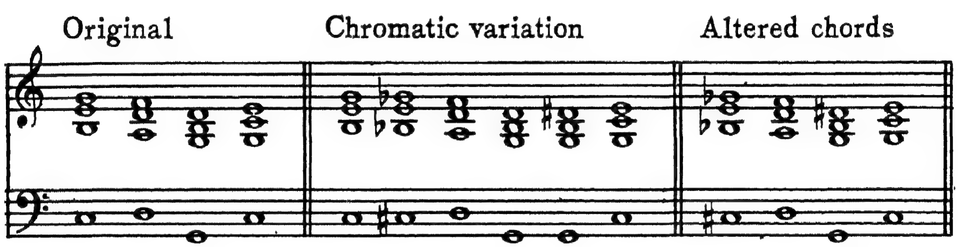


Figure 285. Alternating altered and regular chords.

CHAPTER 16

AUTOMATIC-CHROMATIC CONTINUITIES

AUTOMATIC chromatic continuity may be devised by means of *semitonal motion* in which one direction is followed by whatever voice or voices happen to be moving.

A. In Three-Part Harmony

In three-part harmony, any form of S(5) may be selected as a starting point; this alone offers 4 forms for one voice moving at a time:

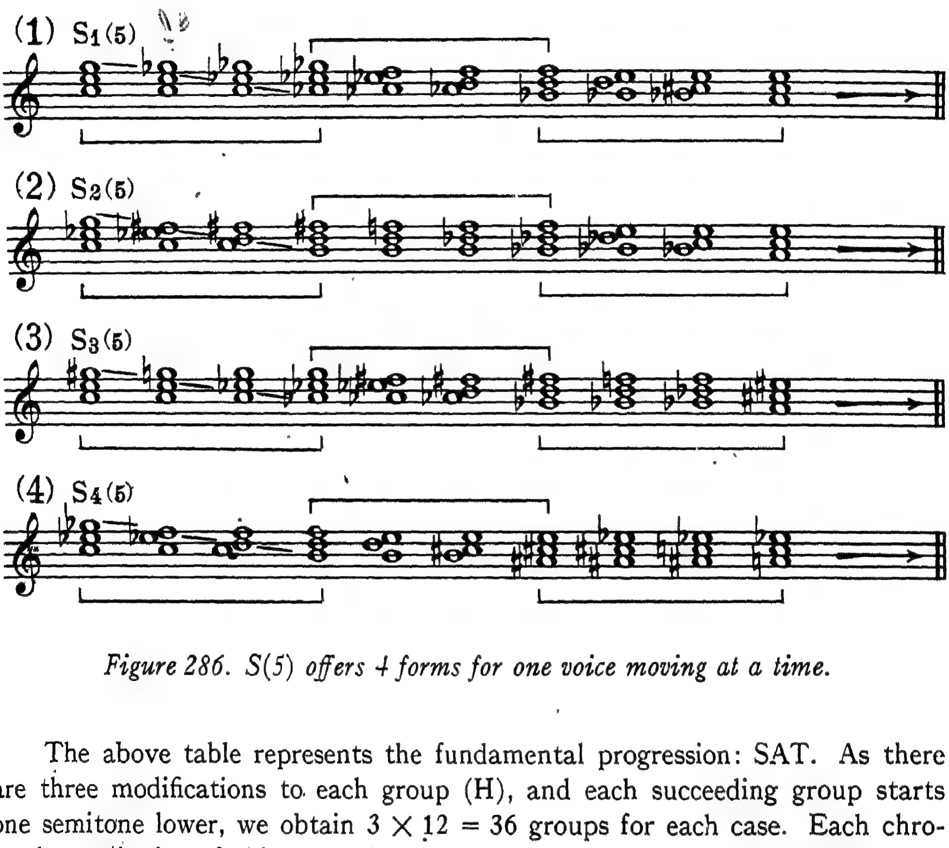


Figure 286. S(5) offers 4 forms for one voice moving at a time.

The above table represents the fundamental progression: SAT. As there are three modifications to each group (H), and each succeeding group starts one semitone lower, we obtain $3 \times 12 = 36$ groups for each case. Each chromatic continuity of this type closes at $36 + 1$, i.e., on the 37th chord of the progression.

Further possibilities develop from variations of the part-sequence, of which there are 6 in this case:

SAT, STA, TSA, AST, ATS, TAS.

Thus, the total number of progressions moving in one direction is $4 \times 6 = 24$, each moving in the same direction. By reversing the direction of each progression, we double the quantity.

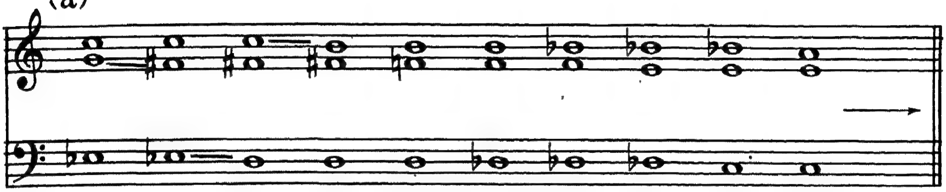
Summarizing the total number of possible forms for three-part automatic chromatic continuity, we arrive at the following figures for all cases in which one voice moves at a time.

4S produce 4 forms of intonation
Chromatic scale produces a sequence of 36 chords
SAT produce 6 variations of the sequence for each form of intonation
The 2 directions (upward and downward) double the quantity of all forms of sequence and intonation

Total: $4 \times 6 \times 2 = 48$ forms

(a) $S_2(5)$: ATS ↓ (b) $S_3(5)$: TSA ↑

(a)



(b)

Figure 287. (a) $S_2(5)$: ATS ↓ . (b) $S_3(5)$: TSA ↑

The technique of semitonal motion makes it possible to move two voices at a time. Both voices must move in one direction.

The number of combinations out of three elements, taken two at a time, is mathematically:

$$_3C_2 = \frac{3!}{2!\,(3-2)!} = \frac{6}{2 \cdot 1} = 3$$

In the simultaneous (vertical) arrangement they are:

	S	S
A		A
T	T	

Simultaneous motion of two parts requires the use of all three combinations arranged in any of the 6 possible forms of succession. Otherwise, there would be no way to arrive at the original structure and the entire sequence would become more and more distorted. Some cases produce consecutive seconds, and these, if felt to be undesirable, may be omitted.

An example of two-part chromatic motion in all six permutations of the original combination.

Original Structure: $S_1(5)$
Direction: ↑

(a) $\genfrac{}{}{0pt}{}{A}{T} + \genfrac{}{}{0pt}{}{S}{T} + \genfrac{}{}{0pt}{}{S}{A}$

(b) $\genfrac{}{}{0pt}{}{A}{T} + \genfrac{}{}{0pt}{}{S}{A} + \genfrac{}{}{0pt}{}{S}{T}$

(c) $\genfrac{}{}{0pt}{}{S}{A} + \genfrac{}{}{0pt}{}{A}{T} + \genfrac{}{}{0pt}{}{S}{T}$

(d) $\genfrac{}{}{0pt}{}{S}{T} + \genfrac{}{}{0pt}{}{A}{T} + \genfrac{}{}{0pt}{}{S}{A}$

(e) $\genfrac{}{}{0pt}{}{S}{T} + \genfrac{}{}{0pt}{}{S}{A} + \genfrac{}{}{0pt}{}{A}{T}$

(f) $\genfrac{}{}{0pt}{}{S}{A} + \genfrac{}{}{0pt}{}{S}{T} + \genfrac{}{}{0pt}{}{A}{T}$

Figure 288. Two-part chromatic motion.

Another form of automatic chromatic continuity applies to the variable number of parts participating in simultaneous moves. In three-part harmony, it is possible to alternate the two simultaneous parts and to use the remaining one to produce compensation.

Thus the following forms are available:

(1)		(2)		(3)	
S		S			S
A	;		A ;	A	,
	T	T		T	

as well as their reciprocals:

(1)		(2)		(3)	
	S		S	S	
	A ;	A	;		A .
T			T		T

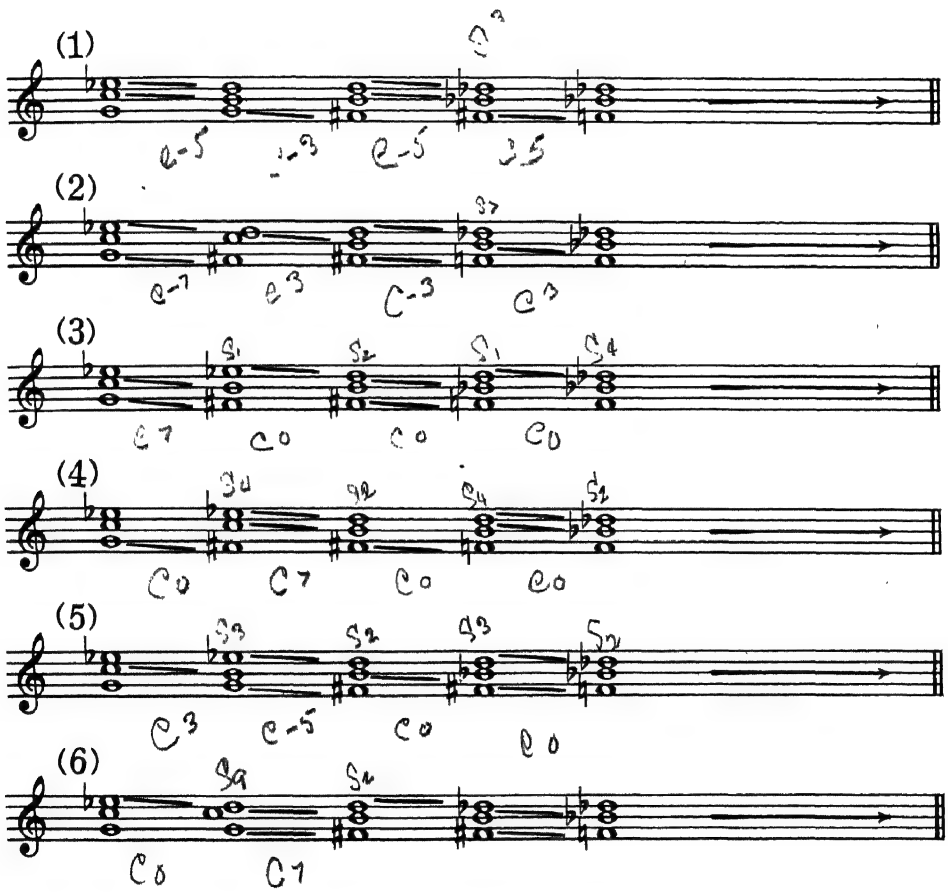


Figure 289. Alternating two simultaneous parts.

The above combinations may be further combined into continuously varying groupings.

$$(1)\quad \begin{pmatrix} S & \\ A & \\ & T \end{pmatrix} + \begin{pmatrix} & S \\ A & \\ & T \end{pmatrix} + \begin{pmatrix} & S \\ A & \\ T & \end{pmatrix}$$

$$(2)\quad \begin{pmatrix} & S \\ & A \\ T & \end{pmatrix} + \begin{pmatrix} & S \\ A & \\ & T \end{pmatrix} + \begin{pmatrix} S & \\ & A \\ & T \end{pmatrix}$$

Figure 290. Combining foregoing (figure 289) with continously varying groups.

The final form of three-part continuity consists of variations of the single and double moves, and variations of the sequence in which the latter appear in both descending and ascending directions.

Example:

$$\begin{array}{ccccccccccc} S| & & S\downarrow & & S\uparrow & & S| & & & S\uparrow & \\ A\downarrow & A| & & A\uparrow & & A| & A\downarrow & & & & A\uparrow \\ & T\downarrow & T\uparrow & & & T\downarrow & & T\downarrow & T\uparrow & & \end{array}$$

Figure 291. Varying the sequence in ascending and descending directions.

It is important to note that the *direction can be changed only after all three voices have performed their moves.*

A. In Four-Part Harmony

In four-part harmony any form of S(7) may be selected as a starting point. This offers 7 forms for one voice moving at a time. Considering the sequence SATB to be one group, we obtain $4 \times 12 = 48$ chords for the descending and as many for the ascending progressions.

As there are 7 forms of intonation to each progression, we obtain: $7 \times 24 \times 2 = 336$ forms, i.e., 7 forms times 24 variations of the SATB sequence, times 2 for the descending and the ascending directions.

The number of combinations out of four elements taken two at a time is:

$$_4C_2 = \frac{4!}{2!\,(4-2)!} = \frac{24}{4} = 6$$

These combinations by two refer to two simultaneous moves to be used independently, i.e., *without compensation.*

S ;	S ;	S			
A		;	A ;	A	
	T		T	;	T.
		B		B	B

In using these combinations by free choice, remember that the direction may be changed only after the participation of all four voices in equal quantities and regardless of the combination selected.

For example:

S ↑			S ↑	
A	A ↑			
	T		T	
		B ↑		B ↑

represents a group in which all voices participate twice.

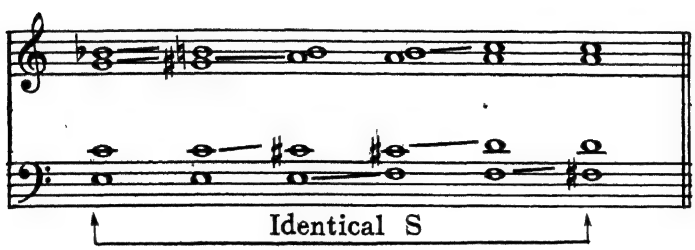


Figure 292. All voices participate twice.

Another form of four-part automatic chromatic continuity is based on the compensation of pairs of the simultaneously moving voices.

The following forms are available:

(1)		(2)		(3)	
S		S		S	
A ;			A ;		A ;
	T	T			T
	B		B	B	

(4)		(5)		(6)	
	S		S		S
A ;		A ;			A.
T			T	T	
	B	B		B	

As the first of the two combinations in each group is compensated by another combination *by two*, this includes the reciprocals as well.

Still another form of automatic chromatic continuity is produced by simultaneous motion of three voices (in any combination) followed by motion of the one remaining voice.

The following groups are available:

(1)	(2)	(3)	(4)
S A T ; B	S A ; T B	S ; A T B	S A T . B .

$$ {}_4C_3 = \frac{4!}{3!\,(4-3)!} = \frac{24}{6\cdot 1} = 4 $$

Single, double and triple movements may be arranged in any desirable sequence, provided that there is no conflict with the principle that the number of moves in the adjacent groups must equalize in one direction.

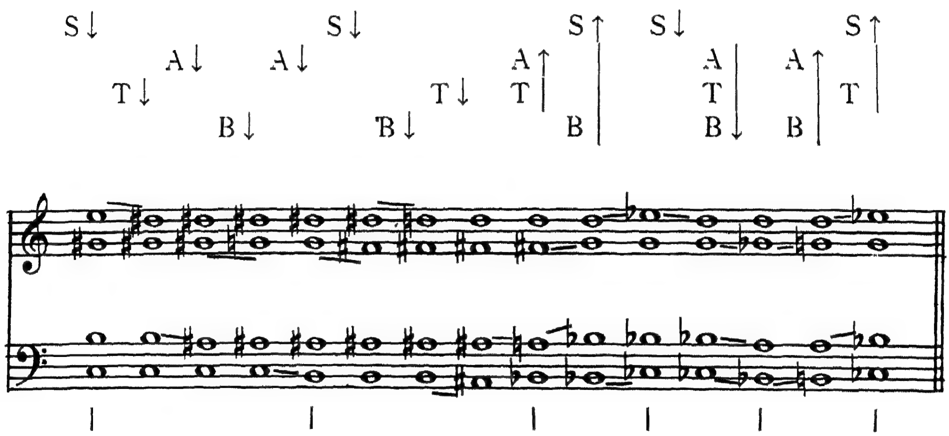


Figure 293. Combined devices in automatic chromatic continuity.

(In the above, the identical structures are marked.)

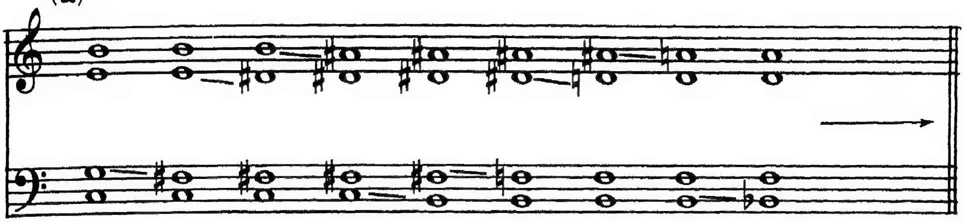
In many cases of four-part continuity, especially with the single moves, classical forms of *suspensions* and *anticipations* take place.

(a) Original Structure: S(7) = 4 + 3 + 4.
Sequence: TASB

(b) Original Structure: S = 3 + 3 + 5.
Sequence: TSAB

See the corresponding music examples on the following page.

(a)



(b)

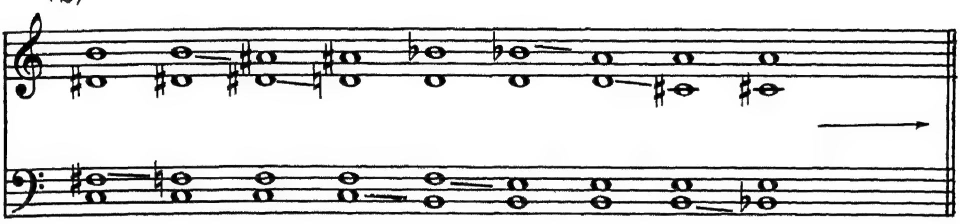


Figure 294. Four-part continuity with single moves.

CHAPTER 17

HYBRID HARMONIC CONTINUITIES

PURITY of harmonic style is more inherent in the music of those composers who were born at a time when they could crystallize past experiences along identical technical lines. Palestrina, J. S. Bach, Wagner, Chopin, Scriabine, Ravel, Debussy, Hindemith—all have sufficient unity in their harmonic expressions.

For practical purposes, however—especially in the field of "arranging" when re-harmonization of a song is desirable—it is sometimes necessary to produce harmonic styles that are intentionally hybrid.

We shall consider that *the mixture of diatonic, symmetric and chromatic forms is hybrid.*

This type of harmonic continuity requires quick changes from one type of harmony to another. The reason for this is that our ears get used to one type very quickly; an instantaneous change to another type, when the habit is already formed, often produces an undesirable disturbing effect. The diatonic type conflicts strongly with the symmetric. It becomes necessary to separate the two one from another by means of the chromatic type, which is more neutral in character.

The **first** necessary condition for successful mixing of harmonic types is *the insertion of the chromatic type between the diatonic and the symmetric.* This can be expressed by the following diagram:

$$\overleftrightarrow{\text{diatonic — chromatic — symmetric}}$$

Hybrid harmonic continuity may be of any desirable length, providing that the diatonic and the symmetric have no immediate contact. For example: di + ch + + sy + ch + sy + ch + di + ch + di.

The **second** requirement for the successful execution of the hybrid continuity concerns the ratios in which the three different types appear. As the chromatic type neutralizes the effect of the preceding type (whether diatonic or symmetric), it is necessary to have more of it.

The most desirable of the simple ratios for this purpose is: di + 2ch + sy.

The **third** requirement concerns the quantities expressing the ratio. In moderate tempo, approximately two or three chords are a desirable unit. In fast tempo the quantity should be increased accordingly. Thus, the average form of hybrid harmonic continuity ($\overrightarrow{Hy}$) can be expressed as follows:

$$\overrightarrow{Hy} = \text{di}3H + \text{ch}6H + \text{sy}3H$$

When the above requirements are actually fulfilled, the resulting music may achieve a very high quality.

The inclusion of one more refinement guarantees the utmost smoothness to such progressions. This becomes particularly important when music is intended for mass consumption—as in dance music, for instance.

The refinement consists of maintaining an *identical intervallic root-relation* between *the last two chords of the preceding chromatic group* and between *the last chord of the chromatic group and the first chord of the following symmetric group*. Further relations of the symmetric group are not influenced by this.

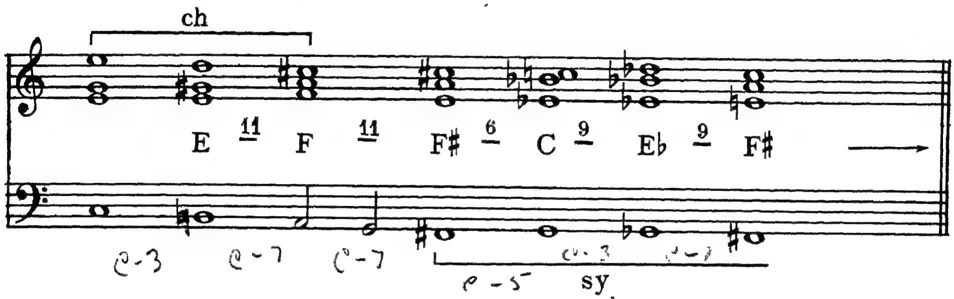


Figure 295. Maintaining identical root relations.

In the above example, identical steps occur at two successive points: between E and F (the last two chords of ch) and between F and F♯ (the last chord of ch and the first chord of sy). As the figure shows, the subsequent relations of the symmetric group (6 + 9 + 9 semitones) are not influenced by the preceding identity of steps.

Figure 296. Hybrid harmonic continuity: di+ch+sy+ch+sy+ch+di+ch+di

CHAPTER 18

LINKING HARMONIC CONTINUITIES

WHEN contrasts between analogous portions of harmonic continuity are desirable, the latter may be bridged by harmonic *connections* of a *different* type. The degree of contrast between the continuity and the connections ("bridges") depends on the type of progressions used in both. One can easily recognize a composer by the type of continuity and connections he uses. For instance, it is typical of Wagner to make *symmetric* connections between the portions of *diatonic* continuity. The starting point of each consecutive section is in the $\sqrt[4]{2}$ relation to the preceding section.

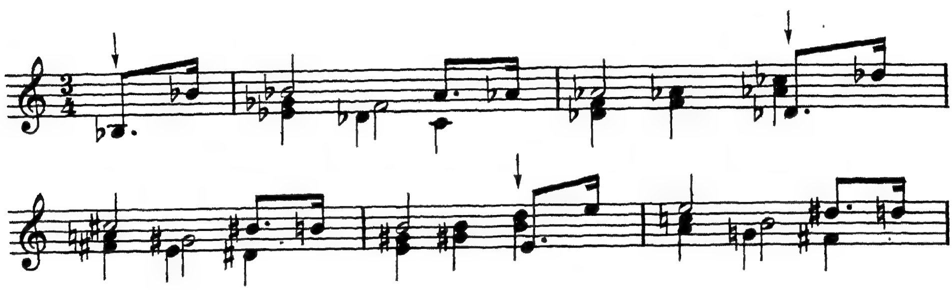


Figure 297. The "Pilgrims' Chorus" from Tannhäuser.

Assuming that any form of harmonic progression is used either as continuity or as connection, we obtain the following nine forms of *combined harmonic continuity.*

(1) diatonic progressions, diatonically connected;
(2) diatonic progressions, symmetrically connected;
(3) diatonic progressions, chromatically connected;
(4) symmetric progressions, diatonically connected;
(5) symmetric progressions, symmetrically connected;
(6) symmetric progressions, chromatically connected;
(7) chromatic progressions, diatonically connected;
(8) chromatic progressions, symmetrically connected; and
(9) chromatic progressions, chromatically connected.

Each form of combined continuity provides a certain amount of variation within its own limitations.

In the first case above, *progressions* may be developed through diatonic cycles (No. 1), whereas the connections will be developed through groups with passing chords. This can be done in reverse as well by connecting one form of group with a cycle *alien* to the group. For instance, $G_{\frac{9}{4}}$ (in which C_{-5} and C_5 participate) can be connected to the following $G_{\frac{9}{4}}$ through C_3 or through C_7.

When diatonic progressions of cycles are connected through G_6 or G_4^6, there is one extra chord within the connection, i.e., the first chord of the group is the last chord of the preceding progression, the middle chord of the group is the extra chord and the last chord of the group is the first chord of the succeeding progression.

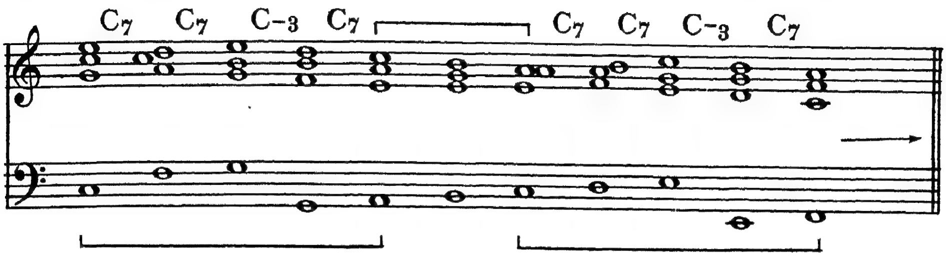


Figure 298. Diatonic progression connected through G_6.

This form of combined continuity requires the *exact recurrence of the cycle group*; however, the *positions* of chords as well as their *forms of tension* may be varied in each subsequent progression of one continuity.

When groups are connected by a cycle, there is no extra chord to be gained.

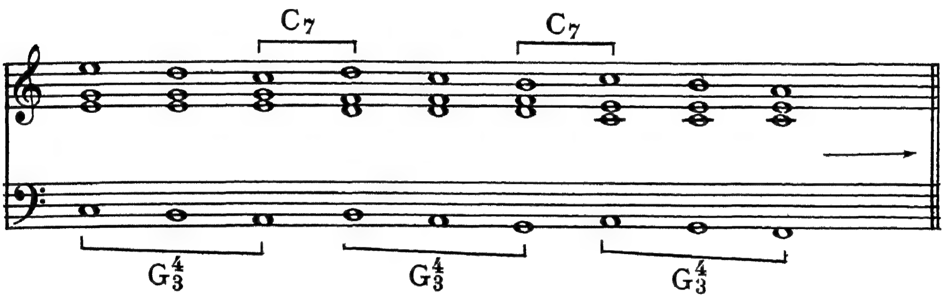


Figure 299. Groups connected by a cycle.

The diatonic connection of symmetric progressions (No. 4) may be accomplished by assuming that *the last chord of the symmetric progression belongs to a certain key*. Thereupon, through one cycle connection the harmony affirms the assumed key in which the subsequent symmetric progression then begins. There is no extra chord appearing during the connection. All forms of symmetry may be used.

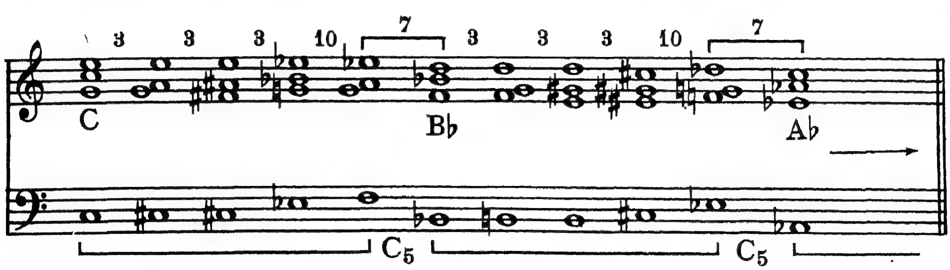


Figure 300. Diatonic connection of symmetric progressions.

Symmetric connection of symmetric progressions (No. 5) must be based on selection of such forms of symmetry as do not appear in the progression itself.

Figure 301. Symmetric connection of symmetric progressions.

A symmetric connection of diatonic progressions (No. 2) does not produce an extra chord, but rather an interval ($\sqrt{2}, \sqrt[3]{2}, \sqrt[4]{2}, \sqrt[6]{2}, \sqrt[12]{2}$). Such a connection may be planned *either* in relation *to the first* or *to the last chord* of the diatonic progression. In Figure 290 the connection through $\sqrt[4]{2}$ referred to the first chord of each diatonic progression.

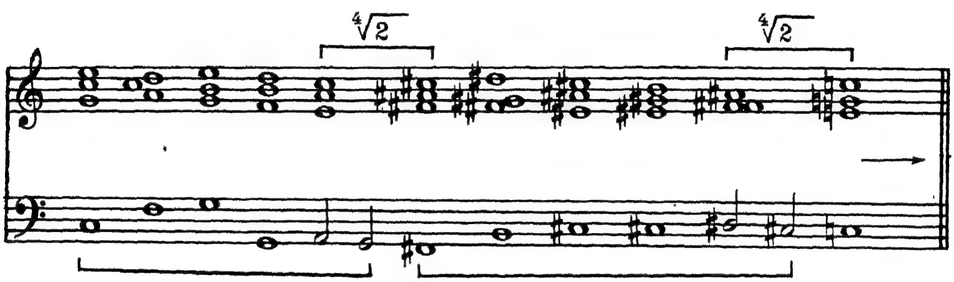


Figure 302. Symmetric connection of diatonic progressions.

Diatonic progressions may be connected through any form of chromatic group (No. 3) (parallel or contrary). An extra chord is gained through such a connection. This type of combined continuity, incidentally, usually sounds like diatonic harmony with modulations.

Figure 303. Chromatic connection of diatonic progressions.

The chromatic connection of symmetric progressions (No. 6) introduces one extra chord. Any form of chromatics can be used. It is desirable that the interval between the extreme chords of the chromatic connection should not duplicate any steps of the symmetric progression.

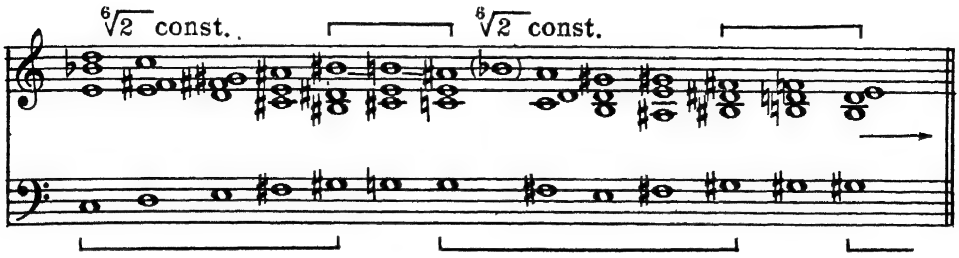


Figure 304. Chromatic connection of symmetric progression.

The diatonic connection of chromatic progressions (No. 7) is achieved by assigning the last chord of a chromatic progression to the key in which such a chord may exist. The latter is connected by a diatonic cycle with some other chord in the same key. Thereupon the chromatic progression is resumed. There is no extra chord gained in the cycle connection. Chromatic progressions (consisting of one or more chromatic groups of any type) may be varied after each diatonic connection.

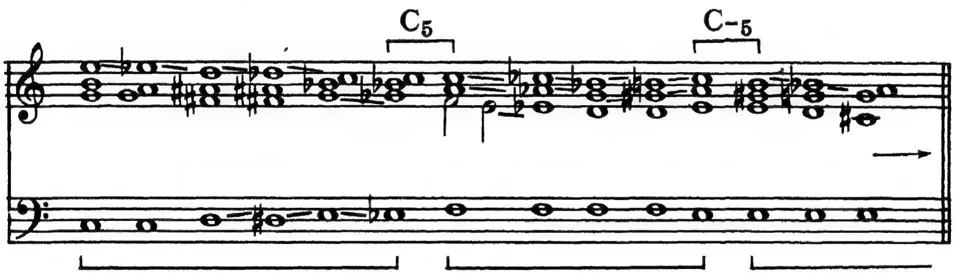


Figure 305. Diatonic connection of chromatic progressions.

Symmetric connection of chromatic progressions (No. 8) is achieved through the selection of a root which does not produce chromatic steps in any voice. There is no gain of an extra chord.

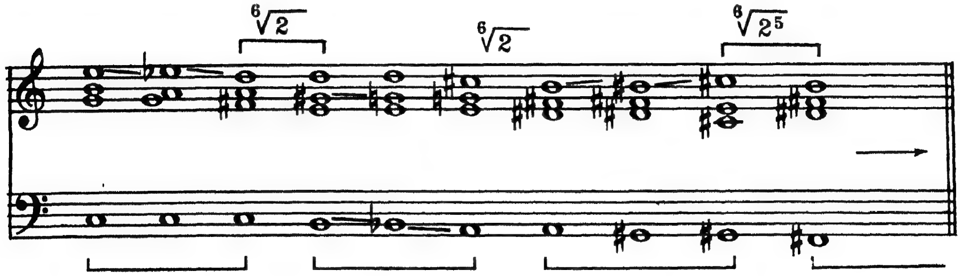


Figure 306. Symmetric connection of chromatic progressions.

The chromatic connection of chromatic progressions (No. 9) may be accomplished by introducing contrasting forms of chromatics. Contrasts may be achieved by the juxtaposition of parallel and contrary chromatics, or by the juxtaposition of chromatics and enharmonics. An extra chord is gained by such connections.

Figure 307. Chromatic connection of chromatic progressions.

The above nine forms of combined harmonic continuity can be further combined *by varying the forms of connection between each set of progressions of any one particular type.*

CHAPTER 19

A DISCUSSION OF PEDAL POINTS

PEDAL Point or Organ Point (P.P.) is a primary axis about which chord progressions are formed. The various patterns of motion which the remaining voices produce in relation to the pedal point consequently result in different effects corresponding to the axial combinations, including the 0-axis.*

P.P. is primarily conceived as a sustained bass, but by means of vertical rearrangement of parts, one can achieve the appearance of a P.P. in any desirable voice. We shall compose pedal points first as basses.

P.P.$\frac{0}{0}$, i.e., a pedal point with a more or less stationary or a slightly revolving pattern for the motion of upper voices, produces either the effect of *accumulation* or *discharge of energy*—the first resulting from a *crescendo*; the second, from a *diminuendo*. In such a form, P.P. is used either *at the beginning* of a composition, mostly as an *introduction* ("take-off") or at the end of it, mostly as a *coda* ("landing"). The next stage of dramatic expression is obtained through the use of several secondary axes against P.P. The following may be considered as fundamental combinations.

$$\text{P.P.}\ \frac{(a+b)+\cdots}{0} \quad \text{and} \quad \text{P.P.}\ \frac{(b+a)+\cdots}{0}$$

In some cases, the pedal point leads to a climax. Then the entire P.P. serves as a form of accumulation of energy followed by discharge (climax). In such cases P.P. is associated with *crescendo* and requires a *prolonged a-axis* for the upper parts of harmony. The climax itself is the ultimate forte.

$$\text{P.P.}\ \frac{a}{0}$$

After such a vigorous climax, an anti-climax—i.e., *moving toward ultimate balance*—is often necessary. Being usually a coda or an episode preceding recapitulation, this requires a gradual dissipation of energy which can be expressed through the use of an *extended b-axis* of upper voices in relation to P.P. The dynamic form for this is *diminuendo*.

$$\text{P.P.}\ \frac{b}{0}$$

The devices of this theory of harmony supply all the necessary forms by which the patterns of harmonic motion—as expressed through tonal cycles in relation to the quantity of voices—may be obtained at will.

For instance, an a-axis for three parts may be obtained through C_3 const., as well as through some techniques of ascending chromatics. For a gradual ascent, $C_3 \circlearrowleft$ and $C_5 \circlearrowright$ may be used. For ascent in leaps, $C_3 \circlearrowright$ is the corresponding technique. All the negative forms of continuous S(7) or any other structures in the hybrid five-part harmony produce the same axis.

*The *axes* referred to are those of the theory of melody, i.e., a, b, c, d and 0. (Ed.)

It often happens that the number of parts in harmony determines the patterns of motion under the same cycle. For example, four tonics S(9) const., i.e., C_3, are stationary in five parts but climb quite decisively in four (the three upper parts of the hybrid four).

A vigorous alternating (ascending-descending) motion may be achieved through the use of $C_7 \circlearrowright$ in three parts (see S(5) in C_7).

An extended descent, i.e., b-axis, may be obtained through the use of continuous S(7) in four-part harmony, through the $\circlearrowleft C_7$ (thus including the six and the twelve tonics), through $\circlearrowright C_5$, as well as by means of descending chromatics.

An oscillating pattern, which may be considered to be an 0-axis in that it has but limited amplitude, may be achieved through the alternate use of positive and negative forms in any type of harmony where parts move through limited melodic intervals. The technique of continuous S(7), alternating the positive and the negative forms under limited coefficients, produces such patterns. For example: $3C_5 + 2C_{-5} + 2C_7 + 3C_{-7} + \ldots$ etc.

The *reversal* of all of the above-described considerations may still serve the purpose, *provided that dynamics becomes a more influential factor than harmony.*

The effect of "vanishing" or "flying away" may be accomplished by what we consider dissipation of energy or moving toward balance. However, we have conditioned associations with the quantity of sound and volume. The use of an a-axis combined with a crescendo leads to a climax—yet the very same axis combined with diminuendo associates itself in our perception with an object growing smaller and flying away in an upward direction. This is due to the influence of the concepts "high" and "low" in reference to frequencies, transformed into spatial analogies. Flying away in an upward direction, which to us as human beings is associated with extreme tension (overcoming of gravity, and the effort necessary to accomplish it), corresponds to the increasing tension of our vocal chords, which tension we all mimic *sympathetically* in infinitesimal degrees while listening to music. Thus, vanishing in mid-air through visual association with the gradual diminution of an object corresponds to the *0-axis combined with diminuendo*; whereas, vanishing into the ground from above requires the *b-axis combined with diminuendo.* This analysis throws light on the many different effects achieved in music by means of pedal point.

An obvious example of the effect of "vanishing" in an upward direction is the brief pedal point in Rimsky-Korsakov's *Scheherazade*, accomplished by the most humble harmonic means.

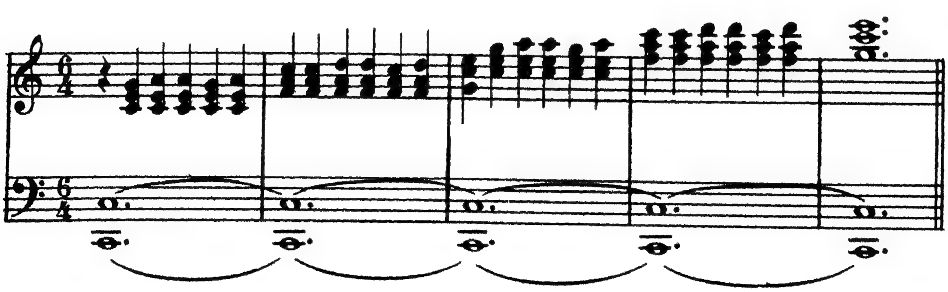


Figure 308. "Vanishing" in upward direction.

In the first movement of Beethoven's Sonata No. 8 (the *Pathetique*), the first four measures of the *allegro con brio* produce a "take-off" by means of $\frac{a}{0}$ axis; this establishes a firm foundation for the music to follow. In the same movement, the third theme begins with a two bar $\frac{0}{0}$ axis pedal point (e♭), accumulating energy for the following diverging texture of broken chords: $\frac{a}{d}$ axis. In the same movement, there is a climactic pedal point preceding a recapitulation. This pedal point on g (the dominant of the key), being of intensely revolving character, accumulates a tremendous energy (due to the crescendo), which energy is dissipated in a duly extended descending (b-axis) passage leading into the recapitulation.

A. Classical Pedal Points

It is regrettable that the manner in which classical composers used the various forms of pedal point is not now generally known to the more prominent contemporary composers; the evidence of this consists of the misplaced pedal points in their own works. In order thoroughly to understand the *classical* approach to this problem, it is first important to classify and define the traditional forms of pedal point.

The two fundamental forms of the classical pedal point are: P.P.T., i.e., *pedal point on the tonic*, and P.P.D., i.e., *pedal point on the dominant.*

P.P.T. affirms the harmonic axis of the composition, i.e., it *establishes the key.* Technically, P.P.T. may usefully be defined as an extension of the ecclesiastic plagal cadence.

The cadence itself consists of *Tonic* + *Subdominant* + *Tonic*, which usually appears as I + IV$_4^6$ + I (T + S_T + T). Here T is the tonic, S_T—the subdominant with a tonic characteristic. This form has for long been used in many sung prayers of the Christian church of different denominations and is usually associated with the intonation formula for "Amen," although it appears very frequently, too, at the beginnings. The device was undoubtedly used in order to help the singers "to tune-up"; it is most prominent in music of the Russian-Orthodox Greco-Catholic church.

Figure 309. Ecclesiastical plagal cadence.

This form of plagal cadence later developed into the form using II S $\binom{6}{5}$ and VII S$\binom{4}{3}$. It often appears as a pedal point on the tonic.

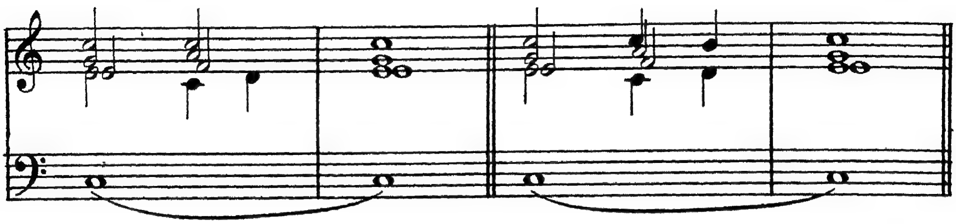


Figure 310. Classical pedal point on tonic.

P.P.T. is either an *initial* (to be used at the very beginning) or the *final* (to be used at the very end) pedal point. It is a sort of airdrome, we might say, from which the flight has to be begun. Thus the initial P.P.T. corresponds to the "take-off", and the final P.P.T. corresponds to the "landing."

But, P.P.D., on the other hand, is a *climactic* pedal point; it corresponds to the *apex* of a flight. Technically P.P.D. may be defined as an extension of the compound authentic cadence or, more specifically, as *an extension of the cadential I S$\binom{6}{4}$).*

The cadence from which such a pedal point derives is as follows:

$$T + S + T_D + D + T$$

Here T is the tonic, S is the subdominant, T_D is the tonic with a dominant characteristic, D is the dominant.

It can be symbolized in the following form:

$$\underset{(II)}{I + IV} + I_{\substack{6\\4}} + \underset{(III)}{V} + I$$

The actual P.P.D. starts on $T_D = I_{\substack{6\\4}}$.

P.P.T. begins with the tonic and ends with a plagal cadence (S + T). P.P.D. is prepared by a subdominant (S), starts with T_D ($I_{\substack{6\\4}}$) and ends with an authentic cadence (D + T). The development of the progressions which occur above P.P. will be discussed later.

As P.P.T. signifies either the beginning or the ending, it usually appears in the introduction or at the beginning of the first theme, or in the conclusive theme or as a coda. P.P.D., which signifies climax, usually appears before the recapitulation.

More than one pedal point can be used in sequence in the course of one composition or one movement. Let us take the most typical scheme of *thematic distribution*—$A + A_1 + B + A_1$—where A and A_1 are the modified expositions of one theme, B is the contrasting theme (middle strain in a song), and A_1 (at the end) is the recapitulation of the first theme (usually in an abbreviated form).

Consider A to include the introduction and the second A_1 to include the coda. We may then chart all the possible combinations of pedal points to be used in a typical scheme of thematic distribution.

	A	A_1	B	A_1
(1)	P.P.T.			
(2)			P.P.D.	
(3)				P.P.T.
(4)	P.P.T.		P.P.D.	
(5)	P.P.T.			P.P.T.
(6)			P.P.D.	P.P.T.
(7)	P.P.T.		P.P.D.	P.P.T.

Figure 311. Traditional location of pedal points.

In the last two cases, P.P.D. is often immediately followed by P.P.T.

The two predominant types of classical pedal point are the *diatonic* and the *chromatic* pedal points.

B. Diatonic Pedal Point

Diatonic P.P.D. or P.P.T. consists of the free use of the diatonic cycles in both the positive and the negative form. It must satisfy all the requirements as to the proper start and proper cadences.

P. P. D.

P. P. T.

Figure 312. P.P.D. + P.P.T. in diatonic type.

The three parts above the pedal point in the above figure are devised by means of classical and hybrid transformations. It is useful to know the structural specifications for the chords appearing above the pedal point and devised in three parts. They are:

5	3	7	7	9
3	1	3	5	7
1	1	1	1	1
S(5)	S(5)	S(7)	S(7)	S(9)

Figure 313. Structural specifications of chords in figure 312.

As to their transformations, they have to be treated as abc, which corresponds practically to a mixture of classical and hybrid techniques.

If all chords above the P.P. are S(5) in three parts, then the classical transformations cover the field completely (↻, ↺ and const. 3).

Continuous four-part setting above the P.P. corresponds to the technique of S(7) const. Note that this device produces very expressive pedal points reminiscent of those of J. S. Bach and Händel, particularly when modes, harmonic major, melodic major or melodic minor are used. All that is necessary is the addition of a stationary bass to the upper four parts moving as seventh-chords.

Figure 314. Continuous 4-part setting above P.P. corresponds to technique of S(7) const.

C. Chromatic (Modulating) Pedal Point

Classical modulating pedal points (P.P.D. and P.P.T.) consist of a rapid succession of key-to-key transitions. The latter are usually performed by means of the chord on the VII or V of the following key; such a limitation is not necessary, however, and any other intermediate chords may be used.

The most important—and heretofore unsolved—problem is that of the particular key selection to be evolved above the pedal point.

As P.P.D. is associated with the *authentic cadence* ($I_{4}^{6} \xrightarrow{C-5} V$), its natural tendency is to modulate through a chain of dominants, i.e., through the keys in C_{-5} relation—which amounts to moving *toward sharps.*

The natural tendency of P.P.T. is to modulate through the chain of *subdominants*, i.e., through the keys in C_5 relation—i.e., in the direction of *flats.* This is due to the fact that P.P.T. is associated with the plagal cadence ($I \rightarrow IV_{4}^{6}$). (Small letters represent the *minor mediants*: lower and upper).

Table of natural key tendencies for modulations in P.P.T. and P.P.D.

	e — a — d — g — c — f — b♭ — e♭
P.P.T.:	C — F — B♭ — E♭ — A♭ — D♭ — G♭ — C♭
	a — d — g — c — f — b♭ — e♭ — a♭
	e — b — f# — c# — g# — d# — a# — e#
P.P.D.:	C — G — D — A — E — B — F# — C#
	a — e — b — f# — c# — g# — d# — a#

Figure 315. Natural key tendencies for modulations in P.P.T. and P.P.D.

However, the natural tendency has only a partial influence in the selection of keys for modulating pedal points.

The main factor, usually neglected in academic musical theories, is *the sonority of the tonic (I) S(5)* of the respective key *in its relation* to pedal point. This can be defined in the form of a requirement, which is: *only those keys may be selected for the classical type of pedal point modulation in which I S(5) taken together with P.P. produces a crystallized structure acceptable in the established four-part harmony.* For instance, it is *wrong* to modulate to the key of D-minor on a P.P.D. in the key of C-major (or C- minor), for the unit g in the bass produces—together with the upper parts—a structure of 1, 5, 7, 9 for S(9), which is not the accepted form, 1, 3, 7, 9. In this case, even though the key tendency is correctly carried out, the result is not satisfactory in sonority.

On the other hand, a key selection which may be contrary to the natural tendency, such as F-minor above the P.P. on g, is perfectly satisfactory as I S(5) in that key, for together with the bass it produces an accepted form of S(11): 1, 7, 9, 11.

The above structural requirement *excludes* the following keys in relation to a pedal point on *c*:

F♯ — minor
G — major
G — minor
G♯ — minor
A — major

All other keys are fully acceptable. An allowance is made for F♯ major because the sonorous quality of the $\sqrt{2}$ is very desirable. The best sounding position for f♯ $S_1(5)$ above *c* is when the pitch-unit nearest to the bass is *a*♯. The latter produces an equivalent of *b*♭ and furnishes a perfect acoustical support for *c*♯ and *f*♯.

Figure 316. Chromatic (modulating) pedal point.

D. Symmetric Pedal Point

Progressions of types II and III and the generalized symmetric progressions may be included in this group. All these types have more or less the same characteristics when used above the pedal point. Any forms of three and four-part harmony may be used.

The pedal point itself is the main (original) tonic. In cases of the generalized symmetric progressions, it is the root-tone of the chord with which the pedal point begins.

(1)

(2) $\sqrt[4]{2}$

(3) 2+2+1

Figure 317. P.P. with harmonic progressions of type II, III and generalized.

A more extensive form of the symmetric pedal point (type III) may be devised through a group of pedal points, each tonic becoming a pedal point in succession. The remaining parts form progressions through the same system of symmetry.

$\sqrt[3]{2}$

Figure 318. An extended symmetric (type III) P.P.

Special Case

There is a special case in which diatonic alternate pedal points on the tonic and the dominant merge with the symmetry of the $\sqrt[12]{2}$ on $S_1(5)$. This is an equivalent of the entire chromatic scale versus tonic-dominant. $S_1(5)$ is *the only* satisfactory form of sonority. Note the coincidence of the tonic and the dominant as S(5).

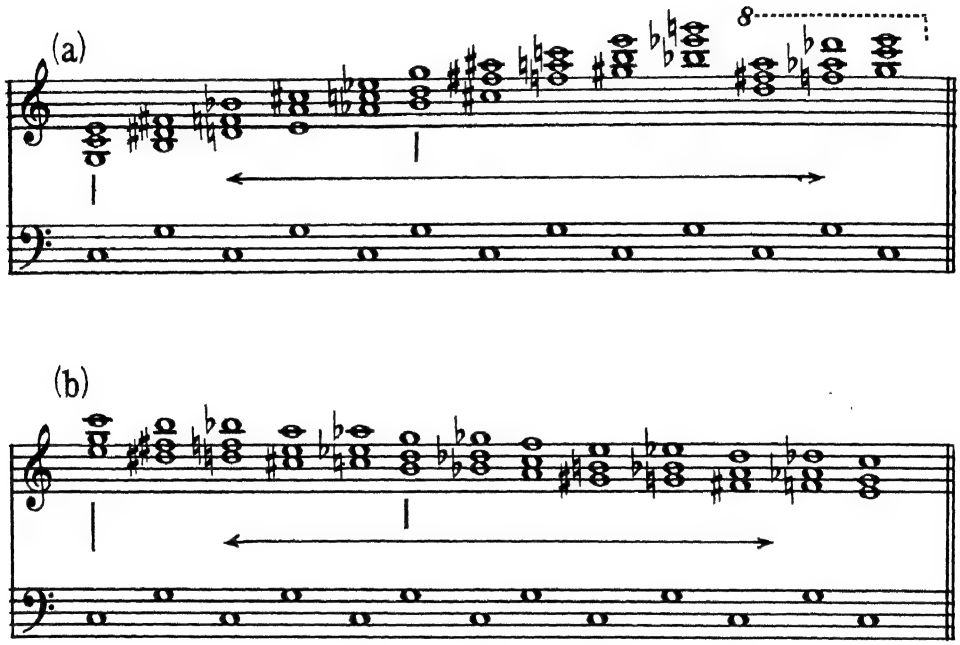


Figure 319. Alternate diatonic P.P. on tonic and dominant merge with symmetry of $\sqrt[12]{2}$ in $S_1(5)$.

CHAPTER 20

MELODIC FIGURATION

Preliminary Survey of the Techniques

THE technique of melodic figuration* consists mainly of the process of evolving *leading tones* for chordal tones in a given harmonic continuity. When leading tones move into chordal tones, they produce *directional units.* Melodic figuration can be defined as a process of transforming *neutral* units (chordal tones) into *directional* units.

A. FOUR TYPES OF MELODIC FIGURATION

There are three types of leading tones satisfying such a definition:

Type one: *suspended tones* (suspensions), i.e., tones belonging to the preceding chord and held over; such tones must be moved into an adjacent chordal tone.

Type two: *passing tones*, i.e., pitch-units inserted between two other pitch-units moving in sequence and constituting chordal tones. Passing tones may, or may not, belong to the same scale as that in which the harmonic continuity has been evolved. In the first case, they are *diatonic* passing tones; in the second, *chromatic* passing tones.

Chromatic passing tones were discussed earlier under their own heading; here, only diatonic passing tones will be discussed.

Type three: *auxiliary tones*, i.e., unprepared leading tones selected with no regard to basic pitch scales. They too, can be either *diatonic* (in which case they have an "ecclesiastic" flavor) or *chromatic* (in which case they add an extreme lyrical expressiveness, due to sudden intensifications, to the music). Chromatic auxiliary tones are one of the most powerful resources of expression in the music of Mozart, Schubert, Chopin, Chaikovsky,** Scriabine and, in some instances, Wagner—as in *Tristan and Isolde.* Contemporary popular songs dealing with love or despair are overloaded with this device.

The fourth type of melodic figuration is based on a technique different from the evolution of leading tones: it introduces certain chordal tones (one or more) of the *following* chord into the *preceding* chord. This device is known as *anticipated tones* or anticipations. It has long been neglected because composers, for some reason, associate anticipations with antiquated harmonies. But it becomes a very important source of harmonic expression when used in harmonic continuity of a more developed type.

*By melodic figuration is meant the process of converting a harmonic continuum into a partially melodic continuum, the melodic characteristics being introduced into the continuum itself—in contrast to the *melodization of harmony*, which means the fabrication of a melody to *go with* a given $H^{\rightarrow}$. (Ed.)

**Schillinger, Russian-born, preferred this simplified spelling to the more commonly accepted form, "Tschaikowsky." (Ed.)

B. Development of Suspensions

The effect of a suspension is to intensify the chord by means of common tones which, while being suspended, rise in rank as a chordal function after which rise they are then released. Every suspension consists of three consecutive phases: preparation, suspension, and resolution. Our ears, due to heredity and habits, accept a suspension only on a strong beat. The source of this habit is strict counterpoint, in which dissonances were only permitted on weak beats and on strong beats, by suspending ("tying over") a common tone. Classical harmonic structures had not been fully crystallized at the time suspensions were used in counterpoint this way, and so these suspended tones produced antiquated harmonic structures resembling those of the old *organum* type.

One of the most common suspensions was the 7 → 11, which, at the moment of suspension, produces the structure: S(11) = 1, 5, 11. Naturally, such a structure fails to conform to the later classical form—and, although Mozart had already felt the need of a more nearly perfected structure at the moment of suspension, theories of harmony even today continue to advocate this most antiquated form.

The following figure illustrates the evolution of structure under suspension. It has been gradually realized that it is necessary to support the eleventh by the ninth; and the ninth, by the seventh.

Figure 320. Evolution of structure under suspension.

The historical crystallization of S(7) as an *independent* structure goes back to the 18th century; the crystallization of S(9) goes back to the middle of the 19th century.

This analysis may well lead us to the conclusion that it is essential that the structures under suspension conform to these crystallized forms of S(7), S(9) and S(11).

A single suspension requires an S(7) in which the suspended tone is a prepared 7th.

A double suspension requires an S(9) in which the suspended tones are the prepared 7th and 9th.

A triple suspension requires preparation of the 7th, 9th and 11th in an S(11).

Similar considerations require that passing tones (diatonic) conform to crystallized chord-structures. This means that the single passing tones must be *passing 7ths*, double passing tones must be *passing 7ths and 9ths*, and triple passing tones must be *passing 7ths, 9ths and 11ths.*

In addition, some groups with passing chords give other double passing tones in parallel (G^4_3) or contrary (G^6_4) motion.

All other cases are crude and antiquated; they create harsh and empty-sounding gaps when orchestrated.

It should not be forgotten that the best composers of the 18th century, such as Mozart and Scarlatti, through constant use of correct suspensions, helped to crystallize the structures of the future, such as S(9).

Figure 321. Use of suspensions in the 18th century.

As classical theory offers suspended and passing tones under positive forms which are always descending, it would be important to have as secure a system for devising *ascending* resolutions of suspensions and for ascending passing tones. Such a system of melodic figuration would exist as a normal one under cycles of consistently negative form. For practical purposes, it is expedient to invert the positive form into ⓓ, either the geometrical or the tonal (i.e., without any changes of accidentals of the original), rather than to think of the 1 as "a negative 7th," the 3rd as "a negative 5th," etc.

The techniques of melodic figuration are applicable to all types of harmonic progression in close, open or mixed positions in four- and five-part harmony.

CHAPTER 21

SUSPENSIONS, PASSING TONES AND ANTICIPATIONS

ALL the elements of melodic figuration may be classified according to direction (ascending, descending), according to chordal functions employed (1 - 13), according to their adherence to scale (diatonic, chromatic)—and, in addition, according to the number of elements simultaneously employed.

The elements	Number of elements employed:	Direction:	Chordal functions employed:	Adherence to scale:
Suspended Tones	Single	↘	↑ 7	Diatonic
	Double	↘ ↘	↑ 9 ↑ 7	Diatonic
	Triple	↘ ↘ ↘	↑ 11 ↑ 9 ↑ 7	Diatonic
Passing Tones	Single	↘	↘ 7	Diatonic
	Single, Double and Triple	Parallel and contrary motion ↗ and ↘	Whole tone interval	Chromatic
	Double	↘ ↘	↘ 9 ↘ 7 | G^6_4 ↘ ↗ | G^4_3 ↘ ↘	Diatonic
	Triple	↘ ↘ ↘	↘ 11 ↘ 9 ↘ 7	Diatonic
Auxiliary Tones	Single	↗ ↘	any function	Diatonic Chromatic
	Double	↗↗ | ↘↘ | ↗↘ | ↘↗	any functions	Diatonic Chromatic
	Triple	↗↗↗ | ↘↗↗ | ↗↘↗ | ↗↗↘	any functions	Diatonic
		↘↘↗ | ↘↗↘ | ↗↘↘ | ↘↘↘		Chromatic
Anticipated Tones	Single		any function	
	Double		any functions	
	Triple		any functions	

Figure 322. Elements of melodic figuration.

A. Types of Suspensions

Single Suspensions: Single suspended tones may be obtained by making the 1, the 3 or the 5 become a prepared 7. The functions, 1, 3 and 5, serve as the preparation; the 7th, as a suspension; and the nearest function one step lower, as a resolution.

Figure 323. Single suspensions.

Double Suspensions: Double suspended tones may be obtained by making the 1 and 3, or the 3 and 5, or the 5 and 7 become a prepared 7 and 9. In a continuity of double suspensions, one of the suspended voices may appear in the *bass*, thus producing an inversion of S(9). The voice-leading in such a case remains usual, i.e., the remaining two voices must furnish 1 and 3.

Figure 324. Double suspensions.

Triple Suspensions: Suspending the 1, 3 and 5, and the 3, 5 and 7 as 7, 9 and 11 produces triple suspensions.

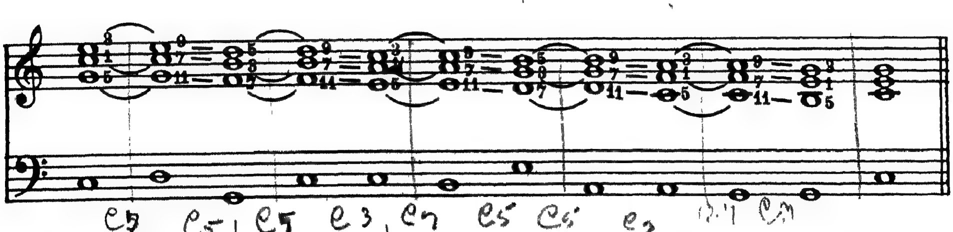


Figure 325. Triple suspensions.

Mixed Suspensions: By combining single, double and triple suspensions, one can achieve considerable variety:

Figure 326. Mixed forms of suspensions.

Ascending Suspensions: We may also obtain *ascending* suspensions by inverting the positive form into position ⓓ.*

Geometrical Inversion.

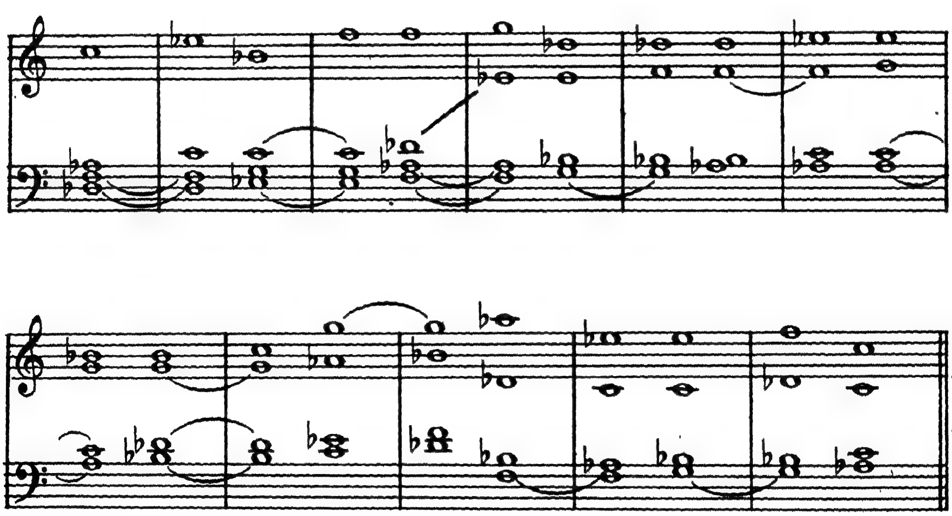


Figure 327. Geometric inversion: figure 326 in position ⓓ.

*As explained in Book III concerning geometrical and tonal inversions—position ⓓ, the reader will recall, is the original upside down; a geometrical inversion preserves the exact intervals, whereas a tonal inversion alters the intervals to conform to the original or some other key. (Ed.)

Tonal Inversion: By canceling the accidentals or by readjusting them, we obtain the same, but in the original—or any other—key.

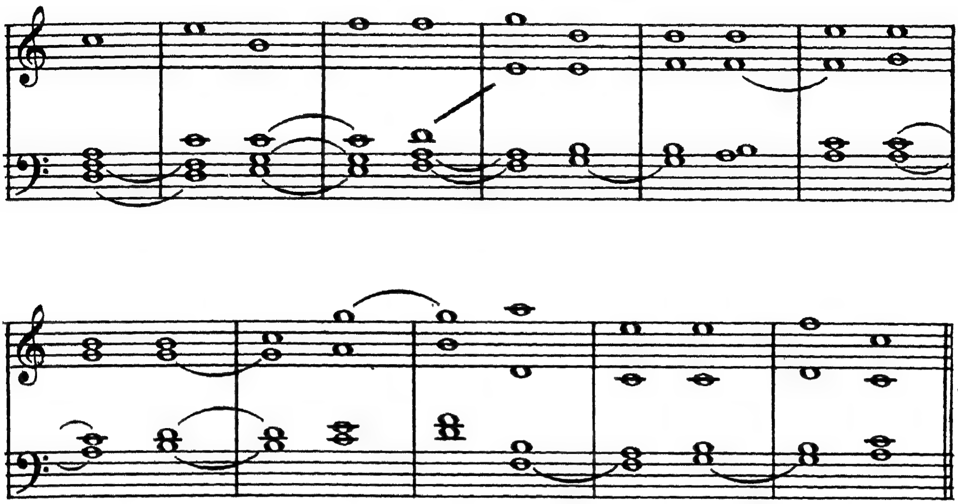


Figure 328. Tonal inversion.

B. Passing Tones

Single Passing Tones: Single passing tones may be produced by moving the 1 downward to 7, stepwise. The particular sequence of voices in which the passing tone will then appear depends upon the particular cycles and the voice-leading.

If passing tones are desirable in one continuous voice, the procedure should be carried out through the procedure I have already described in *generalization of the passing seventh.** A progression of $S(5)C_3$ const. must be written first; the passing sevenths are then inserted afterward. When this procedure has been completed, the bass may then be placed into any other voice (geometrical variation of positions).

Figure 329. Single passing tones.

*See pages 534-6. [Ed.]

Double Passing Tones: Double passing tones are either the passing 7 and 9 obtained by means of downward motion from the 1 and 3, or the 3 and 5, or the 5 and 7—or are parallel and contrary passing tones of the group G^6_4 and G^4_3.

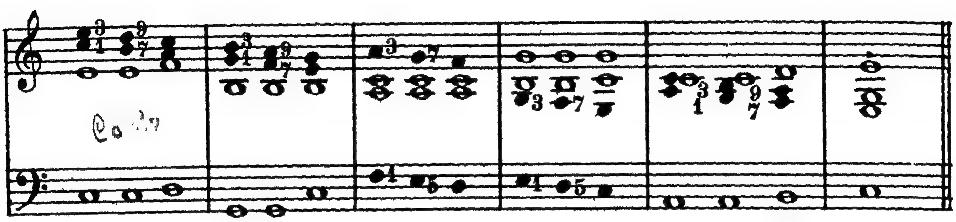


Figure 330. Double passing tones.

Triple Passing Tones: Triple passing tones are 7, 9 and 11 obtained by means of downward motion from the 1, 3 and 5; this corresponds to a preparation of S(11) in C_0.

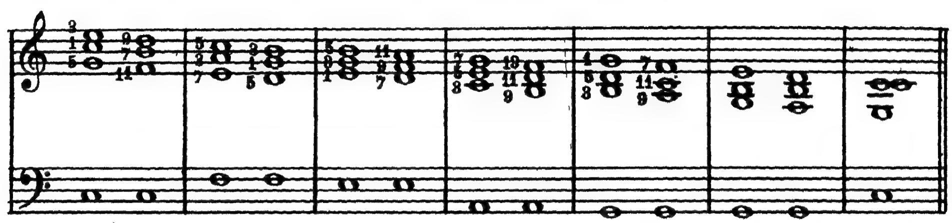


Figure 331. Triple passing tones.

Mixed Forms: Single, double and triple passing tones can be combined in one harmonic continuity.

Figure 332. Mixed forms of passing tones.

Since passing chromatic tones were discussed previously, there is no need for additional illustrations. Chromatic passing tones may be used in combination with diatonic passing tones.

Ascending Passing Tones: Ascending passing tones may be obtained through geometrical or tonal inversions ⓓ. Here are two inversions of Figure 332.

Figure 333. Geometrical inversion of figure 332.

Figure 334. Tonal inversion of figure 332.

Suspended Tones in a Given Chord Progression: So far we have discussed the techniques of suspended and passing tones evolved during the process of composing harmonic continuity—i.e., the H$^{\rightarrow}$ was not already set.

Now, however, we shall develop the technique of producing suspensions in a *given* harmonic continuity. In addition to the standard forms of suspensions, we shall use *delayed resolutions* for this technique—that is, suspensions in which the dissonant (higher numbered) functions become temporarily consonant (lower numbered)—and then resolve in the customary way. Root, third and fifth may also be suspended if the seventh is held. As long as the structures are properly represented during the period of suspension* any functions can be suspended.

Figure 335. Harmonic progression serving as a theme.

*That is, so long as the tones at the moment of suspension comprise altogether an acceptable (5) of some kind. (Ed.)

Figure 336. Variation by means of suspensions.

The simplest way to obtain *ascending* suspensions is to write the harmonic progression first, then to evolve suspensions, and finally to re-write the result into position ⓓ. Otherwise, the original harmonic progression must be written in a consistently *negative* form, and the suspensions must be evolved through 1, 1 and 3, or 1, 3 and 5—resolving upward.

Passing Tones in a Given Chord-Progression: By combining diatonic and the chromatic passing tones, a corresponding variation can be evolved. Using Figure 335 again as a theme, we obtain the following:

Figure 337. Variation by means of passing tones (Diatonic and chromatic).

C. Anticipations

Anticipated tones may be evolved from any chordal function of the following chord, provided that such a function is not the same in pitch as the voice in which the anticipation occurs. The nomenclature I use is: anticipated root, $\rightarrow 1$; third, $\rightarrow 3$; etc.

Single anticipated tones may be evolved to the root, to the third, to the fifth, to the seventh—or to any higher chordal function which is actually present in the following chord. Such forms may be called *anticipations of a constant chordal function.*

In addition to this form, *anticipations of variable chordal functions* may be used, and these may be selected at random. They provide greater variety in the quality of tension, whereas the first form provides a unity of tension. Both forms may be evolved for any harmonic continuity, as shown in the example:

Anticipated I.

Theme: Type I

Variation: $\rightarrow 1$

Figure 338. Anticipation of a constant chordal function.

Theme: Type II

Figure 339. Anticipation of a constant chordal function (continued).

Variation: ⟶ 1

Figure 339. Anticipation of a constant chordal function (concl ıded).

Theme: Type III

Variation: ⟶ 1

Figure 340. Anticipation of a constant chordal function

Anticipated 3.

Type I: Variation: ⟶ 3

Type II: Variation: ⟶ 3

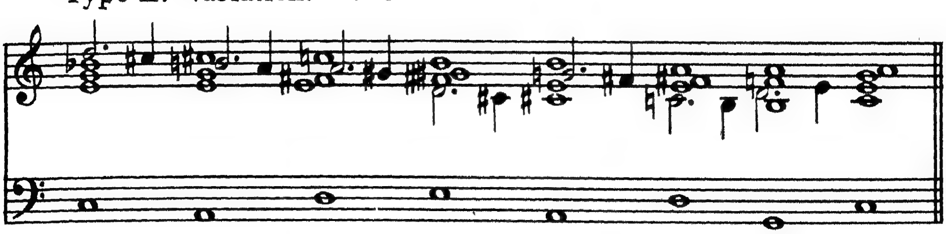


Figure 341. Anticipation of a constant chordal function (continued).

Type III: Variation: → 3

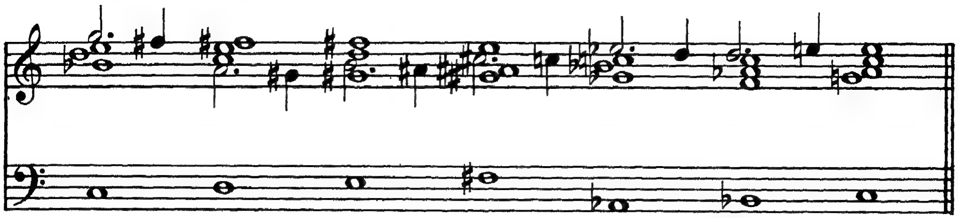


Figure 341. Anticipation of a constant chordal function (concluded).

Anticipated 5.

Type I: Variation: → 5
(common tone) due to cycles and structures

Type II: Variation: → 5
(common tone) due to cycles and structures

Type III: Variation: → 5

Figure 342. Anticipation of a constant chordal function.

Anticipated 7.

Type I: Variation: → 7
(common tone)

Type II: Variation: → 7
(common tone)

Type III: Variation: → 7

Figure 343. Anticipation of a chordal tone.

2. Anticipation of a Variable Chordal Function

Theme: Figure 335.

Variation: variable anticipated chordal functions

Figure 344. Anticipation of a variable chordal function.

Double and Triple Anticipations

Theme: Type III: $\sqrt[6]{2}$, S(9) const.

Variation: Anticipation of variable chordal functions (double and triple)

Figure 345. Double and triple anticipations.

Combined devices: suspended, passing and anticipated tones.

Theme: figure 335.

Variation: evolving combined devices to a given harmonic progression.

Figure 346. Combined devices.

CHAPTER 22

AUXILIARY TONES

AUXILIARY tones, being harmonically unmotivated, may be evolved in a given harmonic continuity. Any chordal function may be preceded by an upper or lower auxiliary tone. The interval *between* the auxiliary and the chordal tone depends on the type of harmony.

In diatonic progressions, auxiliary tones may be either diatonic—i.e., based entirely on those pitch units which produced the harmony itself and hence based on one definite diatonic scale—, or chromatic, i.e., arrived at by free selection among the leading tones which do not exist in the given scale.

As I noted earlier, diatonic auxiliary tones accentuate the diatonic character of the harmony and, because of previous associations, produce in us an impression of "ecclesiastic" music. This is particularly true when such scales as natural major are used. At the same time, some of the derivative scales of the same natural major, when they are supplied with diatonic auxiliary tones, may not produce this "ecclesiastic" impression but rather suggest such styles of harmonic writing as are to be found in compositions by Ravel and, particularly, by Debussy when these composers do express themselves diatonically.

Because of our previous experiences, we have established many auditory habits, among them an especially keen, critical perception of auxiliary tones. When chord progressions evolve diatonically from familar scales, we anticipate *a priori* certain definite forms of auxiliary tones. For example, in $S_2(5)$ the $\searrow 3$—i.e., an upper auxiliary tone (descending) to the third of a minor triad—must be $i = 2$, i.e., an interval of two semitones. If, instead, the movement is over one semitone, an ordinary listener will regard such an auxiliary tone as a "wrong note." The same is true for $S_2(5) \searrow 1$; our ears want the i (interval) to equal 2. The real cause of these reactions is the fact that ordinary listeners are familiar with the harmonic minor in which scale such auxiliary tones are diatonic. What seems wrong to the listener when $i = 1$ would sound perfectly natural if he were familiar, by ear, with d_2 of harmonic major; in such a case, in the key of C ($c - d - e - f - g - a\flat - b$), the root triad would be $e - g - b$ and the discussed auxiliary tones would then be $f \rightarrow e$ and $a\flat \rightarrow g$, with $i = 1$.

In harmonic progression of Types II and III, the auxiliary tones are governed by the master-structure (Σ) of each chord.* This means that the auxiliaries are diatonic during each individual chord. If a certain Σ produces $\nearrow 5$, $i = 1$, and such a Σ is used throughout, then all cases of $\nearrow 5$ must have $i = 1$. On the

*A Σ (the Greek letter, *sigma*) is an expansion of a scale; in Schillinger's *special* harmony, as in the present discussion, it is the first tonal expansion, E_1, of some 7-note scale, as explained by him in his discussion of pitch scales. A Σ of the C major scale would be, for example, C – E – G – B – D – F – A reading upwards. This Σ is the master-structure from which the substructures S(5), S(7), S(9), etc., are derived. The Σ concept is the source of many of the brilliant harmonic and orchestral effects which Schillinger pupils—and *only* Schillinger pupils, so far as we are aware—use in their music and in their arrangements. (Ed.)

other hand, in progressions based on more than one Σ, the respective differences will affect the intervals of the auxiliary tones. For example, if we compare two Σ's—$c - d - e - f\sharp - g - a - b\flat$, with $c - d - e - f - g - a - b\flat$—we find that in $\Sigma_1 \nearrow^5$, $i = 1$ (c-chord: $f\sharp \rightarrow g$), whereas in $\Sigma_2 \nearrow^5$, $i = 2$ (c-chord: $f \rightarrow g$).

If chromatic auxiliary tones are used in progressions of Types II and III, they have to be pre-set in definite relations to the pitch-units of the given Σ. For instance, in Σ_1 ($c - d - e - f\sharp - g - a - b\flat$) of the preceding example, we may introduce a $\nearrow^{13}$, $i = 1$ (i.e., $g\sharp \rightarrow a$ of the c-chord), which is not in the Σ. In this case, one would have to transpose such an auxiliary tone to each chord of identical Σ, whenever the auxiliary tone is to be used. (See page 824, footnote.)

A melodic form containing directional units may start either on a chordal or on an auxiliary tone. However, *it must end with a chordal tone.* Taking this into consideration, we may now evolve many melodic forms of different complexity and character.

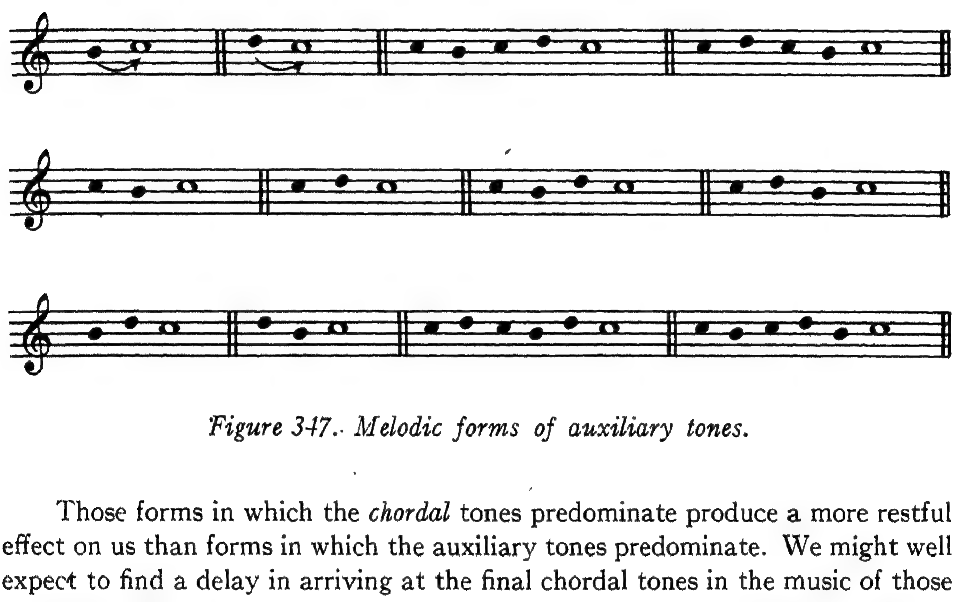


Figure 347. Melodic forms of auxiliary tones.

Those forms in which the *chordal* tones predominate produce a more restful effect on us than forms in which the auxiliary tones predominate. We might well expect to find a delay in arriving at the final chordal tones in the music of those composers who express (intentionally or unintentionally) "longing," "restlessness" and "dissatisfaction." And, indeed, Chopin and Chaikovsky have each the same style of handling auxiliary tones; the difference between their respective styles in this regard lies mostly in the particular intervals between the chordal tones and in the predominance of the *a* axis in Chaikovsky and the *b* axis in Chopin.* Mozart's music already had developed some of the chromatic auxiliary tones which became prominent later in Chopin, Schumann and Chaikovsky. Beethoven, whose music suggests to us a more masculine character, uses a decided predominance of chordal tones in figures containing auxiliaries; the latter

*The *a* axis is, of course, the secondary melodic axis leading upward *from* the primary axis; the *b* axis is the secondary axis leading downward *to* the primary. (Ed.)

usually conform to those well-known melismatic developments commonly known as *gruppetti.* Scriabine uses delays still more exaggerated than in the music of Chaikovsky or Chopin. And a Wagnerian characteristic is his simple directional units used with chromatic auxiliaries superimposed upon chromatic harmonic continuity.

J. S. Bach

Mozart

Beethoven

Chopin

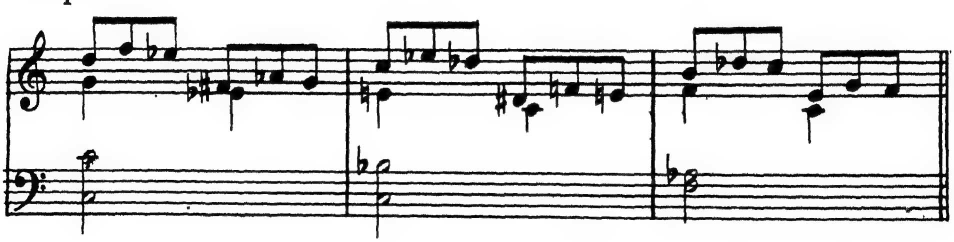


Figure 348. Melodic forms produced by auxiliary and chordal tones, typical of different composers (continued).

Chaikovsky

Scriabine

Wagner

Figure 348. Melodic forms produced by auxiliary and chordal tones, typical of different composers (concluded).

The auxiliary tones we wish to use in any case may be pre-selected either as (a) auxiliaries to a definite chordal function, or (b) auxiliaries to a group of chordal functions, and may appear in one or more voices simultaneously. We shall consider such forms to be *thematic*.

We could classify all ascending and descending forms of auxiliary tones for one, two, three and four voices, but practically speaking, we have a choice of direction (ascending or descending) depending on the case.

Classification of single auxiliary tones

↗ 1	↘ 1	↗ 3	↘ 3	↗ 5	↘ 5	↗ 7	↘ 7

Classification of double auxiliary tones

↗ 3	↘ 3	↗ 3	↘ 3	↗ 5	↘ 5	↗ 5	↘ 5
↗ 1	↗ 1	↘ 1	↘ 1	↗ 1	↗ 1	↘ 1	↘ 1

↗ 7	↘ 7	↗ 7	↘ 7	↗ 5	↘ 5	↗ 5	↘ 5
↗ 1	↗ 1	↘ 1	↘ 1	↗ 3	↗ 3	↘ 3	↘ 3

↗ 7	↘ 7	↗ 7	↘ 7	↗ 7	↘ 7	↗ 7	↘ 7
↗ 3	↗ 3	↘ 3	↘ 3	↗ 5	↗ 5	↘ 5	↘ 5

This table can, of course, be extended to include higher chordal functions.

Classification of triple auxiliary tones

↗ 5	↘ 5	↗ 5	↗ 5	↘ 5	↘ 5	↗ 5	↘ 5
↗ 3	↗ 3	↘ 3	↗ 3	↘ 3	↗ 3	↘ 3	↘ 3
↗ 1	↗ 1	↗ 1	↘ 1	↗ 1	↘ 1	↘ 1	↘ 1

↗ 7	↘ 7	↗ 7	↗ 7	↘ 7	↘ 7	↗ 7	↘ 7
↗ 3	↗ 3	↘ 3	↗ 3	↘ 3	↗ 3	↘ 3	↘ 3
↗ 1	↗ 1	↗ 1	↘ 1	↗ 1	↘ 1	↘ 1	↘ 1

↗ 7	↘ 7	↗ 7	↗ 7	↘ 7	↘ 7	↗ 7	↘ 7
↗ 5	↗ 5	↘ 5	↗ 5	↘ 5	↗ 5	↘ 5	↘ 5
↗ 1	↗ 1	↗ 1	↘ 1	↗ 1	↘ 1	↘ 1	↘ 1

↗ 7	↘ 7	↗ 7	↗ 7	↘ 7	↘ 7	↗ 7	↘ 7
↗ 5	↗ 5	↘ 5	↗ 5	↘ 5	↗ 5	↘ 5	↘ 5
↗ 3	↗ 3	↗ 3	↘ 3	↗ 3	↘ 3	↘ 3	↘ 3

Figure 349. Single, double, and triple auxiliary tones.

This table can also be extended to include higher chordal functions. A table for four simultaneous functions can be devised in a similar fashion. These tables are to be used as guides in the choice of pre-selected groups of directional units.

For instance: ↗ 1 + ↘ 5 + ↘ 7;
↘ 3 + ↗ 3 + ↘ 5;
↗ 5 + ↘ 3 + ↗ 1; etc.

Coefficients of recurrence of any type and form are also applicable to this problem. Examples:*

$r_{3\div 2}$ ↗ 1 + ↗ 1 + ↘ 1 + ↗ 1 + ↘ 1 + ↘ 1
$r_{3\div 2}$ ↗ 1 + ↘ 1 + ↗ 3 + ↘ 1 + ↗ 3 + ↘ 3
$r_{4\div 3}$ ↗ 1 + ↘ 1 + ↗ 1 + ↗ 3 + ↘ 5 + ↘ 5 + ↗ 7 + ↗ 7 + ↘ 3 +
+ ↘ 1 + ↗ 1 + ↘ 1

Each directional unit in the example above applies to one chord.

Another way of selecting the sequence of auxiliary tones is by the parts. The sequence of soprano (S), alto (A), tenor (T) and bass (B)—SATB—or any variation thereof (of which there are 24 for four-part harmony) permits us to have full control over the order in which the auxiliary tones appear. When such a harmonic continuum is orchestrated (vocally or instrumentally), the *sequence of definite voices or instruments* as they enter with a certain figure becomes a matter of considerable importance. We shall consider these forms to be *neutral.* A more detailed specification is possible through the assignment of directions to the sequence of parts: for intance: ↗ T + ↘ B + ↘ A + ↗ S.

These groups are, of course, subject to variation by means of permutations or by means of coefficients of recurrence. Example:

SATB + ATBS + TBAS + BSAT
2 SATB + BTAS + SATB + 2 BTAS
4 TSAB + 2 ABTS + 2 TSAB + ABTS

There is still another way of selecting the sequence of auxiliary tones through several parts, following the principle of "reciprocity" or free choice.

Examples of reciprocity:

	S			S		S		
A		;		A	;		A	; . . .
T			T			T		
	B		B				B	

	S		S			S		
A		;	A		;	A		; . . .
T			T				T	
B				B		B		

Examples of free selections:

S			S	S		S			S		
A	A	;	A		;		A	;	A	A	; . . .
	T			B		B	B		T	T	
										B	

*In the first example the $r_{3\div 2}$ (2+1+1+2) is applied in turn to ↗ and to ↘; in the third example, the $r_{4\div 3}$ is applied to the 1, 3, 5 and 7, in turn, with the ↗ and ↘ following a 1+1+2+2+2+1+1 pattern. (Ed.)

Application of Auxiliary Tones

Diatonic Progressions,
Diatonic and Chromatic
Auxiliary Tones.

(1) Single auxiliary tones- constant chordal function

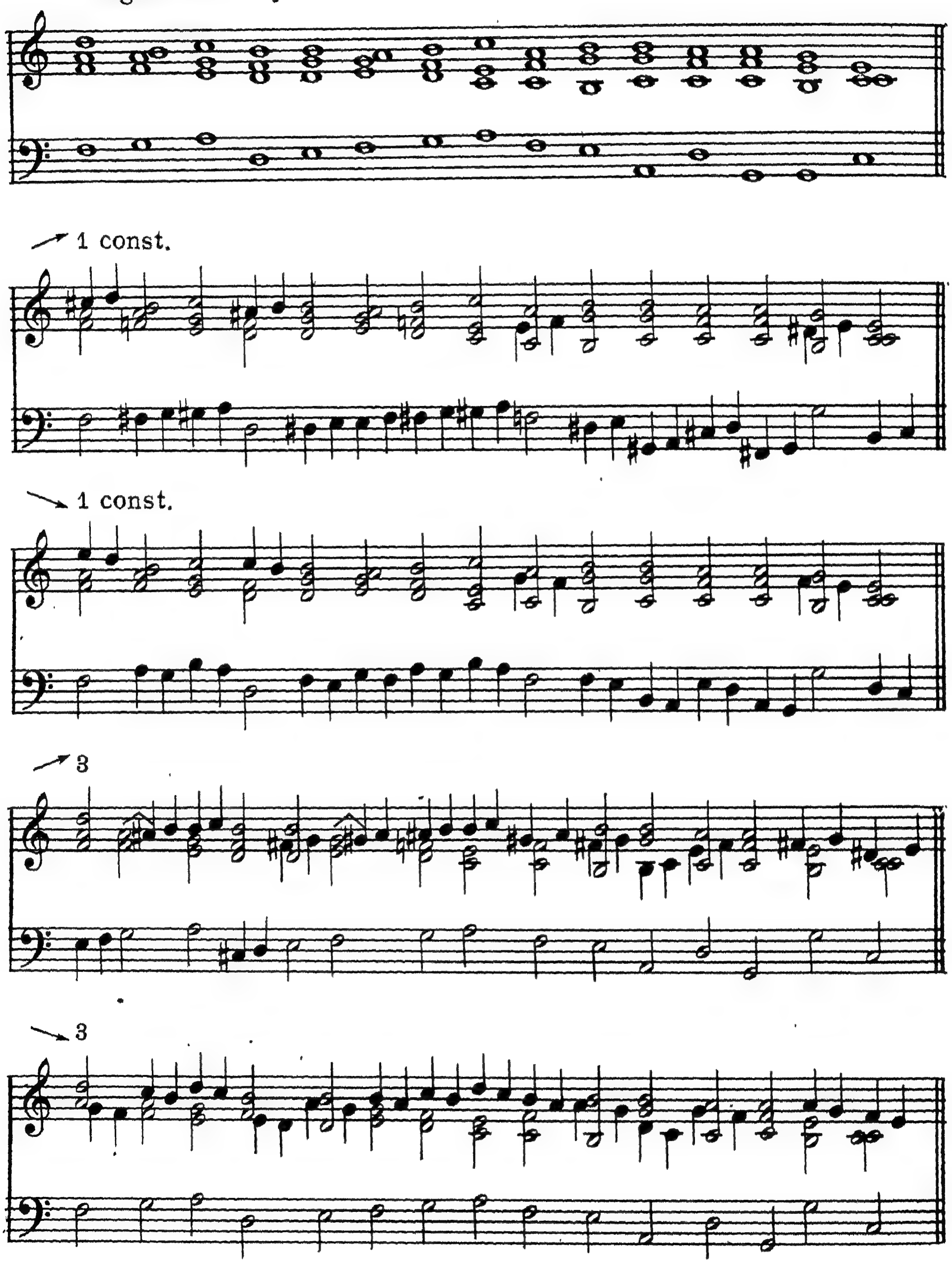


Figure 350. Single auxiliary tones; constant chordal function (continued).

Figure 350. Single auxiliary tones; constant chordal function (concluded).

Variable Chordal Functions. (A) Neutral Selection Through the Sequence of Parts.

↗ 3 + ↘ 5 + 1. (B) Thematic Selection through Pre-set Chordal Functions.

Figure 351. Variable chordal function.

(2) Double Auxiliary Tones. (A) Neutral Selection through the Sequence of Parts.

(B) Thematic Selection through Pre-set Chordal Functions. 3 1 / 5 3

Figure 352. Double auxiliary (continued).

Figure 352. Double auxiliary (concluded).

(3) Triple Auxiliary Tones. (A) Neutral Selection through the Sequence of Parts.

(B) Thematic Selection through Pre-set Chordal Functions. $\begin{matrix} & 9 \\ 5 & 7 \\ 3 & 3 \\ 1 & \end{matrix}$

Figure 353. Triple auxiliary.

Diatonic-Symmetric Progressions,
Diatonic and Chromatic Auxiliary Tones.

Theme: Type II

Variation: → 9

Variation: → 9 → 3

Variation: SATB (Diatonic auxiliaries)

Figure 354. Auxiliary tones in diatonic-symmetric progressions.

Symmetric Progressions,
Diatonic and Chromatic Auxiliary Tones.

Theme: Type III: $\sqrt[6]{2}$ S (9) const.

Variation: ↗ 5 (Diatonic auxiliary)

Variation: ↘ 7 ch, ↗ 5 d, ↗ 3 ch

Variation: BTASB

Figure 355. Auxiliary tones in symmetric progressions.

CHAPTER 23

NEUTRAL AND THEMATIC MELODIC FIGURATION

BY combining all the devices using the *suspended*, the *passing*, the *anticipated*, and the *auxiliary* tones, we attain the final—and fully versatile—form of melodic figuration.

We shall distinguish the two forms of it:

(1) Neutral melodic figuration.
(2) Thematic melodic figuration.

Neutral melodic figuration may be effected in the following forms:

(a) Free development of resources *without* preliminary planning; this corresponds to that technique, the best examples of which are to be found in J. S. Bach's 371 chorals.
(b) Free selection of resources, but with *preliminary planning* of the sequence of parts in which the figuration is to appear.

Theme: Type I

(A)

Figure 356. Neutral melodic figuration (continued).

(B) TABS + ABST + BSTA + STAB

(B)

Figure 356. Neutral melodic figuration (concluded).

Thematic melodic figuration, however, presupposes that the motif to be used throughout the different parts of the harmonic continuity will be selected in advance. A motif to be put to such use must be approached, first, as a *group* of both chordal and non-chordal tones. Ascertaining which tones are in fact the chordal tones is a process based on the principle that, in every seven-unit scale, either the *first* or the *second* pitch-unit is a chordal tone. This gives us two possible definitions to any scale—

(1) c d e f g a·b

—in which c, e, g, b are selected as the chordal tones;

or—

(2) c d e f g a b c

—in which d, f, a, c are to be the chordal tones.

The non-chordal tones then become either auxiliary or passing tones.

Once the chordal functions are designated, their assignment must be performed from the *axis* of the motif. If this axis is not sufficiently prominent, then any arrangement of units by thirds may suggest the position of chordal tones in the motif.

In other cases, the chordal tones may best be detected by elimination of all the accidentals which do not belong to the (real) key signature. Analysis of an actual motif and assignment of the chordal tones will illustrate this process:

Figure 357. Analysis of a motif.

In this case the grouping of thirds is quite apparent for a, c and e are obviously the chordal functions; g# is the lower auxiliary tone to a, and f is the upper auxiliary tone to e. It is understood, in this example, that the entire motif must be superimposed on one chord. The grouping of chordal tones is as follows:

The next step is to assign any one of the seven possible systems to our reading of chordal functions; we may select from the following:

Figure 358. Assigning chordal functions.

Inasmuch as the axis of this motif obviously falls on e, we have to bear in mind that whatever chordal function we select for the motifs, starting chordal function must also be present in the chord itself. For example, if e becomes assigned to function as the ninth, we must start on the fifth.

This assignment of chordal functions in a motif may be either constant or variable. In the first case, chords of a certain tension are required. If the starting point becomes a ninth, all chords must be S(9), as a minimum form of tension. Assigning the axis, in the above case, to a seventh, the starting point in the chord will be a third.

In the variable assignment of chordal functions, the sequence in which the motif appears in the different parts is controlled by the SATB arrangement (24 fundamental forms).

In the constant assignment of chordal functions, the *sequence* in which the motif appears in the different parts is controlled by voice-leading which will necessitate the appearance of the assigned chordal function in some specified voice.

In using thematic melodic figuration, it is advisable to have open positions of the chords so as to provide sufficient range for the motif to move.

Examples of Thematic Melodic Figuration
(Diatonic Progressions)

(1) Constant assignment of chordal functions.
  (a) 1, 3, 5 (axis placed on the fifth) operation from the root.
  The missing functions are compensated (marked with cross) and the original voice-leading resumed.
  (b) 3, 5, 7 (axis placed on the seventh) operation from the third.
  (c) 5, 7, 9 (axis placed on the ninth) operation from the fifth.

See the corresponding music examples on the following pages.

Figure 359. Thematic melodic figuration. Diatonic progressions (continued).

(c)

Figure 359. Thematic melodic figuration (concluded).

(2) Variable assignment of chordal functions

(a) SATB
(b) BTAS
(c) ABST

2 (a)

(b)

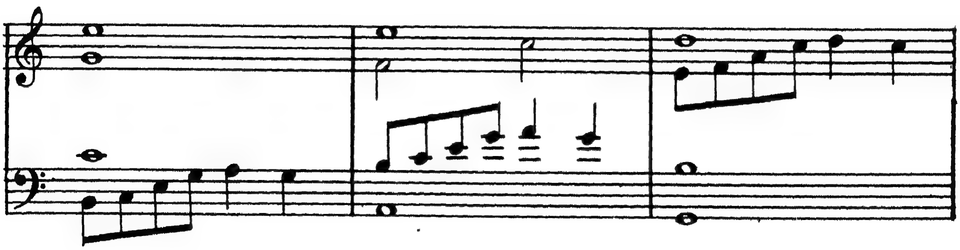


Figure 360. Variable assignment of chordal functions (continued).

Figure 360. Variable assignment of chordal functions (concluded).

In progressions of Types II and III, *at least* the chordal tones of the thematic motif must conform to the particular Σ (13) to be carried out through the harmony. Example:

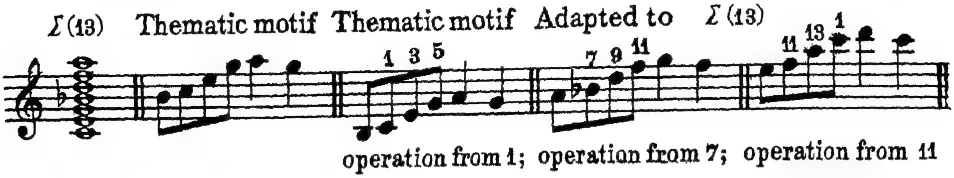
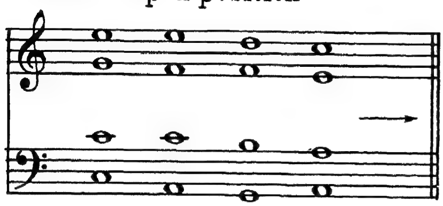


Figure 361. Chordal tones must conform to Σ in progressions of Types II and III.

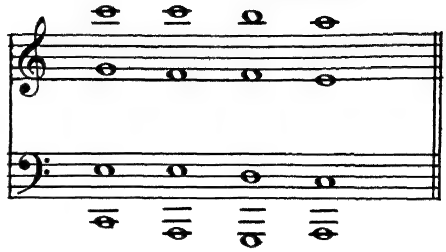
Any crossing of adjacent voices by the thematic motif is undesirable. Compensation of the missing tones during the period of figuration is desirable but unnecessary, particularly in fast tempi.

The range of some thematic motifs is so great that they are bound to cross adjacent voices; in such a case the harmonic continuity has to be rearranged into *extra-open* position, with the original voice-leading preserved. Example:

Theme: Open position



Theme: Extra-open position



Thematic motif

Variation

Figure 362. Using extra-open position to avoid crossing of adjacent voices.

CHAPTER 24

CONTRAPUNTAL VARIATIONS OF HARMONY

WHEN different parts of a harmonic continuity enter and drop out at different time intervals, the continuity acquires contrapuntal* characteristics. This effect arises from greater independence of the voices; it can be accomplished by operations upon any type of harmonic continuity. The sequence in which the different parts may enter or drop out is naturally subject to permutations.

Any three-part harmony offers us six variations for either entering or dropping out, making a total of 12 variations:

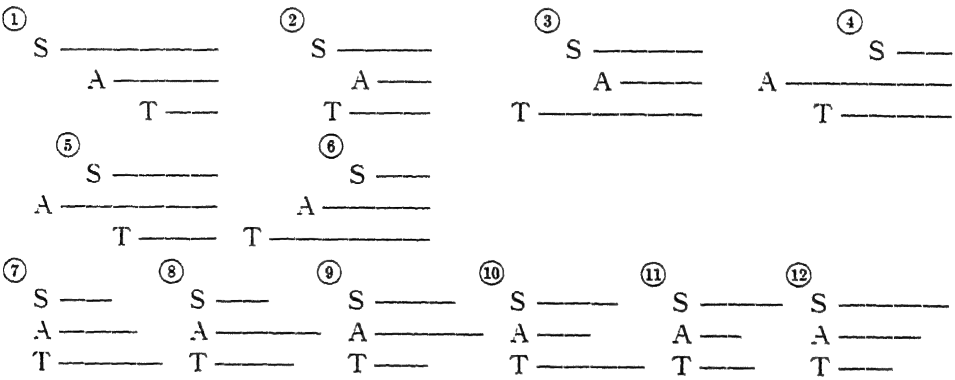


Figure 363. Contrapuntal variations of three-part harmony.

This table can be reduced** to three variations each way (through circular permutations) making a total of 6 variations.

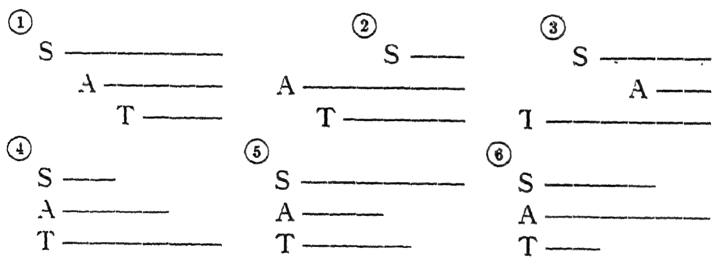


Figure 364. Contrapuntal variations of three-part harmony.

*Indeed, it is Schillinger who gives to this matter of sequence and interval of entrance and dropping out its proper emphasis in counterpoint itself, as will be seen later, in contrast to the customary emphasis on the techniques of simultaneous melodic lines. Consequently, the usefulness of the techniques described in this chapter cannot be overestimated. (Ed.)

**Reduction becomes desirable when the full set of 12 general permutations provide too much raw material, and when some *casual* selection of fewer than 12 would lack the logic of the 6 circular permutations—a lack that would be reflected in the resulting music. (Ed.)

Likewise, any four-part harmony affords 24 permutations each way, making a total of 48 variations. This can be reduced through circular permutations to 4 variations each way, making a total of 8 variations.

In five-part harmony, general permutations produce 120 variations each way, making a total of 240 variations. This can be reduced through circular permutations: 5 variations each way, making a total of 10 variations.

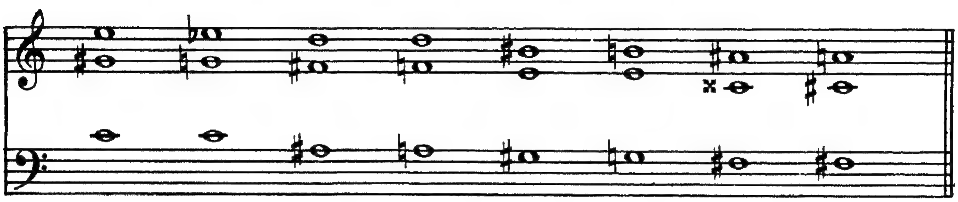
(1) Three-Part Harmony.

(a) Theme: Type III: $\sqrt[6]{2}$, $[S_3(5) + S_2(5)]\ C_0$.

(b) Variation:

S —— S ——————
A ———— A ————
T —————— T ——

(1)(a)



(b)

Figure 365. Contrapuntal variation of three-part harmony.

(2) Four-Part Harmony.

(a) Theme: Type III: $\sqrt{2}$, $S_1(5) + [S_3(5) + S_2(5)]\ C_0$.

(b) Variation:

S ———————— S ————————
A ———— A ———
T ——— T ——
B —— B —————

(2)(a)

Figure 366. Contrapuntal variations of four-part harmony (continued).

(b)

Figure 366. Contrapuntal variations of four-part harmony (concluded).

(3) Five-Part Harmony.

(a) Theme: Type II.

(b) Variation:

S ——

A ——

T ——

B_I ——

B_{II} ——

(3) (a)

(b)

Figure 367. Contrapuntal variation of five-part harmony.

Deciding upon the number of attacks after which the next voice will enter or will drop out may be a matter of free selection and distribution. Or the number of attacks for each voice may be rhythmically arranged. Attack-groups may be composed either with or without interference in relation to the part-sequence group. For example, the part-sequence group might be distributed in one-to-one correspondence to the attack-group:

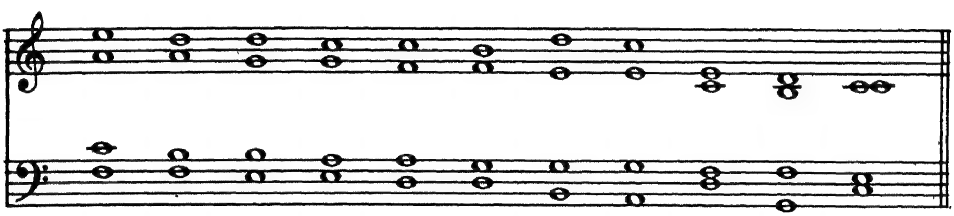
S ————————
A ——————
T ————
B ——

$A = 4a + 3a + 2a + 2a$

Then: $S4a + S3a + S2a + S2a$
$A3a + A2a + A2a$
$T2a + T2a$
$B2a$

Theme: Type I: 11 H.*
Variation.

Theme



Variation

Figure 368. Correlation of part-sequences and attack groups.

*The "11 H" we may read, of course, as "eleven harmonies"—harmonies rather than chords, for each H might be subjected to various attack patterns producing more than one "chord" for each H. (Ed.)

When several entrances produce different part-sequence groups, their interference against the attack-group offers the possibility that each voice may have a different number of attacks at each of its consecutive entrances. Example:

Part-sequence group:

S ——
A ————————
T ———————

$A = 3a + 2a$

The synchronized part-sequence group would then be:

S3a S2a
A3a + A2a + A3a + A2a + A3a + A2a
T2a + T3a + T3a + T2a

(The bass is excluded in the variation)

Theme: 15 H: Chromatic.
Variation.

Theme

Variation

Figure 369. Synchronizing part-sequences and attack groups.

It is this technique which enables us to obtain vocal or instrumental orchestration comparable to that found in the scores of the best composers of the past (Palestrina, Bach, Händel, Wagner, and others).

When this technique for the contrapuntal variation of harmony is applied to harmony that has already been subjected to melodic figuration (neutral or thematic), many more developed forms of counterpoint (including imitations) may be derived from harmony.

One of the advantages that "contrapuntalized" harmony has over counterpoint proper is that it permits complete control over the style or type of harmony *a priori*. Another advantage lies in the fact that this technique is incomparably easier than any purely contrapuntal technique. Still another advantage comes from the fact that it is possible to use such a contrapuntal variation against its own harmonic theme, the theme functioning as a harmonic background; the latter may take on, by means of patterns of attack, any instrumental form, such as (1) sustained chords, (2) staccato chords, (3) broken chords (arpeggio).

In all these cases, the counterpoint stands out against its own harmonic background (accompaniment), particularly when the background is sounded by instruments (or voices) different from the counterpoint itself. When these devices are applied to arranging (when the thematic motif is a fragment of a given piece), they produce very effective introductions, transitions, and conclusions (codas).

The following techniques for melodic or contrapuntal development of harmonic continuity may be suggested:

A. Neutral or thematic melodic figuration carried out in one voice. This, combined with other voices, produces a melody-with-accompaniment:

Theme: Figure 359.
Variation: Thematic Melodic Figuration in Soprano.

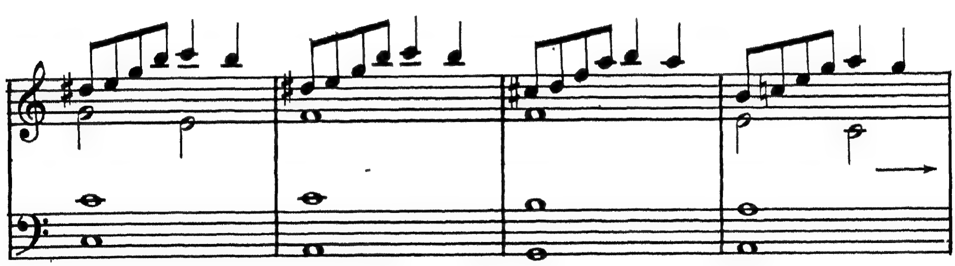


Figure 370. Melodic figuration in soprano.

B. Neutral or thematic melodic figuration carried out through all voices and assigned either to a sequence of chordal functions (Fig. 359, 1) or to a sequence of parts in which the motif appears (Fig. 360).

C. Neutral or thematic melodic figuration (as in B) with gradual entrances or gradual dropping out of voices. When such a form is based on thematic figuration, the result is a *fugato*, i.e., a group of imitations.

SATB

Figure 371. Melodic figuration in all voices.

It is easy to achieve the opposite effect by using the inverted position ⓑ.*

When a fugato is to be used as an introduction which is to have a duration of 4T, all that is necessary is to compose a 4H continuity so that the last chord will lead directly to the following exposition (an equivalent of an exposition in a song is the "chorus"). A fugato used as a modulating interlude between successive expositions (choruses) should be developed from a harmonic continuity that effects the desired modulation. A 4T introduction or interlude may also be constructed by making a three-part fugato with a cadence on the last chord (H_4).

When an 8T introduction or interlude is desirable, the thematic motif emphasizing one chord (H) should occupy a duration rhythmically to 2T.

Likewise, a 6T introduction or interlude may be constructed from 2T-per-H motifs in three-part Fugato with a 2T cadence at the end.

Theme: *Honey-Suckle Rose,*** by Thomas Waller.

Theme

Var. 1

Figure 372. Introductions or interludes on a given theme (continued).

*Inversion ⓑ is the original right side up but backwards. (Ed.)
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Var. 2

Var. 3

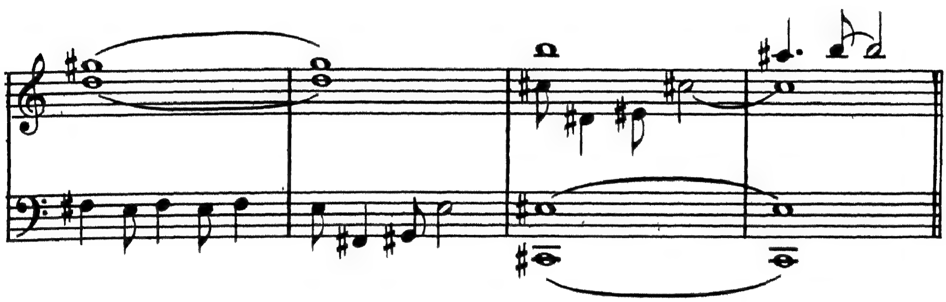


Figure 372. Introductions or interludes on a given theme (concluded).

D. Accompanied Fugato with constant or variable *density** in the harmonic accompaniment.

(1) Constant density in the harmonic accompaniment. Example shown on the following page.

(2) Variable density in the harmonic accompaniment; decreasing density in the accompaniment. Example shown on the following page.

(3) Variable density in the harmonic accompaniment; decreasing density in fugato, increasing density in the accompaniment. [Reverse the procedure of (2)].

**Density* is a term which will be explained more fully at a later point in the text; it is enough to say that it has to do with the total number of parts sounding, in relation to their distribution over the total practical range of instrumentation. (Ed.)

Figure 373. Accompanied fugato with constant or variable density.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK VI

THE CORRELATION OF HARMONY AND MELODY

BOOK SIX

THE CORRELATION OF HARMONY AND MELODY

CHAPTER 1

MELODIZATION OF HARMONY

THE composition of melody with its harmonic accompaniment can be accomplished either (a) by correlating the melody with a chord progression, or (b) by composing the melody to such a progression. While the former procedure is the one more commonly known—and attempts have even been made to develop a theory to this effect—it is the second procedure which has in fact brought forth music of unsurpassed harmonic expressiveness; many composers, particularly the operatic ones (among them, Wagner), composed the melodic parts of their music to harmonic progressions.

So far as my theory is concerned, the technique of harmonization of melody can be developed only if the opposite process is known. If melody can be expressed in terms of harmony, i.e., as a sequence of chordal functions and their respective tensions, then a scientific and universal method for the harmonization of melody can well be formulated by reversing the whole system of operations.

The process of composing melody to chord progressions thus becomes what I shall call the *melodization of harmony*.* The word "melodization" cannot now be found in English dictionaries, but we may be certain it will be found there soon, for the discovery of a new technique necessitates the introduction of a new operational concept.

At this point, I shall apply my theory of melodization to those particular harmonic progressions which satisfy the definition given earlier for the *Special Theory of Harmony*,** as distinct from *general* harmony*** which will be discussed considerably later. According to this definition, all chord-structures are based on E_1, the first expansion of those seven-unit scales which contain seven musical names without any identical intonations. So approached, any pitch unit of melody can be only one of these seven functions: 1, 3, 5, 7, 9, 11, or 13. These seven functions produce that manifold which I call the *scale of tension*. By arranging this scale of tension in a circular fashion, one obtains two harmonic directions: the clockwise, and the counterclockwise. See Figure 1 on the following page.

*This phrase, which designates one of Schillinger's most brilliant discoveries, refers to the construction of a melody to go with an $\overrightarrow{H}$ already constructed, in contrast to melodic figuration, which does not add any additional voice to those of which the $\overrightarrow{H}$ (harmonic continuum) is composed. (Ed.)

**See Book V.

***See Book IX.

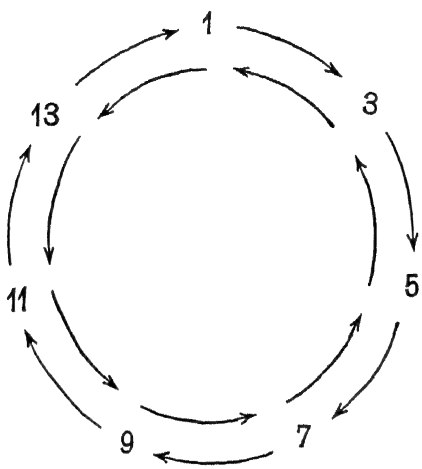


Figure 1. Scale of tension

Clockwise functioning of the consecutive pitch-units of a melody obtains the positive form of tonal cycles.

Counterclockwise functioning of the consecutive pitch-units of a melody obtains the negative form of tonal cycles.

If we assume, for example, that *all pitch-units* of a melody are *stationary and identical* and that we may therefore select any pitch-unit that is stationary, we may choose *c* as such a unit, for illustration. By assigning clockwise functioning to such a unit, the positive form of harmonic progressions is obtained:

	1	3	5	7	9	11	13	1	
Melody:	c	+ c	+ c	+ c	+ c	+ c	+ c	+ c	C_3
Chords:	C	+ A	+ F	+ D	+ B	+ G	+ E	+ C	

By reading the above progression backwards, the negative form is obtained.

Omission of certain of these chordal functions for the consecutive pitch-units of the melody will result in a change of *cycles* but not of direction.

	1	5	9	13	3	7	11	1	
Melody:	c	+ c	+ c	+ c	+ c	+ c	+ c	+ c	C_5
Chords:	C	+ F	+ B	+ E	+ A	+ D	+ G	+ C	

Likewise:

	1	7	13	5	11	3	9	1	
Melody:	c	+ c	+ c	+ c	+ c	+ c	+ c	+ c	C_7
Chords:	C	+ D	+ E	+ F	+ G	+ A	+ B	+ C	

It follows, from the above, that ***every chord has seven forms of melodization,*** insofar as the 1, 3, 5, 7, 9, 11 or 13 can be added as a melodic tone to the chord

itself. Reduction of the scale of tension decreases this quantity accordingly.

Let us consider all the reduced forms of the scale of tension to be the *ranges of tension*. When each chord is melodized by but one attack (or one pitch-unit), the range of tension can be entirely under control.

The minimum range of tension that is possible may be secured by causing but one chordal function to appear in the melody. Let us assume that such a function is the root-tone of the chord. Then, if harmony consists of three parts, the melody so obtained will sound like the bass of progressions of S(5) const.

For example:

$$2C_5 + C_3 + C_5 + 2C_7$$

Melody:	c	+ f	+ b	+ g	+ c	+ d	+ e	+ . . .
Chords:	C	+ F	+ B	+ G	+ C	+ D	+ E	+ . . .

Figure 2. Minimum range of tension: one chordal function in melody.

It is clear that the particular pattern of melody in such a case is conditioned by the cycles through which the roots of the chords move. Predominance of C_7 produces scalewise steps or leaps of the seventh. Other cycles influence the melodic pattern accordingly.

If we assign any *other* chordal function (but still *one* function for the entire progression), the resulting melodic pattern does not change, although the *form of tension* does vary. This time we shall use the 7 to melodize the same chord progression.

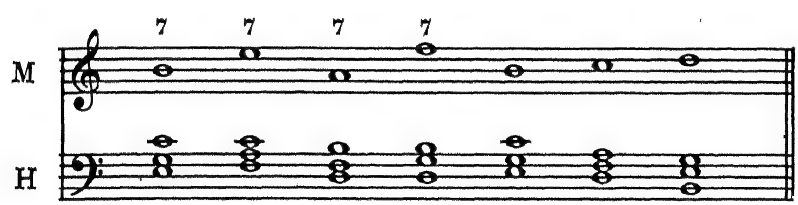


Figure 3. Using seventh to melodize chord progression of Figure 2.

Different ranges of tension produce different styles of melodization. Historically, melodization progresses clockwise through the scale of tension. A narrow range, confined to the lower functions, produces the more archaic or more conservative styles, and the resulting melodization may suggest Haydn or other early forms (I say "early," since in most cases such styles later become hackneyed);

but when a narrow range of tension is confined to higher rather than lower functions the result is melodization that suggests stylistically Debussy or Ravel. An intermediate form may produce characteristics of Wagner, or Franck, or Delius. And when the entire scale is used as a range of tension, i.e., from 1 to 13, the resulting melodization becomes highly flexible, indeed, in its expressiveness.

A. Diatonic Melodization

It follows from the preceding exposition that any chordal function may participate in melodization. The only procedure that remains to be effected is to assign chordal functions for melodization with regard to actual chord-structures. Let us denote melody as M and harmony as H. In terms of attacks, when one pitch-unit has been assigned to melodize each chord, the attack formula is $\frac{M}{H} = 1$. Under such conditions it is possible to evolve seven forms of melodization. For example, a C– chord may be melodized by c(1), or by e(3), or by g(5), or by b(7), or by d(9), or by f(11), or by a(13).

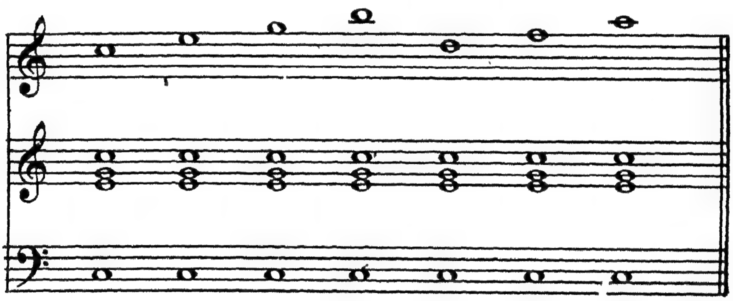


Figure 4. Seven forms of melodization when $\frac{M}{H} = 1$.

The majority of these pitch-units of M are satisfactory; two of them (d and f), however, do not result in satisfactory melodization. This is because such high functions, without support from the immediately preceding function in harmony are not ordinarily acceptable. Similarly, the presence of lower functions in the melodization of high-tension chords is equally unacceptable. The 13 is fully satisfactory, however, as melodization of S(5) because by sonority it converts an S(5) into S(7).*

*As a by-product of these circumstances, a special technique devised by Schillinger may be mentioned. It has been shown (1) that *any* triad will harmonize any tone in the same scale except the 9 and 11; (2) that the 9 is acceptable when two or more melody tones occur per H; (3) that a 9 or 11 not preceded by, respectively, a 7 or 9, is statistically rare in any combination of H and M; and (4) that the undesirable effects of an unsupported 9 or 11 are minimized in fast tempi when three or more melodic tones occur to each H. Now it happens to be so that *any* diatonic melody, provided it moves at $\frac{M}{H} = 3$ or more, may be harmonized by *any* progression of triads, S(5)—and, when S(7)'s are used, the results are still better. In this way, a 16-measure diatonic M^{-} where $\frac{M}{H} = 3$ may be constructed separately, and a 16-measure H^{-} of S(7) may also be constructed independently—and the two will "fit." (Ed.)

We can now construct the table of melodization for the fifth voice above four-part harmony when both melody and harmony are *diatonic.*

Table I: $\frac{M}{H} = 1$.

M	7, 13	9, 13	5, 11, 13	5, 13	5, 11	5, 9
		7	9	11	13	13
	5	5	7	9	9	11
H	3	3	3	7	7	7
	1	1	1	1	1	1
S	S(5)	S(7)	S(9)	S(11)	S(13)	

Figure 5. Table of melodization for fifth voice when M and H are diatonic.

It follows from the above table that:

(1) classical and hybrid four-part harmony can be used for diatonic melodization;
(2) all chordal tones actually participating in the chord as well as the functions designated as M can be used for diatonic melodization;
(3) by diatonic melodization we mean the participation of pitch units of one diatonic scale, from which scale the chord-progression is itself evolved; .
(4) the use of 13 in the melody with an S(7) is acceptable when the root of the chord is in the bass (i.e., do not use inversions);
(5) the alternatives that exist in the table for selection of functions for the melodization of S(13) arise from *two* forms of structures covered by hybrid four-part harmony.

Assuming that there are, on the average, about *five* practical pitch-units (functions) for the melodization of each chord through the form $\frac{M}{H} = 1$, the number of possible melodizations of one harmonic continuity (under such conditions) equals 5 to that power the exponent of which represents the number of chords. Thus a progression consisting of 8 chords produces $5^8 = 390,625$ possible melodizations!

The two fundamental factors which determine the quality and the character of melodization are:

(a) the range of tension;
(b) the melodic *pattern*, i.e., the axial combinations of melodic structure.

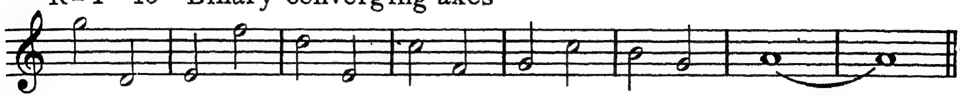
Interest may be concentrated on either one, or on both; attack-interference patterns give additional interest to melodization.

In the following example, R represents the range of tension, and A denotes the axial combination.* All the following examples may be played in any system of accidentals.

Figure 6. Diatonic melodization $\frac{M}{H} = 1$ (*continued*).

*Note that A is used in this portion of the text to denote a quite different aspect of the problem, that of *axial patterns* rather than *axes*. The axes are: *o*, zero axis, unchanging; *a*, *upwards from* the primary axis; *b downward to* the primary; *c*, *downward from* the primary; and *d*, *upward to* the primary axis. (Ed.)

R = 1—13 Binary converging axes



Three attack pattern

Figure 6. Diatonic melodization $\frac{M}{H}$ = 1 (*concluded*).

B. More than One Attack in Melody per H

Increase in the number of attacks of M per H requires a slight remodeling of Table I (Fig. 5). *Any higher function may be supported by the immediately preceding function of immediately preceding rank.* For instance, 9 may be used for melodization of S(5) if two conditions are met:

(1) it must be immediately preceded by 7, *and*

(2) the root of S(5) must be in the bass, a necessary condition for the support of 9. For the same reason, 11 can be used for melodization of S(7) if preceded by 9 *and* if S(7) has a root in the bass.

Additions to Table I:

$7 \rightarrow 9$	$9 \rightarrow 11$
	7
5	5
3	3
1	1
S(5)	S(7)

Figure 7. Table II: $\frac{M}{H} = 2, 3, 4, \ldots$

Figure 8. Diatonic melodization $\frac{M}{H} = 2$.

With the further growth of the number of attacks of $\frac{M}{H}$, greater allowances (particularly in fast *tempi*) can be made. This is particularly true of the use of "unsuitable" functions for melodization when such functions are used *as auxiliary tones moving into chordal tones*, i.e., chordal tones actually present in the harmonic accompaniment. Such styles of melodization (particularly in harmonic minor) may easily be associated with the music of Mozart, Chopin, Schumann, Chaikovsky and Scriabine, i.e., with the sentimental, romantic, lyrical type of melodization.

Examples of Diatonic Melodization.

$\frac{M}{H} = 3$

A = a

Figure 9. Diatonic melodization $\frac{M}{H} = 3$ (*continued*).

A = b

A = a + b

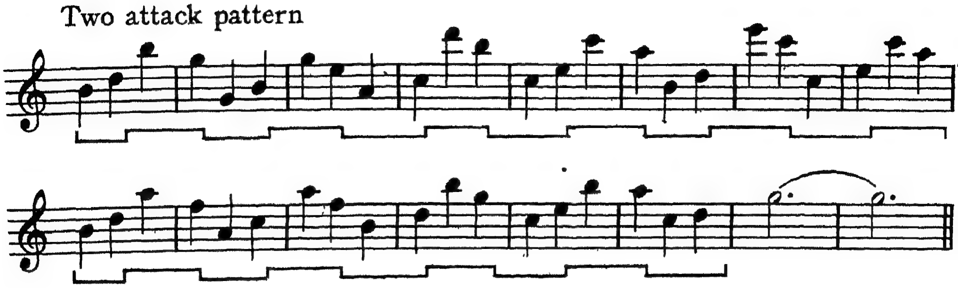
Binary parallel axes

Binary diverging axes

Binary converging axes

Figure 9. Diatonic melodization $\frac{M}{H} = 3$ (*continued*).

Two attack pattern



Four attack pattern

Figure 9. Diatonic melodization $\frac{M}{H} = 3$ (*concluded.*)

Examples of Diatonic Melodization

$\frac{M}{H} = 4$

Figure 10. Diatonic melodization $\frac{M}{H} = 4$ (*continued*).

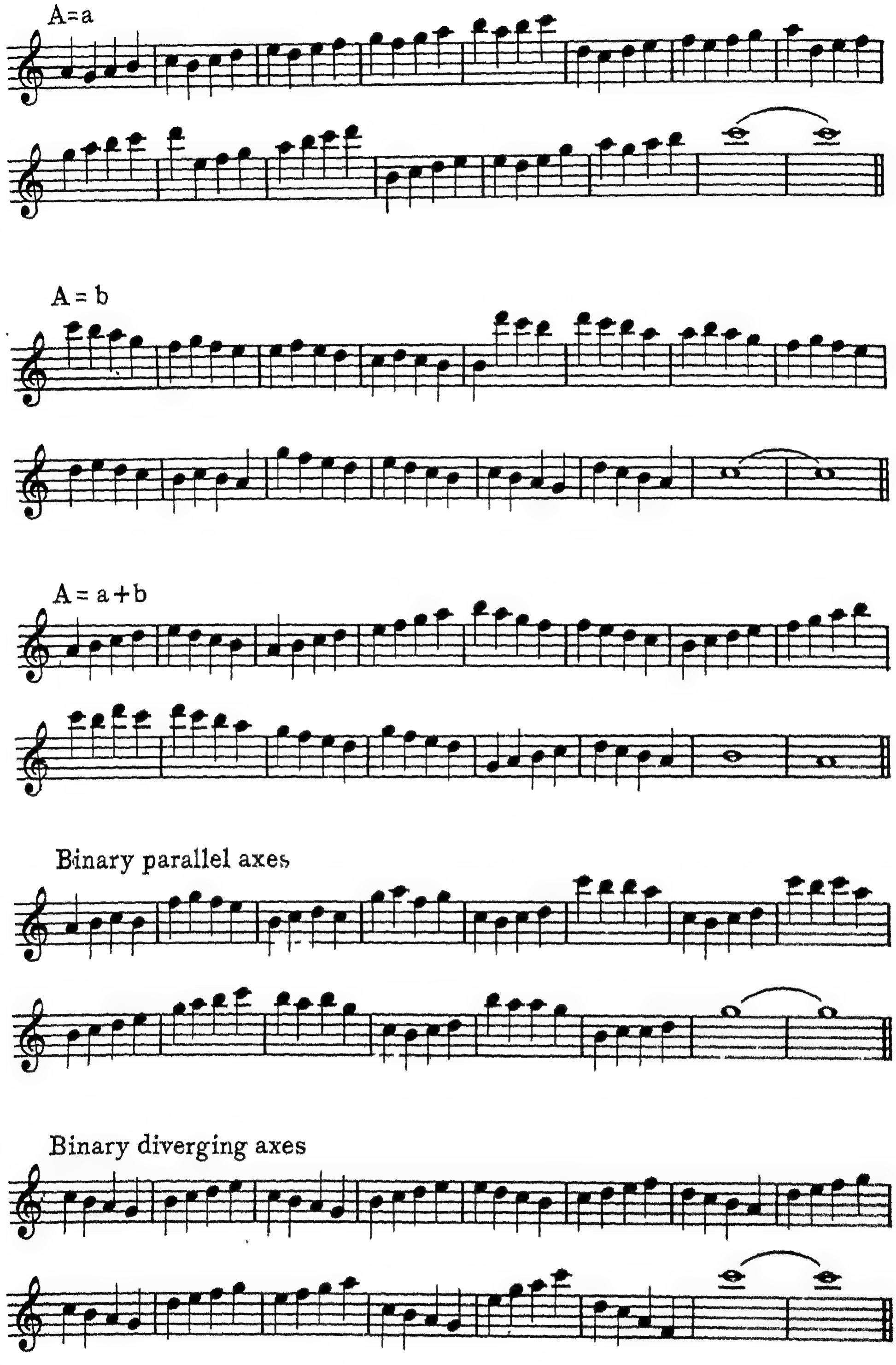


Figure 10. Diatonic melodization $\frac{M}{H} = 4$ *(continued).*

Binary converging axes

Three attack pattern

Five attack pattern

Figure 10. Diatonic melodization $\frac{M}{H} = 4$ *(concluded).*

Examples of diatonic melodization, when $\frac{M}{H} = 5$.

Figure 11. Diatonic melodization $\frac{M}{H} = 5$ *(continued).*

A = a + b

Binary parallel axes

Binary diverging axes

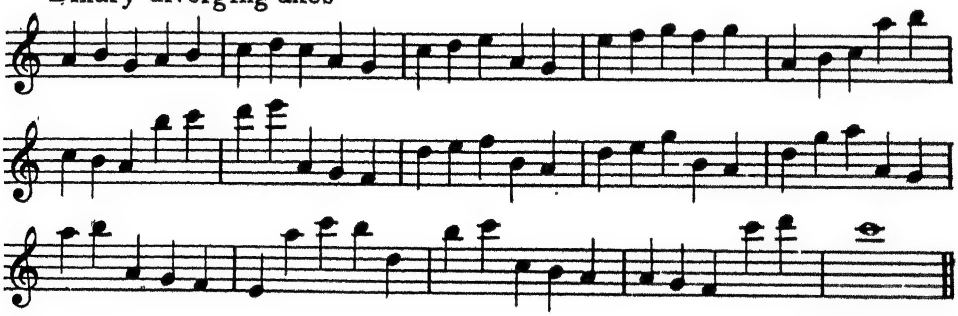


Figure 11. Diatonic melodization $\frac{M}{H} = 5$ (*continued*).

Three attack pattern

Figure 11. Diatonic melodization $\frac{M}{H} = 5$ (*concluded*).

Examples of diatonic melodization, when $\frac{M}{H} = 6$

Compare the following illustrations with Chopin, when the example is played in C – minor.

Figure 12. Diatonic melodization $\frac{M}{H} = 6$ (*continued*).

Binary parallel axes

Binary converging- diverging axes

Figure 12. Diatonic melodization $\frac{M}{H} = 6$ *(continued).*

Attack pattern: 4+2

Figure 12. Diatonic melodization $\frac{M}{H} = 6$ (*concluded*).

Examples of Diatonic Melodization. $\frac{M}{H} = 7$

Figure 13. Diatonic melodization $\frac{M}{H} = 7$ (*continued*).

Figure 13. Diatonic melodization $\frac{M}{H} = 7$ *(continued).*

Attack pattern: (4+3)+(2+3+2)+(3+4)

Figure 13. Diatonic melodization $\frac{M}{H} = 7$ *(concluded).*

Examples of Diatonic Melodization. $\frac{M}{H} = 8$

Compare classical type ($\frac{4}{4}$ series) with jazz ($\frac{8}{8}$ and $\frac{12}{12}$ series) in the following illustrations.

Figure 14. Diatonic melodization $\frac{M}{H} = 8$ *(continued).*

A = b + a

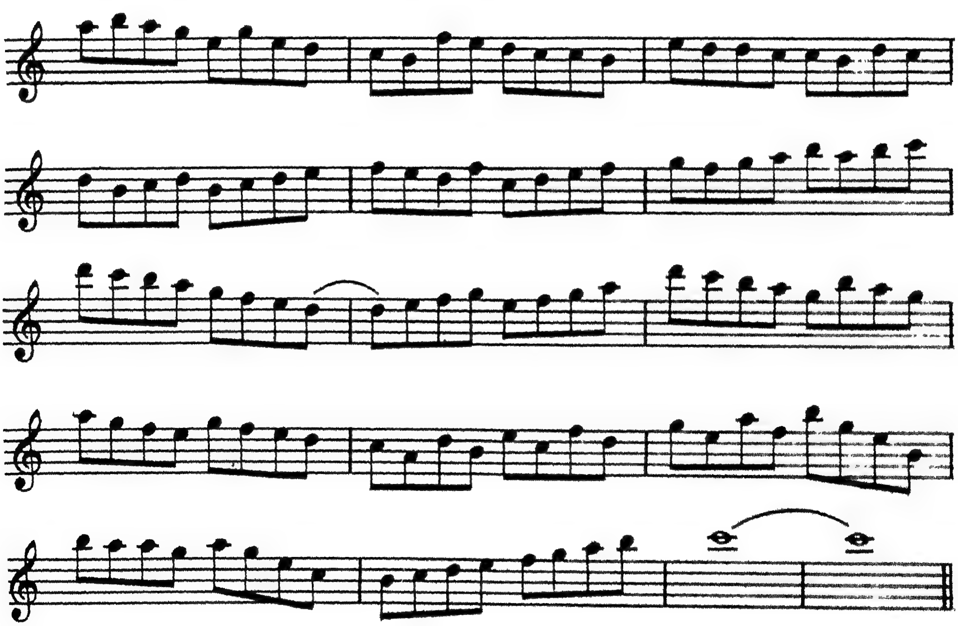


Figure 14. Diatonic melodization $\frac{M}{H} = 8$ *(continued).*

Binary diverging- converging axes

Attack pattern 8 (3)+3+2+3

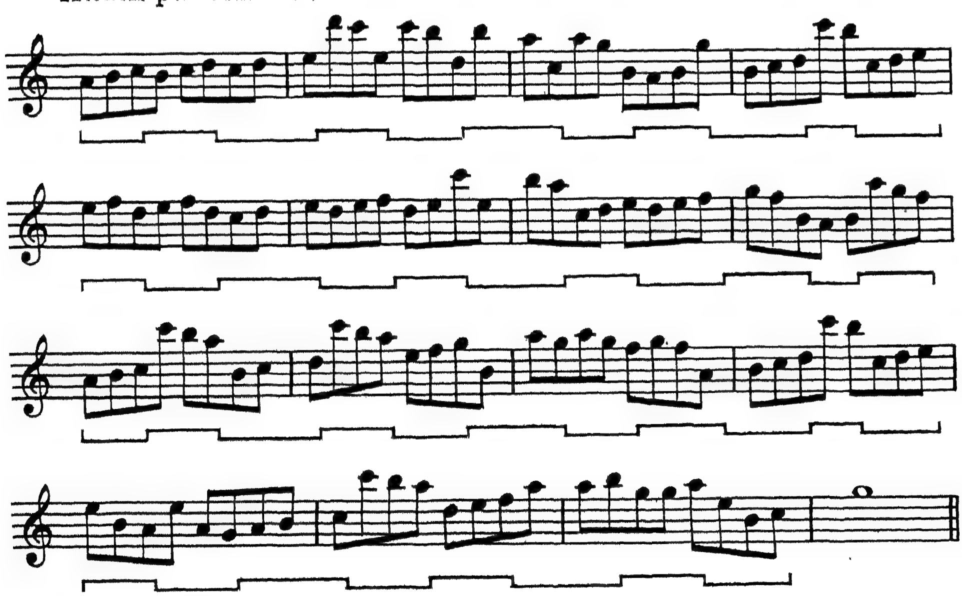


Figure 14. Diatonic melodization $\frac{M}{H} = 8$ (*concluded*).

Examples of Diatonic Melodization. $\frac{M}{H} = 12$

Figure 15. Diatonic Melodization. $\frac{M}{H} = 12$ *(continued).*

Figure 15. Diatonic melodization $\frac{M}{H} = 12$ *(concluded).*

CHAPTER 2

COMPOSING MELODIC ATTACK-GROUPS

IN ALL the forms of melodization previously discussed, the attack-group of M was constant in relation to H. Any preselected quantity of attacks per chord (H) was carried out consistently. The monomial attack group (A) in all these cases was an integer remaining constant throughout $H^{\rightarrow}$. This monomial form of an attack-group can be expressed as $\frac{M}{H} = A$, where A can be any integer from one to infinity.

Now, however, we are to consider binomial attack-groups for the melody. This situation may be expressed as $\frac{M}{2H} = A_1 + A_2$, i.e., the melody covering two successive chords consists of two *different* attack-groups.*

For instance:

$$(1)\ \frac{M}{2H} = 2a + a; \qquad (2)\ \frac{M}{2H} = 3a + 2a;$$
$$(3)\ \frac{M}{2H} = 5a + 3a; \qquad (4)\ \frac{M}{H} = a + 8a; \ldots$$

These expressions can be further deciphered as:

$$(1)\ \frac{M}{H_1} + \frac{M}{H_2} = 2a + a; \qquad (2)\ \frac{M}{H_1} + \frac{M}{H_2} = 3a + 2a;$$
$$(3)\ \frac{M}{H_1} + \frac{M}{H_2} = 5a + 3a; \qquad (4)\ \frac{M}{H_1} + \frac{M}{H_2} = a + 8a; \ldots$$

The main technical significance of a binomial attack-group is that it introduces contrast between the two successive portions of M. The greater the contrast required, the greater the difference between the two number-values of the binomial. This proposition can be reversed as follows: the contrast between the two terms of a binomial decreases when their values approach equality.

Thus, $\frac{M}{2H} = a + 6a$ contrasts more than $\frac{M}{2H} = 2a + 6a$; $2a + 6a$ possesses more contrast than $3a + 6a$; and $3a + 6a$ has more contrast than the *least* contrasting, $5a + 6a$. With further balancing we return to a monomial, $\frac{M}{H_1} + \frac{M}{H_2} = 6a + 6a$ which means that $\frac{M}{H} = 6a$.

If permutation takes place in a binomial attack-group, it results in a *second order* binomial attack group. For instance: $\frac{M}{2H} = 4a + 2a$; in the course of $H^{\rightarrow} = 4H$, this becomes: $\frac{M}{4H} = \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} + \frac{M}{H_4} = 4a + 2a + 2a + 4a$.

*$\frac{M}{2H} = A_1 + A_2$ is not the same situation as $\frac{M}{H} = 2$; the latter means two uniform attacks of melody per H, while the former means two *groups* of melody attacks per H, and one group need not be the same as the other. (Ed.)

What is true of binomial attack-groups is true of any polynomial; the latter, too, are subject to permutations.

Examples of trinomial attack-groups:

$$(1)\ \frac{M}{3H} = 3a + 2a + a; \quad \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} = 3a + 2a + a;$$

$$(2)\ \frac{M}{3H} = 4a + a + 3a; \quad \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} = 4a + a + 3a;$$

$$(3)\ \frac{M}{3H} = a + 2a + 4a; \quad \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} = a + 2a + 4a;$$

$$(4)\ \frac{M}{3H} = 3a + 5a + 8a; \quad \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} = 3a + 5a + 8a.$$

Figure 16. Trinomial attack-groups.

Examples of polynomial attack groups based on the resultants of interference:

(1) $r_{4 \div 3}$:

$$\frac{M}{6H} = \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} + \frac{M}{H_4} + \frac{M}{H_5} + \frac{M}{H_6} = 3a + a + 2a + 2a + a + 3a.$$

(2) $r_{\underline{3 \div 2}}$:

$$\frac{M}{7H} = \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} + \frac{M}{H_4} + \frac{M}{H_5} + \frac{M}{H_6} + \frac{M}{H_7} =$$
$$= 2a + a + a + a + a + a + 2a.$$

(3) $r_{9 \div 8}$:

$$\frac{M}{16H} = 8a + a + 7a + 2a + 6a + 3a + 5a + 4a +$$
$$+ 4a + 5a + 3a + 6a + 2a + 7a + a + 8a.$$

Figure 17. Polynomial attack-groups.

The effect produced by such composition of attacks as (3) is that of counterbalancing the original binomial; the melody starts with excessive animation over H_1 (8a) and complete lack of it over H_2(a); it follows into that state which is closest to balance, after which the counterbalancing begins, ultimately reaching its converse: a + 8a.

In all cases of $r_{a \div b}$, maximum animation takes place at the beginning and at the end. When the *opposite* effect is desired (minimum animation at the beginning and at the end), use the permutation of binomials (which is possible when the number of terms in the polynomial is even). For instance: (3) can be transformed into $\frac{M}{16H} = a + 8a + 2a + 7a + 3a + 6a + 4a + 5a + 5a +$
$+ 4a + 6a + 3a + 7a + 2a + 8a + a.$

In addition to resultants, involution (power) groups, various series of variable velocities (natural harmonic series, arithmetical and geometrical progressions, summation series), may be used as the forms of attack-groups.

For instance: $(2 + 1)^2$: $\frac{M}{4H} = 4a + 2a + 2a + a$;

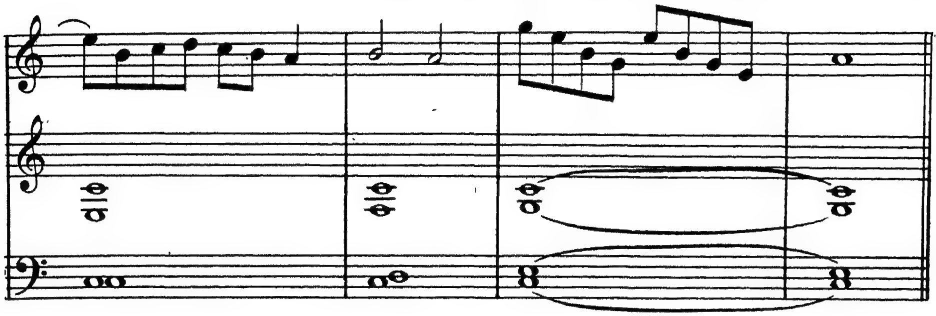
$(1 + 3)^2$: $\frac{M}{4H} = a + 3a + 3a + 9a$;

$\frac{M}{\text{[illegible]}} = 2a + 3a + 5a + 8a + 13a.$

In the present examples, I shall use the simplest duration-equivalents of attacks, as this subject is to be a matter of further analytical investigation later in our text.

Examples of Diatonic Melodization with
Variable Quantity of Attacks of M over H:
$\frac{M}{H}$ = A var.

Figure 18. Diatonic melodization with $\frac{M}{H}$ = A variable. (*continued*).



$\frac{M}{H}$ = Var.-a: (1+4+2) p

$\frac{M}{H}$ = Var.-a: 4+2+2+1; H→ : Hybrid 4 part harmony

Figure 18. Diatonic melodization with $\frac{M}{H}$ = A variable (concluded).

The ties in the above examples were added after the completion of the melodization.

A. How the Durations for Attack-Groups of Melody are Composed

Durations for the attack-groups of melody may be composed by means of the techniques previously discussed as evolution of style in rhythm.* Every attack-group—monomial, binomial, trinomial, quintinomial, etc.—can be expressed through the different numerical series. For instance, a binomial of $\frac{3}{3}$ series is $2 + 1$, or its converse; a binomial of $\frac{4}{4}$ series is $3 + 1$, or its converse; a binomial of $\frac{8}{8}$ series is $5 + 3$ or its converse. Likewise, a trinomial of $\frac{4}{4}$ series is either $2 + 1 + 1$ or one of its permutations; a trinomial of $\frac{6}{6}$ is $4 + 1 + 1$ or one of its permutations; and a trinomial of $\frac{8}{8}$ series is $3 + 3 + 2$ or one of its permutations.

By selection of the durations for the attack-groups according to the different series, we may *translate* a piece of music from one rhythmic style into another.

When a choice is to be made as to the use of a binomial or a trinomial, the form of balance (unbalancing, balancing) becomes the decisive factor.

Of the two binomials, $3 + 1$ and $1 + 3$, the former is the more suitable at the beginning of melody; the latter, at the end. As to a trinomial in $\frac{4}{4}$ series: we might well use $2 + 1 + 1$ at the beginning, $1 + 2 + 1$ somewhere about the center, and $1 + 1 + 2$ at the end. Likewise, in $\frac{8}{8}$ series, $3 + 3 + 2$ at the beginning, $3 + 2 + 3$ about the center and $2 + 3 + 3$ at the end. Four attacks can be achieved, among other ways, by splitting one of the terms of a trinomial. Splitting the terms serves as a general technique for acquiring more terms for low determinants.

Here are examples of the composition of durations for the attack-groups of melody where each term of an attack-group corresponds to one chord: $\frac{M}{H} = A$.

$$A^{\rightarrow} = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7$$

$$A_1 = a;$$

$$A_2 = a + b;$$

$$A_3 = a + b + c;$$

$$A_4 = a + b + c + d + e;$$

$$A_5 = a + b + c;$$

$$A_6 = a + b;$$

$$A_7 = a$$

$$A^{\rightarrow} = a + 2a + 3a + 5a + 3a + 2a + a$$

Series: $\frac{3}{3}$

$$T = 3H_1 + (2+1)H_2 + (1+1+1)H_3 + (\tfrac{1}{2} + \tfrac{1}{2} + 1 + \tfrac{1}{2} + \tfrac{1}{2})H_4 + \\ + (1+1+1)H_5 + (1+2)H_6 + 3H_7.$$

*See Book I, Chapter 13.

$\frac{3}{3}$ series

Figure 19. Series $\frac{3}{3}$.

Series: $\frac{4}{4}$

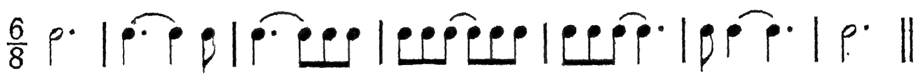
$$T = 4H_1 + (3+1)H_2 + (2+1+1)H_3 + (1+1+\tfrac{1}{2}+\tfrac{1}{2}+1)H_4 + \\ + (1+1+2)H_5 + (1+3)H_6 + 4H_7.$$

$\frac{4}{4}$ series

Figure 20. Series $\frac{4}{4}$.

Series: $\frac{6}{6}$

$$T = 6H_1 + (5+1)H_2 + (4+1+1)H_3 + (1+1+2+1+1)H_4 + \\ + (1+1+4)H_5 + (1+5)H_6 + 6H_7.$$



or

(Waltz or Mazurka)

$\frac{6}{6}$ series

Figure 21. Series $\frac{6}{6}$.

Series: $\frac{8}{8}$

$$T = 8H_1 + (5+3)H_2 + (3+3+2)H_3 + (2+1+2+1+2)H_4 + \\ + (2+3+3)H_5 + (3+5)H_6 + 8H_7.$$

(Fox-trot, Rhumba, Charleston)

$\frac{8}{8}$ series

Figure 22. Series $\frac{8}{8}$.

The final and most refined technique of coordination of attack with duration-groups occurs when the attack-groups are constructed independently of T. This results in an interference between the attack-groups and the duration-groups, and the duration of the individual chords coincides neither with the bar-lines nor with their simplest subdivisions.

A simple case for our illustration: let us choose $A = r_{5\div4} = 4a + a + 3a + 2a + 2a + 3a + a + 4a = 20\,a$.

Execute the durations as $T = r_{4\div3}$. As T in this case has 10a and A has 20a, the interference is a simple one.

$$\frac{a\ (A)}{a\ (T)} = \frac{20}{10} = \frac{2}{1}; \quad \frac{1}{2}\ \frac{(20)}{(10)}$$

Hence, $T' = 16t \cdot 2 = 32t$.

Let $T'' = 8t$, then:

$$N_{T''} = \frac{32}{8} = 4$$

The duration of each consecutive H equals the sum of durations during the time of attacks corresponding to such an H. H_1 corresponds to 4a, the durations of which constitute $3t + t + 2t + t$, and so H_1 will last 7t. Likewise, the next chord, i.e., H_2 will last t—since melodization at this point consists of one attack, and that attack corresponds to one unit of duration.

Here is the final solution* of the case:

$$(1)\quad \frac{a\ (M)}{a\ (H)} = \frac{4}{1} + \frac{1}{1} + \frac{3}{1} + \frac{2}{1} + \frac{2}{1} + \frac{3}{1} + \frac{1}{1} + \frac{4}{1} = 4aH_1 + aH_2 + 3aH_3 + 2aH_4 + 2aH_5 + 3aH_6 + aH_7 + 4aH_8$$

$$(2)\quad \frac{T\ (M)}{T\ (H)} = \left(\frac{3+1+2+1}{7} + \frac{1}{1} + \frac{1+1+2}{4} + \frac{1+3}{4}\right) + \left(\frac{3+1}{4} + \frac{2+1+1}{4} + \frac{1}{1} + \frac{1+2+1+3}{7}\right) = \left[\left(\frac{3t+t+2t+t}{7t}\right) H_1 + \left(\frac{t}{t}\right) H_2 + \left(\frac{t+t+2t}{4t}\right) H_3 + \left(\frac{t+3t}{4t}\right) H_4\right] + \left[\left(\frac{3t+t}{4t}\right) H_5 + \left(\frac{2t+t+t}{4t}\right) H_6 + \left(\frac{t}{t}\right) H_7 + \left(\frac{t+2t+t+3t}{7t}\right) H_8\right]$$

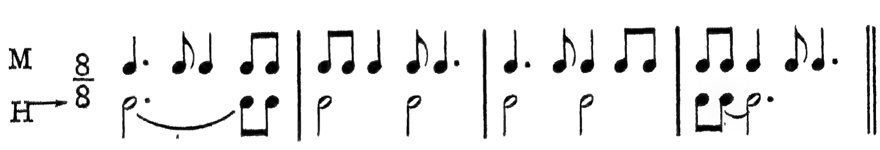


Figure 23. $\frac{a(M)}{a(H)}$ *and* $\frac{T(M)}{T(H)}$ (*continued*).

*This technique is one which the reader who does not remember the antecedent techniques may understand more rapidly if we give this parallel explanation: the number of attacks in the melody for each H is controlled by $r_{5\div4}$, which is $4 + 1 + 3 + 2 + 2 + 3 + 1 + 4$—that means, 4 melody notes against the first H, 1 against the second, 3 against the third H, and so on. But of what *duration* shall each attack of melody be? That is controlled by $r_{4\div3}$—which is $3 + 1 + 2 + 1 + 1 + 1 + 1 + 2 + 1 + 3$ — meaning that the first melody note will last for 3 units, the second for one unit, the third for two units, etc. When the end of the pattern is reached, it begins all over again until the two, $r_{5\div4}$ and $r_{4\div3}$, come to and end at the same point. Knowing, then, that there are to be 4 melodic attacks for the first H, and that the duration of these attacks is to be, respectively, 3 and 1 and 2 and 1, we see that the duration of the H must be the sum—i.e., 7. This process is carried out until the two resultants, $r_{5\div4}$ and $r_{4\div3}$, close. (Ed.)

Figure 23. $\frac{a(M)}{a(H)}$ *and* $\frac{T(M)}{T(H)}$ *(concluded).*

B. Direct Composition of Durations Correlating Melody and Harmony

Time-rhythm of both melody and harmony can be set simultaneously by means of a *proportionate distribution of durations for a constant quantity of attacks of* $\frac{M}{H}$. This can be achieved by synchronizing a polynomial (consisting of the corresponding number of terms, representing attacks) with its square, or by synchronizing the square of a polynomial with its cube, etc. For instance, we might assume that we would like to have 4 attacks per chord with the duration in the style of the $\frac{4}{4}$ series. Let us take a quadrinomial from that series, $3 + 1 + 2 + 2$, and square it.

$$(3+1+2+2)^2 = (9+3+6+6) + (3+1+2+2) + (6+2+4+4) + (6+2+4+4)$$

This distributive square represents T (M). The T (H) is the original quadrinomial, synchronized with the distributive square:

$$8\,(3+1+2+2) = 24 + 8 + 16 + 16$$

We obtain:

$$\frac{T\ (M)}{T\ (H)} = \frac{9t + 3t + 6t + 6t}{24t} + \frac{3t + t + 2t + 2t}{8t} + \frac{6t + 2t + 4t + 4t}{16t} + \frac{6t + 2t + 4t + 4t}{16t}$$

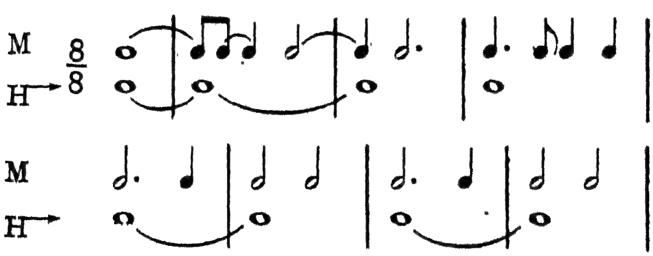


Figure 24. Correlating melody and harmony: direct composition of durations (continued).

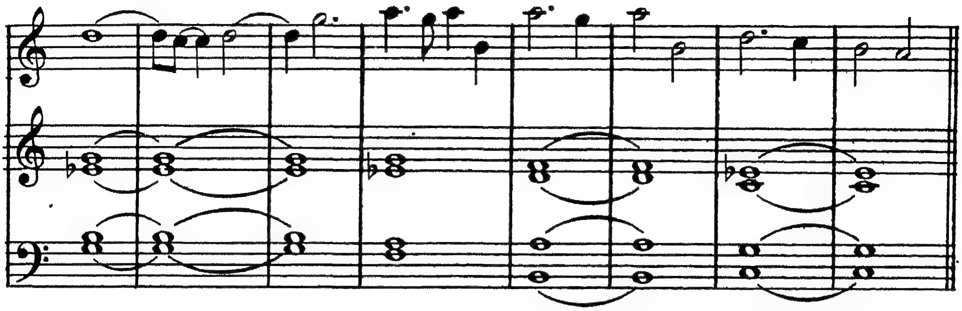


Figure 24. Correlating melody and harmony: direct composition of durations (concluded).

Likewise, a synchronization of the distributive square with the distributive cube of the same polynomial may be used for melodization of harmony. The group arising from the square furnishes durations for the chords; the group arising from the cube furnishes durations for the melody.

$$\frac{T\ (M)}{T\ (H)} = \frac{(2+1+1)^3}{4(2+1+1)^2} = \frac{8t+4t+4t}{16t} + \frac{4t+2t+2t}{8t} + \frac{4t+2t+2t}{8t} + \frac{4t+2t+2t}{8t} + \frac{2t+t+t}{4t} + \frac{2t+t+t}{4t} + \frac{4t+2t+2t}{8t} + \frac{2t+t+t}{4t} + \frac{2t+t+t}{4t}.$$

This produces harmony: $H^{\rightarrow} = 9H$, and melody: $M = 27a$, with constant 3 attacks per chord.

Figure 25. Synchronization of distributive square with distributive cube (continued).

Figure 25. Synchronization of distributive square with distributive cube (concluded).

For still greater contrast in quantity of attacks between M and $H^{\rightarrow}$, use the synchronized first power group for $H^{\rightarrow}$, and use the distributive cube for M.

In addition to distributive powers, coefficients of duration can be used. For instance:

$$\frac{M}{H^{\rightarrow}} = \frac{(3+1+2+1+1+1+1+2+1+3) + (3+1+2+1+1+1+1+2+1+3)}{6+2+4+2+2+2+2+4+2+6}$$

C. Chromatic Variation of Diatonic Melodization

It is expedient to construct a chromatic melody for a diatonic chord progression by using two successive operations:

(1) Diatonic melodization of the harmony; and then

(2) Chromatization of the diatonic melody.

The first technique has been fully covered in the preceding explanation.

The second, chromatization, can be accomplished by means of passing or auxiliary chromatic tones. The most practical way to perform this rhythmically is by means of *split-unit groups*, as discussed earlier in the *Theory of Rhythm* under "Variations."* This splitting does not change the character of durations with respect to their style; it merely increases the degree of animation of the melody.

*See Book I, Chapter 9.

Diatonic melodization

Chromatic variation

Figure 26. Chromatization of diatonic melody.

D. Symmetric Melodization:

The Σ (13) Families

Each style of symmetric harmonic continuity (the Type II, the Type III and the generalized) is governed by the Σ (13) families. Pure styles are controlled by any one Σ(13); hybrid styles are based usually on two, sometimes on as many as three, Σ (13).

The complete manifold of Σ (13) families corresponds to the 36 seven-unit pitch-scales which contain the seven names of non-identical pitches; the Σ (13) is the first expansion (E_1) of such scales.

We shall classify all forms by considering 1, 3, 5 and 7 to be the *lower* structure [as S(7)], with 9, 11 and 13 constituting the *upper* structure [as S(5)], eliminating all enharmonic coincidences and eliminating all those adjacent thirds which do not satisfy i = 3 or i = 4. These limitations are necessitated by the restricted scope of the special theory of harmony.

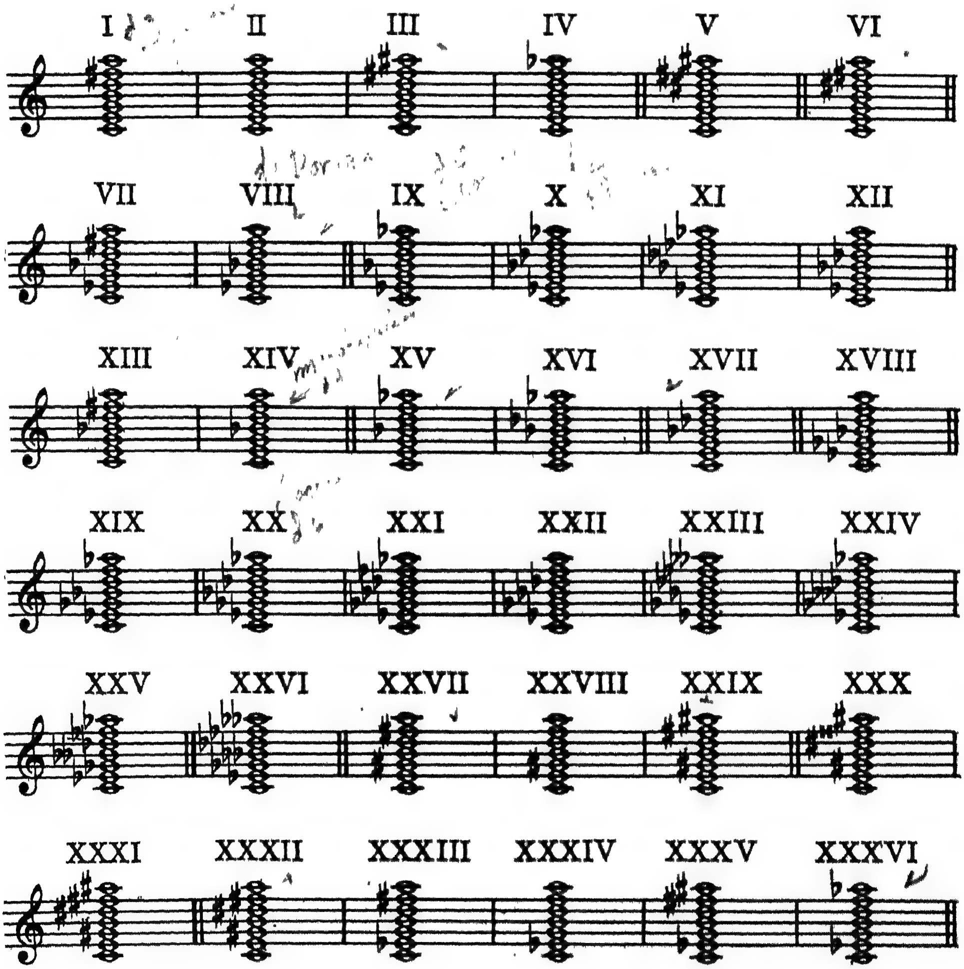


Figure 27. Complete table of Σ 13.

Symmetric melodization provides the composer with resources particularly suitable for equal temperament ($\sqrt[12]{2}$). In the diatonic system some chord-structures, particularly those of high tension, produce harsh-sounding harmonies; but in the symmetric system both the chord-structures and the intonations of the melody are entirely under control—they are subject to choice. The technique of symmetric melodization makes it possible to surpass the refinements of Debussy and Ravel. And whereas it took any important composer many years to crystallize his own original style, this technique of melodization offers us 36 styles to choose from if one Σ (13) is used at a time. The number of possible styles grows enormously with the introduction of blends based on *two* Σ (13). Thereupon the number of styles becomes $36^2 = 1{,}296$. Likewise, by blending *three* Σ (13), which is a reasonable limit of mixing, we acquire $36^3 = 46{,}656$ styles!

We should note, too, that only four of the 36 master-structures have been explored to any extent; the rest are virgin territory, packed with the most expressive resources of melody and harmony.

In offering the following technique, I shall use symmetric progressions of Type II, Type III and the generalized form in four- and in five-part harmony. The main difference between the four- and the five-part type of harmony is *density*. For massive accompaniments, use five; for lighter ones, use four-part harmony.

When all *substructures* [S(5), S(7), S(9), S(11)] derive from *one master-structure* [Σ (13)], they derive all their intonations from that master-structure. The easiest way to acquire a quick orientation in any Σ (13) is to prepare a chromatic table of the master-structure. Taking one Σ (13) [XIII] from Figure 27, we obtain the following table of transpositions:

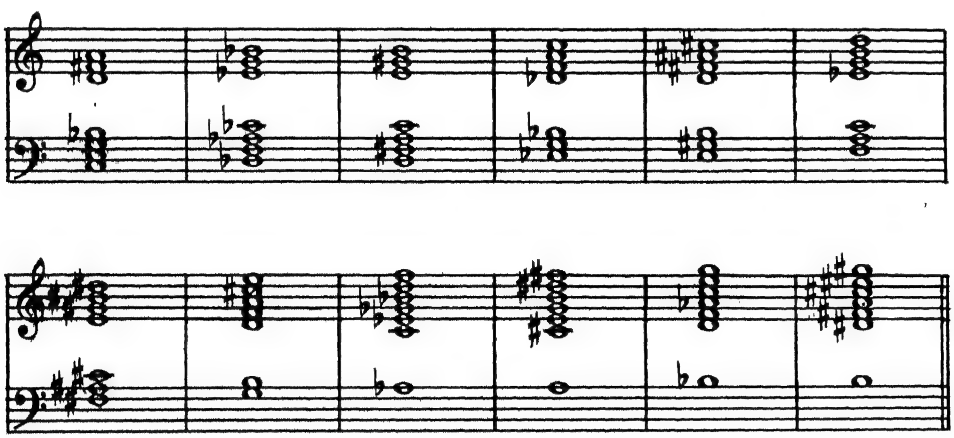


Figure 28. Substructures of one Σ (13).

Such a table is very helpful; in it one can find all intonations of both melody and harmony for any symmetric progression. Each Σ (13), being E_1 of a seven-unit scale, corresponds to E_0 of the same scale.

The remainder of the procedure of melodization is based on the same *principle of tension* as in diatonic melodization. Those functions which are added to the respective tensions of chords are the most desirable ones for use as axes of the melody. Thus, the axis of the melody above S(5) in four-part harmony is either 7 or 13. Actually such a choice creates *polymodality*, as S(5) d_0 serves as an accompaniment to melody which is d_6 or d_5 respectively.* It is *polymodality* that makes such music expressive.

There follows a table of melodic axes for the respective structures in four and five-part harmony. In some cases there is a choice of more than one. Some of the forms are admitted because there has been practical use of them already—for example, S(5) in five-parts with the melodic axis on d_1 ($= 9$). It is interesting to note that Σ (13) [XIII] is used most of all, and that it is the most obvious of the master-structures, as it consists of a large S(7) and a major S(5).

Master-Structure: Σ (13) [XIII]

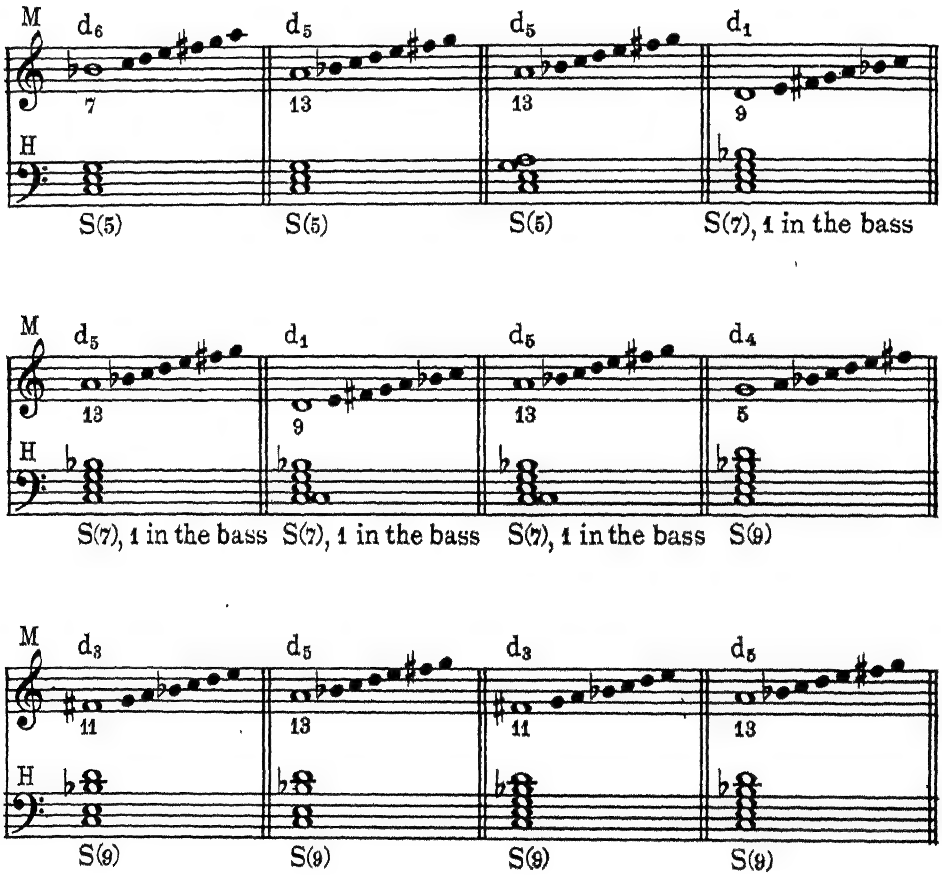


Figure 29. Table of melodic axes in relation to tension of II (continued).

*That is to say: S(5) as a triad in d_0 (that mode which starts on the same tone the d_0—as that on which the key itself starts) serves to accompany a melody the axis of which is located at the 7 (a third *above* the 5 of the 1, 3, 5) or at the 13 (a third *below* the 5 of the 1, 3, 5), thus putting the melody in d_5 (that mode which starts on the *la* of the key) or in d_6 (that mode which starts on the *ti* of the key). (Ed.)

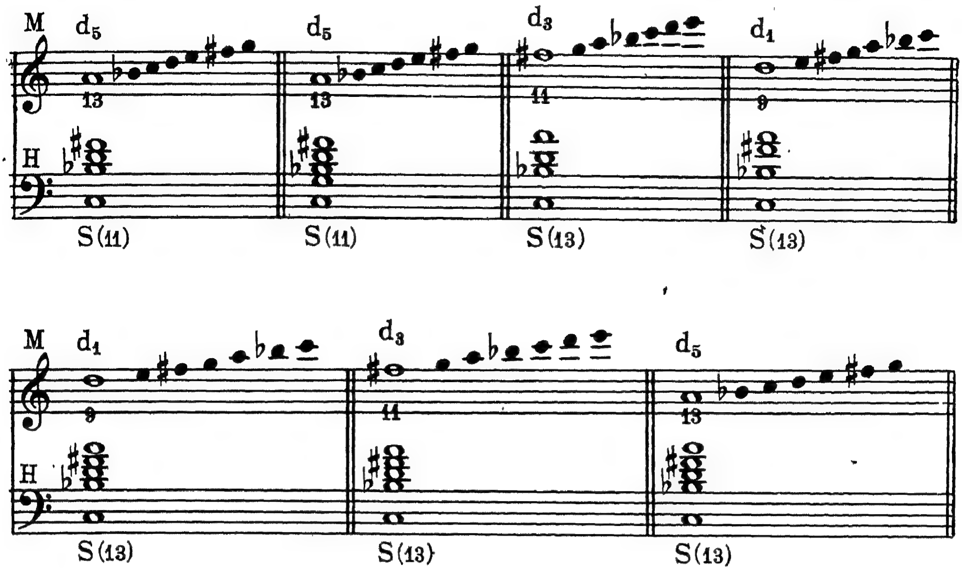


Figure 29. Table of melodic axes in relation to tension of H (concluded).

Using this $\Sigma(13)$ [XIII] we shall melodize a generalized symmetric progression in four parts in $\frac{M}{H} = a$.

Theme: $2 + 2 + 2 + 1$; tension: $S(5) + 2S(7) + S(9) + 2s(13)$

Σ (13): XIII

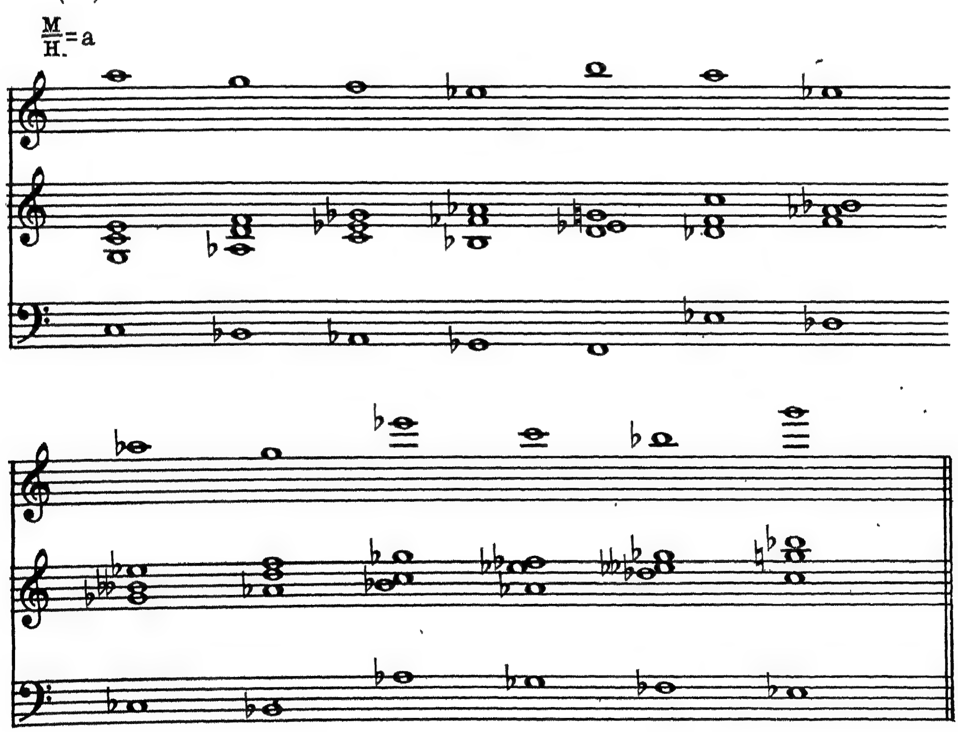


Figure 30. Melodizing a generalized symmetric progression in four parts $\frac{M}{H} = a$.

Theme: Type II: $\zeta = 2C_5 + C_{-7} + 2C_3 + C_{-5}$
tension: $S(5) + S(7) + 2S(9) + S(11)$

Σ (13): XIII

$\frac{M}{H} = a$

Figure 31. Theme of Type II.

With *more than one attack* of M per H, the quality deriving from the transitions in melody during the chord changes becomes more and more noticeable.

In melodizing each H with more than one attack of M, *it becomes necessary to perform modulations in melody*. Such modulations are equivalent to *polytonal-unimodal* and *polytonal-polymodal* transitions. The technique for this is based on *common tones*, on *chromatic alterations*, or on *identical motifs* and a full explanation has been provided in the *Theory of Pitch-Scales* (the first group).*

$\frac{M}{H} = 2 + 4 + 3$; $\frac{4}{4}$ series of T.

Figure 32. Symmetric melodization (continu

*See Book II.

$\frac{M}{H} = 2+6+3+5+4$; $\frac{6}{6}$ and $\frac{3}{3}$ series of T.

Figure 32. Symmetric melodization (continued).

Figure 32. Symmetric melodization (concluded).

With this kind of saturated harmonic continuity, the melody often gains in expressiveness by being more stationary than would be desirable in simple diatonic melodization; greater *stability of tension* is another desirable characteristic.

When mixing the different master-structures for one harmonic continuity, it is desirable to *alter either the lower part of the* Σ *(13)*, i.e., 1, 3, 5, 7, without altering the upper, or the *upper part* of it, i.e., 9, 11, 13, without altering the lower.

Let us now produce such a mixed style of master-structures, confining the latter to two—Σ (13) [XIV] and Σ (13) [XVII[. After such a selection has been made, the master-structures may be called simply: Σ_1 and Σ_2. In devising the style, we resort to coefficients of recurrence, for a predominance of one Σ over another is the chief stylistic determinant.

Let us assume the following recurrence-scheme: $2\,\Sigma_1 + \Sigma_2$.

$$\frac{M}{H} = a + 4a;\ \tfrac{4}{4}\text{ series of T.}$$
$$H^{\rightarrow} = 2C_7 + C_5 + C_3 \text{ (type II).}$$
$$S^{\rightarrow} = 2S(9) + S(13).$$

Figure 33. Recurrence scheme: $2\Sigma_1 + \Sigma_2$ *(continued).*

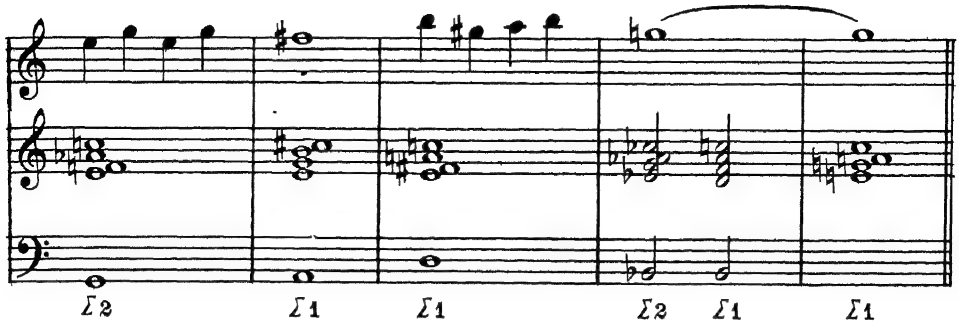


Figure 33. Recurrence scheme: $2\Sigma_1 + \Sigma_2$ *(concluded).*

E. Chromatic Variation of a Symmetric Melodization

Any melody, once it has been evolved by means of symmetric melodization, may be converted into the chromatic type by means of passing and auxiliary chromatic tones. Such chromatic tones do *not* belong to the master-structure. The rhythmic treatment of the durations is to be accomplished by means of split-unit groups.

Theme: figure 33

Figure 34. Chromatic variation of symmetric melodization.

All rhythmic devices—such as composition of attack and duration-groups—are applicable to all forms of symmetric melodization.

F. Chromatic Melodization of Harmony

The chromatic melodization of harmony serves the purpose of melodizing all forms of chromatic continuity. This includes techniques already explained in my discussion of the chromatic system, modulation, enharmonics, altered chords and of hybrid harmonic continuity. Such melodization is applicable to all forms of symmetric progressions; but from this approach we have nothing to gain, for symmetric melodization is itself a more general technique than the technique now being considered.

There are two fundamental forms of chromatic melodization. One of them produces melodies either of the *chromatic* type, or of the *extensively chromatized* type. The other form produces melodies of a purely *diatonic* type from chromatic harmony.

The first technique consists of *anticipating chordal tones and using them as auxiliary tones.* In a sequence, $H_1 + H_2 + H_3 + \ldots$, the chordal tones of H_2 are the auxiliaries and the chordal tones of H_1 are chordal tones while this chord sounds. In the next chord, (H_2), the chordal tones of H_3 are the auxiliaries and the chordal tones of H_2 are chordal tones while this chord sounds. This procedure may be extended *ad infinitum.*

As all of the "disturbing" pitch-units are harmonically justified as soon as the next chord appears, the listener is not aware that nearly every chromatic unit of the whole octave is used against each chromatic group, especially when there are enough attacks of M against H.

The auxiliary tones should be written in the proper manner, i.e., by raising the lower (ascending) auxiliary and by lowering the upper (descending) auxiliary, even if they have a different enharmonic notation when they occur in the following chord.

Figure 35. Chromatic melodization by means of anticipated chordal tones (continued).

Figure 35. Chromatic melodization by means of anticipated chordal tones (concluded).

G. Statistical Melodization of Chromatic Progressions

The second technique derives from the method of constructing a *quantitative scale.* Such a scale may be evolved by a purely *statistical* method. Although it is not obvious even to the most discriminating ear, it is easy to find by plain addition the quantity in which each chromatic pitch-unit appears during the course of a harmonic continuity. To find a quantitative scale, write out a full chromatic scale from *any note* (I do it usually from *c*).

The next procedure is to add up all the *c*-pitches in a given harmonic progression (doubled tones to be counted as one and enharmonics to be included). Then proceed with all of the c♯-pitches, the d-pitches, etc., until we sum up the entire chromatic scale. This produces a quantitative analysis of the full chromatic scale. Now, by eliminating some of those units which have lower marks, we obtain a *quantitative* (diatonic) *scale.*

The unit having the highest total becomes the root-tone of the scale and, possibly, the axis of the future melody. If more than one unit has a high mark, it is up to the composer to select one of them as the axis.

In the chromatic progression of Fig. 35, a quantitative analysis would be:

Figure 36. Quantitative analysis of chromatic scale in figure 35.

By excluding all values below 4, we obtain the following nine-unit scale with the root-tone on *e* (maximum value).

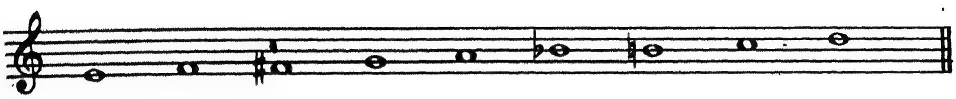


Figure 37 Reduced to nine-unit scale with root-tone on e.

If such a scale seems to be too chromatic, further exclusion of the tones with lower marks will reduce it to a scale of fewer units.

By excluding all the marks below 5 in this case, the scale will be reduced to one of five units and will have a purely diatonic appearance.

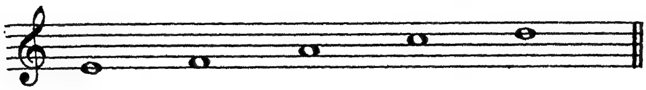


Figure 38. Reduced further by excluding all marks below 5.

The next procedure is the actual melodization, performed according to the diatonic technique. By this method the tones which predominate quantitatively during the course of chromatic continuity (and which affect us as such physiologically, i.e., as excitations) become the units *some of which satisfy every chord.* They attribute great stylistic unity to the entire product of melodization.

The number of attacks of M against H largely depends on the possibilities of melodization.

Figure 39. Chromatic melodization by means of quantitative diatonic scale.

These two techniques of chromatic melodization may be combined in sequence. This results in contrasting groups of first a diatonic and then a chromatic nature. The quantity of H covered by one method can be specified by means of the coefficients of recurrence.

For example: 2H di + H ch.

Figure 40. Combining two techniques of chromatic melodization.

CHAPTER 3

THE HARMONIZATION OF MELODY

THE usual approach to the problems of harmonization of melody seems entirely superficial when we consider that the very task of "finding a suitable harmonization" is expected to solve the problem in its entirety. Looking back at music which has already been written, we find a great diversity of styles of harmonization. In some cases the melody has a predominantly diatonic character while the chords seem to form a chromatic progression; in other cases the melody has a predominantly chromatic character while the accompanying harmony is entirely diatonic. Operatic works by Rimsky-Korsakov and Borodin illustrate the first type; music by Chopin, Schumann and Liszt supply examples of the second type. This raises the whole question of an accurate and systematic classification of the styles of harmonization.

By the pure method of combinations, we arrive at the following forms of harmonization:

(1) Diatonic harmonization of a diatonic melody.
(2) Chromatic harmonization of a diatonic melody.
(3) Symmetric harmonization of a diatonic melody.
(4) Symmetric harmonization of a symmetric melody.
(5) Chromatic harmonization of a symmetric melody.
(6) Diatonic harmonization of a symmetric melody.
(7) Chromatic harmonization of a chromatic melody.
(8) Diatonic harmonization of a chromatic melody.
(9) Symmetric harmonization of a chromatic melody.

In addition to these styles, various hybrids may be formed intentionally—and such hybrids do exist in music written on an intuitive basis. The necessity of handling these hybrid forms of harmonic continuity—which is inevitable not only in popular dance music, but frequently in music of composers who are considered "great" and "classical"—in special arrangements or transcriptions requires a thorough knowledge of all pure, as well as hybrid, forms of harmonization.

A. Diatonic Harmonization of a Diatonic Melody

There are two fundamental procedures required for this method of harmonization:

(a) The distribution of the number of attacks in melody and harmony, i.e., the number of attacks of melody to be harmonized by one chord, or the number of chords harmonizing one attack in melody.
(b) Selection of the range of tension.

Let us take a melody consisting of 12 attacks. Such a melody may be harmonized by 12 different chords, each attack in the melody acquiring its individual chord. But it may offer, as well, *two* attacks of a melody harmonized with each chord; in this case, 6 different chords will constitute the harmonic progression. Further, each 3 attacks of a melody may acquire a chord, making 4 chords necessary for the entire melody. Proceeding in similar fashion, one may ultimately arrive at one chord harmonizing the entire melody—this is quite possible, because no pitch-unit in a diatonic scale may exceed the function of 13th and will merely require an 11th chord for its harmonization, in order to support the 13th as an extreme function in a melody in which all the remaining units of the scale may be present as well.

Let us take, for example, the following melody:

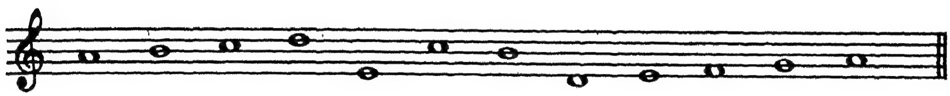


Figure 41. Melody.

In order to harmonize this melody with 12 different chords it is necessary to assign each pitch-unit of the melody to one chord. Such an assignment is based on a selection of the range of tension.

Let us suppose that we decide to make our range of tension from the 5th to the 13th. Having a considerable choice in the assignment of pitch-units as chordal functions, we will give *preference* to those forming a *positive* cycle of roots for the chords.

Examples of assignment of the above melody:

$\frac{M}{H} = 1$ Range of tension: 5 − 13

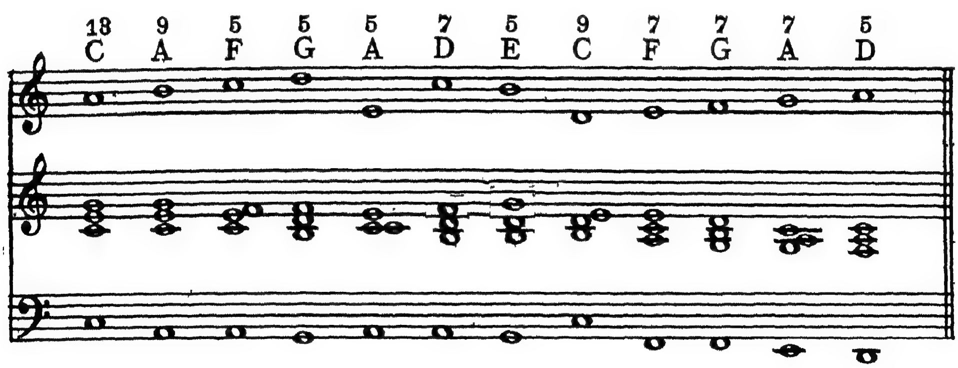


Figure 42. $\frac{M}{H} = 1$ *Range of tension: 5 − 13 (continued).*

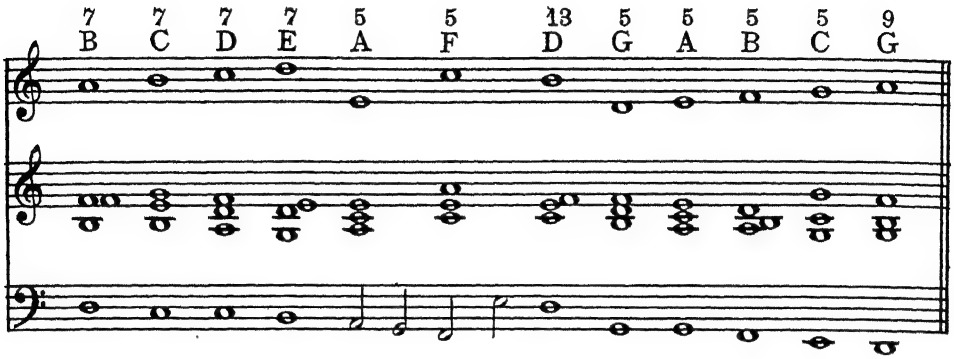


Figure 42. $\frac{M}{H} = 1$. *Range of tension: 5 — 13 (concluded).*

But if we now decide to assign two attacks in the melody against 1 chord, it is necessary to conceive of the two adjacent melodic pitches as being both in a scheme of chordal functions—thirds in this case. Thus, the first 2 units, a + b, have to be translated into $\frac{a}{b}$, which may, of course, assume any one of the following assignments:

a	9	11	13
b	3	5	7

Likewise, the pair, c + d, transforms itself into:

c	9	11	13
d	3	5	7

The next two units produce:

e	5	7	9	11	13
c	3	5	7	9	11

The next two units produce:

d	5	7	9	11	13
b	3	5	7	9	11

The next two units produce:

e	9	11	13
f	3	5	7

The next two units produce:

g	9	11	13
a	3	5	7

This group of assignments offers a considerable variety of harmonization, even if we preserve only the *positive* system of progressions.

$\frac{M}{H} = 2$ Range of tension: 3 — 13

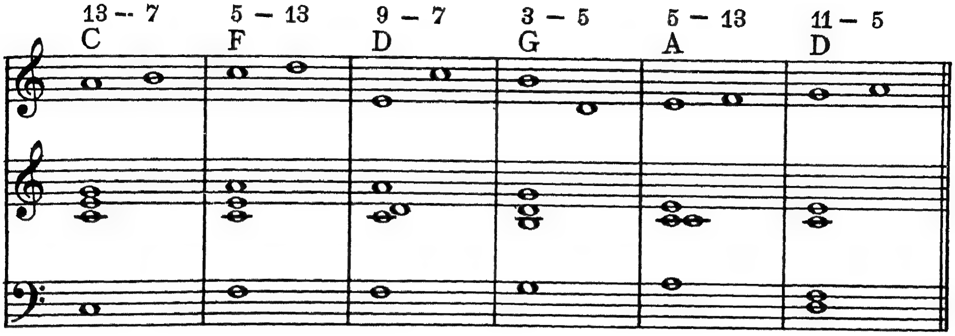


Figure 43. $\frac{M}{H} = 2$ *Range of tension: 3 — 13.*

When we come to assign every three pitch-units of the melody to one chord and to distribute them through the scheme of chordal functions we acquire the following table:

$\frac{M}{H} = 3$ Range of tension: 1 — 13

a.

b.

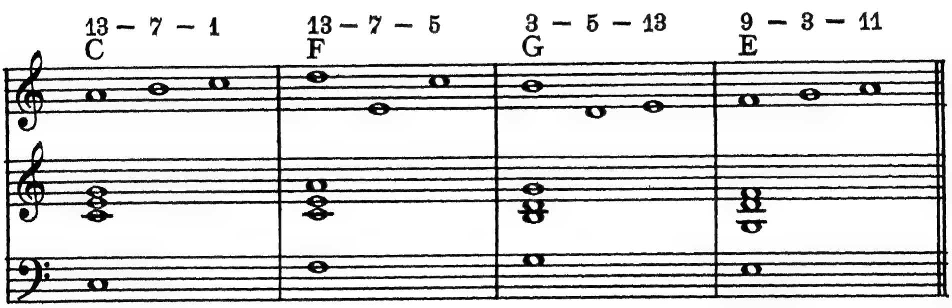


Figure 44. $\frac{M}{H} = 13$ *Range of tension: 1 — 13.*

And here are examples of the same procedure as applied to harmonization of the 12 melodic tones by three and by two chords respectively:

c.

$\frac{M}{H} = 4$ Range of tension: 1 — 13

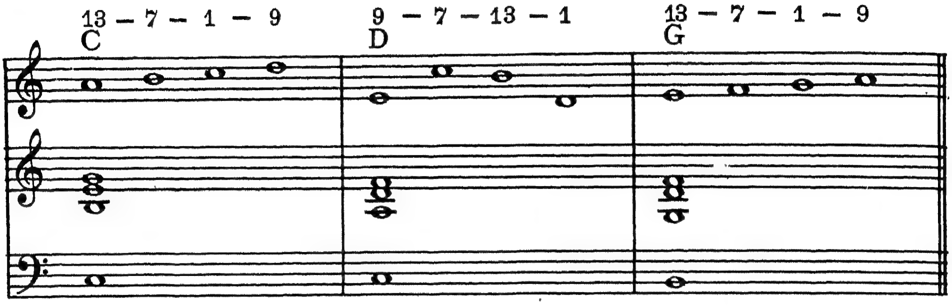


Figure 45. $\frac{M}{H} = 4$ *Range of tension: 1 — 13.*

$\frac{M}{H} = 6$ Range of tension: 1 — 13

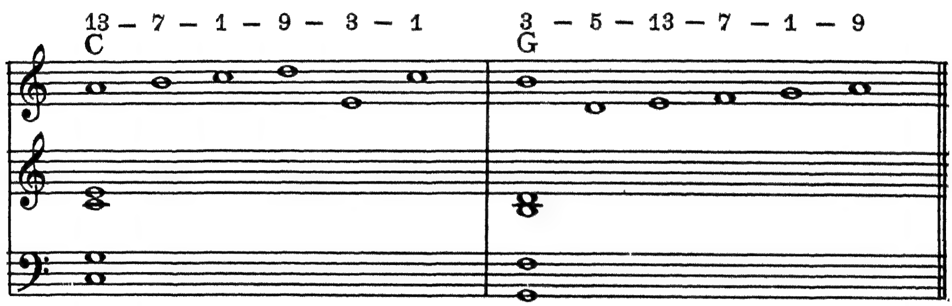


Figure 46. $\frac{M}{H} = 6$ *Range of tension: 1 — 13.*

d.

$\frac{M}{H} = 12$ Range of tension: 1 — 13

Figure 47. $\frac{M}{H} = 4$ *Range of tension: 1 — 13 (continued).*

Figure 47. $\frac{M}{H} = 12$. *Range of tension: 1 — 13 (concluded).*

Likewise, a *non-uniform* group distribution of the pitch-units of a melody may be devised. Rhythmic resultants, or any other material from the procedures already set forth in my theory of rhythm,* may be used as schemes for such distributions.

$\frac{M}{H} = r_{4 \div 3}$ Range of tension: 1 — 9

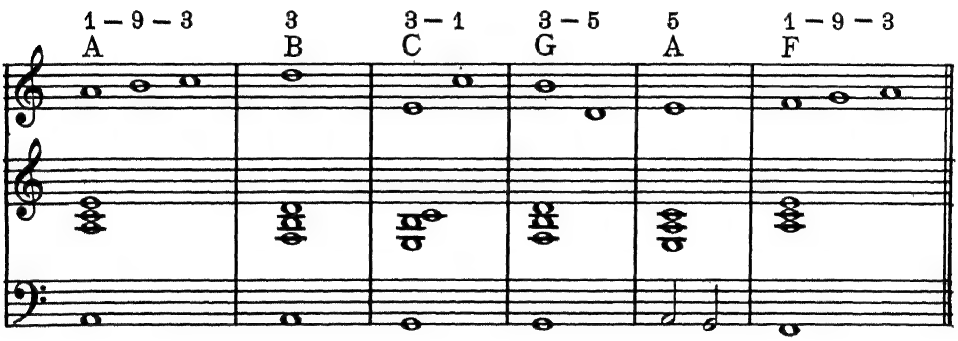


Figure 48. $\frac{M}{H} = r_{4 \div 3}$ *Range of tension: 1 — 9.*

B. Chromatic Harmonization of a Diatonic Melody

To harmonize a diatonic melody chromatically, it is necessary to obtain first a diatonic harmonization, then to insert passing and auxiliary chromatic tones. These inserted tones must not conflict with any of the pitch-units in the melody. For example, in a c 7th chord, if a melody has b, auxiliary tones may be devised on any of the *remaining* chordal functions, i.e., c, e, or g. Such a harmonization will acquire a particularly chromatic appearance if the tones of the figuration are written out together with the chord, thus forming altered chords. The following chromatic harmonization is merely a variation of Figure 44—b, obtained through insertion of the passing and auxiliary chromatic tones.

*See Book I.

*Figure 49. Chromatic harmonization of figure 44*b.

C. Symmetric Harmonization of a Diatonic Melody

Symmetric harmonization of a diatonic melody may be desirable when a certain type of chord structure is preferred to the casual selection that comes when the shapes of the chords are controlled by the diatonic scale. Such chord structures are derivatives of some Σ (13), which may be selected from the complete table of 13th chords. It is usually sufficient to limit the harmonized group to one Σ. In some unavoidable cases, an additional Σ (13) of the same family may be added. A preselected Σ (13) implies a definite harmonic style and brings the structural chord characteristics into prominence. On the other hand, the procedure helps eliminate undesirable or weak sonorities that are inevitable in the purely diatonic system. Any portion of melody consisting of one or more pitch-units may be assigned to be part of a preselected Σ (13) with a definite placement in such a Σ (13). For example, if we take Σ (13) = c − e − g − b♭ − − d − f♯ − a, a melody, the structure of which is in conformity with an incomplete minor S(7), (with omitted 5th), such as c − e♭ − b♭ may be placed on the above Σ as 3 − 5 − 9 or 13 − 1 − 5. No other location of this melodic form is possible with the above Σ.

Figure 50. Melody placed on Σ 13.

After all the melodic forms of one continuous melody are thoroughly analyzed as to their harmonic structure (as in the above case), and after the quantities of attacks of the melody against individual chords are distributed, the next step is to make sure that all such melodic forms will fit a particular Σ (13) selected to satisfy the entire melodic continuity.

To arrive at a practical decision, it is important to verify *all the individual melodic forms* to be harmonized; if necessary, make a corresponding alteration in Σ (13), so as to find a Σ which will satisfy all the forms. Cases in which more than one Σ are needed are comparatively rare, as most of the Σ (13) forms absorb all the partial forms.

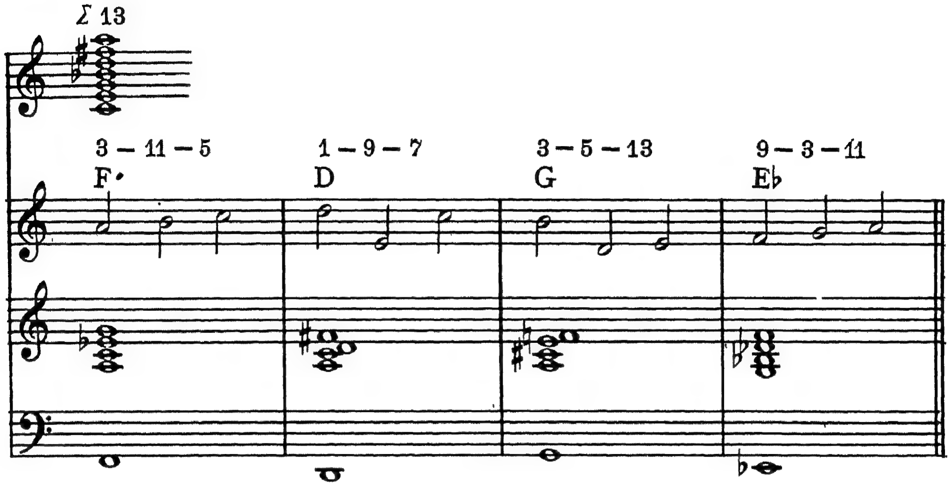


Figure 51. Melodic forms of a given Σ 13.

To convert a *symmetric* harmonization of a *diatonic* melody into *chromatic* harmonization, the principle presented in my discussion of how to convert diatonic harmonization into chromatic may be applied, i.e., auxiliary and passing tones are applied within the symmetric chord progressions.

1. More Than One H Against One M

To harmonize one pitch-unit diatonically with *more than one* chord, use a consistent increase of tension. For example, if a melodic note is c, and the chord originally planned is a, thus making c a third, one may add any quantity of chords in addition to that, so that the third eventually becomes a fifth, a ninth, etc. The cycle progression of such chords does not necessarily have to be C_3. For example, the c note in a melody may be 3 — 5 — 9 which requires the following chords: A — F — B.

Chromatization of the diatonic harmony will not require any changes in the above principle; simply supplement the diatonic harmonization by the auxiliary and passing chromatic tones.

In symmetric harmonization of a diatonic melody, when several chords appear against one pitch-unit of a melody, the identical Σ may satisfy the usual requirements of the symmetric harmonization of a diatonic melody. For example,

harmonizing with the Σ (13) used in Figure 51, the note c may be satisfied by the following chords:

C, B♭, A♭, G♭, F, E♭, D

arranged in any desirable sequence.

Figure 52. Harmonizing note c with Σ13 of figure 51.

Here is an example of the symmetric harmonization of a complete song, *My Own** (Lyric by Harold Adamson, music by Jimmy McHugh); the entire harmonization is based on one Σ 13th (the first chord in parenthesis). To transform this harmonization into a chromatic one, insert passing auxiliary chromatic tones.

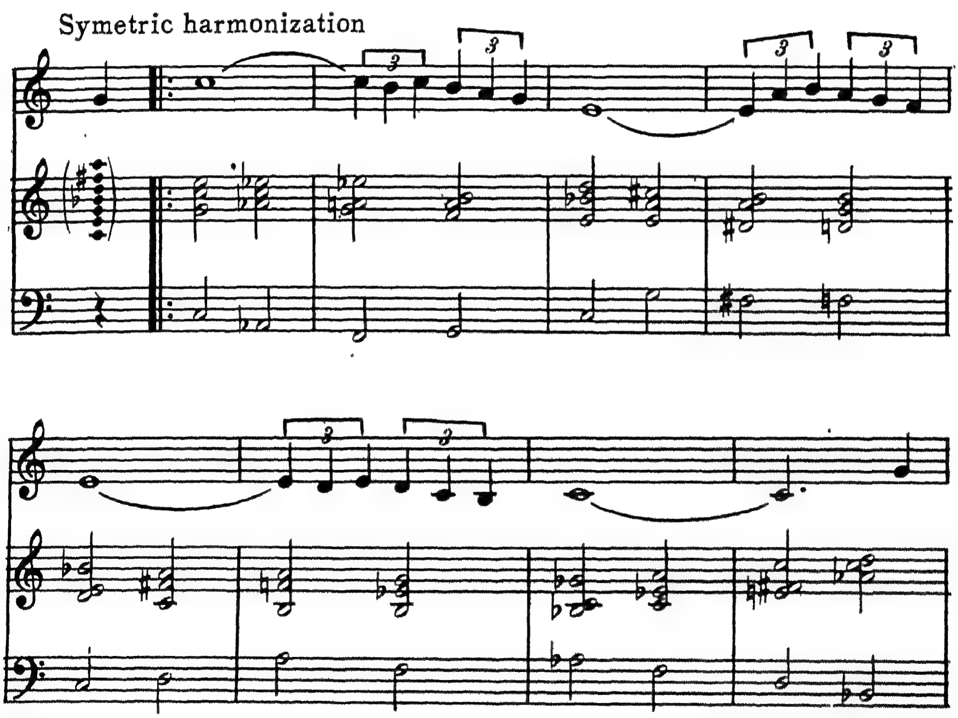


Figure 53. Symmetric harmonization of "My Own" (continued).

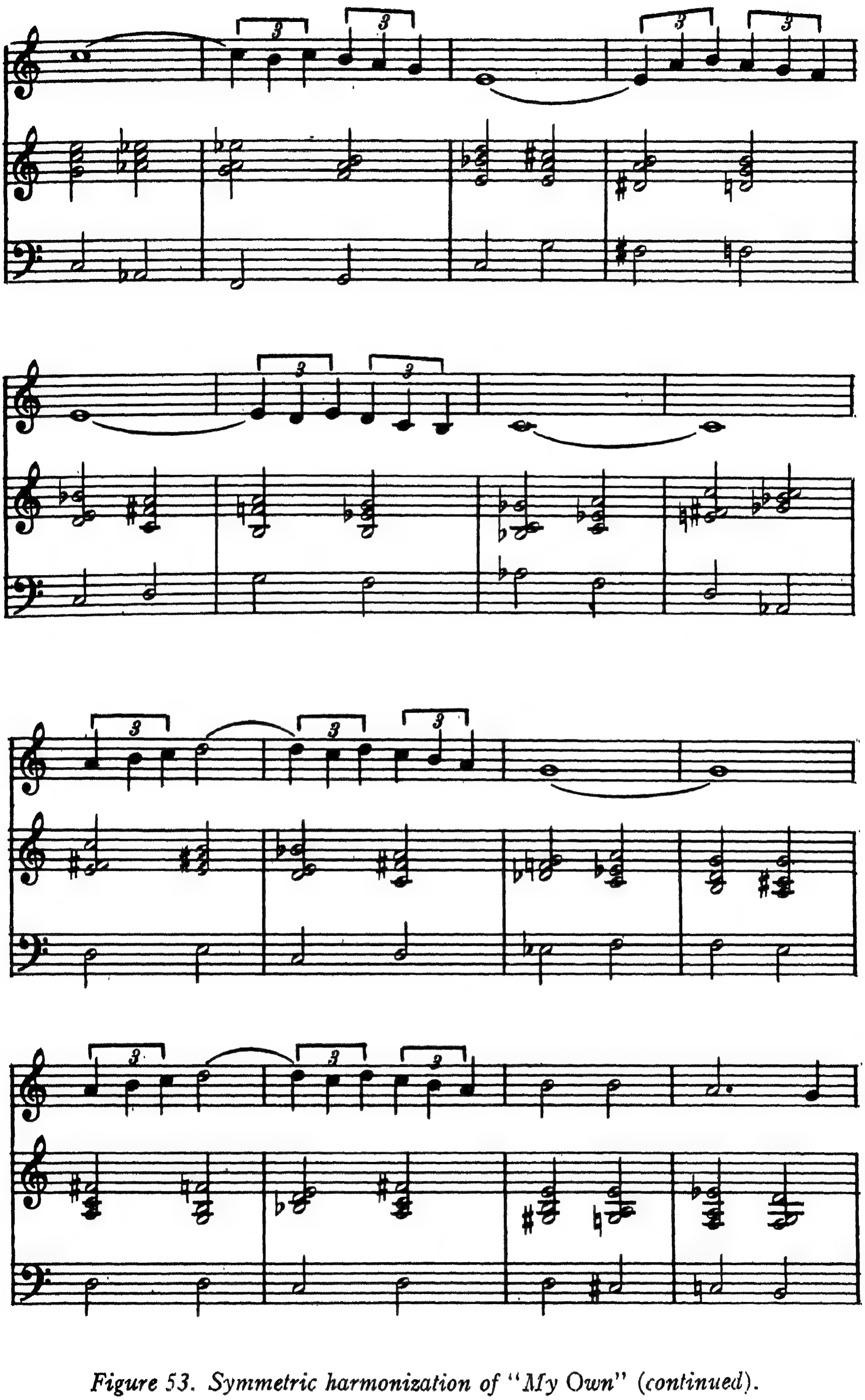


Figure 53. Symmetric harmonization of "My Own" (continued).

Figure 53. Symmetric harmonization of "My Own" (concluded).

D. Symmetric Harmonization of a Symmetric Melody

There is small probability that any melodies composed from symmetric scales exist outside this system, for the whole conception of symmetric scales itself has hitherto been unknown to the musical world. The problem of harmonization of melodies composed from symmetric scales first requires, therefore, the existence of such melodies. As has been explained in discussing the third and fourth groups of symmetric pitch-scales, melodies may be composed through permutation of pitch-units in the sectional scales (each starting with a new tonic). After the complete melodic form is achieved, the final step consists of superimposition of the rhythm of durations on the continuity of melodic forms.

Let us take a scale based on 12 tonics, each sectional scale having a structure 3 + 4; let us limit the entire scale to the first 3 tonics. As scales of the 12-tonic system have a wide range, it is desirable, in many cases, to reduce the range by means of octave-contraction.

Figure 54. Reducing range of $\sqrt[12]{2}$

The next step is to select a melodic form based on circular permutations of the pitch-units in the above scale, and to select a rhythmic form based on synchronization of $3(2+1)$ and $(2+1)^2$.

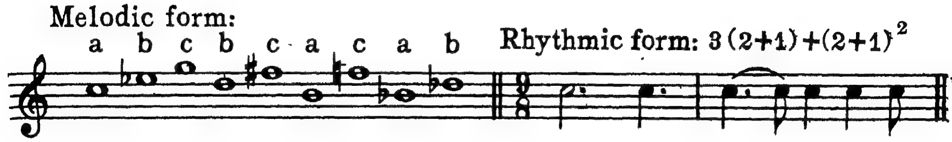


Figure 55. Selecting melodic and rhythmic forms.

By superimposing this rhythm of durations on the melodic form, we obtain an interference between the number of attacks in the melodic form (9) and the number of attacks in the rhythmic form (6). This means that the melodic form will appear twice, and the rhythmic form will appear three times.

Composition of Melodic Continuity

Melodic form consists of 9 attacks

$$\frac{9}{6} = \frac{3}{2} \qquad \begin{matrix} 2 & (9) \\ 3 & (6) \end{matrix}$$

Rhythmic form consists of 6 attacks

Melodic Continuity

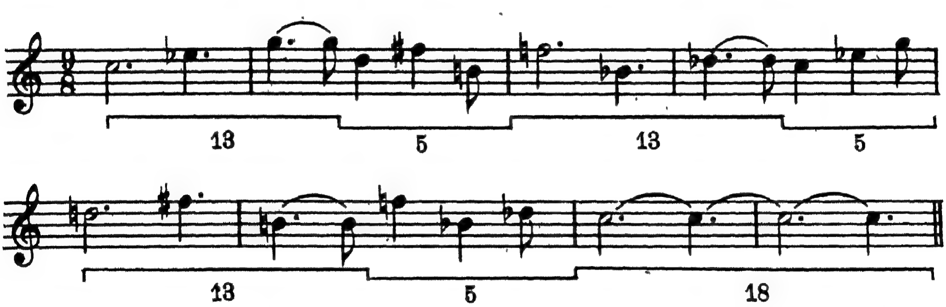


Figure 56. Interference between number of attacks in melodic and rhythmic forms.

For this melody a sequence of chords will be assigned to each tonic. Thus, the first sectional scale emphasizes 13t; the second, 5t; the third, 13t; the second recurrence of the first, 5t; the second recurrence of the second, 13t; the second recurrence of the third, 5t; and an axis (= 18t) is added to complete the whole.

There are two practical methods of symmetric harmonization of melodies constructed on symmetric pitch scales. The first provides an extraordinary variety of devices—the second is limited to a considerably smaller number of harmonizations.

1. The First Method

The first method assigns the important tones (all pitch-units in this case) of a sectional scale to the three upper functions of a $\Sigma(13)$, adding the remaining functions downward through any desirable selection. The first sectional scale in the sample melody has three pitch-units (c, e♭, g) which we shall originally conceive as 13 — 11 — 9, downwards. The continuation of this chord downwards will produce pitch-units with the following names: a, f, d, b. In the following $\Sigma(13)$ a certain structure is offered as a special case of many other possible Σ's.

Figure 57. $\Sigma 13$.

The upper three functions of the chord (denoted in black note-heads in the figure) may produce their own chord in harmony. Thus, the functions 9 — 11 — 13 of the Σ may actually become 1 — 3 — 5. All pitch-units of melody and harmony are identical in this case. (See Figure 58-a). By assigning the same three pitch-units as 3 — 5 — 7 we have to add one function down., (See Figure 58-b).

All further assignments of the three pitch-units, namely 5 — 7 — 9, 7 — 9 — 11, 9 — 11 — 13, 11 — 13 — 1, 13 — 1 — 3 are the c, d, e, f, g, respectively, on Figure 58. This figure offers a complete transposition of all assignments through the three tonics employed in the melody.

C-group

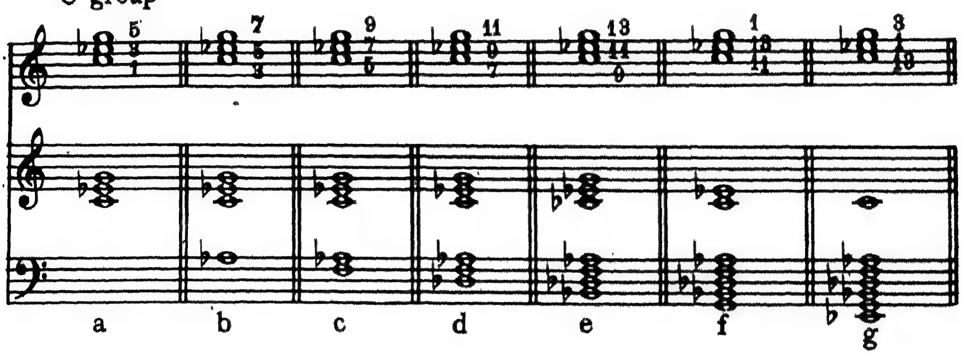
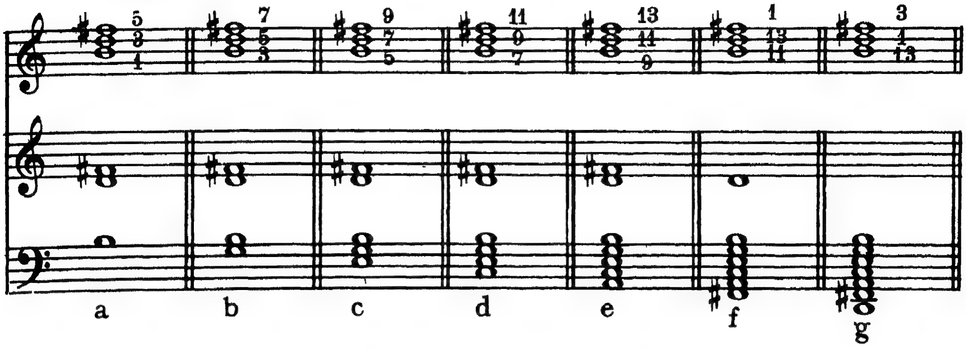


Figure 58. Melodic structures (continued).

B-group



B♭-group

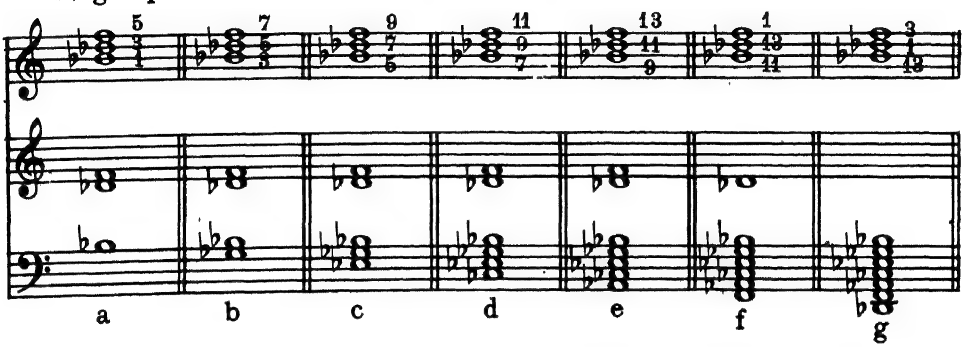


Figure 58. Melodic structures (concluded).

As Figure 58 exhausts all possibilities under the given group of chords, it is possible to exhaust all forms of harmonization for the given melody through various forms of constant and variable assignment of functions. The melody consists of 3 groups; so the sequence of chords with regard to these 3 groups can be read directly from Figure 58. The letters on Figure 59 represent the respective bars of Figure 58 in such a fashion that the first letter refers to the first group of the melody, the second to the second, and the third to the third.

aaa bbb ccc ddd eee fff ggg

aab	aba	baa	cca	cac	acc	eea	eae	aee	gga	gag	agg
aac	aca	caa	ccb	cbc	bcc	eeb	ebe	bee	ggb	gbg	bgg
aad	ada	daa	ccd	cdc	dcc	eec	ece	cee	ggc	gcg	cgg
aae	aea	eaa	cce	cec	ecc	eed	ede	dee	ggd	gdg	dgg
aaf	afa	faa	ccf	cfc	fcc	eef	efe	fee	gge	geg	egg
aag	aga	gaa	ccg	cgc	gcc	eeg	ege	gee	ggf	gfg	fgg
bba	bab	abb	dda	dad	add	ffa	faf	aff			
bbc	bcb	cbb	ddb	dbd	bdd	ffb	fbf	bff			
bbd	bdb	dbb	ddc	dcd	cdd	ffc	fcf	cff			
bbe	beb	ebb	dde	ded	edd	ffd	fdf	dff			
bbf	bfb	fbb	ddf	dfd	fdd	ffe	fef	eff			
bbg	bgb	gbb	ddg	dgd	gdd	ffg	fgf	gff			

Figure 59. Total number of possible harmonizations (continued).

abc	bcb	cde	def	efg
abd	bce	cdf	deg	
abe	bcf	cdg	dfg	
abf	bcg	cef		
abg	bde	ceg		
acd	bdf	cfg		
ace	bdg			
acf	bef			
acg	beg			
ade	bfg			
adf				
adg				
aef				
aeg				
afg				

Figure 59. Total number of possible harmonizations (concluded).

The total number of possible harmonizations to be derived from Figure 59 is as follows: 7 cases with constant tension: aaa, bbb, etc. $18 \times 7 = 126$ cases on a tension that is constant for 2 of the three groups. $35 \times 6 = 210$ cases with variable tension for all 3 groups. Thus, the total number of harmonizations offered for the melody is $7 + 126 + 210 = 343$.

2. The Second Method

The second method is based on a random selection of a Σ(13) based entirely on the composer's preference with regard to sonority. As any Σ (13) has definite substructures, often in limited quantities, the possibilities of harmonization are less varied than through the first method. If one selects Σ(13) with b♭ and f♯ on a c scale (see Figure 60) the possibilities of accommodating a sectional scale 3 + 4 (minor triad) becomes limited to only two assignments, namely, 5 − 7 − 9 and 13 − 1 − 3.

Figure 60. Σ(13) with b♭ and f♯ on c scale.

Retransposing these functions to the melody assigned for harmonization, we obtain the following results:

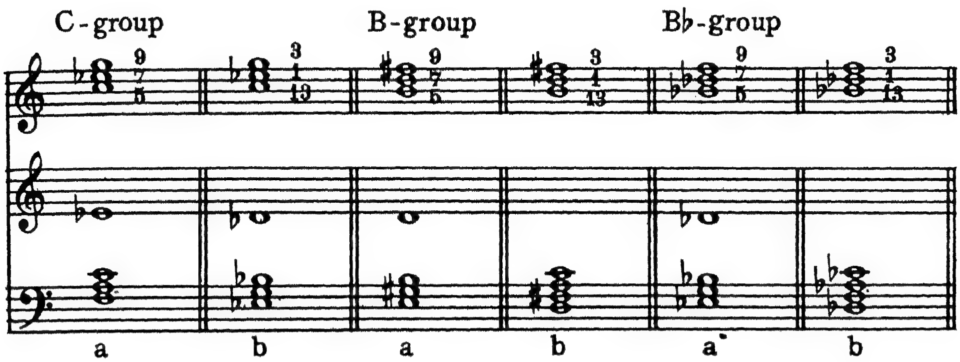


Figure 61. Harmonizing the melody.

It follows from this figure that each sectional scale of melody permits only two versions of chords. By either a constant or variable assignment of the two possible versions, a complete table of possible harmonizations is obtained.

aaa	bbb
aab	bba
aba	bab
baa	abb

Figure 62. Table of possible harmonizations.

The total number of possible harmonizations amounts to 8.

When the sectional scales are too complete, assignment of only certain tones as chordal functions is necessary. For example, in the following scale based on 3 tonics and 5-unit sectional scales, it is sufficient to assign the white notes as chordal functions, then in the melody derived from such a scale, black notes become the auxiliary and passing tones.

Figure 63. Scale based on 3 tonics and 5-unit sectional scales. White notes are chordal. Black are auxiliary.

In some symmetrical scales, the structure of individual sectional scales is such that the sonority of certain pitch-units does not conform to the structures of special harmony (i.e., the harmony of thirds). Some of the units of such sectional scales may be disturbing, and although they may fit as passing tones in some chord structures other than those used in this special harmony, they

decidedly do not fit as passing tones in any Σ (13). In such a case, each pitch-unit in such sectional scale of a compound symmetric scale must be selected either as a chordal function or as an auxiliary tone with a definite direction. These pairs—i.e., the chordal tone and its auxiliary tone—are then *directional units*.

In composing melodic forms from scales containing such directional units, permute the directional units and not simply the individual pitch-units. After all the units are assigned, the above-described procedure of harmonization (the second method) may be applied.

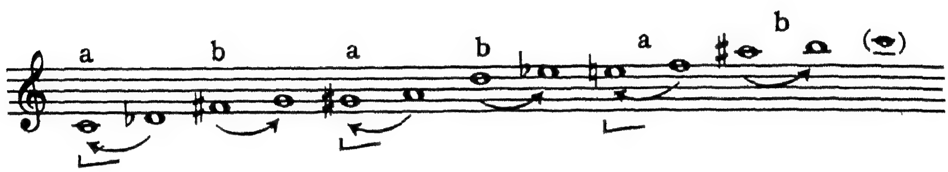


Figure 64. Applying the second method of harmonization.

The arrows on the above figure lead from an auxiliary tone to a chordal function.

E. Chromatic Harmonization of a Symmetric Melody

Chromatic harmonization of a symmetric melody is based on the same principle as chromatic harmonization of a diatonic melody. The procedure consists of inserting passing and auxiliary chromatic tones into symmetric harmonic continuity. As a result of the insertion of passing or auxiliary chromatic tones, altered chords may be formed as independent forms.

This type of harmonization may sound to the listener's ears either as chromatic continuity or as symmetric continuity with passing chromatic tones.

*Deep in a Dream**
by Jimmy Van Heusen and Eddie DeLange.

Figure 65. Four-part hybrid with chromatic harmonization (continued).

Figure 65. Four-part hybrid with chromatic harmonization (continued).

Figure 65. Four-part hybrid with chromatic harmonization (concluded).

If the composer or arranger finds that certain passing or auxiliary tones in the above example sound unsatisfactory, he may eliminate them. The greater the allowance made for altered chords, the greater are the possibilities for giving a chromatic character to a symmetric harmonic continuity.

F. Diatonic Harmonization of a Symmetric Melody

Melodies constructed from symmetric scales cannot be harmonized by a purely diatonic continuity. The style that has the most nearly diatonic characterization is in reality a hybrid of diatonic progressions *symmetrically connected.* This type of harmonization is possible when the melody that has been evolved within the scope of an individual sectional scale is one that can be harmonized by several chords belonging to one key. The relationship of symmetric sectional scales defines the form of symmetric connections between the diatonic portions of harmonic continuity. The diatonic portions of harmonization are brought into conformity with one key.

Symmetrical tonics do not necessarily represent the root chords of a key. For example, a note, c, in a melody scale may be the 1, or the 3, or the 5, etc., of any chord. In most cases, in music of the past, such harmonizations usually pertained to identical motifs in symmetric arrangement—as in the first announcement of a theme by the celli in Wagner's overture to *Tannhäuser*, where identical motifs are arranged through $\sqrt[4]{2}$, and the diatonic portions appear, the first in B minor making a progression IV — I — V — III, the following sections as exact transpositions through the $\sqrt[4]{2}$, i.e., in D minor and F minor respectively.

Figure 66. Identical motifs in symmetric arrangement for "Overture" to Tannhäuser.

In the following example of harmonization, the melody is based on a symmetric scale with three pitch-units (2 + 1) connected through $\sqrt[3]{2}$.

Scale

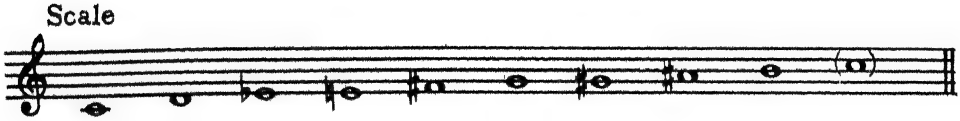


Figure 67. Melody based on symmetric scale with 3 pitch-units connected through $\sqrt[3]{2}$

Each bar comprises one sectional scale utilizing the melodic form, abcb. As there are many ways of harmonizing such a motif, I shall give here one of them which produces $C_0 + C_7 + C_5$ for each group. All the following groups are identical reproductions of the original group, connected through $\sqrt[3]{2}$.

Figure 68. Harmonizing the motif of figure 67. $C_0 + C_7 + C_5$.

Music by Rimsky-Korsakov, Borodin and Moussorgsky has abundant examples of such forms of harmonization.

In order to transform the above harmonization into a chromatic one, all that is necessary is to insert passing and auxiliary chromatic tones. A diatonic harmonization of those symmetric melodies which have not been composed on the sequence of identical motifs, and in which different portions pertaining to individual sectional scales are connected symmetrically, is possible as well. The latter form is not as obvious and it may seem somewhat incoherent to the ordinary listener.

G. Chromatic Harmonization of a Chromatic Melody

A melody which is to be harmonized chromatically must be a chromatic melody consisting of long durations. Each group of three units of melody must then be assigned to a chromatic operation in a chromatic group of harmony. The usual sequence d — ch — d refers to every three notes where the middle note is a chromatic alteration. In the following melody, the chromatic groups of harmony will be assigned as follows:

Group 1: c — c# — d
Group 2: d — d# — e
Group 3: a — a♭ — g
Group 4: g — g# — a
Group 5: a — s# — b

Figure 69. Chromatic melody.

The process of harmonization of a chromatic melody chromatically, consists of two procedures, once the pitch-units have been assigned to some number combinations. As our technique of chromatic harmony deals with 4-part harmony, the melody must become some one of the four parts. Let us assign the chromatic groups to the above melody as follows:

Group 1: 1 — 1 — 1
Group 2: 1 — 1 — 5
Group 3: 5 — 5 — 3
Group 4: 3 — 1 — 1
Group 5: 1 — 1 — 1

In group 3, a♭ is a lowered fifth. In group 5, a♯ is a raised root tone. The following example represents the above melody in a 4-part setting.

Figure 70. Melody of figure 69 in a four-part setting.

The final procedure in chromatic harmonization of a chromatic melody consists of *isolating* the melody; placing it above the harmony; and melodizing the *remaining 3-part harmony* with an additional voice. This additional voice is devised according to the fundamental forms of melodization, i.e., it may double any of the functions present in the chord, or it may add the function next in rank.

In the following example, the notes in parenthesis represent such an added voice. The functions of this voice are:

g — 5	e — 9
b — 13	c♯ — 13
a — 5	d — 5
b — 9	e — 5
b — 7	g — 7
c — 7	a — 7

Figure 71. Chromatic harmonization of a chromatic melody.

H. Diatonic Harmonization of a Chromatic Melody

A chromatic melody may be diatonically harmonized when it has a considerable degree of animation (short durations). In such a case, some of the tones are treated as chordal functions and some become auxiliary or passing chromatic tones. The process of determining which functions are to be diatonic then takes place.

The following example is the same melody that was used as an illustration in the preceding section; here it is used in its most animated form.

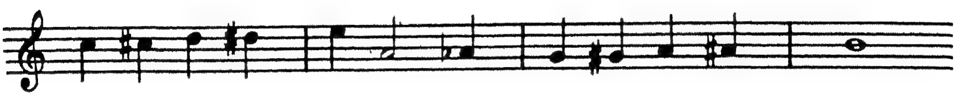


Figure 72. Chromatic melody in animated form.

By assigning $\begin{bmatrix} c-5 \\ d-13 \end{bmatrix}$ we acquire an F chord. In the next bar, by assigning $\begin{bmatrix} a-5 \\ e-9 \end{bmatrix}$ we obtain a D chord. By assigning $\begin{bmatrix} g-1 \\ a-9 \end{bmatrix}$ we obtain a G chord; and by assigning b 1 − 5, we obtain the B and E chords. In this way the entire melody can be placed in a certain desirable key (C major in this case). The units a♯ and c♯ in the second bar are auxiliary tones to the third and fifth respectively of the G chord. The entire harmonization has a Phrygian character.

Figure 73. Diatonic harmonization.

Another example of harmonization of the same melody will be found on the following page; by assigning the melodic tones to operate as the following functions, we obtain another harmonization:

c − 5	e − 13	g − 5	
d − 13	a − 9	a − 13	b − 3

Figure 74. Another harmonization.

I. Symmetric Harmonization of a Chromatic Melody

Symmetric harmonization of a chromatic melody is used for melodies of long durations. In most cases each pitch-unit of a melody has to be harmonized by a different chord. The advantage of the symmetric method of harmonization is that, if a melody is partly diatonic, there is an opportunity to use one chord against more than one pitch-unit of a melody. Any symmetric harmonization, as in the cases above, must be based on a preselected $\Sigma(13)$.

Let us assign the following $\Sigma(13)$ and use it for harmonization of the same melody as that used in the previous examples. The important considerations in the following procedure are (1) the variation of tension, and (2) the utilization of enharmonics as participants of $\Sigma(13)$; a♭ supplies an equivalent of g♯ for the 13th of a B chord.

Figure 75. Symmetric harmonization is based on a preselected $\Sigma(13)$.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK VII

THEORY OF COUNTERPOINT

The Technology of Correlated Melodies

BOOK VII

THEORY OF COUNTERPOINT

CHAPTER 1

THE THEORY OF HARMONIC INTERVALS

ANY sequence of two pitch-units produces a *melodic interval.* A simultaneous combination of two pitch-units produces a *harmonic interval.* The technique of correlation of simultaneous melodies depends entirely on the composition of harmonic intervals. Any number of simultaneous parts (voices) in counterpoint is formed by the pairs. These pairs may be conceived as voices immediately adjacent in pitch, or in any other form of vertical arrangement (i.e., over 1, over 2, etc.).

The degree of harmonic versatility achieved in counterpoint depends on the manifold of harmonic intervals used in a certain style. A limited number of harmonic intervals results in limited forms of harmonic versatility in counterpoint. The study of harmonic intervals is an important prerequisite to the study of counterpoint.

Harmonic intervals have a *dual* origin:

1. physical
2. musical.

The *physical* origin of harmonic intervals goes back to the simplest ratios. The *musical* origin of intervals is based on selective and combinatory processes. All semitones—i.e., the units of the equal temperament of twelve—are the structural units of all other harmonic intervals available in such equal temperament. As they occur in our hearing, they take the following forms:

$$i = 1, i = 2, i = 3, i = 4,$$
$$i = 5, i = 6, i = 7, i = 8$$
$$i = 9, i = 10, i = 11, i = 12$$

The above, of course, includes the entire selection available within one octave range. The addition of an interval to an octave produces a musically identical interval over one octave, for the similarity of different pitch-units within the ratio of 2 to 1 is so great that they even have identical musical names. The present system of musical notation involves—among other forms of confusion—a dual system of interval nomenclature. An interval containing three semitones, for example, may be called either a minor third or an augmented second.

A. Some Acoustical Fallacies

The simple ratios of acoustical intervals are merely *approximate* equivalents of harmonic intervals in equal temperament. It is not scientifically correct to think—as the majority of acousticians do—that a 5 to 4 ratio is the equivalent of a major third; or a 6 to 5, of a minor third; or a 7 to 4, of a minor seventh, etc. These intervals *deviate* considerably from their equivalents in equal temperament.

It is utterly impossible to follow some acousticians in the comparative relations they establish between the type and quality of intervals in the equal temperament of twelve and the equivalents of these intervals in simple acoustical ratios.* So-called "consonance" is a *totally different* type of interval relationship depending on whether it is considered musically or acoustically. If music actually had to use acoustical consonances only, while being confined to the equal temperament of twelve, the only real consonance available would be the octave; no two pitch-units bearing different names would ever be used, and we would have neither harmony nor counterpoint; for no intervals other than an octave (or a perfect fifth, with a certain allowance) are consonances within equal temperament. All other intervals are quite complicated ratios. The art of music in fact, however, has its own possibilities based on the limitations within the given manifold constituted by our tuning system.

Now, the acoustical consonances produce the so-called "natural harmonic scale," which consists of a fundamental tone with all its partials appearing in the same sequence as a natural harmonic series—1, 2, 3, 4, 5, 6, 7, 8, 9, etc. The ratios of acoustical consonances are equivalent to the ratios of vibrations producing pitches. For example, a $\frac{3}{2}$ ratio means that if the actual quantities representing both the numerator and the denominator were multiplied by a considerable number value, they would actually sound as pitches. While $\frac{3}{2}$, as such, sounds to our ear as the resultant of an interference of 3 to 2, $\frac{300}{200}$ cycles per second sounds to our ear as a perfect fifth.

*Figure 1. Acoustical scale of natural harmonics.***

Our ears accept pitch-units and their ratios in the form in which they reach our ears and our auditory consciousness—and not as they are asked to do according to the traditional musical schooling. For example, a melody played simultaneously in the key of c and in the key of b next to it, or a seventh above, sounds decidedly disturbing to musicians of our time. Yet an interval that is

*Indeed, despite the specific warning of the great acoustician, Helmholz, against careless application of his discoveries to music, the "acoustical fallacy" has vitiated endless quantities of musical theorizing. So we find Sir Donald Francis Tovey—by no means an undistinguished writer on music—lamenting that no "true" harmonic ideas are based on equal temperament, a statement which he can make directly in the face of the best that Western music has produced for more than 400 years. (Ed.)

**This scale is necessarily given in the notation used for equal temperament; the intervals in the acoustical scale—save for the octaves—are, of course, *not* identical with the same intervals in equal temperament. (Ed.)

musically identical is acoustically so different that, being placed three octaves apart, it produces a musically *consonant* impression.* The reason for this is that such an absolute interval as the seventh three octaves apart approximates the 15 to 1 ratio, i.e., the sound of a 15th harmonic in relation to its fundamental—and when the pitches are so far apart, the deviation from equal temperament becomes less obvious in our discrimination of pitch. The following tables offer a group of examples illustrating musically consonant intervals which are usually classified as dissonances, together with their correspondence to the proper location of harmonics. In all these cases, no octave substitution can be made without affecting the actual state of consonance.

Figure 2. Musically consonant intervals usually classified as dissonant.

Likewise, when musical consonances are placed in a wrong pitch register—such as low register—they produce upon our ears the effect of musical dissonances. The reason for this is that, being an approximation of simple ratios, they require the placement of their fundamentals at such low frequencies that they are below the range of audibility. For example, a major third—being associated with $\frac{5}{4}$ ratio—would require that its fundamental be located two octaves below the fourth harmonic; when music is played in major thirds in the contra-octave, the physical existence of such a fundamental is impossible.

*When Schillinger played the fourth example here (a melody coupled to its 7th at a 7 plus 3 octave interval), any number of capable musicians thought it was a 4-octave coupling. (Ed.)

**These numbers correspond to the numbers appearing on the acoustical scale of natural harmonics (Figure 1); they refer to the pitch of the first note. (Ed.)

The following tables offer three examples of the low setting of intervals.

Figure 3. The low setting of intervals.

With these thoughts in mind, we can see that no serious theory of the resolution of dissonant intervals may be devised without specifications as to the *exact octave location* of the intervals. In studying my theory of resolution of intervals, bear in mind that I offer it for the purpose of giving the composer a versatile treatment of progressions of harmonic intervals—not for the purpose of eliminating dissonances. Esthetically as well as physiologically, all of us desire sequences of tension and release. And, as different harmonic intervals produce different degrees of tension, the versatility of the sequence of intervals will satisfy such requirements.

It has often been the case that music written according to the "rules and regulations" of dogmatic counterpoint does not sound esthetically as convincing as real counterpoint in the 16th or 17th centuries. This inferior quality is due to the limited number of harmonic intervals and the forms of treatment of the latter.

B. Classification of Harmonic Intervals within the Equal Temperament of Twelve

Any harmonic interval may be classified in one of two ways:

1. With regard to its *density*, i.e., the fullness of sonority;
2. With regard to its *tension*, i.e., the degree of dissonance.

Classification of *density* evolves from the intervals producing the "emptiest" effect upon our ears up to the intervals producing the "fullest" effect. The table on the following page is only a general one; nevertheless, it serves the purpose with a certain degree of approximation—the first few intervals sound decidedly empty; the last few, decidedly full; in the middle, there are some intermediate intervals.

*See the footnote on the preceding page with regard to these numbers. (Ed.)

Figure 4. Classification of intervals according to density.

Classification of intervals according to *tension* is based on a separation of consonances from dissonances—and upon a separation of intervals which are consonances or dissonances *by name* from those which are consonances or dissonances *by sonority*. Every case in which a consonance and a dissonance correspond both in name and sonority is a case implying *diatonic* intervals; all cases in which the consonances and dissonances do *not* correspond with their original names produce *chromatic* intervals. The group of diatonic consonances includes perfect unisons, perfect octaves, perfect fifths, perfect fourths, major thirds, minor thirds, major sixths, minor sixths. The group of diatonic dissonances includes major and minor seconds, major and minor sevenths, major and minor ninths. All the chromatic intervals are classified into *augmented* and *diminished.*

The Augmented Intervals:
Unison, 2nd, 3rd, 4th, 5th, 6th.

The Diminished Intervals:
Octave, 7th, 6th, 5th, 4th, 3rd.

Tension

Figure 5. Diatonic and chromatic dissonances and consonances.

The augmented unison is equivalent to minor 2nd by sonority.
,, ,, 2nd ,, ,, ,, minor 3rd ,, ,,
,, ,, 3rd ,, ,, ,, perfect 4th ,, ,,
,, ,, 4th ,, ,, ,, no diatonic interval.
,, ,, 5th ,, ,, ,, minor 6th by sonority.
,, ,, 6th ,, ,, ,, minor 7th ,, ,,

The diminished octave ,, ,, ,, major 7th ,, ,,
,, ,, 7th ,, ,, ,, major 6th ,, ,,
,, ,, 6th ,, ,, ,, perfect 5th ,, ,,
,, ,, 5th ,, ,, ,, no diatonic interval.
,, ,, 4th ,, ,, ,, major 3rd by sonority.
,, ,, 3rd ,, ,, ,, major 2nd ,, ,,

The following "dissonant" intervals are actually consonances by sonority: the augmented 2nd, 3rd, 5th; the diminished 7th, 6th, 4th. All other chromatic intervals will be treated as dissonances, with resolutions corresponding to those of either diatonic or chromatic dissonances.

C. Resolution of Harmonic Intervals

The need for varying the tension results in the procedure known as the resolution of intervals. It is important to realize that the variation of tension may be gradual quite as well as sudden; the transition from a more dissonant harmonic interval to a less dissonant one, and finally into a fully consonant one, is just as desirable as a direct transition from extreme tension to full consonance.

In the following tables, intervals such as the perfect 4th and 5th are included along with the dissonances—not for the purpose of relieving them of tension, but for the purpose of devising different useful manipulations for contrapuntal sequences. The actual number of resolutions known to any composer has a definite effect on the harmonic versatility of his counterpoint. For example, if a composer knows only *four* resolutions of a major 2nd (which is the usual case) as compared to the *twelve* possible resolutions, the number of musical possibilities open to him is considerably restricted. Thinking in terms of variations one can see that the number of permutations available from four elements differs so much from those afforded by twelve elements that they cannot be compared, the first giving twenty-four variations and the second giving 479,001,600 variations. It is easy to see that when a composer suffers such losses as to the quantity of resolutions for each harmonic interval, the loss in the total versatility of his counterpoint is incalculable.

There is no need to memorize all the details for the resolution of intervals, as there are general underlying principles evolved over the centuries:

1. All diatonic intervals resolve through outward, inward, or oblique motion. Each moving voice moves by a semitone or whole tone.*

*An $i = 3$ is also correct when such an interval represents two adajcent musical names (c — d#, for example). (J.S.)

2. A resolution obtained through oblique motion may be replaced by one in which the formerly sustained voice leaps by a melodic interval of a perfect 4th, either up or down.

3. All intervals known as *2nds* have a tendency to expand. All intervals known as *7ths* have a tendency to contract. All 7ths are the exact equivalent of 2nds in the octave inversion (i.e., pitch-units are identical with those of the 2nds). All the *9ths* have a tendency to *contract*. All the *4ths* and *5ths* are "neutral," i e., they either *expand* or *contract*.

Thus, the entire range of permutations of semitones and whole tones, with their respective directions, constitutes the entire manifold of resolutions.

The reader may refer to the "Chart of Resolution of Diatonic Intervals" below.

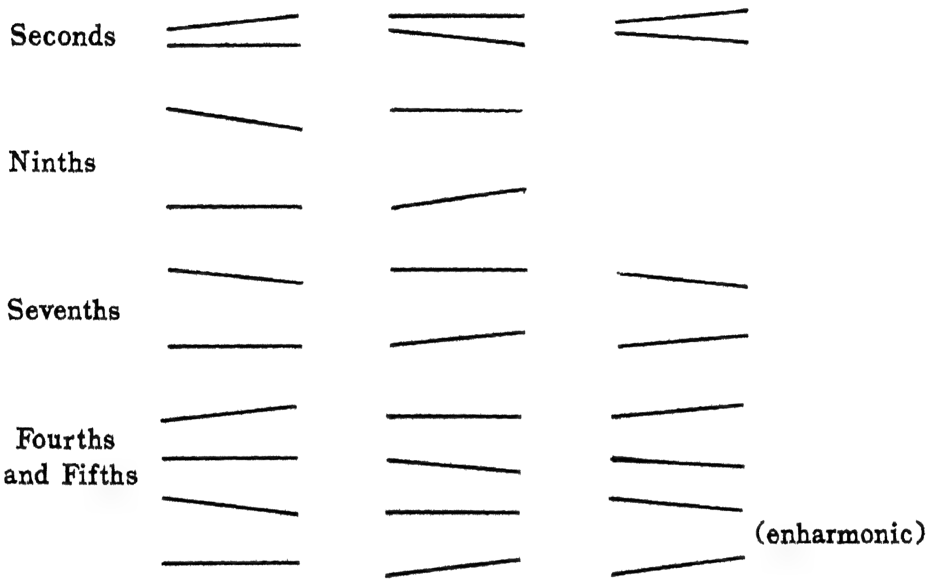


Figure 6. Resolution of diatonic intervals.

The following is a complete table of resolutions of diatonic intervals. The intervals in parentheses are the secondary resolutions, used in all cases in which the first resolution produces a dissonance:

Figure 7. Resolution of seconds and sevenths (continued).

Figure 7. Resolution of seconds and sevenths (concluded).

Fourths and Fifths

Figure 8. Resolution of fourths and fifths.

Figure 9. Resolution of ninths.

D. Resolution of Chromatic Intervals

All chromatic intervals which are augmented have a tendency toward expansion, and all chromatic intervals which are diminished have a tendency toward contraction. The logic of the resolution of augmented or diminished intervals is as follows: $^{d\sharp}_{c}$ is a 2nd derived through augmentation of a major second, through either altering the d to d♯, or the c♯ to c♮. Originally it could only have been a 2nd $^{d}_{c}$ or $^{d\sharp}_{c\sharp}$. Considering the dual origin of such interval, we find the respective resolutions: if d♯ is the alteration of d, its inertia makes it move further in the same direction, to e; or if c♮ is the alteration of c♯, it moves by inertia* to b♮. These two steps taken individually or simultaneously constitute the fundamental resolutions. An analogous procedure must be applied to diminished intervals; the diminutions are produced through inward alteration.

The following is a complete table of resolutions of chromatic intervals. When a chromatic interval resolves into a consonance by sonority, the sign "enh." is placed above it meaning "enharmonic." When the interval of resolution is surrounded by parentheses, the interval of resolution is a dissonance:

Figure 10. Resolution of chromatic intervals (augmented).
[Continued on following page].

*Note that inertia scientifically refers to the tendency of moving bodies to keep on moving in the same direction, as well as the tendency of motionless bodies to remain motionless. (Ed.)

Figure 10. Resolution of chromatic intervals (augmented) [concluded].

Figure 11. Resolution of chromatic intervals (diminished).
[Continued on following page].

Figure 11. Resolution of chromatic intervals (diminished) [*concluded*].

In the old counterpoint we often find a type of resolution different from those described above. They were known as *cambiata** resolutions and were conceived of as a melodic step of a 3rd instead of a 2nd. No good explanation has ever been given of the use of such resolutions; I offer an hypothesis to explain these resolutions, which I believe is the correct one.

As the tradition of old counterpoint was developed while the pentatonic (5-unit) scales were in use, some of the pitch-units of full diatonic (heptatonic, 7-unit) scales were absent. If we find that in resolving an interval $\genfrac{}{}{0pt}{}{d}{c}$, d moves to e, while c moves to a (instead of to b), a *cambiata* takes place simply because the scale is a pentatonic scale and the unit, b, does not exist.

This approach offers us a definite principle for resolution of intervals in scales which have not been in use in classical traditional music confining all resolutions merely to the step with the succeeding musical name. For example, in harmonic *a*-minor, the interval $\genfrac{}{}{0pt}{}{a}{g\sharp}$ may be resolved through movement of the lower voice only to f♮, as no other pitch-unit with the name f exists in the scale.

* *Nota cambiata*, i.e., a class of "changing" note. In addition to the requirements of resolution, the classical *cambiata* also observed certain temporal considerations with respect to the accent. (Ed.)

CHAPTER 2

THE CORRELATION OF TWO MELODIES

AS counterpoint represents a system of correlation of melodies in simultaneity and continuity, it is absolutely essential that the composer be thoroughly familiar with the constitution of melody. Only through complete familiarity with the material discussed in my exposition of the *Theory of Melody** is the successful accomplishment of such a task possible. The correlation of melodies is usually considered to be one of the most difficult of procedures; this is because the structural constitution of even one melody is unknown in ordinary theory; hence the combination of two unknown quantities is an entirely fantastic task to undertake.

The problem is not only that of putting two voices together, but one of either combining two melodies already made, or making a composition of two melodies with distinct individual characteristics. As *each* melody consists of several components—such as the rhythm of durations, attacks, melodic forms, the forms of trajectorial motion, etc.,—the correlation of *two* melodies adds one more component to those just mentioned: harmonic correlation. Counterpoint can be defined briefly as *a system of correlation of rhythmic, melodic, and harmonic forms in two or more conjugated melodies.*

I shall assume that the forms applying to one individual melody are known through the previous material; we will now cover that field of harmonic correlation which is based on the theory of harmonic intervals in Chapter 1 of this section. After covering this particular subject, I shall then discuss other forms of correlation—so that the composer may be capable of using the complete resources offered by contrapuntal technique.

A. Two-Part Counterpoint

The fundamental technique in writing two-part counterpoint is based on the writing of a new melody to a given melody. A given melody is usually *abstracted* from its rhythm of durations, thus producing a melodic form which may be taken from a choral, as well as from a popular song. The usual way of presenting such an abstracted melodic form is in whole notes, and this is usually called the *cantus firmus* ("firm chant," canonic, or established, chant). The abbreviation we shall use for cantus firmus will be "C.F."; for the melody written to it, counterpoint or "C.P." The first forms of counterpoint will be classified according to the number of attacks in C.P. occurring against one attack in C.F. All of these fundamental forms of counterpoint are devised as follows:

$$\frac{CP}{CF} = 1, 2, 3 \ldots \ldots n$$

*See Book IV.

B. $\boxed{\frac{CP}{CF} = a}$

This form of counterpoint—through international agreement for a number of centuries—implies the usage of consonances only. As we shall have four fundamental forms of harmonic correlation, and as some of these forms will be polytonal (i.e., there will be two different keys used simultaneously), we will have to use consonances by *name* and by *sonority*.

The positive requirements for harmonic correlation in 2-part counterpoint are:

a. A variety of types of interval (i.e., intervals as expressed by different numbers).
b. A variety of density.
c. Well-defined cadences, expressed through the use of leading tones moving into their axes.
d. Crossing of C.P. and C.F. is permissible *when necessary*.

The negative requirements are:

a. Elimination of consecutive intervals which are perfect unisons, octaves, 4ths and 5ths. Dissonances may not be used consecutively; the only intervals to be used in parallel motion are thirds and sixths.
b. There may be motion toward such intervals as unison, octave, 4th, or 5th—only through contrary (outward or inward) directions.
c. There may not be repetition of the same pitch-unit in CP unless it is in a different octave.

The forms of harmonic relations previously used in time continuity (see my earlier discussion of the theory of pitch scales)* will be used in counterpoint as the forms of simultaneous harmonic correlation.

C. Forms of Harmonic Correlation

1. U. — U. Unitonal — Unimodal: (identical scale structure and key signature).
2. U. — P. Unitonal — Polymodal: (a family scale with identical key signature).
3. P. — U. Polytonal — Unimodal: (identical scale structure, different key signature).
4. P. — P. Polytonal — Polymodal: (different scale structure, different key signature).

In the 14th century, in the music of Guillaume de Machault,** we find a fully developed type 2, and, in some cases, an undeveloped type 3. Only the ignorance and vanity of some contemporary composers make them believe that

*See Book II.

**Phonograph records of a Mass written by this composer for the coronation of Charles V are available. (*Les Paraphonistes de St. Jean des Matines* and Brass Ensemble, conducted by Van). A reconstruction of Machault's 2- and 3- part chansons in modern musical notation was published by the Deutsche Musikgesellschaft in 1926, in the edition of Friedrich Ludwig.

they are the discoverers of polytonal counterpoint; the joke being especially good on those modern French composers who make claim to priority, being unaware that it is their own direct musical ancestors who were the originators of this style centuries ago.

It is also unfortunate that the idea of polytonality is commonly associated with so-called "dissonant counterpoint", i.e., the counterpoint of continuous tension without release. Music based on polytonality *with* resolutions is a very fruitful, highly promising, and almost undiscovered field.

The usual length of a C. F. is about 5, 7, 9, or more bars, preferably in odd numbers—this requirement being traditional. The selection of different key signatures for types 3 and 4 is entirely a matter of choice. Any two scales—the root tones of which produce a consonance—may be used for this type of counterpoint. The best way to construct these exercises is to place the C.F. on a central staff, with two staves below and two staves above, assigning a different type of counterpoint to each staff.

In the following group of exercises, each part must be played *individually* with C.F. Each example produces four types of counterpoint with a historical perspective of eight centuries, for the first and second types were considerably developed during the Middle Ages, and the third and the fourth types are mostly used—when at all—in the music of today.

It is important to realize that all forms of traditional contrapuntal writing were based on the conception of *each melody being in a different mode.* One can even trace polytonal forms (although in their embryonic form) as far back as the 13th century.

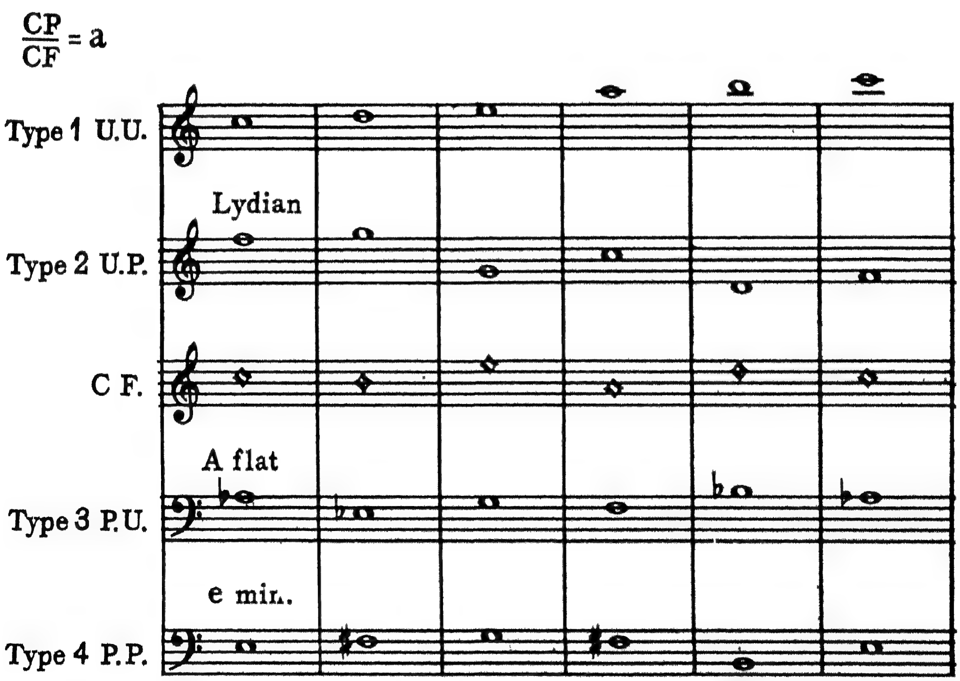


Figure 12. Two-part counterpoint.

As a temporary device for harmonic accompaniment, a double pedal point may be used in addition to the 2-part counterpoint. The root tones of both contrapuntal parts become the axes which must be assigned as chordal functions of a double pedal point. For example, in counterpoint of type 1 (giving the same pitch-units for both voices) the single root tone may be assigned as the root, or 3rd or 5th, etc., of a simple chord structure. Then, inasmuch as c is the axis for both contrapuntal parts in the example, the pedal point will become $\frac{g}{c}$ or $\frac{e}{a}$, $\frac{c}{f}$, etc.

This device is applicable to all four types of counterpoint. For example, in type 2, if one contrapuntal part were in Ionian c and the other in Aeolian a, the two might represent a root and a 3rd, or a 3rd and a 5th, etc., respectively. The pedal point in such a case would be $\frac{e}{a}$ or $\frac{c}{f}$, etc. In types 3 and 4, with any two such axes as c and a♭, we may use $\frac{e\flat}{a\flat}$ or $\frac{c}{f}$, etc., as pedal points. Each double pedal point must last through the entire contrapuntal continuity.

More flexible forms of harmonization of the 2-part counterpoint will be offered later.

D. $\boxed{\frac{CP}{CF} = 2a}$

In devising two attacks of the counterpoint against one attack of the C.F., the following combinations of harmonic intervals are possible:

(c = consonance; d = dissonance)

c – c

c – d

d – c

d – d*

In old counterpoint, all these cases were used in both *strict* and *free* style, with the exception that a dissonance was not supposed to occur on the first of the two attacks.

Each bar may start with either a consonance or a dissonance, and, in the case of $\frac{CP}{CF} = 2$, all dissonances require immediate resolution. The following pages contain a few examples of such contrapuntal exercises.

*In scalewise contrary motion only. (J.S.)

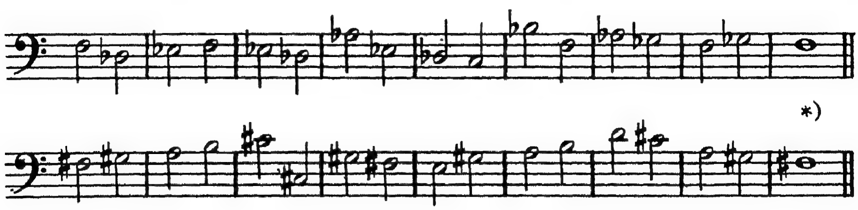
*) Aeolian axis caused by necessity of having a consonance for the ending. (J. S.)

Figure 13. Two attacks of C.P. to one of C.F.

Figure 14. Two attacks of C.P. to one of C.F. (continued).

CP
Type 4

CP
Type 4



*)Allowance is made for a weak dissonance. (J. S.)

Figure 14. Two attacks of C.P. to one of C.F. (concluded).

Figure 15. Two attacks of C.P. to one of C.F. (continued).

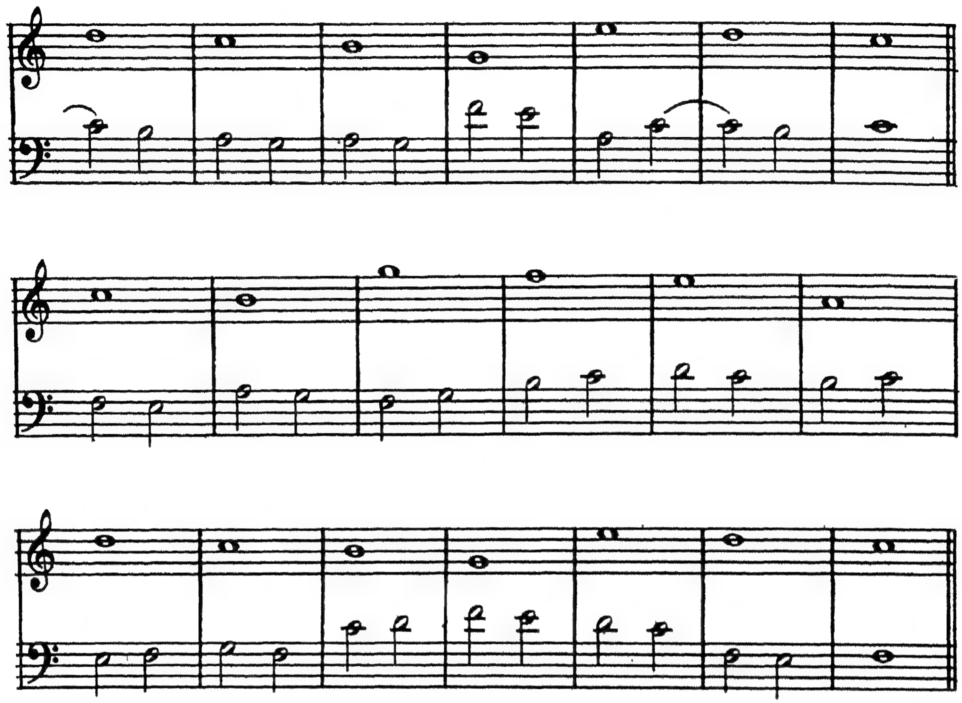


Figure 15. Two attacks of C.P. to one of C.F. (concluded).

E. $\boxed{\frac{CP}{CF} = 3a}$

Three attacks of CP against one attack of CF offer us the following combinations of harmonic intervals:

c — c — c
c — d — c
d — c — c
c — c — d resolution
d — c — d resolution
d — d *— c
c — d — d*

The d — c — c combination offers a new device which only becomes possible with three or more attacks; we shall call it a *delayed* (or indirect) *resolution.* Instead of resolving a tense interval at once, we move it to another consonance, after which we resolve the dissonance.

*In scalewise contrary motion only. (J.S.)

This procedure accomplishes two things:

(1) it produces psychological suspense, thus making the music more interesting;

(2) it produces *ipso facto* a more expressive melodic form.

Figure 16. Examples of delayed resolutions.

Figure 17. $\frac{CP}{CF} = 3a.$

F. $\boxed{\frac{CP}{CF} = 4a}$

Four attacks of CP against one attack of CF offer still more combinations of harmonic intervals:

c — c — c — c
c — c — c — d resolution
c — c — d — c
c — d — c — c
d — c — c — c
c — c — d — d*
c — d — d* — c
d — d ** c — c
d — c — c — d resolution
c — d — c — d resolution
d — c — d — c

There are wider-possibilities in the field of delayed resolution for $\frac{CP}{CF} = 4$.

Parallel axes, centrifugal and centripetal forms now become more prominent among the devices by which the composer may construct the second melody.

Figure 18. Examples of delayed resolutions.

It is also useful to know all the advantageous starting points for those scalewise passages which end with a consonance:

Figure 19. Examples of passages ending with a consonance.

*In scalewise contrary motion only. (J.S.)

**Either the same as in *, or two independent dissonances, both of which are resolved by the following c — c in any order. (J.S.)

Figure 20. $\frac{CP}{CF} = 4a.$

G. $\boxed{\frac{CP}{CF} = 5a}$

It is no longer necessary to tabulate all the possible combinations of c and d.

The best melodic quality in the CP will result from extensive use of delayed resolutions. Combined with a variety of interval and with scalewise passage, delayed resolutions make available the most versatile forms of melody.

The devices for delayed resolution, impossible for fewer attacks than five, are as follows:

$d_1\ d_2\ c\ d_1\ c$—the first dissonance is followed by a second dissonance with its resolution, then by the repetition of the first dissonance with its resolution.

$d_1\ d_2\ c\ d_2\ c$—the first dissonance is followed by the second dissonance without resolution, followed by the resolution of the first dissonance, then by the second dissonance and its resolution.

Figure 21. Examples of delayed resolutions.

Figure 22. Scalewise passages ending with a consonance.

Figure 23. $\frac{CP}{CF}$ *= 5a (continued).*

Figure 23. $\frac{CP}{CF} = 5a$ (*concluded*).

H. $\boxed{\frac{CP}{CF} = 6a}$

Still other devices for delayed resolutions become possible with six attacks:

$d_1\ d_2\ d_1\ c\ d_2\ c$—the first dissonance, the second dissonance, repetition of the first dissonance with its resolution, repetition of the second dissonance with its resolution;

$d_1\ d_2\ c\ d_2\ c\ c$—the first dissonance, the second dissonance, resolution of the first dissonance, repetition of the second dissonance, the delay, and the resolution of the second dissonance;

$d_1\ d_2\ c\ d_1\ c\ c$—the first dissonance, the second dissonance with its resolution, repetition of the first dissonance, delay, and resolution of the first dissonance;

$d_1\ c\ c\ d_2\ c\ c$—a combination of two groups of three, each consisting of dissonance, delay, and resolution.

Other combinations may be devised in a similar way, for example, $d_1\ c\ d_2\ c\ d_2\ c$—which is the combination, 2 + 4.

In using six attacks against CF, it is easy to devise a great variety of melodic forms and interference patterns, as discussed in the section on melodization of harmony.*

*See Book VI, pages 619-625.

Figure 24. Examples of delayed resolutions

Figure 25. Scalewise passages ending with a consonance.

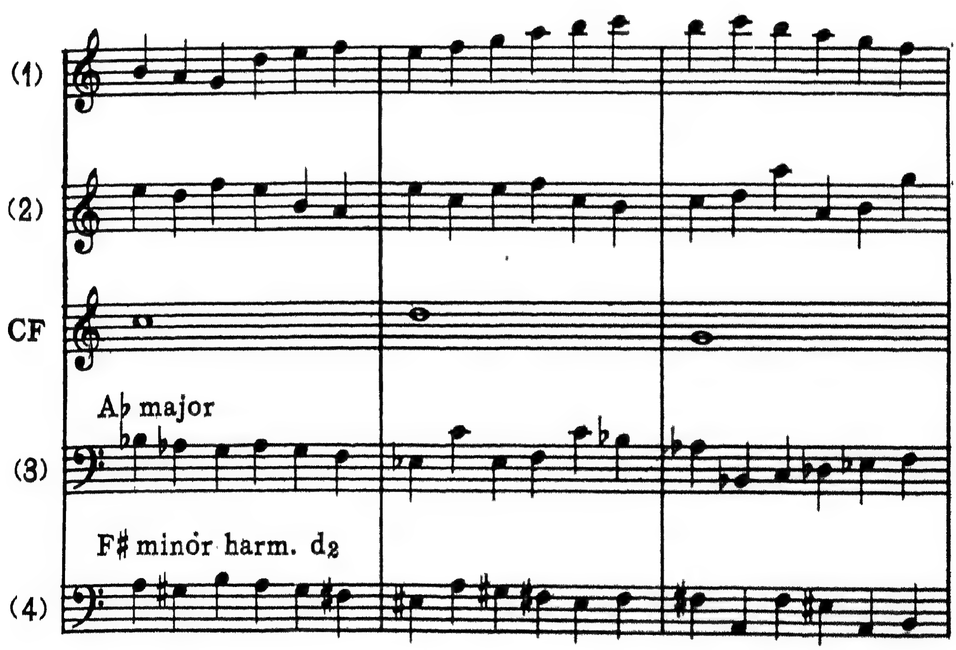


Figure 26. $\frac{\text{CP}}{\text{CF}}$ = *6a (continued).*

Figure 26. $\frac{CP}{CF}$ = *6a (concluded).*

I. $\boxed{\frac{CP}{CF} = 7a}$

Seven attacks of CP against one of CF offer new forms of delayed resolutions. The number of new combinations grows, and it becomes quite easy to develop various melodic forms built on parallel, converging, and diverging axes.

Figure 27. Examples of delayed resolutions.

Figure 28. Scalewise passages ending with a consonance.

Figure 29. $\frac{CP}{CF} = 7a$ *(continued).*

Figure 29. $\frac{CP}{CF} = 7a$ *(concluded).*

J. $\boxed{\frac{CP}{CF} = 8a}$

Eight attacks of a CP against one of CF offer a still greater variety of melodic forms. The latter may be obtained through the technique of delayed resolutions. It is equally fruitful to devise melodic forms by means of attack-groups, for example, by thinking of 8 as $\frac{8}{8}$ series represented through its binomials and trinomials. Interference groups may be carried out in counterpoint in the same way as in the melodization of harmony*, in which technique such groups were used against the attacks of H.

Figure 30. Examples of delayed resolutions.

*See Book VI.

All 8-against-1 scalewise passages ending with a consonance must start and end with the same pitch unit, as this is a property of our seven-name musical system.

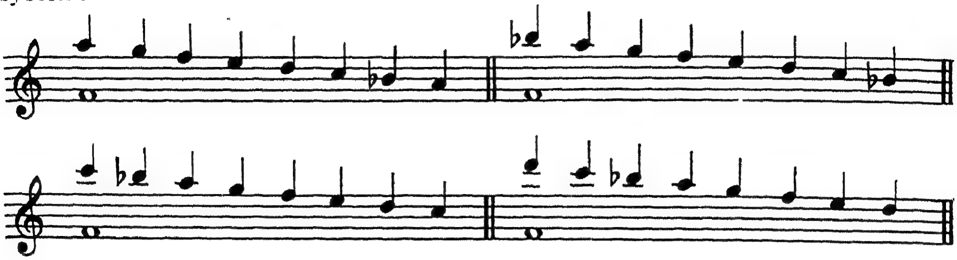


Figure 31. Scalewise passages ending with a consonance.

Figure 32. $\frac{CP}{CF} = 8a$

The $\frac{CP}{CF} = 8a$ gives the composer sufficient technical equipment for an unlimited number of attacks. It would be desirable for the student now to devise such cases as $\frac{CP}{CF} = 12\ a$, and $\frac{CP}{CF} = 16\ a$, as they provide very useful material for animated forms of passage-like obligatos. Under the usual or traditional treatment, such groups with many attacks of CP against CF remain uniform or nearly uniform in durations.

The most important conditions for obtaining an expressive counterpoint are:

(1) an abundance of dissonances;
(2) delayed resolutions; and
(3) interference attack-groups.

CHAPTER 3

ATTACK-GROUPS IN TWO-PART COUNTERPOINT

IN all the forms of counterpoint discussed so far, the attack-group of CP against each attack of CF was constant: $\frac{CP}{CF} = A$ const. The monomial attack group consisted of any desirable number of attacks: $A = a, 2a, 3a, \ldots ma$.

Now, however, we arrive at *binomial* attack-groups for CP. This situation may be expressed as $\frac{CP}{CF} = A_1 + A_2$, i.e., the counterpoint to be written to two successive attacks of the cantus firmus is to consist of two different attack-groups.

For instance:

$$(1)\ \frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{2a}{a} + \frac{a}{a}; \qquad (2)\ \frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{3a}{a} + \frac{2a}{a};$$

$$(3)\ \frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{5a}{a} + \frac{3a}{a}; \qquad (4)\ \frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{a}{a} + \frac{8a}{a}; \ldots$$

The selection of number values for the attacks of CP against the attacks of CF depends on the amount of contrast desired in the two successive attack-groups of CP.

All further details pertaining to this problem are to be found in my earlier discussion of the theory of melodization.*

Binomial attack-groups are subject to permutations. For example, if $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{4a}{a} + \frac{2a}{a}$, this binomial attack-group may be varied further through permutations of a higher order. Suppose CF has 8a; then the whole contrapuntal continuity will acquire the following distribution of attack-groups:

$$\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} + \frac{CP_3}{CF_3} + \frac{CP_4}{CF_4} + \frac{CP_5}{CF_5} + \frac{CP_6}{CF_6} + \frac{CP_7}{CF_7} + \frac{CP_8}{CF_8}, \text{ or}$$

$$\frac{CP_{1-8}}{CF_{1-8}} = \frac{4a}{a} + \frac{2a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{4a}{a} + \frac{2a}{a}.$$

Polynomial attack-groups of CP against CF may be devised in a similar fashion.

The resultants of interference, their variations, involution groups, and series of variable velocities may all be used as material for this purpose.

Examples of polynomial attack-groups of $\frac{CP}{CF}$:

$$(1)\ \frac{CP_{1-6}}{CF_{1-6}} = \frac{3a}{a} + \frac{a}{a} + \frac{2a}{a} + \frac{2a}{a} + \frac{a}{a} + \frac{3a}{a};$$

$$(2)\ \frac{CP_{1-9}}{CF_{1-9}} = \frac{2a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{2a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{2a}{a};$$

$$(3)\ \frac{CP_{1-5}}{CF_{1-5}} = \frac{a}{a} + \frac{2a}{a} + \frac{3a}{a} + \frac{5a}{a} + \frac{8a}{a};$$

$$(4)\ \frac{CP_{1-4}}{CF_{1-4}} = \frac{9a}{a} + \frac{6a}{a} + \frac{6a}{a} + \frac{4a}{a}.$$

*See Book IV.

The simplest duration-equivalents of attacks will be used in the following examples.

$$\frac{CP}{CF} = \frac{a}{a} + \frac{6a}{a} + \frac{3a}{a}$$

Figure 33. $\frac{CP}{CF}$ = A var.

$$\frac{CP}{CF} = \frac{a}{a} + \frac{5a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{3a}{a} + \frac{3a}{a} + \frac{4a}{a} + \frac{2a}{a} + \frac{5a}{a} + \frac{a}{a}$$

Figure 34. $\frac{CP}{CF}$ = A var.

$$\frac{CP}{CF} = (1+2)^3 = \frac{a}{a} + \frac{2a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{4a}{a} + \frac{8a}{a}$$

Figure 35. $\frac{CP}{CF}$ = A var.

A. More than One Attack of CF to CP

At this stage it should not be difficult for the student to develop the technique of writing *one* attack of CP to a group of attacks of CF. In an exercise, CF must be so constructed as to permit the matching of one attack against a given attack-group. In composing a counterpart to a given melody, it is necessary to compose the attack-groups first. This should be done with a view to the possibilities of resolving the harmonic intervals. Whenever the assumed group does not permit one to carry out the resolution requirements (such as expanding of the second, contracting of the seventh or the ninth, etc.), then the attack-group itself must be reconstructed.

As was mentioned previously, it is entirely practical to re-write the given melody into uniform durations first, then to assign advantageous attack-groups. After the counterpoint has been written, the original scheme of durations may then be reconstructed.

With the equipment which I have so far presented, only such melodies may be used as the cantus firmus which is built on one scale at a time; the scale itself must belong to the first group (see my discussion of the theory of pitch scales).*

The procedure of distributing the attack-groups of a given melody is analogous to that used in the technique of the harmonization of melody,** according to which the attacks of a given melody were distributed in relation to the number of chords accompanying the melodic attacks.

*See Book II. **See Book VI, Chapter 3.

The following example is a melody which has been subjected to different attack treatments in the process of writing a counterpart to it.

$$\frac{CP}{CF} = \frac{a}{3a} + \frac{a}{a} + \frac{a}{2a} + \frac{a}{2a} + \frac{a}{a}$$

Figure 36. Single attack of CP to many of CF.

$$\frac{CP}{CF} = \frac{a}{6a} + \frac{a}{3a} + \frac{a}{2a} + \frac{a}{4a} + \frac{a}{3a}$$

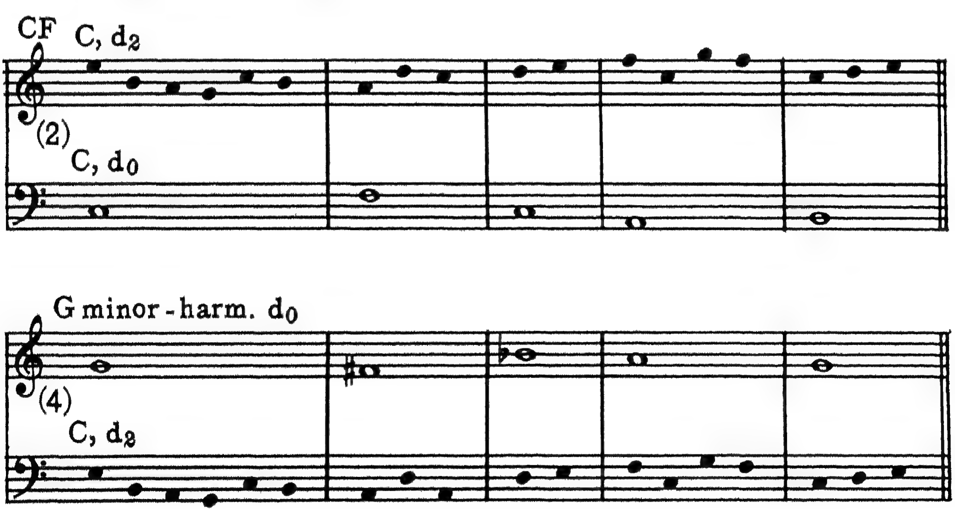


Figure 37. Single attack of CP to many of CF.

In writing a counterpart to a given melody (but *without* consideration of any harmonic accompaniment that may also be given) it is important to consider:

(1) the composition of attacks, and
(2) the composition of durations.

The choice of means for the composition of attacks depends on the degree of animation of the given melody. If a lively melody is to be compensated, then the countermelody should be devised on the basis of reciprocation of attacks and, finally, of durations. All the techniques pertaining to the variation of two elements serve as material for such a two-part compensation (counterbalancing).

If a lively melody is to be contrasted, then the countermelody should be devised by *summing up* groups of attacks together with their durations. The sums of durations of the given melody, with the specified number of attacks against each attack of the countermelody, define the durations of the counterpart.

If a slow melody is to be compensated (counterbalanced) by a slow counterpart, then the technique of reciprocation of attacks and durations should be used. Variations of two elements provide such a technique.

If a slow melody is to be contrasted, then the countermelody should be devised first by defining the number of attacks in the countermelody against each individual attack of the given melody, after which the sum of the attacks of the counterpart will represent the duration, equivalent to the duration of one attack of the given melody.

When one handles melodies which have animated portions alternating with slow ones, or with cadences, it will be found that these are particularly suited for the compensation method. In such a case, when one melody stops, the other moves—and *vice versa*.

Let us analyze the problem, say, of writing a counterpart to a given melody, taking the setting to Ben Jonson's *Drink to Me Only With Thine Eyes.*

The melody is:

Reconstruction of this melody into a CF gives it the following appearance:

Figure 38. C.F. of Drink to Me Only with Thine Eyes.

This is a fairly animated type of melody.

Let us first devise a scheme of durations for CP. One of the simplest solutions for a contrasting CP would be to make each attack of CP correspond to T; we would obtain CP = 4a and a = 6t. For a less moderate contrast, we could assign CP = 8a and a = 3t. To obtain a CP of the counterbalancing type, we would have to assign two contrasting elements, if such can be found in CF. As $T_1 = 2a$ and $T_2 = 6a$, and as $T_3 = 5a$ and $T_4 = a$, this CF provides sufficient material for assigning two elements and for compensating them in CP. There is, of course, no way to counterbalance the original version of this melody. In this way we obtain the following three solutions, each different, but all equally acceptable.

Figure 39. Varying counterpoints to melody of Drink to Me Only with Thine Eyes *(continued).*

Figure 39. Varying counterpoints (continued).

Figure 39. Varying counterpoints (concluded).

B. Direct Composition of Durations in Two-Part Counterpoint

In composing an original two-part counterpoint, it is often desirable first to compose the two counterparts rhythmically. The entire technique of handling binomials and their variations (as set forth in my theory of rhythm)* is applicable in this case.

Counterbalancing (compensation) is achieved through permutation of binomials, and this may follow through the higher orders. For example:

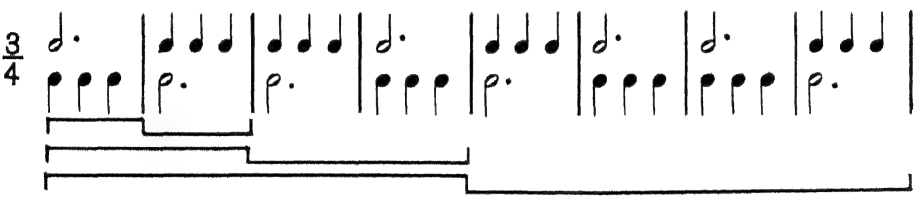


Figure 40. Counterbalancing through permutation of binomials.

It does not matter which part is written first (thus becoming the CF) in such a case. It is essential, however, to write one part *completely*, not section by section. The CP must be written *after* the CF has been completed.

For a more *diversified* rhythmic continuity, resultants with an even number of terms may be used; the binomials constantly reciprocate in such a case. For example, $T = r_{8 \div 7}$ $(+\ 8t)$:

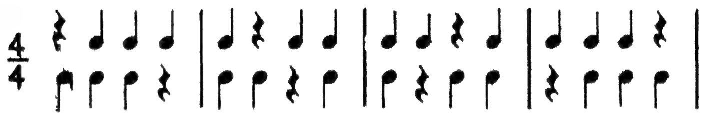
Figure 41. Employing reciprocating binomials.

*Book I.

In all such cases (continuous reciprocation of the variable binomials), the number of attacks of CP against CF remains constant while the durations vary

Homogeneous effects of rhythm in both counterparts may be achieved througl varying the rests or split-unit groups. The groups themselves do not have to be binomials. The two "best" of any polynomial groups are the self-reciprocating members.*

For example: (a) rests



or: (b) tied rests

(c) split-unit groups

Figure 42. Self-reciprocating members.

Any rhythmic group set against its converse provides a satisfactory counterpart. For example: $2\left(r_{\frac{5\div4}{2}}\right)$; T = 4t.

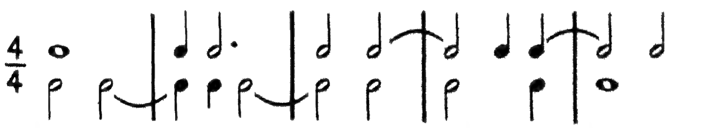


Figure 43. Converse of a rhythmic group provides satisfactory counterpoint.

Any of the series of variable velocities may be used for such a purpose. For example:

Figure 44. Summation series I.

*In variations or circular permutation of three or more elements, it is selective and desirable to choose only pairs of the resultant groups. The self-reciprocating groups, of which there are rarely more than two, are called the best. They will be found in circular permutations of rests, accents and split-unit groups (also non-uniform durations). See Book I. (Ed.)

Adjacent contrasts for two mutually compensating parts may be achieved by synchronized involution-groups placed in a sequence. The two powers supply the *a* and *b* elements, and thus are treated through the permutations of two elements (any order).

For example: $(2+1)^2 + 3(2+1)$.
$a = (2+1)^2$; $b = 3(2+1)$

Or, for example: $4(2+1+1) + (2+1+1)^2$.
$a = 4(2+1+1)$; $b = (2+1+1)^2$.

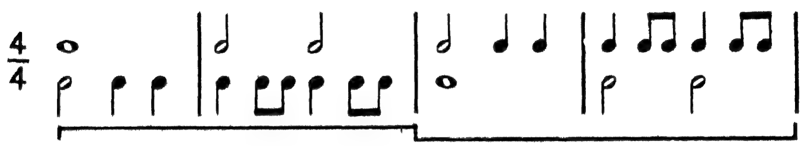


Figure 45. Permutation of two elements.

All the above devices permit one to start work with the composition of either part as the CF, and they all refer to counterbalancing (compensation).

The technique of simultaneous harmonic contrasts between CF and CP is based on the distributive involution of the two synchronized parts used simultaneously. Any number of terms may be used as a group. The limitation of *two parts* corresponds to the *two power-groups* (adjacent or non-adjacent powers). In all such cases, the number of attacks of CP against CF is constant, and such a number equals the number of terms in the polynomial. Thus, a binomial squared gives $\frac{CP}{CF} = 2a$; a trinomial squared gives $\frac{CP}{CF} = 3a$, etc.

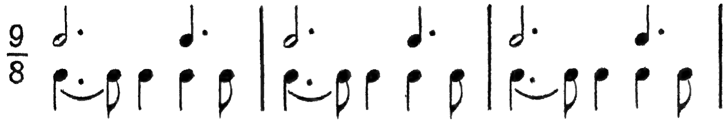
Still greater contrasts may be achieved either by using larger polynomials, or by synchronizing non-adjacent powers. In the latter case a binomial cubed and used against its synchronized first power gives $\frac{CP}{CF} = 4a$, i.e., 2^2; a trinomial cubed and used against its synchronized first power gives $\frac{CP}{CF} = 9a$, i.e., 3^3, etc.

Nothing prevents the composer from using adjacent higher powers—such as cubes against squares, fourth power groups against cubes, etc.

In all these cases the lower power employed represents the CF, as it is easier to match several attacks against a given single attack, than *vice versa.*

Examples:

(a) CF = 3(2+1); CP = $(2+1)^2$.



(b) CF = 9(2+1); CP = $(2+1)^3$.

(c) CF = 8(2+1+2+1+2); CP = $(2+1+2+1+2)^2$.

(d) CF = 16(2+1+1); CP = $(2+1+1)^3$.

Figure 46. Using larger polynomials for contrast.

In addition to involution-groups, *coefficients of duration* may be used, as in $\frac{CP}{CF} = \frac{2\,(r4 \div 3)}{r8 \div 6} = \frac{(3+1+2+2+1+3) + (3+1+2+2+1+3)}{6+2+4+4+2+6}$, as well as the resultants of instrumental interference composed for two parts.

In all the following examples, the intonation of CF was composed first.

Figure 47. Two-part counterpoint with pre-composed duration group (continued).

Figure 47. Two-part counterpoint with pre-composed duration group (continued).

Coefficients of duration = R $\underline{4 \div 3}$

Figure 47. Two-part counterpoint with pre-composed duration group (continued).

The resultants of instrumental interference: R $\underline{3 \div 2}$

Figure 47. Two-part counterpoint with pre-composed duration group (concluded).

C. Chromatization of Diatonic Counterpoint

It would seem to be easy to write a chromatic counterpart to any diatonic melody, for any suitable pitch-unit may be chosen from anywhere in the entire chromatic scale. But such countermelodies have one general defect, a neutral character which comes with a uniform scale. To the average listener such counterpoint sounds as if any pitch-unit would be just as acceptable as those already set.

This peculiarity of musical perception is due to our inherited and cultivated *diatonic orientation.* The average listener hears chromatic units as an ornamental supplement to a diatonic scale. Chromatic units are commonly used as auxiliary tones moving into the diatonic units of a given scale, forming *directional* units. Now, diatonic units are perceived as *independent* pitches (although in a certain grouping in sequence), but chromatic units are perceived as *dependent* pitches *leading into* diatonic pitches.

Music constructed entirely chromatically, i.e., without diatonic dependence, therefore usually belongs to a category different from diatonic music with directional units; it is known under the name of "atonal", or "twelve-tone" music.

For this reason,* we shall use chromatic counterpoint with diatonic dependence only. This kind of counterpoint may be devised at its best by means of inserting passing or auxiliary chromatic units *after* the diatonic counterpoint has been written.

This technique is applicable to all four types of harmonic relations. It is important to note that the conversion of diatonic into chromatic counterpoint does not affect the established forms of resolutions; remodeling of durations may be accomplished by means of split-unit groups, a device allowing us to preserve the character of the rhythm which was originally set.

*Unless, of course, the composer *wants* to write "atonal" music. (Ed.)

Original

Chromatic variation

Original

Chromatic variation

Original

Figure 48. Chromatic variation of diatonic counterpoint (continued).

Chromatic variation (both parts are chromatized)

Figure 48. Chromatic variation of diatonic counterpoint (concluded).

CHAPTER 4

THE COMPOSITION OF CONTRAPUNTAL CONTINUITY

THE extension of any given contrapuntal continuity is based on geometrical mutations.

The fundamental technique of these geometrical mutations, in two-part counterpoint, is the interchange of music assigned to CF and CP. Assuming that CF represents the actual melody, and CP represents the actual counterpart, we obtain two variants for each voice: $\frac{CP}{CF}$ and $\frac{CF}{CP}$, where both CF's and both CP's are identical but change their vertical positions.

In the old systems of counterpoint, this device was known as "vertical convertibility in octave." We shall regard it merely as a device formed by two variants of the exposition for any counterpoint; we shall consider such convertibility to be an inherent property of counterpoint as such.

By applying the principle of variation of two elements *ad infinitum*, i.e., through permutations of the higher orders, we can compose an entire piece of music from a single contrapuntal exposition.

Figure 49. Contrapuntal continuity of the third order produced through permutation of parts of the original exposition (continued).

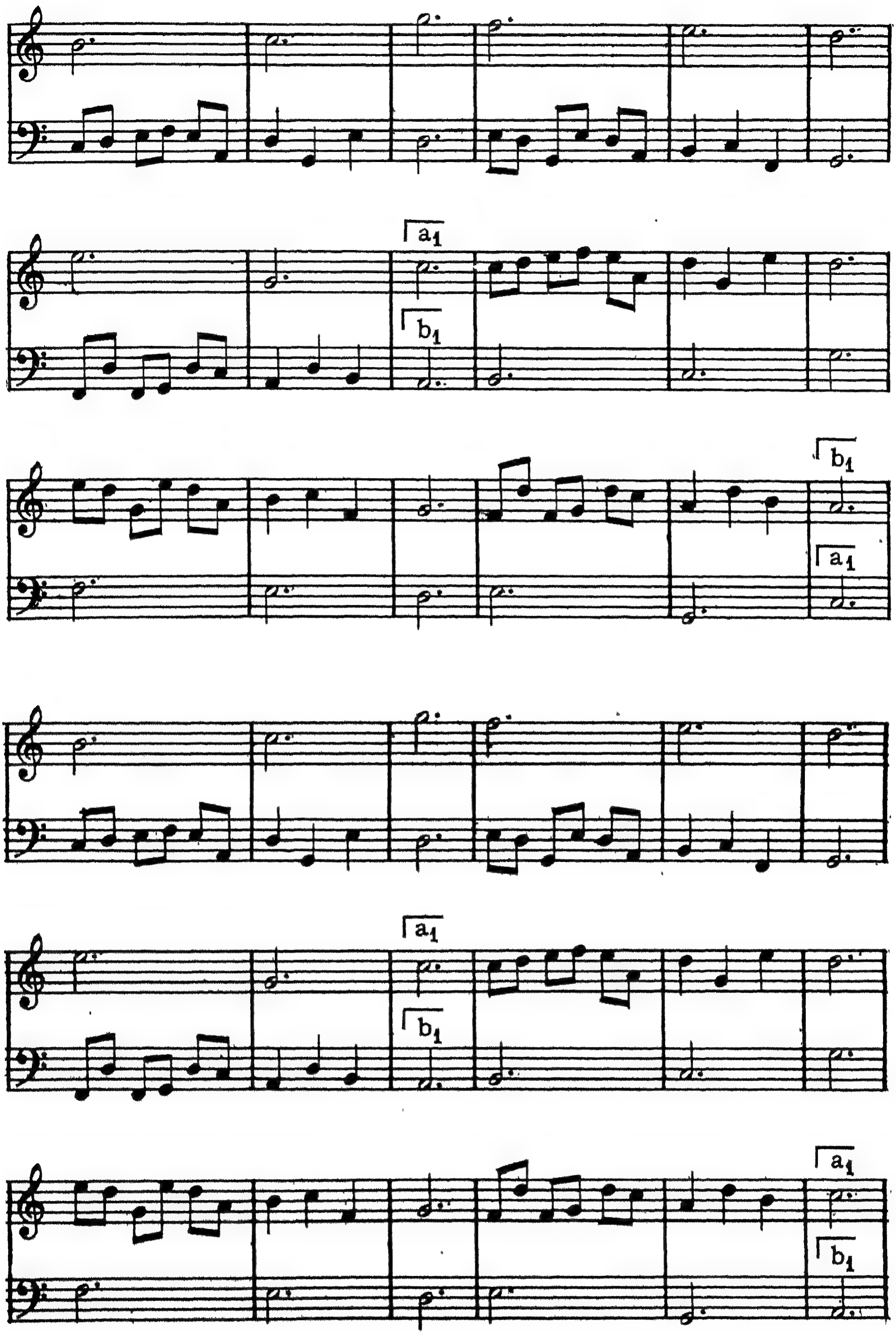


Figure 49. Contrapuntal continuity of the third order produced through permutation of parts of the original exposition (continued).

Figure 49. Contrapuntal continuity of the third order produced through permutation of parts of the original exposition (concluded).

When it is conceived geometrically, any musical exposition becomes subject to *quadrant rotation* (as described earlier in my discussion of geometrical projections of music), yielding the four variations of the geometrical position: ⓐ, ⓑ, ⓒ, ⓓ.*

Through vertical permutation of parts, two-part exposition yields two variants. Each variant has four rotational positions; the total number of variants for one two-part contrapuntal exposition is therefore eight:

$$\frac{CF}{CP}ⓐ,\ \frac{CP}{CF}ⓐ,\ \frac{CF}{CP}ⓑ,\ \frac{CF}{CP}ⓑ,\ \frac{CF}{CP}ⓒ,\ \frac{CP}{CF}ⓒ,\ \frac{CF}{CP}ⓓ,\ \frac{CP}{CF}ⓓ.$$

In making a transition from one form to another in the same part, place the respective pitch-unit in its nearest pitch position. This is true of both the octave and the geometrical inversion. ***The axis of inversion*** for ⓒ and ⓓ is the *axis of CF*, or the part assumed to function as the CF.

*To remind the reader, these geometrical positions are: ⓐ the original; ⓑ the same but backwards; ⓒ the original backwards and upside down; ⓓ the original upside down. See Book III. (Ed.)

Type I

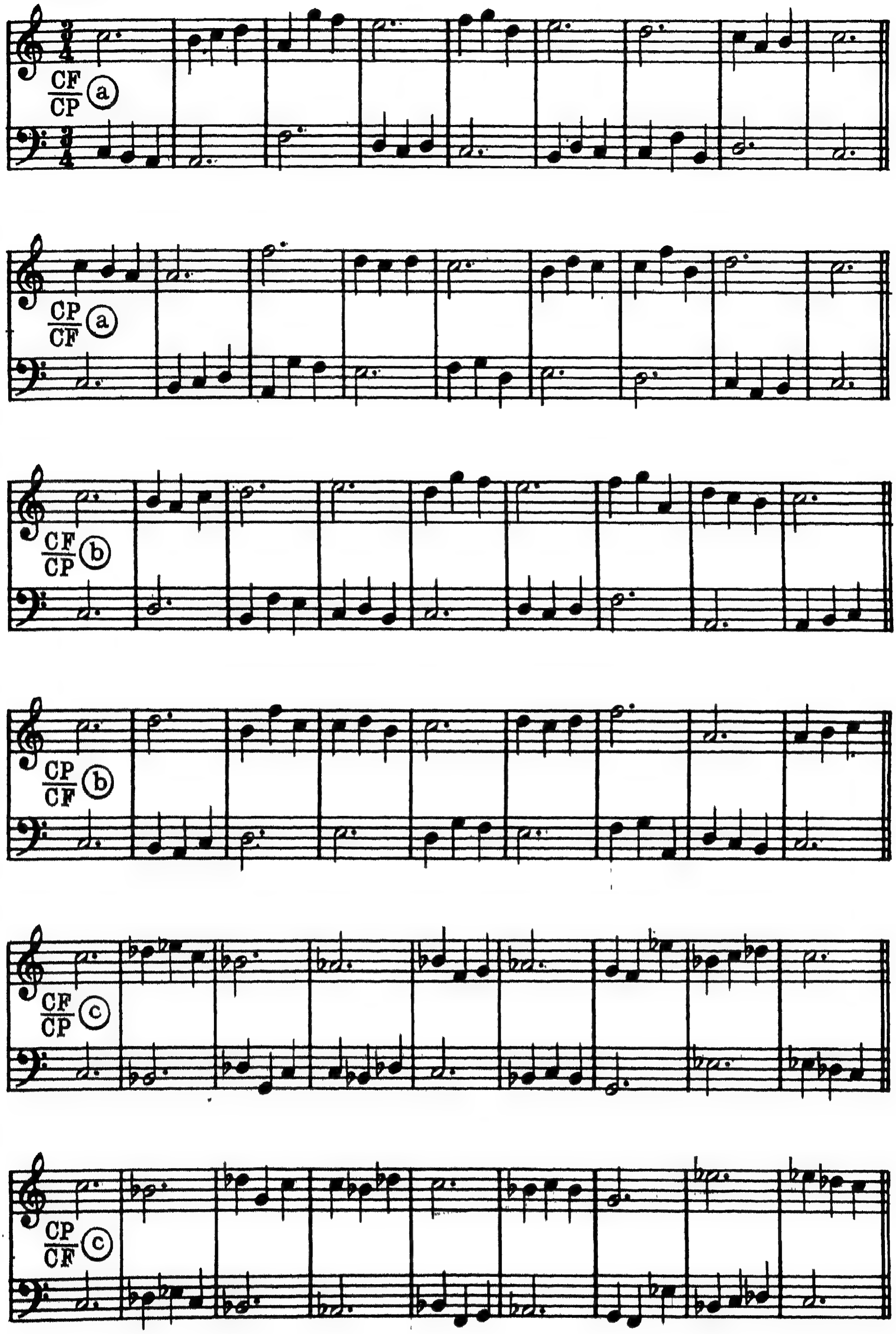


Figure 50. Variants of one exposition. Type I and quadrant rotation (continued).

Figure 50. Variants of one exposition. Type I and quadrant rotation (concluded).

Type II

Figure 51. Type II and quadrant rotation (continued).

Figure 51. Type II and quadrant rotation (concluded).

Type III and/or IV

Figure 52. Type III and/or IV. Quadrant rotation (continued).

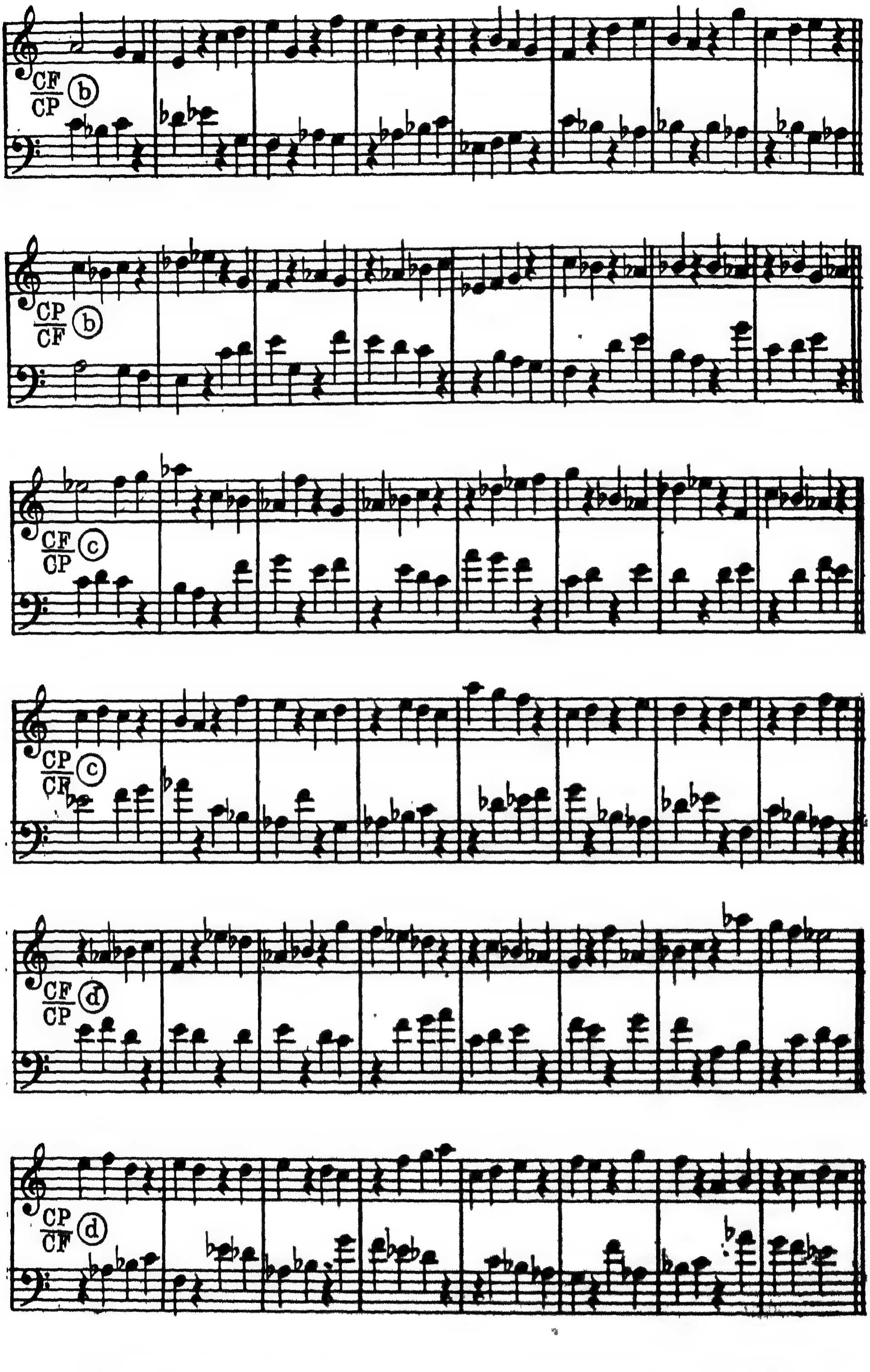


Figure 52. Type III and/or IV. Quadrant rotation (concluded).

These eight variants of contrapuntal exposition may be selected in any desirable combination. Any combination of the selected variants produces a complete form of continuity, i.e., a whole composition.

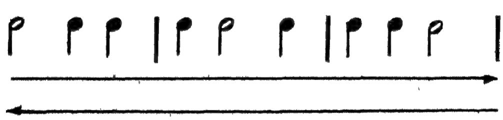
The selection of various geometrical inversions must be guided by a definite tendency with regard to the number and distribution of contrasts; all considerations pertaining to this matter were discussed in the section on geometrical projections of music. *

The most important principles to remember are:

(1) ⓐ and ⓑ are identical in intonation and converse in temporal structure;
(2) ⓒ and ⓓ are identical in intonation and converse in temporal structure;
(3) ⓐ and ⓓ are converse in intonation and identical in temporal structure;
(4) ⓐ and ⓒ are converse in intonation and converse in temporal structure;
(5) ⓑ and ⓒ are converse in intonation and identical in temporal structure;
(6) ⓑ and ⓓ are converse in intonation and converse in temporal structure.

There is a way of developing identical temporal structures for all geometrical inversions: any symmetrical group is identical with its converse; for instance:

(1) $r_{5\div4} = 4 + 1 + 3 + 2 + 2 + 3 + 1 + 4$

(2)


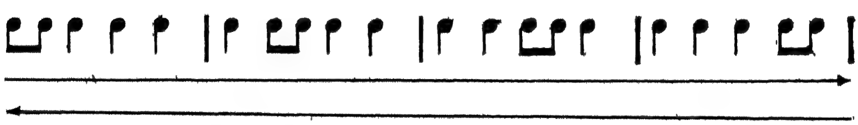
(3)


Figure 53. A symmetrical group is identical with its converse.

There is also a way of developing an identical pitch-scale for all geometrical inversions, when such is desirable. The original scale must be symmetrically constructed (which does not necessarily place it into the third or fourth group). In such a case the pitch units in ⓐ and ⓓ are not identical but the scale structures (that is, the sets of intervals) are identical.

For instance:

ⓐ c – e♭ – f – g – b♭ (3 + 2 + 2 + 3) ↑
ⓑ b♭ – g – f – e♭ – c (3 + 2 + 2 + 3) ↓
ⓒ d – f – g – a – c (3 + 2 + 2 + 3) ↑
ⓓ c – a – g – f – d (3 + 2 + 2 + 3) ↓

Figure 54. Symmetrit scale is identical for all geometrical inversions.

*See Book III.

Here are some examples of complete forms of contrapuntal continuity based on geometrical inversions:

$$(1)\ \frac{CP}{CF}\textcircled{a} + \frac{CF}{CP}\textcircled{a} + \frac{CF}{CP}\textcircled{d} + \frac{CP}{CF}\textcircled{b} + \frac{CF}{CP}\textcircled{a} + \frac{CF}{CP}\textcircled{c} + \frac{CP}{CF}\textcircled{b}\ ;$$

$$(2)\ \frac{CF}{CP}\textcircled{a} + \frac{CP}{CF}\textcircled{b} + \frac{CF}{CP}\textcircled{c} + \frac{CP}{CF}\textcircled{d} + \frac{CF}{CP}\textcircled{a}\ ;$$

$$(3)\ \frac{CP}{CF}\textcircled{c} + \frac{CF}{CP}\textcircled{b} + \frac{CP}{CF}\textcircled{a} + \frac{CF}{CP}\textcircled{d} + \frac{CF}{CP}\textcircled{a}\ ;$$

$$(4)\ \frac{CF}{CP}\textcircled{a} + \frac{CF}{CP}\textcircled{c} + \frac{CP}{CF}\textcircled{b} + \frac{CP}{CF}\textcircled{d} + \frac{CF}{CP}\textcircled{a}\ .$$

Figure 55. Forms of contrapuntal continuity.

We shall apply the first of the above schemes of continuity to the theme based on the exposition in type II, Fig. 51. The theme will be used in its original ST version (i.e., without the added balance).

Figure 56. Scheme 1 applied to exposition in figure 51 (continued).

Figure 56. Scheme 1 applied to exposition in figure 51 (concluded).

As we have seen before, the interchangeability of CF and CP produces two forms for each geometrical position. This property may be utilized for the purpose of producing continuity based on *imitation.* The two reciprocal expositions following one another are planned in such a manner that the first one consists of an unaccompanied CF only, whereas the second has both parts. When CF exchanges its positions, the resulting effect is imitation.

In the following example, Fig. 52, type III, will serve as a theme. The complete continuity will follow this scheme: $CF\text{ⓐ} + \frac{CP}{CF}\text{ⓐ} + \frac{CF}{CP}\text{ⓓ} + \frac{CP}{CF}\text{ⓒ} + \frac{CF}{CP}\text{ⓑ}$

Figure 57. Exposition of figure 52 developed by geometrical inversions (continued).

Figure 57. Exposition of figure 52 developed by geometrical inversions (concluded).

CHAPTER 5

CORRELATION OF MELODIC FORMS IN TWO-PART COUNTERPOINT

THUS FAR, we have been concerned with the harmonic and the temporal correlation of two melodic parts. The melodic forms we have used have been planned in some general way, but many details were merely the outcome of the harmonic treatment of intervals.

Now, however, it is time to consider a systematic method for correlating *melodic* forms. Melody is expressed, fundamentally, by means of an *axial* combination; the correlation of two melodies, then, becomes essentially a problem of coordination between the two axial groups.*

A. Use of Monomial Axes

We shall begin our analytical survey with a glance at *monomial* axes for *both* CF and CP. The following 25 forms become possible:

$$\frac{CF}{CP} = \frac{0}{0};\ \frac{a}{0};\ \frac{0}{a};\ \frac{b}{0};\ \frac{0}{b};\ \frac{c}{0};\ \frac{0}{c};\ \frac{d}{0};\ \frac{0}{d};$$

$$\frac{a}{a};\ \frac{b}{a};\ \frac{a}{b};\ \frac{c}{a};\ \frac{a}{c};\ \frac{d}{a};\ \frac{a}{d};\ \frac{b}{b};\ \frac{c}{b};\ \frac{b}{c};$$

$$\frac{d}{b};\ \frac{b}{d};\ \frac{c}{c};\ \frac{d}{c};\ \frac{c}{d};\ \frac{d}{d}.$$

It is important to note that the various forms of balancing and unbalancing are *inherent* in the above combinations. Analysis of two parts as being parallel or contrary is *not* sufficient, as, under either condition, one voice may be balancing and the other may be unbalancing, both voices may be balancing, or both may be unbalancing.

For example: $\frac{CF}{CP} = \frac{b}{b};\ \frac{d}{b};\ \frac{b}{c};\ \frac{a}{d}.$

In the first case, both voices are parallel and balancing; in the second case, both voices are parallel, but CF is unbalancing, and CP is balancing; in the third case, both voices are contrary, but both are balancing; in the fourth case, both voices are contrary, but both are unbalancing.

It follows from the above considerations that the correct way to achieve continuous motion in two-part counterpoint is to introduce an unbalancing axis in one of the parts when the other part is moving toward balance, unless a cadence is desired. The music of J. S. Bach contains more parallel motion than is usually realized; but he always managed to avoid unintentional cadencing. On the other hand, many academic theoreticians advocate an abundance of contrary motion as being essentially contrapuntal; contrary motion is in itself of little importance,

*See Book IV.

however; it actually becomes a source of monotony unless it is used along with the proper constitution of balance relations between CF and CP.

The selection of axial combinations for the two counterparts (or for one counterpart to a given part) depends on the form of expression.

Axial relations with regard to their directions are: (1) parallel; (2) contrary; (3) oblique.

Axial relations with regard to their balancing tendencies are:

$$(1)\ \frac{U}{U};\ (2)\ \frac{U}{B};\ (3)\ \frac{B}{U};\ (4)\ \frac{B}{B}.$$

In addition, the zero-axis expresses a continuous state of balance.

All further development of the technique of correlating axial combinations of two melodies follows the ratio development of the quantities of axes in one part in relation to those in another.

Under such conditions, all the above described cases refer to one category only: $\frac{CP}{CF} = ax$, i.e., one secondary axis of counterpoint corresponds to one secondary axis of the cantus firmus, *ax* being used as an abbreviation of the word, "axis."

B. Binomial Axial Groups

Coming now to the binomial relations of axial groups of the counterpoint in relation to the cantus firmus, we see that:

$$\frac{CP}{CF} = \frac{2ax}{ax},\ \text{or}\ \frac{ax}{2ax}$$

Under such conditions, a monomial axis of one part corresponds to a binomial axial combination of another. For instance:

$$\frac{CP}{CF} = \frac{0+a}{0};\ \frac{a+b}{b};\ \frac{c+d}{a};\ \frac{b+0}{c};\ \frac{d+a}{0};\ \ldots,\ \text{etc.}$$

$$\frac{CP}{CF} = \frac{0}{0+a};\ \frac{b}{a+b};\ \frac{a}{c+d};\ \frac{c}{b+0};\ \frac{0}{d+a};\ \ldots,\ \text{etc.}$$

It is easy to see that there are 200 such simultaneous combinations, as there are 10 original binomial axial combinations, each having 2 permutations. Twenty combinations are now combined vertically with 5 monomials (0, a, b, c, d). This produces $20 \cdot 5 = 100$. Finally, 100 must be multiplied by 2, as each simultaneous combination can be inverted.

The period of duration of one axis equals the sum of durations of the two axes constituting the binomial. Thus, in a combination:

$\frac{CP}{CF} = \frac{2ax}{ax} = \frac{axmt + axnt}{axpt} = \frac{T}{T} = 1$, the time period for both parts is the same.

Time ratios for binomial axes must be selected in accordance with the series which the monomial axis represents. If, for instance, the duration of ax of CF is 8T, then CP may be matched as any binomial of $\frac{8}{8}$ series. We might select the 5+3 binomial of this series.

Now we can define the simultaneous temporal relations as follows:

$$\frac{CP}{CF} = \frac{ax5T + ax3T}{ax8T}$$

In a simultaneous combination of a binomial-against-a-monomial axial combination, we find that the following is significant: during the period of the monomial axis (balanced, balancing, or unbalancing) its counterpart has two phases, which may be any of these pairs: U+U; U+B; B+U; or B+B. If we single out a continuous balance (0-axis) as an independent form, we obtain 12 forms of balance relations between CP and CF, when one of them is a binomial and the other a monomial.

$$\frac{CF}{CP} = \frac{ax}{2ax} = \frac{0}{U+U};\ \frac{0}{U+B};\ \frac{0}{B+U};\ \frac{0}{B+B};$$

$$\frac{U}{U+U};\ \frac{U}{U+B};\ \frac{U}{B+U};\ \frac{U}{B+B};$$

$$\frac{B}{U+U};\ \frac{B}{U+B};\ \frac{B}{B+U};\ \frac{B}{B+B}.$$

Just as many are available for $\frac{CP}{CF} = \frac{ax}{2ax}$. If the 0-axis participates in a binomial, there are 15 more combinations: O+U, O+B, B+O, O+O multiplied by 3.

Let us select *one* of the many possible combinations. Let it be $\frac{CP}{CF} = \frac{2ax}{ax} = \frac{U+U}{B} = \frac{d+a}{c}$. Suppose that CF = 8T, and suppose that we match the previously selected time-ratio for CP. Then the correlation of $\frac{CP}{CF}$ appears as follows: $\frac{CP}{CF} = \frac{d5T + a3T}{c8T}$. In this case CP unbalances for 5T in the direction below its P.A. (primary axis) and unbalances still further in the direction above its P.A. for 3T. While this happens, CF moves steadily toward its own P.A. in the upward direction during the course of 8T. The graph of this would be:

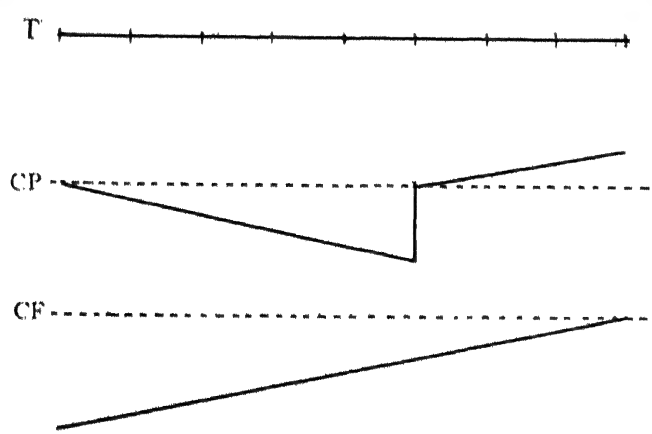


Figure 58. $\frac{CP}{CF} = \frac{d5T + a3T}{c8T}$

C. Trinomial Axial Combinations

In the same fashion, trinomial axial combinations of one part may be correlated with a monomial axis of another. The number of simultaneous combinations equals the number of trinomials times 5.

There are 60 trinomials with two identical terms (as noted in my discussion of the theory of melody)* and 60 trinomials with all terms different. This yields $(120 \cdot 5 =)$ 600 for $\frac{CP}{CF}$, and the same quantity for $\frac{CF}{CP}$.

As the number of axes is three in one part and one in the other part, we may write:

$$\frac{CP}{CF} = \frac{3ax}{ax} \text{ or } \frac{CP}{CF} = \frac{ax}{3ax}.$$

In each case, the trinomial requires three temporal coefficients, the sum of which equals that of the monomial.

$\frac{CP}{CF} = \frac{3ax}{ax} = \frac{axmt + axnt + axpt}{axT}$, where $mt + nt + pt = T$. Let T equal 5.

Then, by selecting $2+2+1$ which is one of the trinomials of $\frac{5}{3}$ series, we obtain:

$$\frac{CP}{CF} = \frac{ax2T + ax2T + axT}{ax5T}$$

The trinomial distribution of the O, U and B yields the following number of the forms of balance.

O+O+U; O+O+B; U+U+O; U+U+B; B+B+O; B+B+U.

Each of the above 6 combinations has 3 permutations, giving a total of $6 \cdot 3 = 18$. When each of these variations is placed against O, U or B in the counterpart, the number of forms becomes tripled: $18 \cdot 3 = 54$. Thus, $\frac{CP}{CF}$ and $\frac{CF}{CP}$ have 54 forms each.

But the above forms contain trinomials with two identical terms. The addition of trinomials without identical terms produces one combination: O+U+B, which has 6 permutations. These 6 forms, placed against the three possible forms of the counterpart, produce $(6 \cdot 3 =)$ 18 combinations.

$\frac{CP}{CF}$ and $\frac{CF}{CP}$ have 18 forms each.

The total of trinomial combinations of balance of $\frac{CP}{CF}$ is $(54 + 18 =)$ 72, and the same number for $\frac{CF}{CP}$.

When secondary axes are substituted for the forms of balance, each case gives more than one solution. For example: if $\frac{CP}{CF} = \frac{U + O + B}{U}$, then—

CP	CF
(1) U = a; U = d;	U = a; U = d.
(2) O = O	
(3) B = b; B = c;	

*See Book IV.

—and the following solutions are available:

$$\frac{CP}{CF} = \frac{a+0+b}{a}\,;\ \frac{a+0+b}{d}\,;\ \frac{a+0+c}{a}\,;\ \frac{a+0+c}{d}\,;$$

$$\frac{d+0+b}{a}\,;\ \frac{d+0+b}{d}\,;\ \frac{d+0+c}{a}\,;\ \frac{d+0+c}{d}\,.$$

Let us assign the previously discussed $\frac{5}{8}$ series trinomial time ratio. We obtain the following solutions:

$$\frac{CP}{CF} = \frac{a2T+02T+bT}{a5}\,;\ \frac{a2T+02T+bT}{d5T}\,;\ \frac{a2T+02T+cT}{a5T}\,;$$

$$\frac{a2T+02T+cT}{d5T}\,;\ \frac{d2T+02T+bT}{a5T}\,;\ \frac{d2T+02T+bT}{d5T}\,;$$

$$\frac{d2T+02T+cT}{a5T}\,;\ \frac{d2T+02T+cT}{d5T}\,;$$

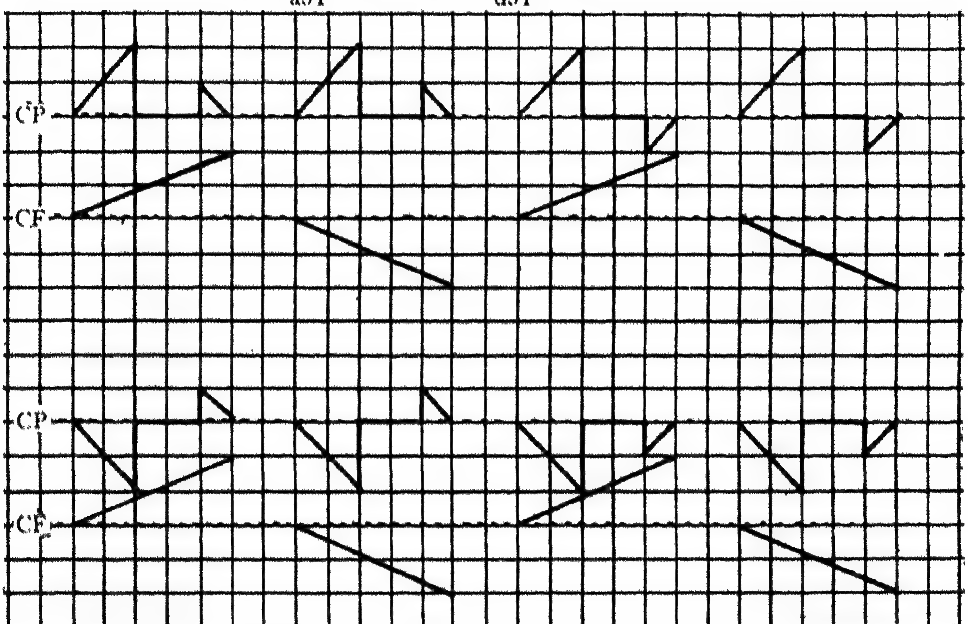


Figure 59. $\frac{5}{8}$ series in trinomial time ratio.

D. Polynomial Axial Combinations

Ultimately, a polynomial axial combination may serve as counterpart to a monomial axis. The effect of such a correlation is instability (polynomial) versus stability (monomial). The selection of forms of O, U and B depends on the effects of balance necessary in each particular case. An abundance of unbalancing axes results in restless, disquieting, unstable melodies—such melodies are often called dramatic, passionate, ecstatic, etc. An abundance of balancing and O-axes produces restful, quiet, stable melodies, usually termed contemplative, epical, or serene.

Examples of compositions of $\frac{CP}{CF} = \frac{max}{ax}$.
Let m = 5; then: $\frac{CP}{CF} = \frac{5ax}{ax}$.

Let us consider our balance-group to be U+B+U+B+U, and assume that the two extreme terms are identical, but different from the middle one. Then the possibilities for the U's are:

(1) a+d+a and (2) d+a+d

In the first combination, let us assume that both B's are identical but on the opposite side of P.A. from the two identical U's. Then we get c+c for the B+B. The entire axial combination for CP appears as follows:

CP = a+c+d+c+a

Let CF be represented by B, and let it be b in order to achieve greater variety of balancing forms of CP in relation to CF.

$$\frac{CP}{CF} = \frac{a+c+d+c+a}{b}$$

Let the duration of the entire group be 16T. Let the temporal coefficients correspond to $\frac{8}{8}$ series on the basis of t = 2T. Then, by selecting a quintinomial (for the five axes of CP), we obtain the following temporal scheme:

$$\frac{CP}{CF} = \frac{a4T + c2T + d4T + c2T + a4T}{b16T}$$

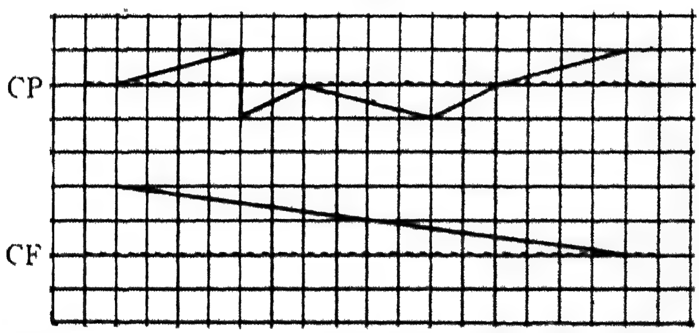


Figure 60. Graph of $\frac{CP}{CF} = \frac{a4T + c2T + d4T + c2T + a4T}{b16T}$

E. Developing Axial Relations Through Attack-Groups

The temporal ratios, discussed so far, referred to the form $\frac{CP}{CF} = 1, 2, 3, \ldots m$.

Such axial relations may be further developed into polynomial groups in both CF and CP:

(1) through the technique previously applied to the composition of attack-groups, as in *Melodization of Harmony*;*

(2) by direct application of ratios producing interference.

*See Book VI.

The first technique makes it possible to match any desirable number of axes of the CP against each axis of the CF.

Let us take a CF with 4 axes. We may match 2, 3, or more axes of CP against each axis of CF and in any desirable sequence.

For example: $\frac{CP}{CF} = \frac{2ax}{ax} + \frac{2ax}{ax} + \frac{2ax}{ax} + \frac{2ax}{ax}$.

By assigning temporal coefficients in such a way that the sum of durations in each 2ax of CP corresponds to the duration of ax of CF, we acquire a synchronized $\frac{CP}{CF}$. With the temporal coefficients based on $r_{5\div4}$, for instance, we obtain the following correlation:

$$\frac{CP}{CF} = \frac{ax4T + axT}{ax5T} + \frac{ax3T + ax2T}{ax5T} + \frac{ax2T + ax3T}{ax5T} + \frac{axT + ax4T}{ax5T}$$

Let 0+b+c+a be the axial combination of CF, and let (0+a) + (0+b) + + (b+0) + (a+0) be the axial combination of CP. Then $\frac{CP}{CF}$ acquires the following appearance:

$$\frac{CP}{CF} = \frac{04T + aT}{05T} + \frac{03T + b2T}{b5T} + \frac{b2T + 03T}{c5T} + \frac{aT + 04T}{a5T}$$

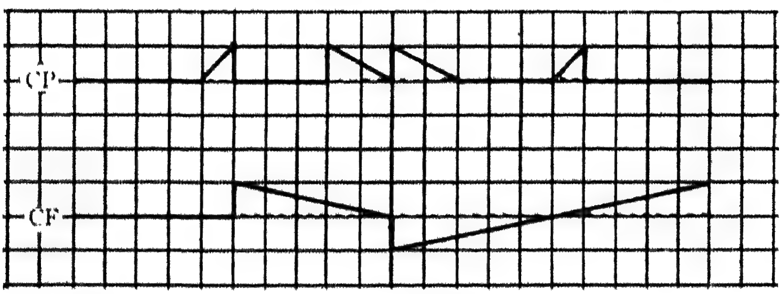


Figure 61. Two axes of CP matched to each one of CF.

When proportionate relations of the temporal coefficients of $\frac{CP}{CF}$ are desirable, and when a constant number of the axes of CP is assigned against each axis of CF, the technique of ***distributive involution*** solves the problem.

For example: $\frac{CP}{CF} = \frac{9ax}{3ax} = \frac{3ax}{ax} + \frac{3ax}{ax} + \frac{3ax}{ax}$.

To carry out this form of correlation in proportions, we may select the square of 2+1+1 of the $\frac{4}{4}$ series.

$$\frac{CP}{CF} = \frac{ax4T + ax2T + ax2T}{ax8T} + \frac{ax2T + axT + axT}{ax4T} + \frac{ax2T + axT + axT}{ax4T}$$

Let the axial combination for both CP and CF be the trinomial a+b+c. Then:

$$\frac{CP}{CF} = \frac{a4T + b2T + c2T}{a8T} + \frac{a2T + bT + cT}{b4T} + \frac{a2T + bT + cT}{c4T}.$$

See musical illustration on following page.

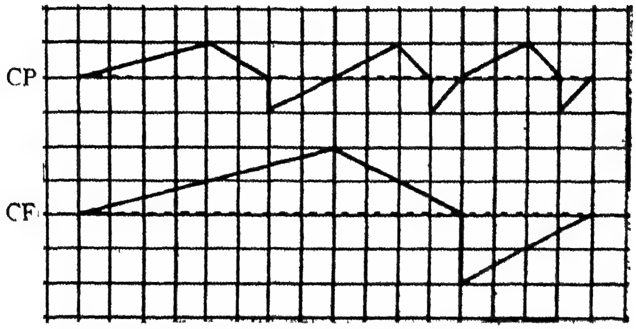


Figure 62. Proportionate relation of temporal coefficients of CP and CF.

F. Interference of Axis-Groups

The most complex temporal relations result when the respective axes in CP and CF produce interference ratios. I shall discuss here only the simplest forms of such interference, those which require uniform temporal coefficients for both CP and CF, and differ only in value. This corresponds to ***binary synchronization*** as described in my earlier discussion of the ***Theory of Rhythm.**** In this sense, an $\frac{a}{b}$ ratio represents the number of secondary axes in the two counterparts.

Let us take the $\frac{3}{2}$ ratio. Under such conditions $\frac{CP}{CF} = \frac{3ax}{2ax}$, or $\frac{CP}{CF} = \frac{2ax}{3ax}$. After synchronization, the first expression appears as follows:

$$\frac{CP}{CF} = \frac{ax2T + ax2T + ax2T}{ax3T + ax3T}$$

Let CF consist of 0+d and CP—of a+d+0. Then:

$$\frac{CP}{CF} = \frac{a2T + d2T + 02T}{03T + d3T}$$

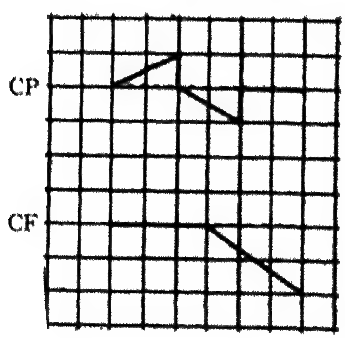


Figure 63. More complex temporal relations of CP and CF.

*See Book I.

Series of accelerations used in their reciprocal directions serve as additional material for the temporal coefficients of $\frac{CP}{CF}$. This technique produces two counterparts in the form of "growth" against "decline."

An example:

$$\frac{CP}{CF} = \frac{axT + ax2T + ax3T + ax5T}{ax5T + ax3T + ax2T + axT}$$

Axial combinations: $\frac{CP}{CF} = \frac{a+b+c+d}{a+b+c+d}$. Hence:

$$\frac{CP}{CF} = \frac{aT + b2T + c3T + d5T}{a5T + b3T + c2T + dT}$$

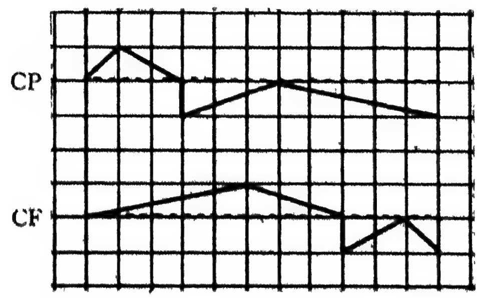


Figure 64. Adding series of accelerations.

This case illustrates the fact that even identical axial combinations in both counterparts may be made contrasting by the reciprocation of temporal coefficients.

An obvious contrast, that of some axial combinations against their own magnified versions, may be achieved by means of coefficients of duration applied to the original group of temporal coefficients.

An example:

$$\frac{CP}{CF} = \frac{2(ax3T + axT + ax2T + ax2T)}{ax6T + ax2T + ax4T + ax4T}$$

Axial combination: $\frac{CP}{CF} = \frac{a+b+c+d}{a+b+c+d}$. Hence:

$$\frac{CP}{CF} = \frac{a3T + bT + c2T + d2T + a3T + bT + c2T + d2T}{a6T + b2T + c4T + d4T}$$

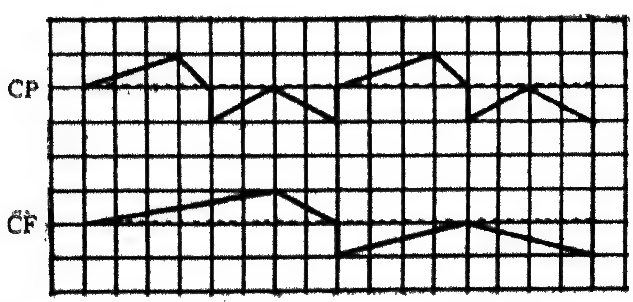


Figure 65. Applying coefficients of duration.

G. Correlation of Pitch-Time Ratios of the Axes

After correlation of temporal coefficients has been established, correlation of pitch ranges of both counterparts is the next step.*

Secondary axes that are otherwise identical may have different rates of speed. In terms of pitch ranges, it means that a greater range in one axis may be covered in the same period of time required by another axis to traverse a smaller range.

Use of identical axes having different pitch-ranges produces a noticeable amount of contrast.

$\frac{CP}{CF} = \frac{axT2P}{axTP}$. Let a be the axis in both parts.

Then: $\frac{CP}{CF} = \frac{aT2P}{aTP}$.

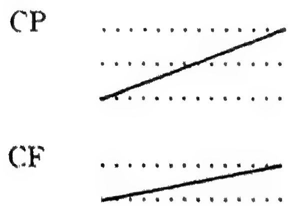


Figure 66. Different pitch ranges for identical axes.

When the two counterparts are represented by axes identical with respect to balance, but non-identical in structure, the contrast becomes still more obvious.

$$(1)\ \frac{CP}{CF} = \frac{B}{B}.$$

$$\frac{CP}{CF} = \frac{b2P}{cP};\ \frac{c2P}{bP};\ \frac{b3P}{cP};\ \frac{c3P}{bP};\ \frac{b3P}{c2P};\ \frac{c3P}{b2P};\ \ldots$$

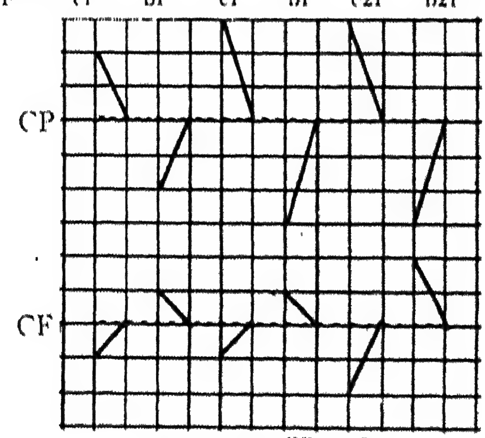


Figure 67. $\frac{CP}{CF} = \frac{B}{B}$

*The student should be cautioned that these—and similar passages in the text as to the *calculation* of music in advance of writing it are not simply mathematical curiosities, but are the very core of Schillinger's system. Maximum efficiency and fluent coordination of all the factors involved in "good" music cannot be achieved without just such exact planning as is being illustrated in these portions of the text. (Ed.)

$$(2)\ \frac{CP}{CF} = \frac{U}{U}\ .$$

$$\frac{CP}{CF} = \frac{a2P}{dP}\ ;\ \frac{d2P}{aP}\ ;\ \frac{a3P}{dP}\ ;\ \frac{d3P}{aP}\ ;\ \frac{a3P}{d2P}\ ;\ \frac{d3P}{a2P}\ ;$$

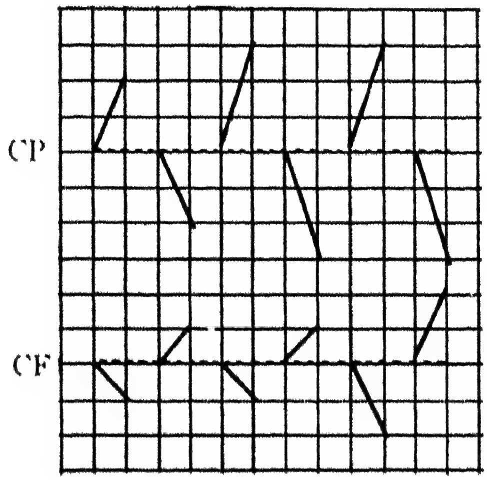


Figure 68. $\frac{CP}{CF} = \frac{U}{U}$

Still greater contrasts result from juxtaposition of pitch ranges of the two counterparts when the axial structures differ with respect to balance.

$$\frac{CP}{CF} = \frac{U}{B}\ .$$

$$\frac{CP}{CF} = \frac{a2P}{bP}\ ;\ \frac{a2P}{cP}\ ;\ \frac{d2P}{bP}\ ;\ \frac{d2P}{cP}\ ;\ \ldots$$

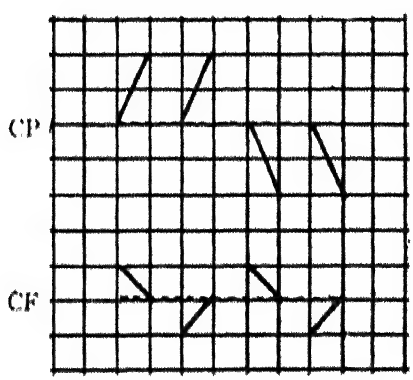


Figure 69. $\frac{CP}{CF} = \frac{U}{B}$

The 0-axis need not detain us in calculations aimed at correlating the pitch-ranges of the two counterparts.

As pitch-ratios may be in direct, oblique or inverse relations with the time-ratios in each part, the correlation of the two counterparts offers the following fundamental possibilities:

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ direct}};\ \frac{T \div P \text{ oblique}}{T \div P \text{ direct}};\ \frac{T \div P \text{ inverse}}{T \div P \text{ direct}}$$

$$\frac{T \div P \text{ oblique}}{T \div P \text{ oblique}};\ \frac{T \div P \text{ inverse}}{T \div P \text{ oblique}};\ \frac{T \div P \text{ inverse}}{T \div P \text{ inverse}}$$

The second, the third, and the fifth forms have another variant, each by inversion. The total number of the above relations is $6+3 = 9$.

Examples:

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ direct}}$$

$$(1)\ \frac{CP}{CF} = \frac{bTP + c2T2P + a4T4P}{d4T4P + b3T3P};$$

$$(2)\ \frac{CP}{CF} = \frac{aTP + b2T2P + a3T3P + d4T4P}{04T + a3T3P + c2T2P + bTP}.$$

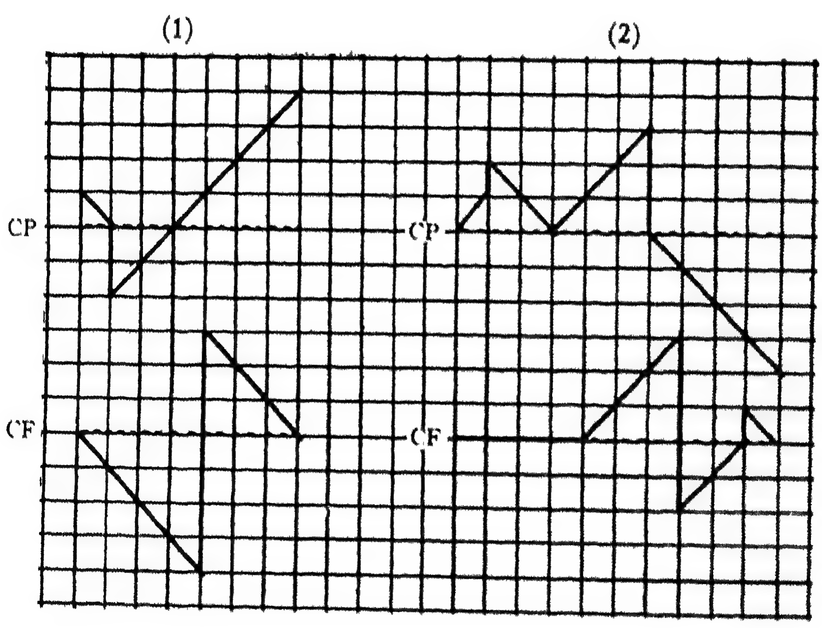


Figure 70. Inverting various forms $\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ direct}}$

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ oblique}}$$

$$(1)\ \frac{CP}{CF} = \frac{a4T4P + c2T2P}{dT3P + c2T2P + d2T1P};$$

$$(2)\ \frac{CP}{CF} = \frac{b3T3P + dTP + c2T2P + a2T2P}{dT4P + b3T3P + c4T1P}$$

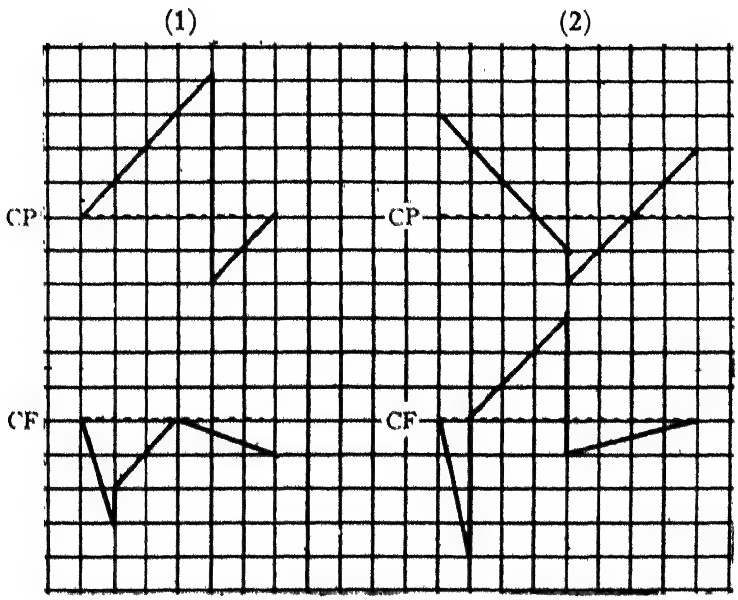


Figure 71. $\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ oblique}}$.

$$\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ direct}}$$

$$(1)\ \frac{CP}{CF} = \frac{a6T2P + b3T4P}{b4T4P + d2T2P + c2T2P + dTP};$$

$$(2)\ \frac{CP}{CF} = \frac{a2T2P + d2T2P + aTP + dTP + a2T2P + d2T2P}{c4T1P + c3T2P + c2T3P + cT4P}$$

See the corresponding illustrations on the following page.

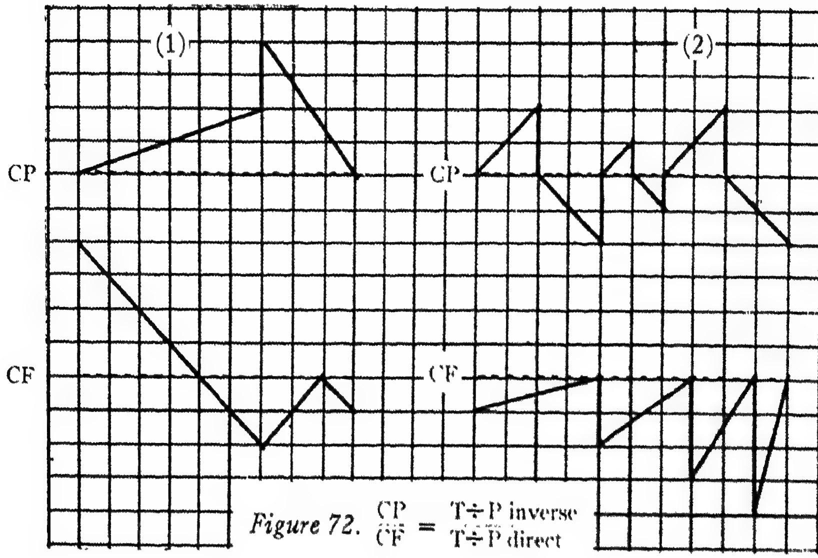


Figure 72. $\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ direct}}$

$$\frac{CP}{CF} = \frac{T \div P \text{ oblique}}{T \div P \text{ oblique}}$$

$$(1)\ \frac{CP}{CF} = \frac{a3T1P + a2T2P + bT3P + b3T1P + b2T2P + aT3P}{c3T5P + d4T4P + c5T3P}$$

$$(2)\ \frac{CP}{CF} = \frac{bT5P + a2T4P + d3T3P + b4T2P + a5T1P}{a7T3P + b5T5P + c3T7P}$$

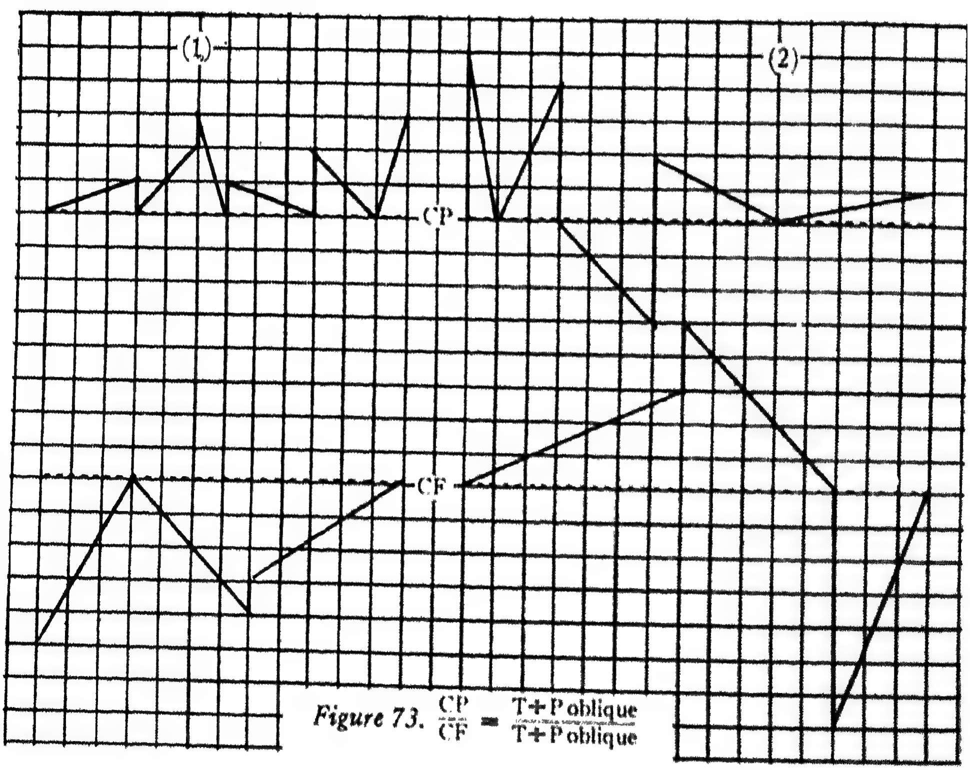


Figure 73. $\frac{CP}{CF} = \frac{T + P \text{ oblique}}{T + P \text{ oblique}}$

$$\frac{CP}{CF} = \frac{T \div P \text{ oblique}}{T \div P \text{ inverse}}$$

$$(1)\quad \frac{CP}{CF} = \frac{b3T2P + c3T3P + b2T3P}{aT2P + b2T1P + c2T1P + d3T1P}$$

$$(2)\quad \frac{CP}{CF} = \frac{a4T3P + d3T3P + a3T4P}{cT4P + b2T3P + b3T2P + c4T1P}$$

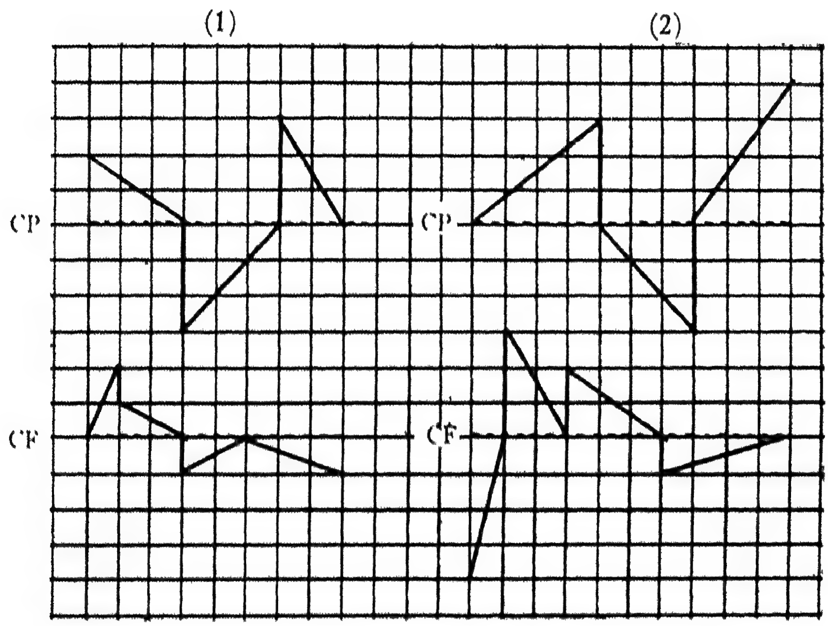


Figure 74. $\frac{CP}{CF} = \frac{T \div P \text{ oblique}}{T \div P \text{ inverse}}$

$$\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ inverse.}}$$

$$(1)\quad \frac{CP}{CF} = \frac{a3T1P + cT3P + c3T1P + aT3P}{a5T3P + b3T5P}$$

$$(2)\quad \frac{CP}{CF} = \frac{cT2P + c2T1P + b2T1P + b4T2P}{d6T3P + d3T6P}$$

See the corresponding illustrations on the following page.

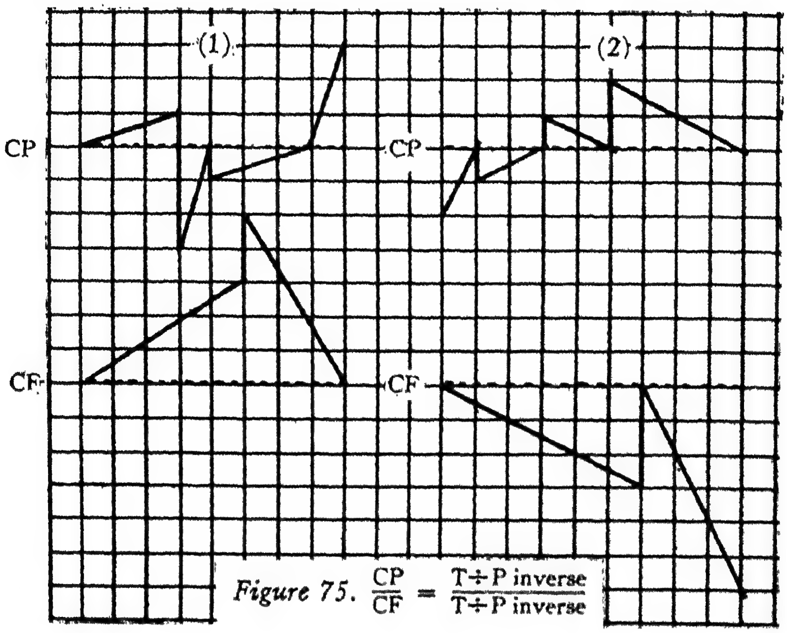


Figure 75. $\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ inverse}}$

Example of Application

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ inverse}}$$

$$\frac{CP}{CF} = \frac{a4T4P + b3T3P + a3T3P + b2T2P}{b8T1P + d4T2P}$$

$$T(CF) = (4+3+3+2)^2 = (16+12+12+8) + (12+9+9+6) + \\ + (12+9+9+6) + (8+6+6+4).$$

$$T(CP) = (\boxed{1}+1+1+1+1+1+1+1+1+1+1+1) \circlearrowright$$

Axial combination of $\frac{CP}{CF}$ in its general form:

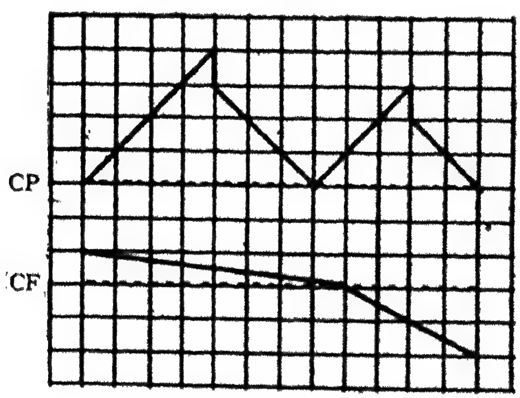


Figure 76. $\frac{T \div P \text{ direct}}{T \div P \text{ inverse}}$

*

Let CF be constructed from C-maj. nat. d_0 scale and CP—from A♭-maj. nat. d_6 scale.* Let P = 5p with approximation. Under such conditions, the range of CF will be about an octave and a half, and the range of CP—about two octaves.

Figure 77. Melody for preceding figure.

**C-maj. nat. d_0 scale* means—to refer to the material on pitch-scales—"key of C, the natural major scale (the "all white keys" scale), zero displacement (i.e., the tonic in C itself)." *A♭-maj. nat. d_6* means "key of A-flat, natural major, sixth displacement (i.e., the mode starting on G as its tonic). (Ed.)

H. Composition of a Counterpart to a Given Melody by Means of Axial Correlation

In order to correlate counterparts by means of axial correlation, it is necessary just to reconstruct the axial group of the given melody.*

After this analysis of the TP ratios of CF has been accomplished, it is important to detect whether the $T \div P$ is of direct, oblique, or inverse form.** After this, the general planning of the CP axial combination must follow first, with respect to the $T \div P$ correlation; second, with respect to the axial combination itself and its own $T \div P$ ratios.

The following graph is a transcription of the first four measures of the common musical setting of Ben Jonson's "Drink to Me Only With Thine Eyes."

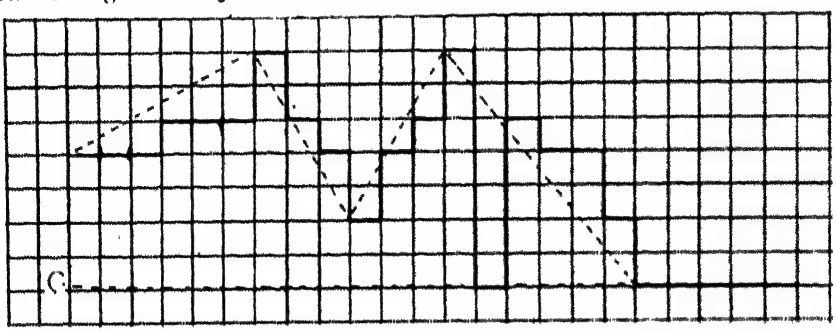


Figure 78. Graph of Drink to Me Only with Thine Eyes.

On analysis, we find that this melody contains a modal modulation, for $P.A._1$ is Phrygian (d_2), and $P.A._2$ is Ionian (d_0). The entire axial group gradually gravitates toward $P.A._2$, where it reaches its absolute balance. If we take into account all the minute crossings, an analysis of the axial group will appear as follows:

$$P.A._1 = a6t + b2t + dt + ct + a2t + b3t + d3t$$
$$P.A._2 = b3t + 05t + \boxed{t}.$$

The modulation here from one mode (d_2) to another (d_0) is performed by establishing a correspondence between d3t ($P.A._1$) and b3t ($P.A._2$). We can say that d3t ($P.A._1$) $\equiv$ b3t ($P.A._2$). As the pitch ranges are approximately equal, the TP ratio may be regarded as constant.

*This is a technique indispensable in modern "arranging" and in virtually all good orchestration of any style. (Ed.)

**The pitch-time ratio ("TP" ratio, or $T \div P$) means just what it says: The duration of the particular axis divided by its "height" or "depth" measured vertically in semitones. Composers seeking to perfect a style based on tastes they have already formed will find it useful to analyze, say, a hundred of their "favorite melodies," noting the axes—0, a, b, c and d—, the sequence of axes in groups, the durations (T) of each axis, the pitch-range (P) of each axis, and the TP ratios involved. (Ed.)

Let us now devise a counterpart in $1 \div 4$ time-ratio, meaning that CP will have only one secondary axis. As the general tendency of the CF is that of gradual gravitation toward balance in the course of two oscillations (which correspond to four directions and eight individual axes), we shall introduce a b-axis for the counterpart.* Then CP will consist of one direction, consistently gravitating toward balance. Under such conditions, $\frac{CP}{CF}$ represents a complete cycle of development.

This counterpart corresponds to case (2) in group (a) of Figure 39, where CP has an Aeolian P.A. (d_5).

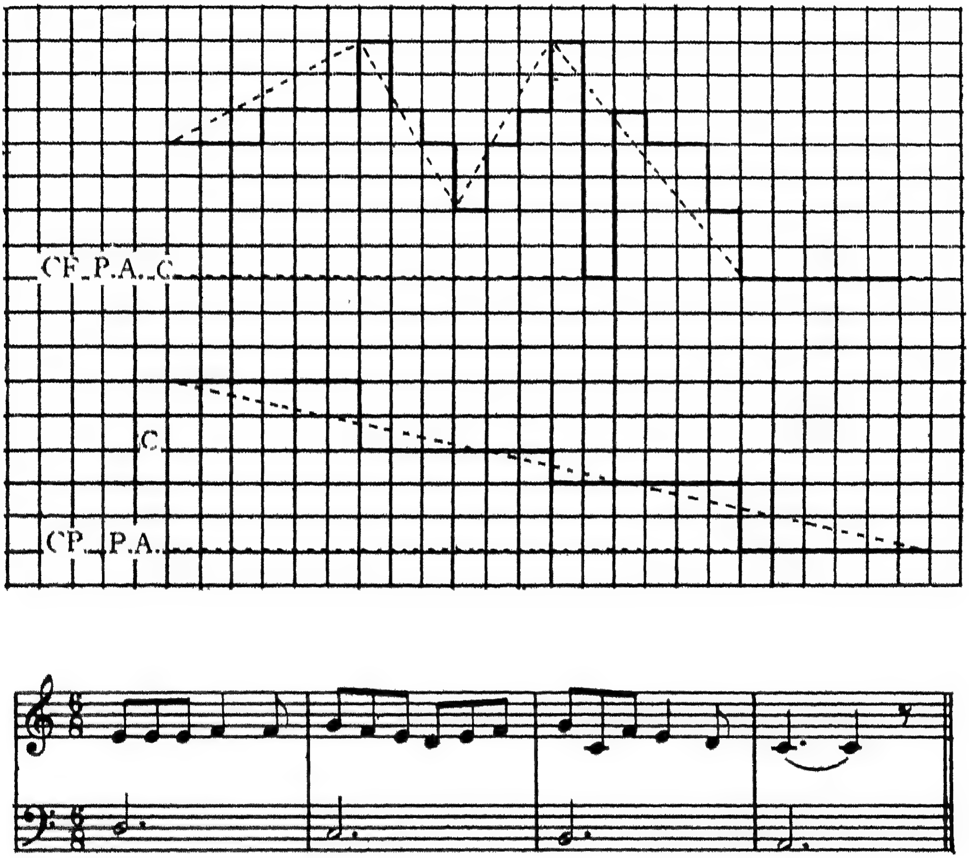


Figure 79. Counterpart in $1 \div 4$ time-ratio.

*That is, in this case, the over-all, general trend of CF, regardless of the oscillations, is in the general form of a b (downwards to the P.A.) axis, which same axis is here chosen as the form for the entire CP. (Ed.)

CHAPTER 6

TWO-PART COUNTERPOINT WITH SYMMETRIC SCALES

UNITY of style requires that both the *cantus firmus* and its counterpart be based on symmetric scales if one of them is.

Scales of the third group and scales of the fourth group, mostly in contracted form, serve as material for counterpoint. It is acceptable to have one counterpart in the third group and another in either the third or the fourth group.* When the two counterparts are in scales which belong to different groups, two cases may be distinguished:

(1) both scales have an *identical set of pitches;*
(2) each scale has a *different set of pitches.*

Example:

$$(1)\begin{cases} \overrightarrow{S_1} = \overset{T_1}{\underline{c}} - f - \overset{T_2}{\underline{ab}} - db - \overset{T_3}{\underline{e}} - a\natural - \overset{T_1}{c} \\ \overrightarrow{S_2} = \overset{T_1}{\underline{c}} - db - \overset{T_2}{\underline{e}} - f - \overset{T_3}{\underline{g\sharp}} - a - \overset{T_1}{c} \end{cases}$$

$$(2)\begin{cases} \overrightarrow{S_1} = \overset{T_1}{\underline{c}} - d - eb - f - \overset{T_2}{\underline{f\sharp}} - g\sharp - a - b - \overset{T_1}{c} \\ \overrightarrow{S_2} = \overset{T_1}{\underline{c}} - d - f - g - \overset{T_2}{\underline{ab}} - bb - db - eb - \overset{T_3}{\underline{e\natural}} - f\sharp - a - b - \overset{T_1}{c} \end{cases}$$

Figure 80. Identical and different sets of pitches.

The relations between the harmonic axes of the two counterparts may be carried out in all four of the forms previously used. Their meaning with regard to symmetric scales appears as follows:

Type I (U.U.) [Unitonal-unimodal]: both scales have the same T_1, the same number of tonics, and an identical set of pitch-units.

Type II (U.P.) [Unitonal-polymodal]: both scales have the same number of tonics, their sets of pitch-units are identical, but their harmonic axes are on different tonics.

Type III (P.U.) [Polytonal-unimodal]: both scales have an identical form of symmetry (the quantity of tonics) and an identical set of pitch-units; none of the tonics of one scale has pitches in common

*Third group scales are one octave in range with 2, 3, 4, 6 or 12 symmetrically arranged tonics; fourth group scales are of *more than* one octave in range, and of 2 or more symmetric tonics. (Ed.)

with the tonics of the other, i.e., the two sets of tonics belong to the mutually exclusive sets of pitches.

Type IV (P.P.) [Polytonal-polymodal]: the two scales belong to either identical or non-identical forms of symmetry; their sectional scales are of non-identical structure, yet they belong to one family (according to the classification offered in my discussion of the first group of scales*); the two sets of tonics belong to mutually exclusive sets of pitches.

Figure 81. Two-part counterpoint in scale of third group. Type I.

Figure 82. Two part counterpoint in scale of third group. Type II (continued).

*See Book II, *Theory of Pitch-Scales.*

Figure 82. Two part counterpoint in scale of third group. Type II (concluded).

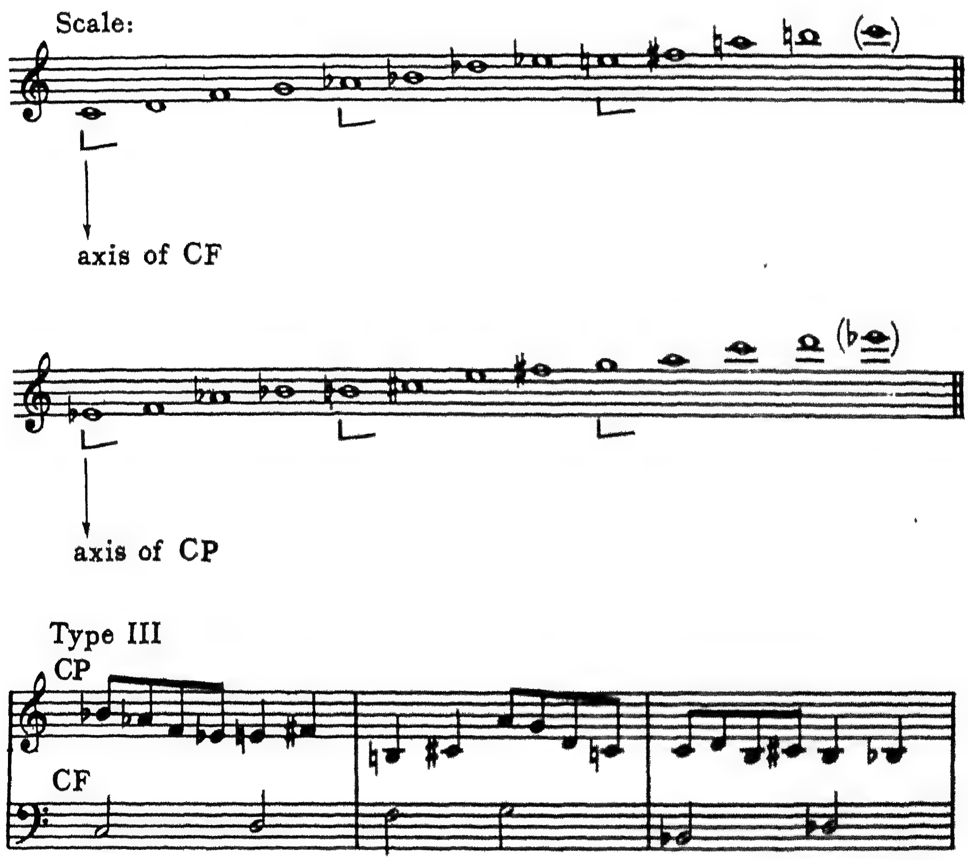
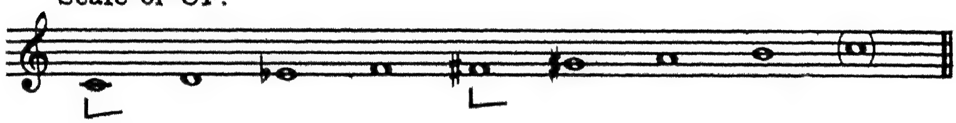


Figure 83. Two-part counterpoint in scale of fourth group. Type III (continued).

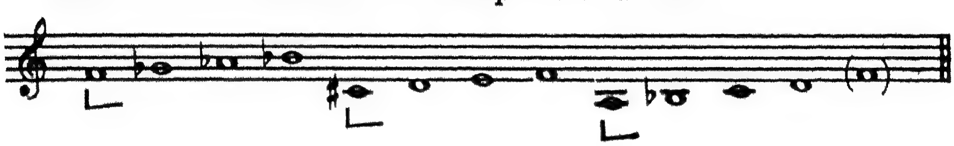
Figure 83. Two-part counterpoint in scale of fourth group. Type III (concluded).

Scale of CF:



Scale of CP:

Scale of CP contracted and transposed to F-axis:



Type IV

Figure 84. Two-part counterpoint in scale of fourth group. Type IV (continued).

Figure 84. Two-part counterpoint in scale of fourth group. Type IV (concluded).

CHAPTER 7

CANONS AND CANONIC IMITATION

THE source of *continuous imitation*, usually known as *canonic*, is the well known phenomenon of acoustical resonance which bears the name of the Hellenic nymph, Echo. Before any composer existed on this planet, nature created, by chance, a quintuple echo—the "Lorelei" (which can be justly called a five-part canon) discovered on the Rhine river. The Russian Admiral Wrangel described a place in Siberia where the river Lena enters a canyon about 600 feet high and where a pistol shot rapidly repeats itself more than a hundred times. How would you like that for a canon?

But music theorists, as is typical of the species, think the canon is a purely esthetic development. Whatever they think, canon is actually a natural phenomenon and is the most ancient form of musical continuity.

The common belief is that it requires great skill to write a canon; but the real cause of whatever difficulty is encountered in writing in this form is simply methodological incompetence. Both the music theorists and the composers are guilty, for neither has been able to formulate the principles of continuous imitation. I shall not discuss the case of Sergei Ivanovich Taneiev, as his interpretation of canon requires knowledge of his work, *Convertible Counterpoint in Strict Style*—a highly complicated system which deals only with the strict style and which fails to bring us any solution to melodic and rhythmic forms; it is preoccupied with vertical and horizontal convertibility of intervals in the harmonic sense.

A *canon* is a complete composition written in the form of *continuous imitation.*

The usual academic approach to this form is such that the student is taught first how to write an "ordinary" imitation (scientifically: *discontinuous imitation*). After not getting anywhere with this form of imitation, the student next begins to struggle with the canon. Inasmuch as, from the very start, the principles of imitation have not been disclosed to him, it does not make any difference whether the imitation is discontinuous or continuous. But once such principles are defined and the technique is specified, it becomes obvious that *discontinuous* imitation is merely a *special* case of *continuous imitation.*

With this in mind, let us now establish the actual principles of continuous imitation. *Continuous imitation consists of one melody coexisting in two or more different parts in its different phases and at a velocity that remains constant in any given part.* This melody, being of identical structure in both parts, may vary in intonation; such variance occurs only when the scale-structure itself varies.

The temporal organization of continuous imitation has no direct influence on the duration of a canon. Longer rhythmic groups are preferable, however, as continuous recurrence of the same rhythmic structure eventually becomes monotonous.

The main source of continuous self-stimulation in a canon is its *melodic form*, i.e., the axial group. With the devices offered in my theory of melody discussed earlier,* it is possible to evolve an axial group of great extension and, if necessary, *without* repetitions. In this way the continuance of the melodic flow may be completely insured.

The correlation of harmonic types and the treatment of harmonic intervals remain the same as for all other forms of contrapuntal technique. This permits us to compose canons in unitonal as well as in polytonal types.

A. Temporal Structure of Continuous Imitation

A complete composition based on continuous imitation is known as a *canon*.

The duration of continuous imitation—or of a canon—is some multiple of its *temporal structure*. The temporal structure of a two-part canon is related to the theme of the canon as $3 \div 1$. The first third of the whole is the announcement; the second third is the imitation of the announcement in the first voice and the counterpoint in the second voice; and the last third is the imitation of the first portion of counterpoint in the second voice and the second portion of counterpoint in the first voice. After the temporal scheme is exhausted, it begins to repeat itself with new intonations.

We shall designate the first entering voice as $P^{\rightarrow}_{I}$ (whether upper or lower), the second entering voice as $P^{\rightarrow}_{II}$, the first announcement as CP_1, the first portion of counterpoint as CP_2, and the second portion of counterpoint as CP_3, etc. The temporal structure of a canon then appears as follows:

$\frac{P^{\rightarrow}_{I}}{P^{\rightarrow}_{II}} = \frac{CP_1 + CP_2 + CP_3}{CP_1 + CP_2}$. The continuation of the temporal structure does not alter the process; it merely increases the subnumerals of CP in the original relation:

$$\frac{P^{\rightarrow}_{I}}{P^{\rightarrow}_{II}} = \frac{CP_1 + CP_2 + CP_3}{CP_1 + CP_2} + \frac{CP_4 + CP_5}{CP_3 + CP_4} + \frac{CP_6 + CP_7}{CP_5 + CP_6} + \dots$$

The temporal structure of any two-part canon is based on two elements, which appear as reciprocal permutations. All forms of variation of two elements are applicable therefore to two-part canons (see my earlier discussion of the *Theory of Rhythm*).** Let a and b be two elements each representing any kind of duration-group. Then,

$\frac{P^{\rightarrow}_{I}}{P^{\rightarrow}_{II}} = \frac{a + b + a}{a + b}$, and the continuation of the temporal structure assumes the following appearance:

$$\frac{P^{\rightarrow}_{I}}{P^{\rightarrow}_{II}} = \frac{a + b + a}{a + b} + \frac{b + a}{a + b} + \frac{b + a}{a + b} + \dots$$

*See Book IV. **See Book I.

The duration of some temporal structure is the factor really controlling the flow of the canon. The longer the structure (not as to speed, but as to the number of attacks), the greater the fluidity of the canon. Duration-groups of all kinds are acceptable as temporal structures for continuous imitation and for the canon.

1. Temporal structures composed from the parts of resultants.

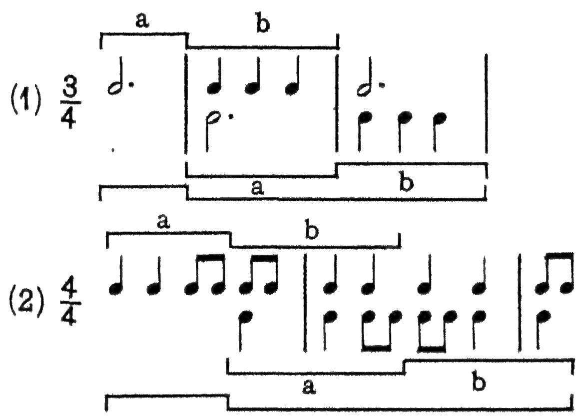


Figure 85. Temporal structures based on resultants.

2. Temporal structures composed from complete resultants.

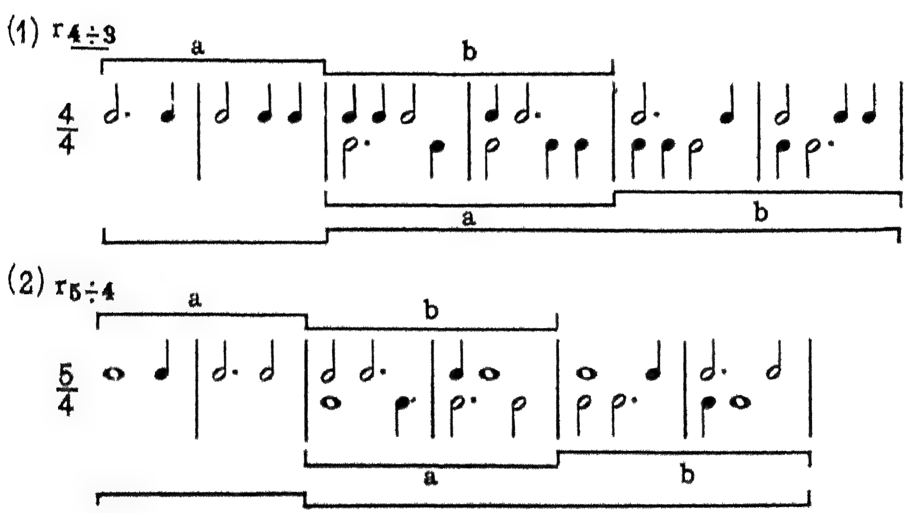


Figure 86. Temporal structures based on complete resultants (continued).

Figure 86. Temporal structures based on complete resultants (concluded).

3. Temporal structures evolved by means of permutations.

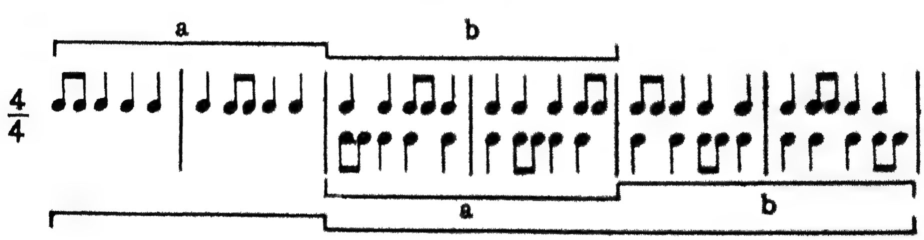
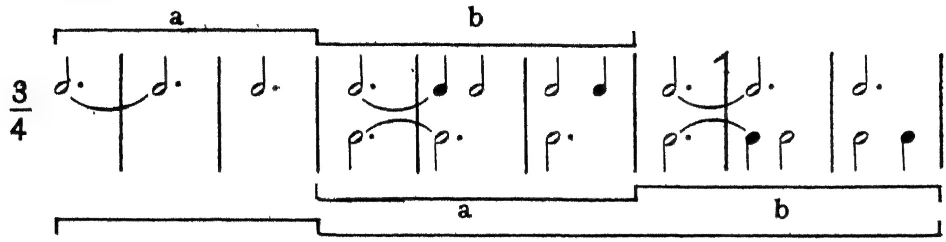


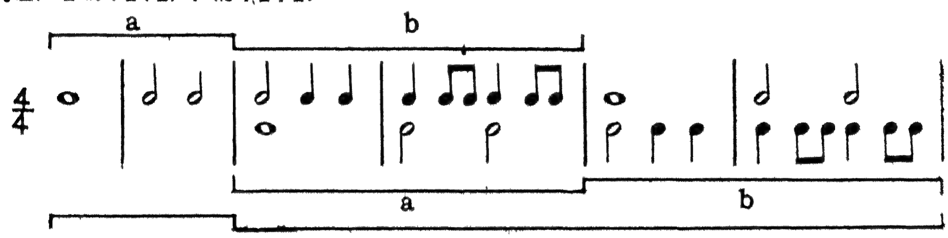
Figure 87. Temporal structures based on permutations.

4. Temporal structures composed from synchronized involution-groups.

(1) $3(2+1)+(2+1)^2$



(2) $4(2+1+1)+(2+1+1)^2$



(3) $(3+1+2)^3+6(3+1+2)^2$

Figure 88. Temporal structures based on involution groups (continued).

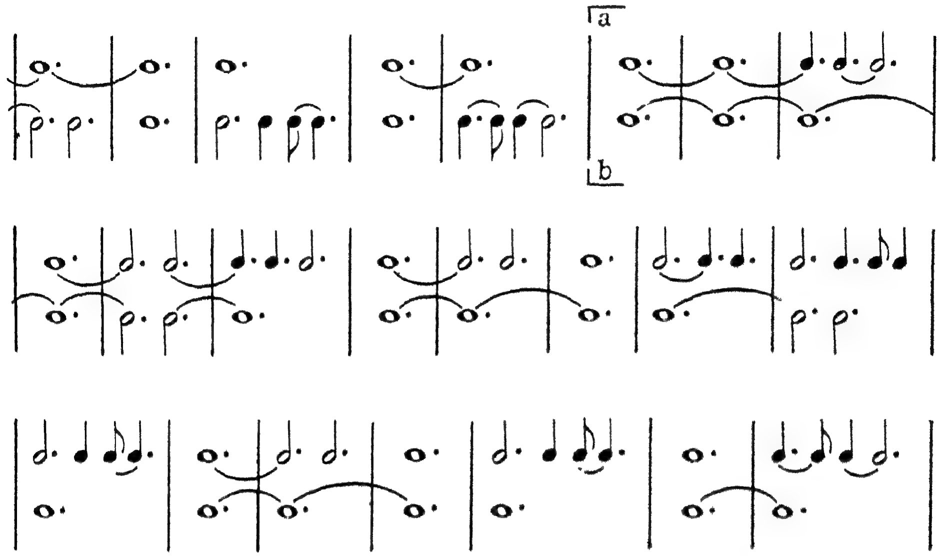


Figure 88. Temporal structures based on involution groups (concluded).

5. Temporal structures composed from acceleration-groups and their inversions.

Figure 89. Temporal structures based on acceleration-groups and their inversions.

B. Canons in All Four Types of Harmonic Correlation

As a canon is a duplication of melody at a certain time interval, differences of intonation in the two counterparts are due to scale-structures. Counterpoint of Type I (U.U.) produces identical intonations; type II (U.P.), non-identical intonations; type III (P.U.), identical intonations; and type IV (P.P.), non-identical intonations. The choice of axes for all four forms of correlation remains based on the original principle, consonance between the two axes of the two counterparts. In Types II and IV, the starting P.A. may be in a dissonant relation with the P.A. of the first voice, but it must end on a consonance.

As continuous imitation can go on indefinitely, we need to know the exact technique of bringing such an imitation to a close. Cadences are produced by leading tones moving into the primary axis. Since in canon, what happens in the first moving voice defines what happens in the second voice, all that is necessary, if we wish to cadence, is to produce a leading tone in the first moving voice. When this portion of melody is transferred to the second voice, the first voice produces its own leading tone, after which both voices close on their respective primary axes.

Symmetric pitch-scales may be used in canons.

Examples of two-part canons in all four types of harmonic correlation

Type I

Type II

Type III

Figure 90. Two-part canon in four types (continued).

Type IV

A, d_8

Figure 90. Two-part canon in four types (concluded).

Type II

Type III

Figure 91. Two-part canon in three types (continued).

Type IV

Figure 91. Two-part canon in three types (continued)

Figure 91. Two-part canon in three types (concluded).

Figure 92. Two-part canon in Type I and Type I Symmetric.

C. Composition of Canonic Continuity by Means of Geometrical Inversions

The original version of a canon may be considerably extended by means of geometrical inversions.

The voice entering first produces an axis of inversion for the positions ⓒ and ⓓ. The final balance of the last cadence must not participate in the sequence of inversions, as this would disrupt the continuous flow of the canon. It must be used only at the very end of the composition if the canon ends in position ⓐ or ⓓ. Otherwise a new balance must be added.

Under such conditions, the canon consists of several contrasting and independent sections of continuous imitation.

Example of a canon developed through the use of geometrical inversions.

Figure 93. A canon developed by geometrical inversion (continued).

Figure 93. A canon developed by geometrical inversions (continued).

Figure 93. A canon developed by geometrical inversions (concluded).

Each geometrical inversion allows the use of two vertical permutations of the counterparts. Octave readjustment of the parts becomes a necessity under such conditions.

CHAPTER 8

THE ART OF THE FUGUE

IN the generalized sense a *fugue* may be defined as a complete composition based on *discontinuous imitation.*

A *fragmentary* (incomplete) composition based on *discontinuous imitation* constitutes a *fugato.* A fugato usually appears as a polyphonic episode in an otherwise homphonic composition.

All other names established in the past—such as sinfonia, invention, praeludium (sometimes), fughetta—refer to the same fundamental form, the fugue. The difference lies mainly in the magnitude of the composition or in the type of harmonic correlation of the counterparts. A fugue which is unitonal-unimodal is called an invention, a praeludium, or a sinfonia—praeludium being the loosest term of all, as in many cases it has nothing in common with the fugue. A fugue (in this general sense) which is unitonal but polymodal (and of a *specified* polymodality) is called a fugue (in the specific sense).

It is my opinion that the presence or absence of polymodality or of polytonality is a matter of harmonic specifications which vary with time and place. Therefore, I feel that any complete composition based on discontinuous imitation may rightly be called a *fugue.*

A. The Form of a Fugue

The temporal structure of a fugue depends on the number of themes ("subjects"). It is customary to call a fugue with one theme a "single fugue"; the fugue with two themes, a "double fugue". Triple fugues are very rare; indeed, a real triple fugue requires many parts (voices) if the idea that each part is a theme is not to become nonsensical.

For this reason it is expedient to confine fugues in two-part counterpoint to fugues with but one theme.

The general characteristic of all fugues is the appearance of the theme in all parts in sequence. This complete thematic cycle is known as an *exposition.* In two-part counterpoint, the first voice entering announces the theme (we shall call it CF, for the sake of uniformity in terminology), after which the second voice enters with the imitation—the imitation is usually called "reply" and might as well be called "echo". In fact, the imitation is the same theme, sometimes with differences caused by the form of harmonic correlation. The *reply* in types I and III is identical with the *theme,* whereas in types II and IV it is non-identical because the scale-structure is modified.

At the time the second voice entering makes its announcement (CF), the first entering voice evolves a counterpart (CP) to it. The form of *the first exposition* (E_1) is—

$$E_1 = \frac{P_I}{P_{II}} = \frac{CF + CP}{CF}$$

—and the form of *any other exposition* (E_n) is—

$$E_n = \frac{P_I}{P_{II}} = \frac{CF + CP}{CP + CF}.$$

In both cases the voice entering first (P_I) and the second voice entering (P_{II}) may be inverted.

In a fugue in which CF and CP represent the only thematic material and no interludes are used, the entire composition acquires the following form:

$$F = E_1 + E_2 + E_3 + \ldots + E_n.$$

In homophonic music this would correspond to a *theme with variations*. In the fugue the variation technique consists of geometrical inversions of the original exposition.

The counterpoint to the theme may be either *constant* (i.e., the CP is carried out through the entire fugue), or *variable* (i.e., a new CP is composed for each exposition). Statistically, the use of constant or variable CP is about 50 percent. In the 17th and 18th centuries a constant CP was something of a luxury, for counterpoint which we may now consider to be general technique was at that time known as "vertically convertible counterpoint," which was believed to be more difficult to execute. On the other hand, the older composers did not know the technique of geometrical inversions; they used tonal inversions instead and merely as a trick on some special occasions.

With the systematic use of geometrical inversions, the fugue becomes greatly diversified, with the result that constant CP becomes merely a practical convenience. Once the theme and the counterpoint are composed (which we will call the preparation of one exposition), one may develop the entire fugue by means of quadrant rotation arranged in any desirable sequence. If rotations refer to the entire E, the fugue assumes the following appearance:

$F = E_1 m + E_2 n + E_3 p + \ldots$, where m, n and p are any of the geometrical inversions.

For example:

(1) $F = E_1 a + E_2 d + E_3 c + E_4 b + E_5 a$

(2) $F = E_1 a + E_2 b + E_3 a + E_4 d + E_5 a + E_6 c + E_7 a$

(3) $F = E_1 a + E_2 a + E_3 c + E_4 b + E_5 b + E_6 d$

Such schemes are subject to variation according to the composers' inventiveness.

Quadrant rotation may affect each appearance of the theme; in that case, the theme and reply appear in different geometrical positions.

For example:

$$(1)\quad E=\frac{P_{I}}{P_{II}}=\frac{CF\,a+CP}{CF\,d}$$

$$(2)\quad E=\frac{P_{I}}{P_{II}}=\frac{CF\,b+CP}{CP\;\;+CF\,a}$$

$$(3)\quad E=\frac{P_{II}}{P_{I}}=\frac{CF\,b}{CF\,c+CP}$$

It is important to note that the position is always identical for two simultaneous parts; CF_{d} means that CP set against it is also in position d.

Quadrant rotation applied to theme and reply produces altogether 16 geometrical forms of exposition.

B. Forms of Imitation Evolved Through Four Quadrants

a / a	b / a	c / a	d / a
a / b	b / b	c / b	d / b
a / c	b / c	c / c	d / c
a / d	b / d	c / d	d / d

Figure 94. Imitation evolved through four quadrants.

All those cases which involve one geometrical position for the entire E form the diagonal arrangement (heavily outlined) on the above table; they are special cases of the general rotary system.

It is easy to see that with this technique a fugue of any length may be composed with little effort.

An example of fugal scheme employing quadrant rotation:

$$\mathrm{F}=\left(\frac{\mathrm{CF}\textcircled{a}+\mathrm{CP}\textcircled{a}}{\mathrm{CF}\textcircled{a}}\right)\mathrm{E}_1+\left(\frac{\mathrm{CF}\textcircled{d}+\mathrm{CP}\textcircled{d}}{\mathrm{CP}\textcircled{d}+\mathrm{CF}\textcircled{d}}\right)\mathrm{E}_2+$$

$$+\left(\frac{\mathrm{CF}\textcircled{a}+\mathrm{CP}\textcircled{d}}{\mathrm{CP}\textcircled{a}+\mathrm{CF}\textcircled{d}}\right)\mathrm{E}_3+\left(\frac{\mathrm{CF}\textcircled{d}+\mathrm{CP}\textcircled{a}}{\mathrm{CP}\textcircled{d}+\mathrm{CF}\textcircled{a}}\right)\mathrm{E}_4+$$

$$+\left(\frac{\mathrm{CF}\textcircled{c}+\mathrm{CP}\textcircled{c}}{\mathrm{CP}\textcircled{c}+\mathrm{CF}\textcircled{c}}\right)\mathrm{E}_5+\left(\frac{\mathrm{CF}\textcircled{a}+\mathrm{CP}\textcircled{c}}{\mathrm{CP}\textcircled{a}+\mathrm{CF}\textcircled{c}}\right)\mathrm{F}_6+$$

$$+\left(\frac{\mathrm{CP}\textcircled{b}+\mathrm{CF}\textcircled{d}}{\mathrm{CF}\textcircled{b}+\mathrm{CP}\textcircled{d}}\right)\mathrm{E}_7+\left(\frac{\mathrm{CP}\textcircled{b}+\mathrm{CF}\textcircled{c}}{\mathrm{CF}\textcircled{b}+\mathrm{CP}\textcircled{c}}\right)\mathrm{E}_8+$$

$$+\left(\frac{\mathrm{CF}\textcircled{d}+\mathrm{CP}\textcircled{b}}{\mathrm{CP}\textcircled{d}+\mathrm{CF}\textcircled{b}}\right)\mathrm{E}_9$$

Figure 95. Quadrant rotation in a fugal scheme.

As this example shows, the CF may appear in the same voice successively when its geometrical position *alters*.

The form of a fugue in which the counterpoint is varied with some, or with each, of the expositions may also be subjected to quadrant rotation.

The general scheme of such a fugue is:

$$\mathrm{F}=\left(\frac{\mathrm{CF}+\mathrm{CP}_1}{\mathrm{CF}}\right)\mathrm{E}_1+\left(\frac{\mathrm{CF}+\mathrm{CP}_1}{\mathrm{CP}_1+\mathrm{CF}}\right)\mathrm{E}_2+\left(\frac{\mathrm{CF}+\mathrm{CP}_2}{\mathrm{CP}_2+\mathrm{CF}}\right)\mathrm{E}_3+$$

$$+\left(\frac{\mathrm{CF}+\mathrm{CP}_3}{\mathrm{CP}_3+\mathrm{CF}_3}\right)\mathrm{E}_4+\ \ldots$$

An example with application of the quadrant rotation

$$\mathrm{F}=\left(\frac{\mathrm{CF}+\mathrm{CP}_1}{\mathrm{CF}}\right)\textcircled{a}\ \mathrm{E}_1+\left(\frac{\mathrm{CF}+\mathrm{CP}_2}{\mathrm{CP}_1+\mathrm{CF}}\right)\textcircled{d}\ \mathrm{E}_2+$$

$$+\left(\frac{\mathrm{CF}\textcircled{b}+\mathrm{CP}_3\textcircled{a}}{\mathrm{CP}_2\textcircled{b}+\mathrm{CF}\textcircled{a}}\right)\mathrm{E}_3+\left(\frac{\mathrm{CF}+\mathrm{CP}_2}{\mathrm{CP}_1+\mathrm{CF}}\right)\textcircled{b}\ \mathrm{E}_4+$$

$$+\left(\frac{\mathrm{CF}+\mathrm{CP}_3}{\mathrm{CP}_2+\mathrm{CF}}\right)\textcircled{c}\ \mathrm{E}_5+\left(\frac{\mathrm{CP}_2\textcircled{b}+\mathrm{CF}\textcircled{c}}{\mathrm{CF}\textcircled{b}+\mathrm{CP}_1\textcircled{c}}\right)\mathrm{E}_6$$

Figure 96. Quadrant rotation.

In the old fugue form, the elimination of monotony was usually achieved by means of *interludes*. An interlude (we shall term it: I) is a contrapuntal sequence of either the imitation or the general type. Statistical analysis of actual fugues shows that about 50 of every 100 interludes are thematic (i.e., based on elements of CF or CP); the others are neutral, i.e., they use thematic elements of their own.

As in the case of counterpoint itself, an I may be composed once and rotated afterwards. In other cases, a new I may be composed each time. In the old classical fugues, the interludes served mainly as bridges between the E's, each I leading into a new key.

In our fugues of types I and II, the I's may serve the same purpose, but in types III and IV the interludes are hardly necessary, for key variety is already inherent in the group of different symmetric tonics. As we shall see later, the fact that we have two parts does not limit the number of tonics.

The general scheme of a fugue with interludes appears as follows:

$$F = E_1 + I_1 + E_2 + I_2 + E_3 + I_3 + \ldots + E_n + I_n.$$

This form is equivalent to the *first rondo* form of homophonic music.

$I_1, I_2, I_3, \ldots$ may be either identical (although in different geometrical positions) or totally different. I_n, i.e., the last interlude, is a rather common feature in the old fugues and has the function of a *conclusion* (coda). By rotating the same interlude, we acquire *new modulatory directions*.

C. Steps in the Composition of a Fugue

The method of composing a fugue by this system consists of the following stages:

(1) Composition of the theme;
(2) Composition of the counterpoint (one or more) to the theme; this is equivalent to the "preparation" of an exposition;
(3) Preparation of the exposition (or of all expositions if there is more than one counterpoint) in four geometrical positions:

$$\frac{CF}{CP}\,ⓐ\,;\ \frac{CF}{CP}\,ⓑ\,;\ \frac{CF}{CP}\,ⓒ\,;\ \frac{CF}{CP}\,ⓓ\,;$$

(4) Composition of the interlude(s), if there are to be any;
(5) Preparation of the four geometrical positions of the interlude(s), if any;
(6) Composition of the scheme of the fugue; and
(7) Assembly of the fugue.

D. Composition of the Theme of a Fugue

In a fugue the theme (or "subject") is of the utmost importance; it constitutes at least one half of the entire composition. It is therefore odd that no one has hitherto defined clearly the requirements for a fugal theme.

A good fugal theme is usually ascribed to the composer's "genius," and this neither helps nor consoles a student of the subject, for what we want to know, precisely, is: what makes the melody a suitable fugal theme? Experience shows that not every "good" or expressive melody makes a suitable fugal theme, and that not every suitable fugal theme is a good melody for any *other* purpose. Many composers who were outstanding melodists nevertheless failed to show any important achievements as contrapuntalists—e.g., Chopin, Schumann, Liszt, Chaikovsky, and others.

The first requirement of a fugal theme is that it be an *incomplete* melodic form. In the best and most typical fugues by J. S. Bach, we find that such *incomplete melodic forms used as themes are succeeded by their completions during the counterpoint* which evolves in the course of the announcement of the theme in the second voice.

An incomplete melodic form in this case means that *at the moment the second voice starts the theme, the first voice has not arrived at its own primary axis.*

For an illustration, take Fugue II, Vol. I, *The Well Tempered Clavichord* (later to be referred to by the abbreviation, "W.T.C.") by J. S. Bach.

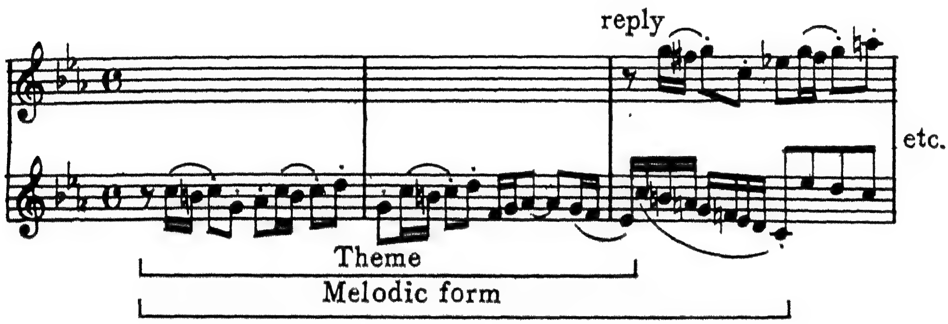


Figure 97. Fugue II of W.T.C.

This particular theme ends on the first sixteenth note of the third measure, while the melodic form completes itself on the third quarter of the same bar. It is interesting to note that the theme (and the whole melodic form) is constructed on the binary axis: $\frac{0}{d}$.

In order to present his announcement clearly, Bach used $\frac{1}{t}$ (= ♪) at the very point where the theme might otherwise have stopped; he reserves the use of the eighth note until the reply is well on its way of development. In this way, Bach eliminates the danger of stopping—which, indeed, had it occurred, would have spoiled the entire fugue. Another important detail is the juxtaposition of the db-axis in CP versus the 0-axis in CF.

All the other requirements of a fugal theme actually derive from the first one: *all such resources of temporal rhythm and axial forms may be used as will demonstrate an unfinished melodic structure in the very process of formation.*

The presence of any one of the following structural characteristics, as well as of any combinations of the latter, will increase its suitability as a fugal theme.

(1) The presence of rests; particularly a decreasing series of rests, combined with an increasing number of attacks; "stop-and-go" effects; the effect of "gaining momentum."

(2) A sequence of decreasing duration-values: rhythmic acceleration in the broadest sense.

(3) "Dialogue" effects achieved by means of *binary* axes, and by means of attack-groups contrasting in form, such as a legato-staccato contrast.

(4) Effects of growth, achieved by means of binary and ternary *diverging* axes.

(5) The presence of resistance forms, including repetition, phasic and periodic rotation, particularly those forms that lead to climaxes.

Combinations of the above techniques applied to one theme make the latter more saturated and tense, which increases its fugal characteristics.

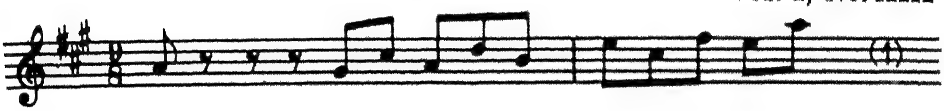
Fugal themes by J. S. Bach--and by "just J.S."

(Numbers in musical examples refer to the preceding classifications).

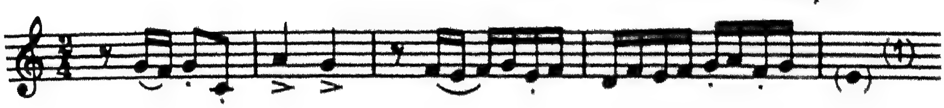
W.T.C. by J.S. Bach

Vol. I, No. IX

Vol. I, No. XIX



Vol. II, No. I



Vol. II, No. XI

Vol. II, No. XXII

Figure 98. Fugal themes of W.T.C.

by J. S.

Figure 99. Fugal themes by J. S.

Figure 100. Fugal themes of W.T.C.

Figure 101. Fugal themes by J.S.

Figure 102. Fugal themes of W.T.C.

Figure 103. Fugal themes by J. S.

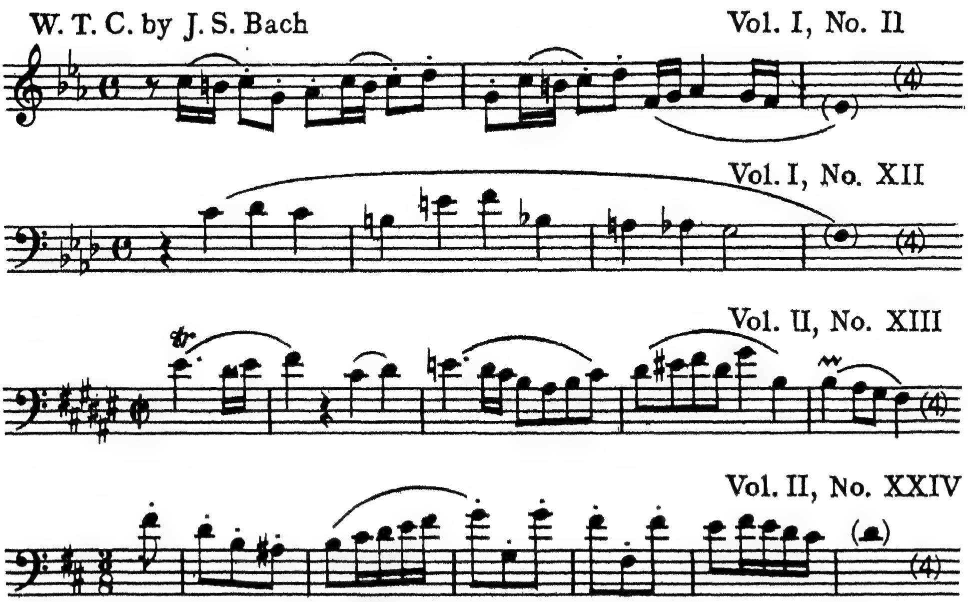


Figure 104. Fugal themes of W.T.C.

Figure 105. Fugal themes by J. S.

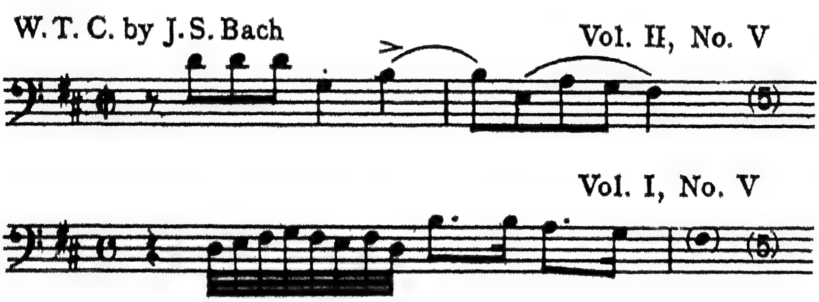
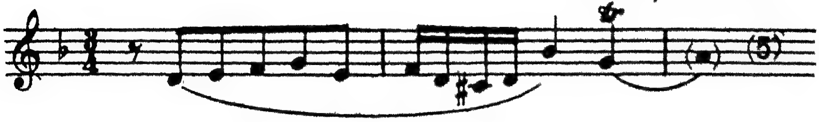


Figure 106. Fugal themes of W.T.C. (continued).

Vol. I, No. VI



Vol. I, No. XV

Vol. II, No. X

Figure 106. Fugal themes of W.T.C. (concluded).

by J.S.
(Symmetric: $\sqrt[3]{2}$)

Figure 107. Fugal themes by J. S.

As indicated by the above examples, the total duration of a theme (in terms of the number of attacks, or in terms of measures) largely depends on the composer's own decision. However, the following generalization is true for most classical fugues: the duration of the fugal theme in inverse proportion to the number of parts. Indeed, the first theme of Fugue IV, Vol. I, W.T.C. has but five attacks; the theme in Fugue XXII, Vol. I, W.T.C. has six attacks. Both of these fugues are written in five parts. On the other hand, Fugue X of the same volume, written in two parts, has a theme of twenty-six attacks.

It is not important that the reply should enter on the same time-unit of the measure as the theme; on the contrary, a difference in the starting moments (in relation to the measure divisions) adds interest to the whole composition as it produces an element of surprise.

Themes which are unsuitable for fugues may be subjected to alterations which will make them suitable.

It can be demonstrated, by reversing the procedure, that the simple addition of a 0-axis to any melodic form will render it suitable as a fugal theme. J. S. Bach's theme from his *Toccata and Fugue in D-minor* for organ, if it is deprived of its 0-axis, loses all its fugal quality. When the 0-axis is taken out, the axial combination becomes b+a+c+a, and the theme seems to have nothing but rotation in relatively narrow range. But the inclusion of the 0-axis produces an effect of growing resistance, and the axial combination becomes:

$$\frac{0}{d+c+c}$$

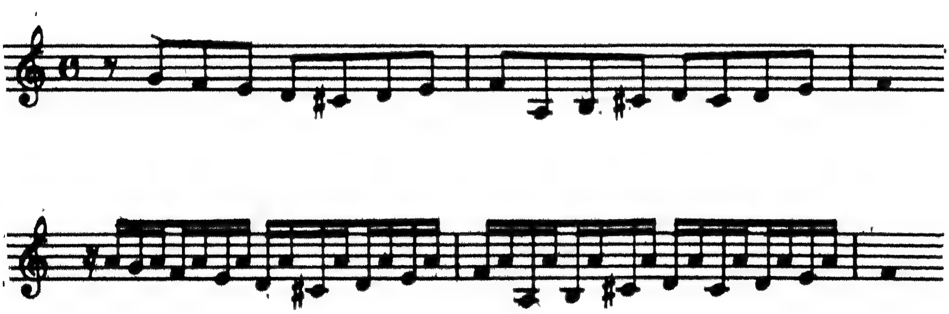


Figure 108. $\frac{0}{d+c+c}$

The number of measures in a fugal theme is optional; it may be even or odd; it may be integral or fractional. Both odd and fractional are preferable to even and integral, because the latter two suggest a cadence at the end of the theme. I believe that one of the factors that influenced Buxtehude and all the Bachs is their awareness of *cantus firmus* (in a strict sense) as a theme—*cantus firmi* usually had an odd number of attacks, as noted earlier.

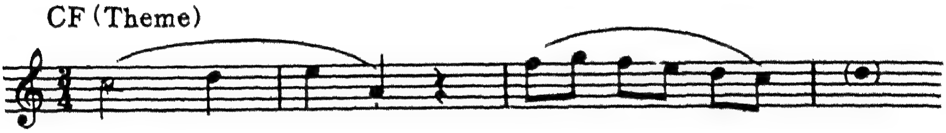
E. Preparation of the Exposition

After selecting the theme, the composer must prepare the fugal exposition.

As it is easy, with this method, to write four types of fugues on one theme, so it becomes desirable to prepare four expositions for future fugues. In a two-part fugue, the entire preparation of E consists simply of writing a CP to the CF. It is advisable that the exposition prepared for each type should be written out in all four geometrical positions; this saves time during the process of assembling the fugue. Fugues of type IV often require preparation of two expositions, for when the axes exchange in $\frac{CP}{CF}$, CP may not fit, and a new counterpoint must be written (CP_{II}).

To make the demonstration of all techniques pertaining to fugue concise, I shall use a very brief theme.

CF (Theme)



Type I: $\frac{CF}{CP}$ ⓐ

Type I: $\frac{CF}{CP}$ ⓑ

Type I: $\frac{CF}{CP}$ ⓓ

Figure 109. A brief theme and the various fugal techniques (continued).

Type I: $\frac{\text{CF}}{\text{CP}}$ (c)

Type II: $\frac{\text{CF}}{\text{CP}}$ (a)

Type II: $\frac{\text{CF}}{\text{CP}}$ (b)

Type II: $\frac{\text{CF}}{\text{CP}}$ (d)

Type II: $\frac{\text{CF}}{\text{CP}}$ (c)

Figure 109. Technique of the fugue (continued).

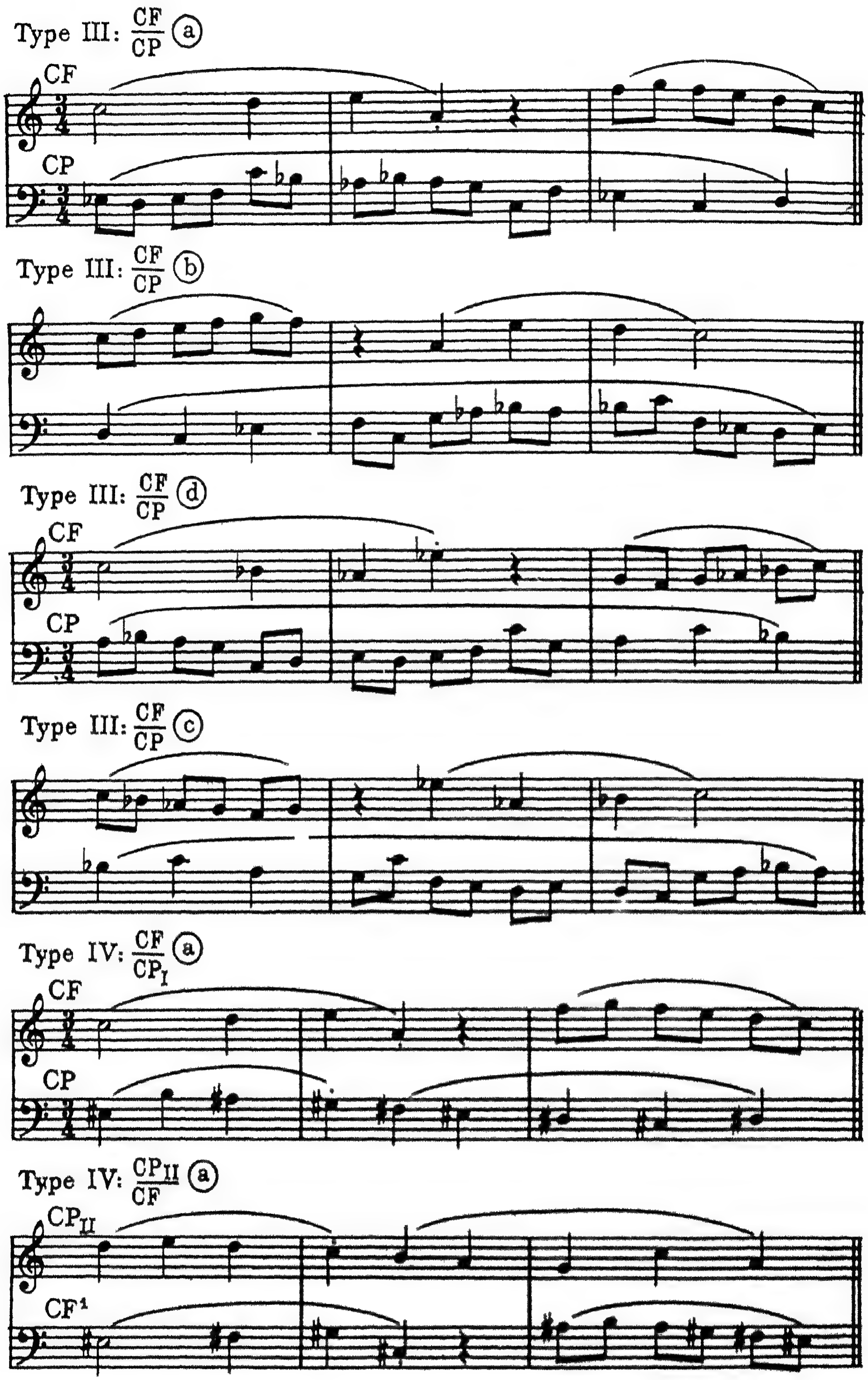


Figure 109. Technique of the fugue (continued).

Type IV: $\frac{CF}{CP_I}$ (b)

Type IV: $\frac{CP_{II}}{CF}$ (b)

Type IV: $\frac{CF}{CP_I}$ (c)

Type IV: $\frac{CP_{II}}{CF}$ (c)

Type IV: $\frac{CF}{CP_I}$ (d)

Type IV: $\frac{CP_{II}}{CF}$ (d)

Figure 109. Technique of the fugue (concluded).

F. Composition of the Expositions

Composition of the expositions in type I involves no special considerations, for both parts have an identical P.A.

In type II, the modal modulations of CF and its respectively related CP must be in one system of modal sequence. For example, if P.A. of CF_1 is c and P.A. of CP_1 is e, the axis of CF_2 ("reply") must be a and CP_2 (counterpoint to "reply") must have P.A. on c in order to retain axial unity in the first part for the course of one exposition, and in order to preserve the vertical relation of $\frac{CP}{CF}$ as it was originally conceived.

The entire structure of the fugue (from the above relations) appears as follows:

$$F = \begin{bmatrix} ((CF_1 + CP_1)\,c \\ CF_2\ a \end{bmatrix} E_1 + \begin{bmatrix} ((CF_3 + CP_3)a \\ CP_2c + CF_4\ f \end{bmatrix} E_2 + \begin{bmatrix} ((CF_5 + CP_5)\,f \\ CP_4a + CF_6\ d \end{bmatrix} E_3 + \ \dots$$

where c, a, f, d, are the primary axes of the respective parts.

Likewise, if $\frac{CF_1}{CP_1} = \frac{c}{f}$, then the sequence of P.A.'s becomes: $\frac{c}{g} + \frac{g}{c + d} + \frac{d}{g + a} + \dots$

In type III, the tonal (key) modulations of CF and of its respectively related CP, must be in one system of symmetric sequence. This sequence preserves its constant $\frac{CP}{CF}$ relation only when CP_2 (the reply) forms its P.A. in symmetric inversion to the original setting.

Let us take the symmetry of $\sqrt[4]{2}$; for example: $\frac{CP}{CF} = \frac{e\flat}{c}$. In order to preserve the axial relation where CP is 3 semitones above CF, the reply must appear from the opposite equidistant point, i.e., from a. This permits a relative stability of both parts, as CP_1 - being three semitones above CF - requires the c-axis.

The structure of such a fugue, evolved on four points of symmetry (tonics), appears as follows:

$$F = \begin{bmatrix} ((CF_1 + CP_1)\,c \\ CF_2\ a \end{bmatrix} E_1 + \begin{bmatrix} ((CF_3 + CP_3)a \\ CP_2c + CF_4\ f\sharp \end{bmatrix} E_2 + \begin{bmatrix} ((CF_5 + CP_5)\,f\sharp \\ CP_4a + CF_6\ e\flat \end{bmatrix} E_3 +$$

$$+ \begin{bmatrix} ((CF_7 + CP_7)\,e\flat \\ CP_6\,f\sharp + CF_8\ c \end{bmatrix}$$

A similar case evolved from three points of symmetry ($\sqrt[3]{2}$), where $\frac{CP}{CF} = \frac{e}{c}$, gives the following sequence of P.A.'s:

$$\frac{c}{a} + \frac{a\flat}{c + e} + \frac{e}{a\flat + c}$$

In type IV, in order to carry out the sequence of P.A.'s in symmetric inversion of the original setting, it often becomes necessary to prepare two independent expositions—

$$E = \frac{CP_I}{CF} \text{ and } E^1 = \frac{CP_{II}}{CF^1}$$

—as CP may be in an intervallic relation to CF_2 different from the relation to CF_1. The difference usually appears in variations of a semitone or whole tone, which results in the most disturbing relations—such as a second instead of a third. For this reason, the example in Figure 109 offers two expositions.

It is easy to see that CP_I is unfit to be a counterpoint to the reply, by exchanging it with P.A. or CF.

The sequence of symmetric P.A.'s in type IV of Fig. 109 would develop on the basis of its pre-set expositions:

$$E = \frac{CP_I}{CF} = \frac{e\sharp}{c} \text{ and } E^1 = \frac{CP_{II}}{CF^1} = \frac{c}{e\sharp}.$$

Considering the enharmonic equality of e♯ and f, a♯ and b♭, etc., and the fact that CF is evolved in natural major d_0 and CF^1 in natural major d_6, we obtain the following structure for the fugue:

$$F = \left[\frac{(CF + CP_{II})c}{CF^1 \quad e\sharp}\right]E_1 + \left[\frac{(CF + CP_{II})f}{(CP_I + CF^1)a\sharp}\right]E_2 + \left[\frac{(CF + CP_{II})b\flat}{(CP_I + CF^1)\, d\sharp}\right]E_3 + \ . \ . \ .$$

In the old classical fugues the reply appears on the dominant (i.e., seven semitones above or five semitones below the theme). If there was a sequence of expositions before the interlude took place, the theme would usually return to the tonic. According to our type II, if $CF_1 = c$ and $CF_2 = g$, CF_3 should have been *d*, CF_4 should have been *a*, etc. However, this was not the case in the fugues of the classical period, and there was a good reason for it: the tuning of *mean* temperament (the two-coordinate system: $\frac{3}{2}$ and $\frac{5}{4}$) developed an aberration in pitches deviating from the tuning center (= 1), and so it was not possible to get satisfactory intonation in the course of a sequence traveling through C_6 or C_{-6} P.A.'s. Although *equal* temperament has since overcome this defect, the habit remained with composers until the end of the 19th century. century.

G. Preparation of the Interludes

Interludes ($I_1, I_2, \ . \ . \ . \ I_m$) serve as bridges between the expositions. The last interlude, if the fugue ends with one, would be called a *postlude* or *coda*.

Interludes serve one or both of two purposes:

(1) to divert the listener's attention away from the persistence of theme;

(2) to produce a modulatory transition from one key-axis to another.

Interludes of the first form are confined to one key but may have any number of successive P.A.'s, thus producing modal modulations (U.-P.) between the two adjacent expositions having the same key-axis (U.-U. and U.-P.). The second form contains different key-axes (P.-U. and P.-P.) and connects the two adjacent expositions having different key-axes (P.-U. and P.-P.).

Both forms of interludes may be either *neutral* or *thematic*. *Neutral* interludes are based on material of rhythm, or intonation, or both, which does not appear in any of the exposition. *Thematic* interludes borrow their material of rhythm, or intonation, or both, from either the CF or the CP of the exposition. Any of the above described types of interludes may be executed either in *general* or in *imitative* counterpoint.

The duration of an interlude depends on the duration of the exposition and the number of interludes. The form of an interlude itself has an influence upon its duration. In order to construct a perfect fugue, the duration of interludes must be put into some definite relationship with the duration of expositions. Assuming that one exposition is the temporal unit (T), we arrive at the following fundamental schemes for the temporal organization of interludes:

(1) T (E) = T (I), i.e., the duration of an interlude equals that of an exposition. This presupposes an equal duration for each of the interludes.

(2) T (E) > T (I), i.e., the duration of an exposition is longer than that of an interlude. An exact ratio must be established in each case.

(3) T (E) < T (I), i.e., the duration of an interlude is longer than that of an exposition. An exact ratio must be established in each case.

(4) $I^{\rightarrow} = I_1T + I_22T + I_33T + \ldots$, i.e., each successive interlude becomes longer. The durations of consecutive interludes may evolve in any desirable type of progression (natural, arithmetic, geometric, involution, summation, etc.). The resulting effect of such fugue-structures is that the interludes in the course of time begin to dominate the theme. Thus, the persistence of the theme diminishes.

(5) $I^{\rightarrow} = I_1nT + I_2(n\text{-}1)T + I_3(n\text{-}2)T + \ldots$, i.e., each successive interlude becomes shorter. The resulting effect is opposite to that of (4); the domination of theme over interludes grows in the course of time.

(6) $I^{\rightarrow}$, i.e., the sequence of interludes develops according to some form of rhythmic grouping.

As convertibility and quadrant rotation are general properties, the same interlude may be used several times during the course of a fugue. This, in combination with key-transpositions, offers an enormous variety of resources—and at the same time conserves the composer's energy.

H. Non-Modulating Interludes

(Types I and II)

Non-modulating interludes may be either neutral or thematic and they can be evolved in either general or imitative counterpoint.

(1) An example of Interlude type II executed in general counterpoint. Non-thematic (Neutral).

(2) An example of Interlude type II executed in imitative counterpoint. This one is thematic with reference to CF of Fig. 109.

See the corresponding musical illustrations on the opposite page.

(1)

(2)

Figure 110. Interlude type II.

I. Modulating Interludes

1. Modulating counterpoint evolved through harmonic technique.

Contrary to the general notion, J. S. Bach's counterpoint is less "contrapuntal" than it is generally believed to be. This is especially true of his tonal (key-to-key) modulations. It is obvious that Bach, as well as many other important contrapuntalists, thought of key-to-key transitions in terms of ***modulating chords;*** see, for example, J. S. Bach's *W.T.C.*, Vol. I, fugue No. X (a two-part fugue) in E-minor—the harmonic background of this fugue is very distinct, and this fugue is typical rather than exceptional.

It is easy to convert any modulating chord-progression written in four-part harmony into two-part harmony.

The chord structures in two-part harmony have the following functions:

(1) S(3) = 1, 3; used instead of the S(5) of three-part structure;
(2) S(5) = 1, 5; used instead of the S(5) of three-part structure;
(3) S(7) = 1, 7; used instead of the S(7) of four-part structure.*

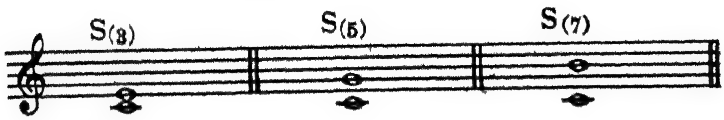


Figure 111. Chord structures in two-part harmony.

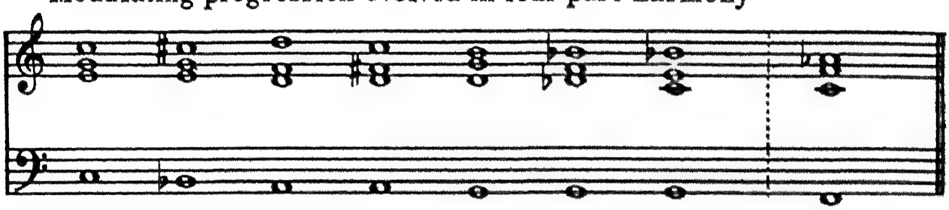
*Or a two-part incomplete S(7) = 3, 7 may be used instead of the S(7) in four-part harmony—as in the fourth chord in the example of translation that follows. (Ed.)

In order to obtain an interlude from a four-part chord-progression, it is necessary to select those corresponding chordal functions which will *translate* the four-part structures into two-part structures. The voice-leading pertaining to two-part harmony will not be discussed here, as all positions of the two functions are equally as acceptable for the present purpose. Both parts are more or less in the vicinity of the four-part harmony range. The final step consists of developing melodic figuration in both parts, with somewhat contrasting rhythms of durations and attacks.

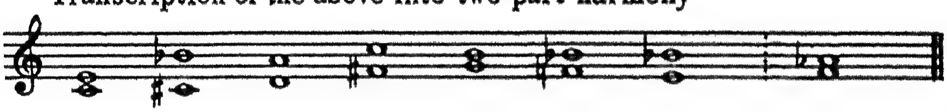
Modulating interludes may be either neutral (general counterpoint) or thematic (imitative counterpoint). In the latter case, thematic material is either borrowed from the CF or the CP of the expositions--or it is entirely independent.

(1) Neutral and (2) Thematic.

Modulating progression evolved in four-part harmony



Transcription of the above into two-part harmony



Interlude (1)

Figure 112. Modulating interludes (continued).

Interlude (2)

Figure 112. Modulating interludes (concluded).

An interlude may be used in the same fugue more than once, appearing in the different geometrical positions. It may also be transposed to any desirable key-axis in any of the four quadrants.

2. Modulating counterpoint evolved through melodic technique.

I offer this new technique in order to enable the composer to compose in pure contrapuntal style even when a key-to-key transition is desirable.

Modulating counterpoint consists of two independently modulating melodies (see my earlier discussion of *modulation* in the book dealing with the *Theory of Pitch-Scales*)* whose primary axes are in a constant, simultaneous relationship at any given key-point of the sequence. After vertical dependence has been established (the harmonic interval between CP and CF), it becomes necessary *to assign to the primary axis of CP the meaning of that tonic which is nearest to CF through the scale of key-signatures.*

Let the exposition end in the key of C, and let CF end on *c* and CP end on *a*. Then *a* becomes a-minor (as the key nearest to the key of C through the scale of key signatures; A- major would be far more remote). Thus, we have established a constant dependence where CP is the minor key three semitones below CF.

The next step consists of planning the modulation of P_I (originally: CF) Let the modulation be to the key of f- minor.

Then:

$$P_I^{\rightarrow} = C + d + G + f$$

Now we assume that in order to retain the original vertical dependence between P_I and P_{II}, each axis of a major key must be reciprocated by a minor

*See Book II.

key, and vice versa. Then:

$\dfrac{\overrightarrow{P_I}}{\overrightarrow{P_{II}}} = \dfrac{C + d + G + f}{a + F + e + A\flat}$, i.e., while P_I modulates from C to d, P_{II} modulates from a to F, and when P_I modulates from d to G, P_{II} modulates from F to e; finally both parts arrive at CF having an A♭-axis and CP having an f-axis.

The period of modulation from key to key in both parts is approximately the same.

(1) Neutral and (2) Thematic

(1)

(2)

Figure 113. Modulating interludes.

The easiest way to compose modulating interludes by contrapuntal technique is through a sequence of procedures:

(1) P_I modulates to the first intermediate key;
(2) P_{II} " " " " " "
(3) P_I " " " second " "
(4) P_{II} " " " " " "

and so on, until the entire modulation is completed.

J. Assembly of the Fugue

The process of assembling a fugue consists of planning the general sequence of expositions, interludes, their geometrical positions and their primary axes (key-axes).

In the following group of fugues only such materials were used as had been prepared in advance (see Fig. 109, 110, 112, and 113).

The first three fugues have interludes (of both harmonic and melodic type), while the fourth has none, as the key-variety is sufficiently great without interludes. The last fugue has independent counterpoints for the theme and the reply. The latter are interchanged in E_6.

The form of Fugue I (Fig. 114)

$$E_1(a) + I_1 + E_2(a) + E_3(d) + I_2(c) + E_4(d)(c) + I_3(b) + E_5(b)$$

The form of Fugue II (Fig. 115):

$$\overset{C}{\overline{E_1(a) + E_2}}\ (a) + I_1 + \overset{F}{\overline{E_3(a)}} + \overset{C}{\overline{E_4(d) + E_5(d)(c)}} + I_2 + E_6(a)(b)$$

The form of Fugue III (Fig. 116)

$$\overset{C}{\overline{(E_1}} + E_2)\ (a) + (E_3 + E_4)\ (d) + I_1 + \overset{A\flat}{\overline{(E_5 + E_6)}}\ (a) + E_7\,\overset{A\flat E\flat C}{\overline{(b)(a)(a)}}$$

The form of Fugue IV (Fig. 117):

$$(E_1 + E_2 + E_3 + E_4 + E_5)\ (a) + E_6\ (c) + E_7\ (d) + E_8\ (b)$$

(1) Fugue Type I

Figure 114. Fugue of type I (continued).

Figure 114. Fugue of type I (continued).

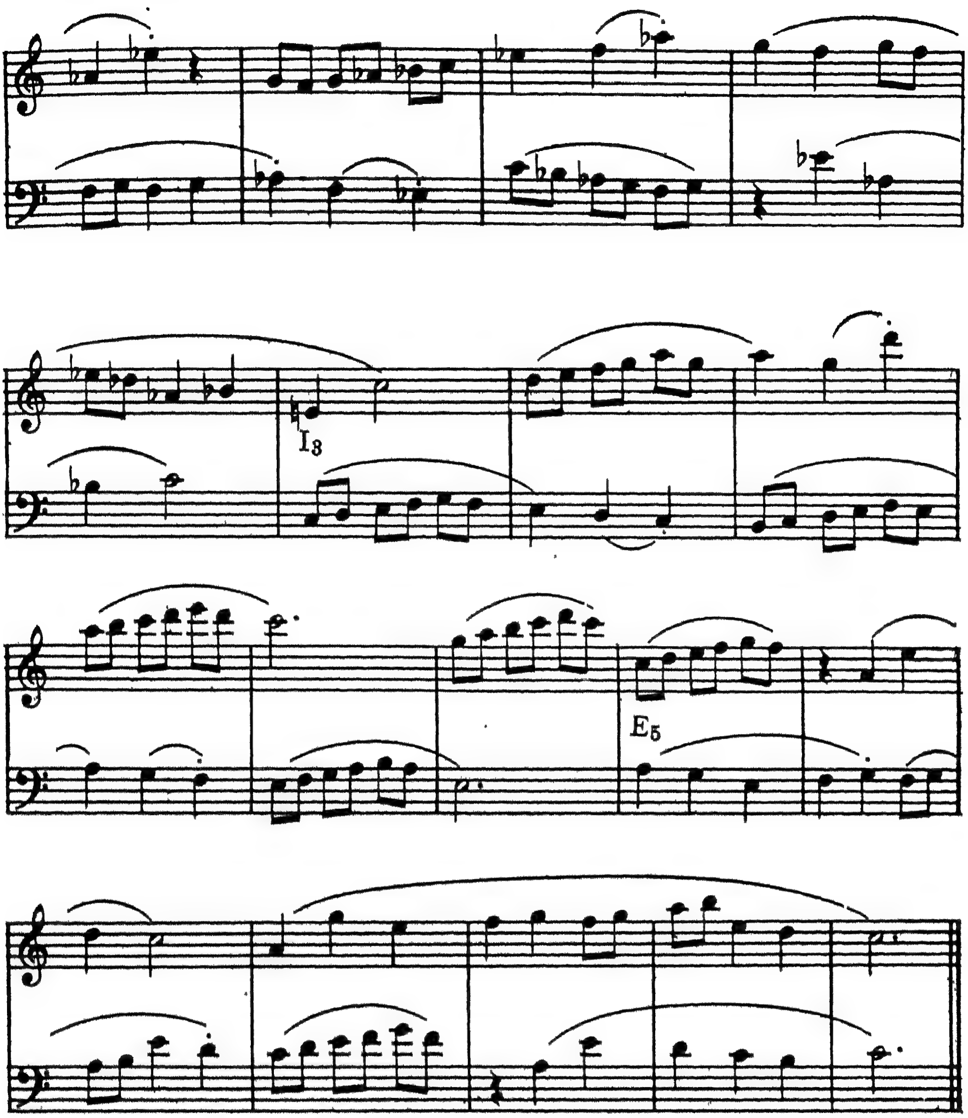


Figure 114. Fugue of type I (concluded).

(2) Fugue Type II

Figure 115. Fugue of type II (continued).

Figure 115. Fugue of type II (continued).

Figure 115. Fugue of type II (concluded).

(3) Fugue Type III

Figure 116. Fugue of type III (continued).

Figure 116. Fugue of type III (continued).

Figure 116. Fugue of type III (continued).

Figure 116. Fugue of type III (concluded).

(4) Fugue Type IV

Figure 117. Fugue of type IV (continued).

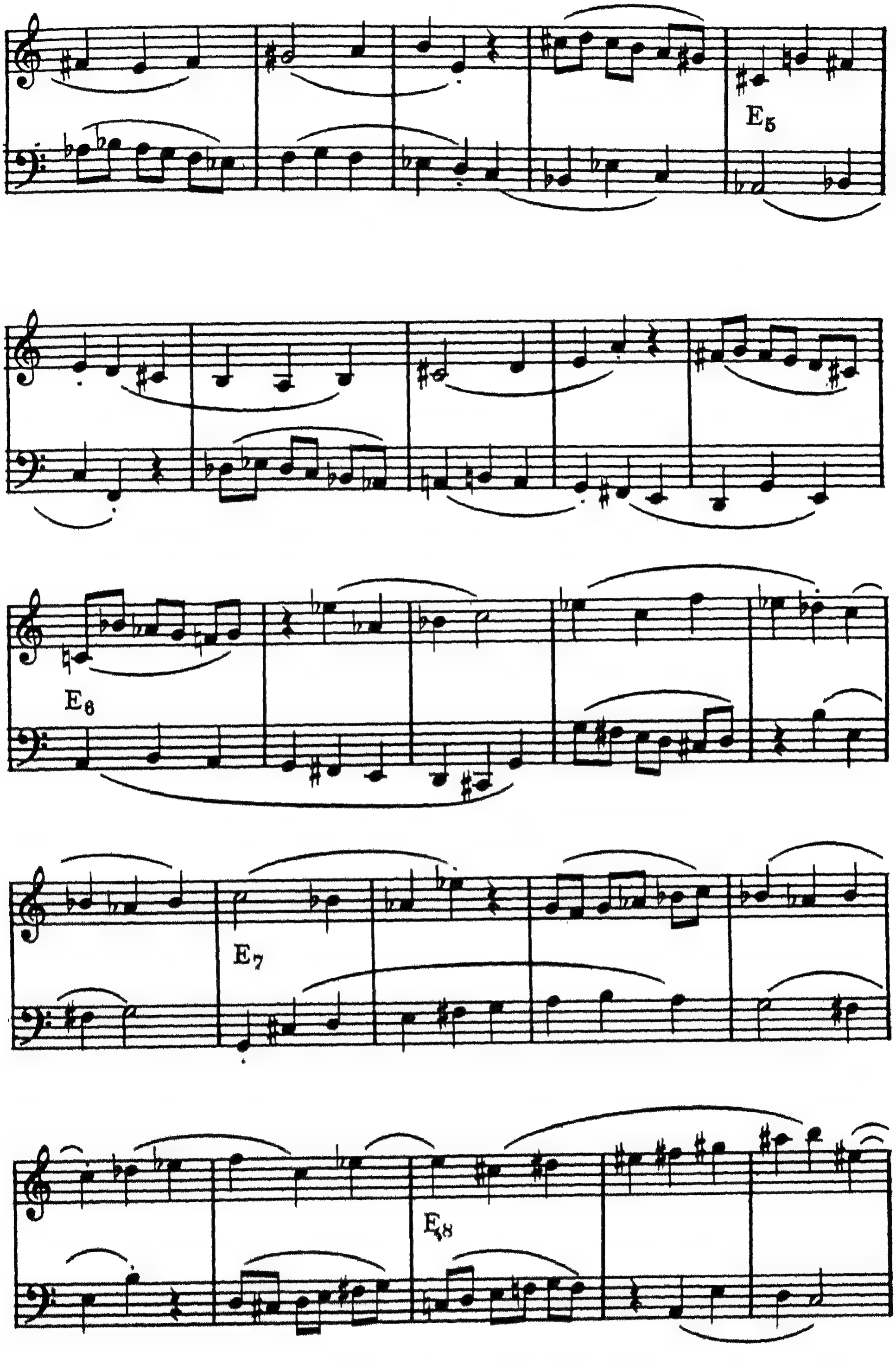


Figure 117. Fugue of type IV (continued).

Figure 117. Fugue of type IV (concluded).

(Editor's note. The original manuscript included material on the generalization of 2-part counterpoint into 3 or more part counterpoint. This is omitted because it was largely in outline form. Various students who studied privately with Schillinger have presented their notes to the editors, and have made it possible to complete Schillinger's outlines. This material will be published at a later date.)

CHAPTER 9

TWO-PART CONTRAPUNTAL MELODIZATION OF A GIVEN HARMONIC CONTINUUM

WE are now to concern ourselves with the technique of writing two correlated melodies (two-part counterpoint) to a given chord-progression. The counterpoint itself must satisfy all the requirements applying to harmonic intervals. Each of the melodic parts (to be designated as M_I and M_{II}, or as CP_I and CP_{II}) must also satisfy the requirements pertaining to melodization of harmony.

The sequence in which such a two-part melodization should be accomplished is:

(1) the writing of $H^{\rightarrow}$;

(2) the writing of the M with a fewer number of attacks per H;

(3) the writing of the M with the greater number of attacks per H.

It does not matter which of the two melodies is selected to be M_I and which is to be M_{II}.

In view of the fact that the natural physical scale of frequencies increases in the upward direction of musical pitch, we shall first produce that melody which has the fewer number of attacks in a position immediately above harmony, and the melody with the greater number of attacks we shall develop *above* the first melody. Such an arrangement will be considered to be fundamental; it may later be rearranged.

We arrive at the two possible settings:

$$(1)\ \frac{\dfrac{M_I}{M_{II}}}{H^{\rightarrow}} \quad \text{and} \quad (2)\ \frac{\dfrac{M_{II}}{M_I}}{H^{\rightarrow}}$$

Octave-convertibility (exchange of the positions of M_I and M_{II}) is possible only when the harmonic intervals of both melodic parts are chosen with an eye to such convertibility—and this is mainly a matter of supporting certain higher functions (such as 11) by the immediately preceding function (such as 9).

All forms of quadrant rotation (ⓐ, ⓑ, ⓒ and ⓓ) are acceptable on the condition that M_I and M_{II} always remain *above* the chord progression, $H^{\rightarrow}$.

Just as melodization of harmony by means of *one* part produced different types of melody in relation to the different types of harmonic progressions, the same possibilities still exist for *two*-part melodization.

It is to be remembered that some types of melody in one-part melodization were the outcome of new techniques. For instance, the technique of a modulating symmetric melody above all forms of symmetric harmony, or the technique of a diatonic melody evolved from a quantitative scale above all forms of chromatic harmony—both are forms not known in music prior to my development of the principles for these procedures. All such new techniques may be applied now to two-part melodization. This, naturally, will result in new types of counterpoint.

The distribution of attacks of M_I, M_{II} and $H^{\rightarrow}$ is a matter of considerable complexity and will be discussed more fully later. For the present, we shall distribute the attacks for all three parts (M_I, M_{II} and $H^{\rightarrow}$) uniformly and by means of multiples.

Some elementary forms of the distribution of attacks.

M_I	a	2a	a	3a	a	4a	a	4a	2a	6a	2a	8a	2a	6a	3a	8a	4a
M_{II}	a	a	2a	a	3a	a	4a	2a	4a	2a	6a	2a	8a	3a	6a	4a	8a
$H^{\rightarrow}$	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H

M_I	9a	3a	12a	3a	12a	4a	15a	3a	16a	4a	. . .
M_{II}	3a	9a	3a	12a	4a	12a	3a	15a	4a	16a	
$H^{\rightarrow}$	H	H	H	H	H	H	H	H	H	H	

Figure 118. Distribution of attacks.

Here the quantities of attacks in $\frac{M_I}{M_{II}}$ are designated as the attacks per chord.

Each original setting of two simultaneous melodies accompanied by a chord-progression offers seven forms of exposition:

$$(1)\ M_I;\ (2)\ M_{II};\ (3)\ H^{\rightarrow};\ (4)\ \frac{M_I}{M_{II}};\ (5)\ \frac{M_I}{H^{\rightarrow}};\ (6)\ \frac{M_{II}}{H^{\rightarrow}};\ (7)\ \begin{array}{c}M_I\\ \hline M_{II}\\ \hline H^{\rightarrow}\end{array}$$

A. Melodization of Diatonic Harmony by Means of Two-Part Diatonic Counterpoint (*Type I and II*)

The melody with the lowest number of attacks and which appears immediately above the harmony must conform to the principles of diatonic melodization. It is desirable not to use higher functions (9, 11) in this melody (we shall call it M_{II}), for the latter should be reserved for use in the melody with the larger number of attacks (we shall call it M_I), so that the higher functions of M_I may be supported by M_{II}. Scales of both melodies must have a common source of derivation; this common source is the diatonic scale of the harmony.* Any derivative scales of the original d may be employed.

The harmony itself may be devised in four or in five parts; four-part harmony is preferable, for the texture of a duet accompanied by five parts is somewhat heavy.

None of the melodies should produce consecutive octaves with any of the harmonic parts.

*That is, the diatonic scale that results from converting the Σ (E_1) of the given chord into its zero expansion (E_0). For example, the chord (reading upwards in thirds) C - E - G - B♭ - D - F♯ - A (Σ XIII) becomes, when contracted to E_0, the third displacement or mode (d_3) of the natural G minor scale, C - D - E - F♯ - G - A - B♭. (Ed.)

M_I should be written as a counterpoint to M_{II} and as a melodization of the given chord-progression.

Identical as well as non-identical scales (which derive through permutation of the pitch-units of d_0) may be used in M_I, M_{II} and $H^{\rightarrow}$. Under such conditions, any d_0 produces 35 possibilities of modal relations between the above-mentioned three components.

As we are employing seven-unit scales, and as—

$$_7C_3 = \frac{7!}{3!\,(7-3)\,!} = \frac{5040}{6\cdot 24} = \frac{5040}{144} = 35$$

—the number of possible two-part melodizations to one chord-progression (written in one definite d) is:

$$_7C_2 = \frac{7!}{2!\,(7-2)\,!} = \frac{5040}{2\cdot 120} = \frac{5040}{240} = 21$$

(1)

(2)

Figure 119. Diatonic two-part melodization.(continued).

Figure 119. Diatonic two-part melodization (continued).

(6)

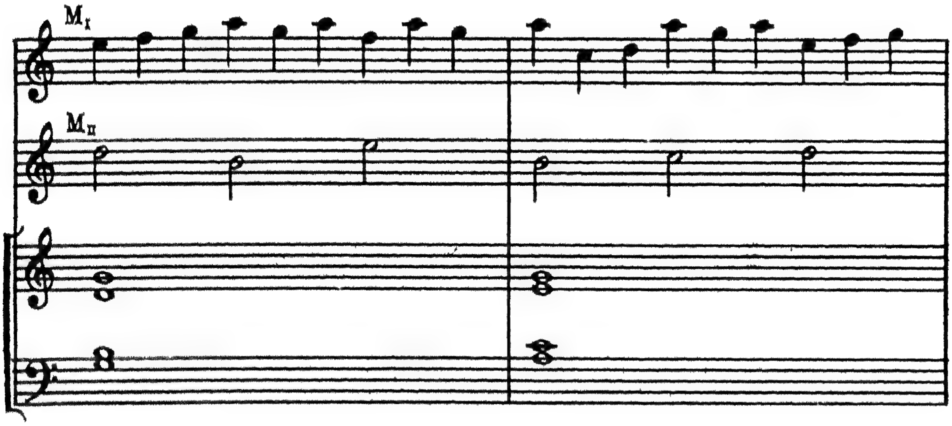


Figure 119. Diatonic two-part melodization (concluded).

B. Chromatization of Diatonic Two-Part Melodization

In order to produce a greater contrast between M_I and M_{II} either one can be subjected to chromatic variation. If desirable, both melodies can be used in their chromatic version.

Chromatic variation is achieved by means of passing or auxiliary chromatic tones.

By means of combining the two variations of Fig. 120, we obtain a new version in which chromatic sections alternate with the diatonic ones.

Theme: Figure 119 (3)

Var. I

Var. II

Figure 120. Chromatic variations. Var. I, II and III (continued).

Figure 120. Chromatic variations. Var. I, II and III (concluded).

C. Melodization of Symmetric Harmony

(Type II, III and Generalized) by means of Two-Part Symmetric Counterpoint

Symmetric melodization is based on the pitch-scale which is a contraction of the particular Σ 13 that corresponds to each individual H. Theoretically, each chord requires a new scale. The quality of the melody, however, depends on the number of tones there are in common among the successive Σ 13's upon which the $S^{\rightarrow}$'s are based; this is true of both M_I and M_{II} of two-part melodization.

The requirements for two-part symmetric melodization may be stated as follows:

(1) Adherence of one M to a particular set of pitch-units, producing a scale.

(2) Graduality of melodic modulation, which is executed by means of common tones, chromatic alterations and identical motifs.

(3) Strict adherence to contrapuntal treatment of the harmonic intervals between M_I and M_{II}.

Figure 121. Symmetric two-part melodization (continued).

Figure 121. Symmetric two-part melodization (concluded).

D. Chromatization of a Symmetric Two-Part Melodization

This technique is identical with chromatization of diatonic counterpoint. The passing and auxiliary chromatic tones are not part of the Σ 13. Either of the two contrapuntal parts may be chromatized. Alternation of chromatic and symmetric sections in both melodies is fully satisfactory.

Theme: Figure 121 (2)

Var. I

Var. II

Figure 122. Chromatization of a symmetric two-part melodization (continued).

Figure 122. Chromatization of a symmetric two-part melodization (concluded).

E. Melodization of Chromatic Harmony by Means of Two-Part Counterpoint

As we know, one-part melodization of chromatic harmony is possible by two distinctly different procedures:

(1) that based on directional units, and
(2) that based on quantitative scale.

Chromatic melodization in two parts is therefore possible in the following combinations of the above techniques:

M_I	di	ch	di	ch
M_{II}	di	di	ch	ch
$H^{\rightarrow}$	ch	ch	ch	ch

where *di* (diatonic) represents the quantitative scale; *ch* of M represents the directional-units method, and *ch* of $H^{\rightarrow}$ stands for chromatic harmonic continuity.

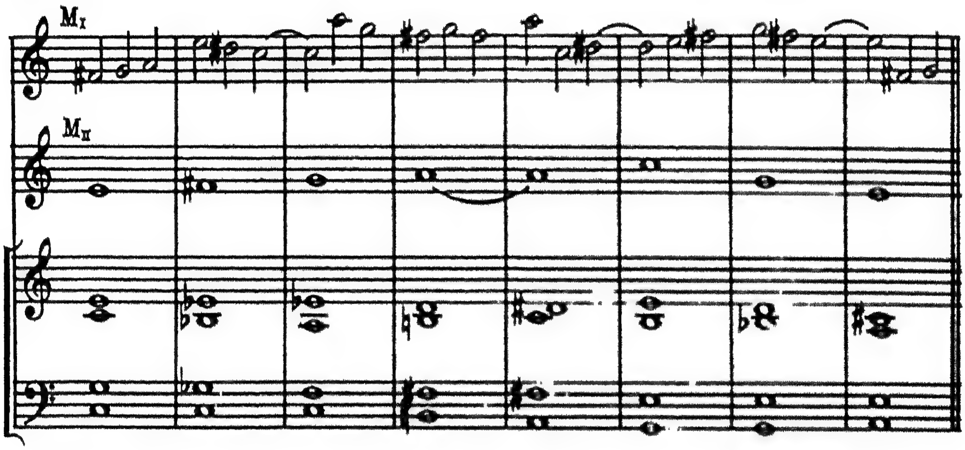
If a contrast is to be achieved between M_I and M_{II}, one of them becomes di; the other, ch.

If a similarity is preferable (contrast may still be achieved by juxtaposition of the quantities of attacks of $\frac{M_I}{M_{II}}$), both melodies are either di or ch. The first has a diatonic character (due to adherence to one particular pitch-scale) and the second has a modulating character which abounds in semitonal directional units.

Progression: H → ch

Quantitative scale: S → di

(1)



(2)

Figure 123. Melodization of chromatic harmony (continued).

Figure 123. Melodization of chromatic harmony (concluded).

CHAPTER 10

ATTACK-GROUPS FOR TWO-PART MELODIZATION

THE number of attacks as among $\frac{M_I}{\frac{M_{II}}{H}}$ may be either constant or variable.

We say it is a *constant* form of the attack-group when each individual H has a definite corresponding number of attacks in M_I and M_{II}, which number remains the same for every consecutive H.

$$\frac{M_I}{\frac{M_{II}}{H}} = A \text{ const.}$$

Constant-A does not necessarily mean an even distribution in $\frac{a\,(M_I)}{a\,(M_{II})}$. An even distribution may be considered as merely a special case of this relationship.

Examples of an even distribution of A:

M_I	4a	6a	6a	8a	8a	9a	12a	12a
M_{II}	2a	2a	3a	2a	4a	3a	3a	4a
H	a	a	a	a	a	a	a	a

Examples of uneven distribution of A:

M_I	2a+3a	4a+2a	4a+2a	4a+6a
M_{II}	a+a	a+a	2a+a	2a+2a
H	a	a	a	a

M_I	4a+2a+3a+6a	6a+3a+6a+4a+2a+9a
M_{II}	2a+a+a+2a	3a+a+2a+2a+a+3a
H	a	a

We have a variable form of the attack-group when A emphasizes a group of chords, and when each consecutive H has a specified number of attacks for a definite number of chords.

For example: $A^{\rightarrow} = A_1 + A_2 + A_3$

$$\text{Let } A_1 = \begin{array}{c} M_I \\ \hline M_{II} \\ \hline H \end{array} \begin{array}{c} = \\ = \\ \end{array} \begin{array}{c} 2a+a \\ \hline a+a \\ \hline a \end{array} \text{ and let } A_2 = \begin{array}{c} M_I \\ \hline M_{II} \\ \hline H \end{array} \begin{array}{c} = \\ = \\ \end{array} \begin{array}{c} 4a+3a \\ \hline 2a+a \\ \hline a \end{array}$$

$$\text{and let } A_3 = \begin{array}{c} M_I \\ \hline M_{II} \\ \hline H \end{array} \begin{array}{c} = \\ = \\ \end{array} \begin{array}{c} 4a+6a+3a \\ \hline 2a+2a+a \\ \hline a \end{array} \quad \text{then:}$$

$$\begin{array}{c} M_I \\ \hline M_{II} \\ \hline H^{\rightarrow} \end{array} \begin{array}{c} = \\ = \\ \end{array} \left(\begin{array}{c} 2a+a \\ \hline a+a \\ \hline a \end{array}\right) H_1 \begin{array}{c} + \\ + \\ \end{array} \left(\begin{array}{c} 4a+3a \\ \hline 2a+a \\ \hline a \end{array}\right) H_2 \begin{array}{c} + \\ + \\ \end{array} \left(\begin{array}{c} 4a+6a+3a \\ \hline 2a+2a+a \\ \hline a \end{array}\right) H_3$$

All other considerations concerning the distribution and the number of attacks are identical with those I have discussed as part of one-part melodization.*

Example of Correlated Attack-Groups in Two-Part Melodization

$$\begin{array}{c} M_I \\ \hline M_{II} \\ \hline H^{\rightarrow} \end{array} \begin{array}{c} = \\ = \\ \end{array} \left(\begin{array}{c} 2a+3a \\ \hline a+a \\ \hline a \end{array}\right) H_1 \begin{array}{c} + \\ + \\ \end{array} \left(\begin{array}{c} 3a+4a \\ \hline a+a \\ \hline a \end{array}\right) H_2 \begin{array}{c} + \\ + \\ \end{array} \left(\begin{array}{c} 4a+3a+2a \\ \hline a+\ a+\ a \\ \hline a \end{array}\right) H_3$$

$H^{\rightarrow} = 6\sqrt[4]{2}$, S(9) const.; $\Sigma 13$ XIII; $S = \frac{3p}{p}$; transformation: $\circlearrowright$

$T'' = 12t$ in $\frac{3}{4}$ time.

Figure 124. Correlated attack-groups in two-part melodization (continued).

*See Book V.

Figure 124. Correlated attack-groups in two-part melodization (concluded).

A. Composition of Durations

Durations and duration-groups which will satisfy the attack-groups composed for two-part melodization may either be selected from the various series of the *evolution-of-rhythm* families (in which case there is no interference between the attacks of the attack-group and the attacks of the duration-group. They may also be based on a *direct composition of duration-groups* (which may, or may not, produce an interference between the attacks of the attack-group and the attack of the duration-group) superimposed upon the attack-groups.

When the respective attack-groups are represented by durations selected from style-series, and the number of individual attacks in the attack sub-groups does *not* correspond to the number of attacks in the duration-groups, it is necessary to split the respective duration-units. This consideration concerns only the first technique, that is, the matching of attack-groups by a series of durations.

The musical example of Figure 125 is a translation of its corresponding attack-group into $\frac{3}{3}$ series, where three types of split-unit groups were used: $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. One exception to the series was made at the cadence, where a musical quarter was split into $\frac{4}{4}$ series binomial, i.e., $3+1$.

The numerical representation of this example of melodization appears as follows:

$$\begin{array}{l} M_{I} \\ \hline M_{II} \\ \hline \overrightarrow{H} \end{array} \begin{array}{c} = \\ = \end{array} \begin{pmatrix} 1/2t+1/2t+1/2t+1/2t+t \\ \hline t \qquad + \quad 2t \\ \hline 3t \end{pmatrix} H_1 \begin{array}{c} + \\ + \end{array}$$

$$\begin{array}{c} + \\ + \end{array} \begin{pmatrix} 1/3t+1/3t+1/3t+1/2t+1/2t+1/2t+1/2t \\ \hline t \qquad + \quad 2t \\ \hline 3t \end{pmatrix} H_2 \begin{array}{c} + \\ + \end{array}$$

$$\begin{array}{c} + \\ + \end{array} \begin{pmatrix} 1/4t+1/4t+1/4t+1/4t+1/3t+1/3t+1/3t+1/2t+1/2t \\ \hline t \qquad + \quad t \qquad + \quad t \\ \hline 3t \end{pmatrix} H_3$$

Figure 125. Numerical representation of Figure 124.

The abundance of split units and split-unit groups in this instance is due to the abundance of attacks over each H and to a relatively low value of the series. With a series of higher value, the splitting of units would be greatly reduced.

We shall next translate the same example into $\frac{9}{8}$ series:

$$\begin{array}{l} M_{I} = \\ \hline M_{II} = \\ \hline H^{\rightarrow} \end{array} \left(\begin{array}{l} t+3t+t+3t+t \\ \hline 4t \quad +5t \\ \hline \quad 9t \end{array}\right) H_1 \begin{array}{c} + \\ + \end{array} \left(\begin{array}{l} t+2t+t+t+2t+t+t \\ \hline 4t \qquad +5t \\ \hline \qquad 9t \end{array}\right) H_2 \begin{array}{c} + \\ + \end{array}$$

$$\begin{array}{c} + \\ + \end{array} \left(\begin{array}{l} t+t+t+t+t+t+t+t+t \\ \hline 4t \qquad +3t \qquad +2t \\ \hline \qquad 9t \end{array}\right) H_3$$

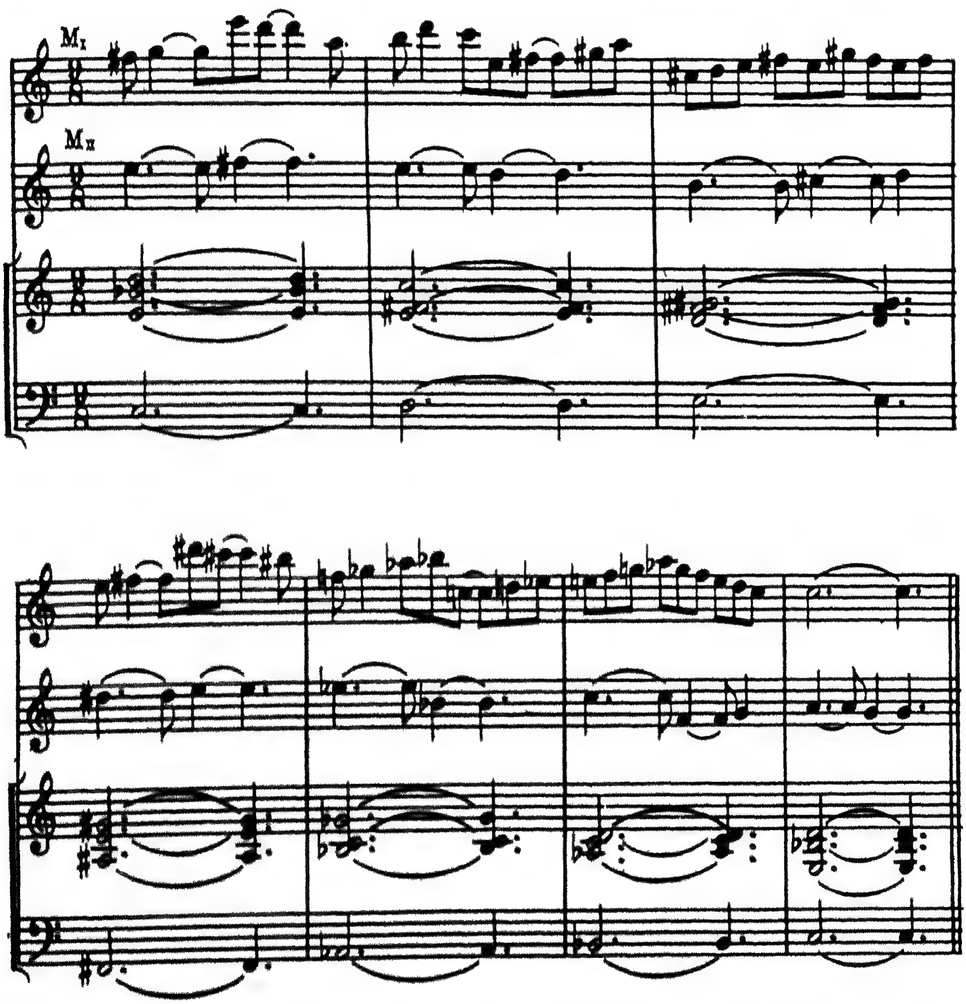


Figure 126. The melody of figure 125 translated into $\frac{9}{8}$ series.

We shall next take a case in which the attack-groups and the duration-groups are composed independently. Let $r_{5\div4}$ represent the number of attacks of M_I to each attack of M_{II}, and let every 2 attacks of M_{II} correspond to one attack of $H^{\rightarrow}$. The distribution of attacks for all three parts will be as follows:

$$\begin{array}{l} \frac{a(M_I)}{a(M_{II})} = \\ \frac{a(M_{II})}{a(H^{\rightarrow})} = \end{array} \begin{pmatrix} \frac{4a+a}{a+a} \\ a \end{pmatrix} H_1 \begin{array}{c}+\\+\end{array} \begin{pmatrix} \frac{3a+2a}{a+a} \\ a \end{pmatrix} H_2 \begin{array}{c}+\\+\end{array} \begin{pmatrix} \frac{2a+3a}{a+a} \\ a \end{pmatrix} H_3 \begin{array}{c}+\\+\end{array} \begin{pmatrix} \frac{a+4a}{a+a} \\ a \end{pmatrix} H_4$$

Superimpose the following duration-group:

$$T = r_{4\div3} = 16t;\ 10a$$

Than: $\frac{a(A)}{a(T)} = \frac{20}{10} = \frac{2}{1}$; $\begin{matrix}1(20)\\2(10\end{matrix}$

Hence, $T' = 16t \cdot 2 = 32t$

Let $T'' = 8t$, then: $N_{T''} = \frac{32}{8} = 4$

Each $a(M_I)$ corresponds to an individual term of T; each $a(M_{II})$ corresponds to the sum of the respective durations of M_I; each $a(H^{\rightarrow})$ corresponds to the sum of 2 durations of M_{II}.

The final temporal scheme of this two-part melodization takes the following form:

$$\begin{array}{l} \frac{M_I}{M_{II}} = \\ \frac{M_{II}}{H^{\rightarrow}} = \end{array} \begin{pmatrix} \frac{3t+t+2t+t+t}{7t\qquad +t} \\ 8t \end{pmatrix} H_1 \begin{array}{c}+\\+\end{array} \begin{pmatrix} \frac{t+t+2t+t+3t}{4t\qquad +4t} \\ 8t \end{pmatrix} H_2 \begin{array}{c}+\\+\end{array}$$

$$\begin{array}{c}+\\+\end{array} \begin{pmatrix} \frac{3t+t+2t+t+t}{4t\quad +4t} \\ 8t \end{pmatrix} H_3 \begin{array}{c}+\\+\end{array} \begin{pmatrix} \frac{t+t+2t+t+3t}{t+7t} \\ 8t \end{pmatrix} H_4$$

Figure 127. Two-part melodization. Attack-groups and duration-groups composed independently (continued).

Figure 127. Two part melodization (concluded).

B. Direct Composition of Durations

Direct composition of durations becomes particularly valuable when one desires a ***proportionate distribution of durations for a constant number of attacks*** among the component parts (M_I, M_{II} and $H^{\rightarrow}$). Distributive involution of three synchronized powers solves this problem.

It follows from my theory of rhythm* that the cube of a binomial produces an eight-term polynomial, the square of a binomial produces a quadrinomial and the first-power group remains a binomial. Thus, the number of attacks in the two adjacent parts $\frac{M_I}{M_{II}}$ and $\frac{M_{II}}{H^{\rightarrow}}$ is two. Cubing of a trinomial gives a twenty-seven-term polynomial, the synchronized square producing nine terms, and the first-power group producing three terms. The number of attacks between the two adjacent parts remains three. The number of terms of the original polynomial thus equals the number of attacks between each pair of adjacent parts.

We shall devise now a correlated proportionate system of duration-groups. The distributive cube will serve as T for M_I, the synchronized distributive square as T for M_{II} and the synchronized first-power group as T for $H^{\rightarrow}$

We shall operate from the trinomial of the $\frac{3}{4}$ series. This yields the following attack-group correlation:

$$\frac{a(M_I)}{\frac{a(M_{II})}{a(H^{\rightarrow})}} = \frac{9a}{\frac{3a}{a}}$$

The entire temporal scheme assumes the form shown on the following page:

*See Book I.

$$\begin{array}{c} \dfrac{T(M_{I})}{\dfrac{T(M_{II})}{T(\overrightarrow{H})}} = \dfrac{[(8t+4t+4t) + (4t+2t+2t) + (4t+2t+2t)]}{\dfrac{(16t \quad + 8t \quad + 8t \quad)}{32tH_1}} \begin{array}{c} + \\ + \end{array} \\ \begin{array}{c} + \\ + \end{array} \dfrac{[(4t+2t+2t) + (2t+t+t) + (2t+t+t)]}{\dfrac{(8t \quad + 4t \quad + 4t \quad)}{16tH_2}} \begin{array}{c} + \\ + \end{array} \\ \begin{array}{c} + \\ + \end{array} \dfrac{[(4t+2t+2t) + (2t+t+t) + (2t+t+t)]}{\dfrac{(8t \quad + 4t \quad + 4t \quad)}{16tH_3}} \end{array}$$

In addition to this technique, ***coefficients of duration*** may be used for correlation of durations in two-part melodization.

Example:

$$\dfrac{M_I}{\dfrac{M_{II}}{\overrightarrow{H}}} = \dfrac{(3t+t+2t+2t)+(3t+t+2t+2t)+(3t+t+2t+2t)+(3t+t+2t+2t)}{\dfrac{(6t+2t+4t+4t) + (6t+2t+4t+4t)}{12tH_1 + 4tH_2 + 8tH_3 + 8tH_4}}$$

Figure 128. Direct composition of durations through distributive involution of three synchronized powers (continued).

Figure 128. Direct composition of durations (concluded).

C. Composition of Continuity

The seven forms of exposition previously classified may be now incorporated into a continuity of two-part melodization. The meaning of these seven forms as applied to composition may be expressed as follows:

(1) M_{I} — Solo melody: theme A;

(2) M_{II} — Solo melody: theme B;

(3) $H^{\rightarrow}$ — Solo harmony: theme C;

(4) $\frac{M_{I}}{H^{\rightarrow}}$ — Solo melody with harmonic accompaniment (theme A accompanied);

(5) $\frac{M_{II}}{H^{\rightarrow}}$ — Solo melody with harmonic accompaniment (theme B accompanied);

(6) $\frac{M_{I}}{M_{II}}$ — Duet of two melodies $\left(\frac{\text{Theme A}}{\text{Theme B}}\right)$

(7) $\begin{array}{c} M_{I} \\ \hline M_{II} \\ \hline H^{\rightarrow} \end{array}$ — Duet of two melodies with harmonic accompaniment $\left(\begin{array}{c} \text{Theme A} \\ \hline \text{Theme B} \\ \hline \text{Theme C} \end{array}\right)$

The above seven forms serve as thematic elements of a composition in which they appear in an organized sequence producing a complete musical whole.

Themes A, B and C must be considered as component parts of the whole in which they express their particular characteristics. The particular characteristics which distinguish A from B and from C are:

(1) High mobility of A (maximum quantity of attacks);
(2) Medium mobility of B (medium quantity of attacks);
(3) Low mobility of C (minimum quantity of attacks) combined with maximum density (four or five parts).

The planning of the continuity must be based on a definite pattern for variation of the density, combined with variation in the quantity of attacks.

The scale of density may be arranged from low to high density, as follows:

$$(1)\quad A,\ \begin{matrix}A\\B\end{matrix},\ C,\ \begin{matrix}A\\C\end{matrix},\ \begin{matrix}A\\B\\C\end{matrix};\ \text{or as}$$

$$(2)\quad B,\ \begin{matrix}A\\B\end{matrix},\ C,\ \begin{matrix}B\\C\end{matrix},\ \begin{matrix}A\\B\\C\end{matrix}.$$

The relatively extreme points of any such scale produce contrasts; for instance:

$$(1)\ \begin{matrix}A\\B\\C\end{matrix}+B+\begin{matrix}A\\B\\C\end{matrix}+A+\begin{matrix}A\\B\\C\end{matrix}+A+C+B+C+\begin{matrix}A\\B\\C\end{matrix};$$

$$(2)\ A+C+B+C+A+\begin{matrix}A\\B\\C\end{matrix}+B+\begin{matrix}A\\B\\C\end{matrix}+A+B$$

Durations corresponding to one individual attack of the component of lowest mobility (mostly $H^{\rightarrow}$) become time-units of the continuity. Such units (we shall call them T) may be arranged in any form of rhythmic distribution.

Correlation of the thematic duration-groups (T's, with their coefficients) with the different forms of density constitutes the composition.

Assuming that there are three forms of density and three forms of mobility, we obtain the following ***combined thematic forms*** (low, medium, high): $3^2 = 9$.

Density	Low	Low	Medium	Medium	Low	High	Medium	High	High
Mobility	Low	Medium	Low	Medium	High	Low	High	Medium	High

For instance: $\frac{\text{Density}}{\text{Mobility}} = \frac{\text{Low}}{\text{Low}} \doteq M_{II}$; $\frac{\text{Density}}{\text{Mobility}} = \frac{\text{High}}{\text{Low}} \doteq H^{\rightarrow}$;

$\frac{\text{Density}}{\text{Mobility}} = \frac{\text{High}}{\text{Medium}} \doteq \frac{M_{II}}{H^{\rightarrow}}$, etc.

Let us now devise a composition in which gradual and sudden variations of both mobility and density will be combined.

It is desirable to use a scheme of two-part melodization which will be *cyclic* and *recapitulating*, i.e., one permitting a correct transition from the end to the beginning for all three components.

For the present, we shall not resort to any additional techniques (such as inversions, expansions, etc.); the complete synthesis of all these and other procedures will be accomplished in my later discussion of *composition** as such.

Let Figure 127 serve as the fundamental scheme for two-part melodization, as this material is both cyclic and recapitulating.

Let us adopt the following scheme of density and mobility:

$$\frac{\text{Density}}{\text{Mobility}} = \frac{\text{Low}}{\text{Low}} + \frac{\text{Low}}{\text{High}} + \frac{\text{Medium}}{\text{High}} + \frac{\text{High}}{\text{Medium}} + \frac{\text{High}}{\text{Low}} + \frac{\text{Medium}}{\text{High}} + \frac{\text{High}}{\text{High}}$$

A sequence of thematic elements and their combinations, corresponding to the seven *forms of expositions* and satisfying the above scheme of *thematic forms*, may be selected as follows:

$$E^{\rightarrow} = M_{II}E_1 + M_IE_2 + \frac{M_I}{H^{\rightarrow}}E_3 + \frac{M_{II}}{H^{\rightarrow}}E_4 + H^{\rightarrow}E_5 + \frac{M_I}{M_{II}}E_6 + \left(\frac{\frac{M_I}{M_{II}}}{H^{\rightarrow}}\right)E_7.$$

Let us make T correspond to H, and establish the following sequence for the T's: $T = r_{5+3}$.

$$T^{\rightarrow} = T_13H + T_22H + T_3H + T_43H + T_5H + T_62H + T_73H$$
$$T^{\rightarrow} = 7T\ 15H.$$

The 7T of $T^{\rightarrow}$ produce no interference in relation to the 7E of $E^{\rightarrow}$. But there is an interference between $T^{\rightarrow}E^{\rightarrow}$ and $H^{\rightarrow}$, however, for $H^{\rightarrow} = 8H$.

$$\frac{T^{\rightarrow}E^{\rightarrow}}{H^{\rightarrow}} = \frac{7}{8};\quad \frac{8(7)}{7(8)}\qquad E^{\rightarrow\prime} = 7\cdot 8 = 56\ TE.$$

As 7 TE corresponds to 15 H, there will be 7 TE·8 = 56 TE and 15 H·8 = 120 H.

The complete composition after synchronization evolves into the following form:

$$T^{\rightarrow\prime}E^{\rightarrow\prime} = 56\ TE\ 120\ H;\ T'' = H;\ N_{T''} = 120.$$

As, in Figure 127, $T'' = TH$, the entire composition consumes 120 measures —which is 15 times the duration of the original scheme of melodization.**

*See Book XI.

**Observe that the original source material plus the formula requires about 2½ pages (MS), whereas the same composition in score requires 8 pages. (Ed.)

Here is the final layout of the composition:

$$T^{\rightarrow}E^{\rightarrow} = [M_{II}\ (H_1 + H_2 + H_3)\ T_1E_1 + M_I\ (H_4 + H_5)\ T_2E_2 + \frac{M_I}{H^{\rightarrow}}(H_6)T_3E_3 +$$

$$+ \frac{M_{II}}{H^{\rightarrow}}\ (H_7 + H_8 + H_1)\ T_4E_4 + H^{\rightarrow}(H_2)\ T_5E_5 + \frac{M_I}{M_{II}}\ (H_3 + H_4)\ T_6E_6 +$$

$$+ \cfrac{M_I}{\cfrac{M_{II}}{H^{\rightarrow}}}\ (H_5 + H_6 + H_7)\ T_7E_7] + [M_{II}\ (H_8 + H_1 + H_2)T_8E_8 +$$

$$+ M_I\ \ (H_3 + H_4)\ T_9E_9 + \frac{M_I}{H^{\rightarrow}}(H_5)\ T_{10}E_{10} + \frac{M_{II}}{H^{\rightarrow}}\ \ (H_6 + H_7 + H_8)$$

$$T_{11}E_{11} + H^{\rightarrow}\ (H_1)\ T_{12}E_{12} + \frac{M_I}{M_{II}}\ (H_2 + H_3)\ T_{13}E_{13} +$$

$$+ \cfrac{M_I}{\cfrac{M_{II}}{H^{\rightarrow}}}\ (H_4 + H_5 + H_6)\ T_{14}E_{14}] + [M_{II}\ (H_7 + H_8 + H_1)\ T_{15}E_{15} +$$

$$+ M_I\ \ (H_2 + H_3)\ T_{16}E_{16} + \frac{M_I}{H^{\rightarrow}}\ (H_4)\ T_{17}E_{17} +$$

$$+ \frac{M_{II}}{H^{\rightarrow}}\ (H_5 + H_6 + H_7)\ T_{18}E_{18} + H^{\rightarrow}(H_8)\ T_{19}E_{19} + \frac{M_I}{M_{II}}(H_1 + H_2)T_{20}E_{20} +$$

$$+ \cfrac{M_I}{\cfrac{M_{II}}{H^{\rightarrow}}}\ (H_3 + H_4 + H_5)\ T_{21}E_{21}] + [M_{II}\ (H_6 + H_7 + H_8)\ T_{22}E_{22} +$$

$$+ M_I\ \ (H_1 + H_2)\ T_{23}E_{23} + \frac{M_I}{H^{\rightarrow}}\ (H_3)\ T_{24}E_{24} +$$

$$+ \frac{M_{II}}{H^{\rightarrow}}\ (H_4 + H_5 + H_6)\ T_{25}E_{25} + H^{\rightarrow}\ (H_7)\ T_{26}E_{26} +$$

$$+ \frac{M_I}{M_{II}}\ (H_8 + H_1)\ T_{27}E_{27} + \cfrac{M_I}{\cfrac{M_{II}}{H^{\rightarrow}}}\ (H_2 + H_3 + H_4)\ T_{28}E_{28}] +$$

$$+ [M_{II}\ (H_5 + H_6 + H_7)\ T_{29}E_{29} + M_I\ (H_8 + H_1)\ T_{30}E_{30} +$$

$$+ \frac{M_I}{H^{\rightarrow}}\ (H_2)\ T_{31}E_{31} + \frac{M_{II}}{H^{\rightarrow}}(H_3 + H_4 + H_5)\ T_{32}E_{32} + H^{\rightarrow}\ (H_6)\ T_{33}E_{33} +$$

$$+ \frac{M_I}{M_{II}}\ (H_7 + H_8)\ T_{34}E_{34} + \cfrac{M_I}{\cfrac{M_{II}}{H^{\rightarrow}}}\ (H_1 + H_2 + H_3)\ T_{35}E_{35}] +$$

$$+ [M_{II}\ (H_4 + H_5 + H_6)\ T_{36}E_{36} + M_I\ (H_7 + H_8)\ T_{37}E_{37} +$$

(Continued on opposite page).

$$+\frac{M_I}{H^{\rightarrow}}(H_1)\,T_{38}E_{38}+\frac{M_{II}}{H^{\rightarrow}}(H_2+H_3+H_4)\,T_{39}E_{39}+H^{\rightarrow}(H_5)\,T_{40}E_{40}+$$

$$+\frac{M_I}{M_{II}}(H_6+H_7)\,T_{41}E_{41}+\frac{M_I}{\frac{M_{II}}{H^{\rightarrow}}}(H_8+H_1+H_2)\,T_{42}E_{42}]+$$

$$+[M_{II}(H_3+H_4+H_5)\,T_{43}E_{43}+M_I(H_6+H_7)\,T_{44}E_{44}+$$

$$+\frac{M_I}{H^{\rightarrow}}(H_8)\,T_{45}E_{45}+\frac{M_{II}}{H^{\rightarrow}}(H_1+H_2+H_3)\,T_{46}E_{46}+$$

$$+H^{\rightarrow}(H_4)\,T_{47}E_{47}+\frac{M_I}{M_{II}}(H_5+H_6)\,T_{48}E_{48}+\frac{M_I}{\frac{M_{II}}{H^{\rightarrow}}}(H_7+H_8+H_1)\,T_{49}E_{49}]+$$

$$+[M_{II}(H_2+H_3+H_4)\,T_{50}E_{50}+M_I(H_5+H_6)\,T_{51}E_{51}+$$

$$+\frac{M_I}{H^{\rightarrow}}(H_7)\,T_{52}E_{52}+\frac{M_{II}}{H^{\rightarrow}}(H_8+H_1+H_2)\,T_{53}E_{53}+H^{\rightarrow}(H_3)\,T_{54}E_{54}+$$

$$+\frac{M_I}{M_{II}}(H_4+H_5)\,T_{55}E_{55}+\frac{M_I}{\frac{M_{II}}{H^{\rightarrow}}}(H_6+H_7+H_8)\,T_{56}E_{56}].$$

Figure 129. Numerical layout of a complete two-part melodization.

Below you will find the complete musical representation of the numerical layout just given:

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation og figure 129 (continued).

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation of figure 129 (concluded).

CHAPTER 11

HARMONIZATION OF TWO-PART COUNTERPOINT

THE main procedure in writing a harmonic accompaniment to the duet of two contrapuntal parts consists of *assigning harmonic consonances to be chordal functions.*

Each combination of two pitch-units producing a simultaneous consonance becomes a pair of chordal functions—this premise concerns all types of counterpoint and all types of harmonization.

Those pitch-units which produce dissonances are perceived by us, through auditory association, as auxiliary and passing tones. When what we might call "justification" of the consonance as a pair of chordal functions takes place, the harmonic accompaniment acquires a proper meaning.

A. Diatonic Harmonization

Under the conditions imposed by Special Harmony, the kind of two-part counterpoint which can be harmonized by Special Harmony must be constructed from seven-unit scales of the first group, not containing identical intonations.

As all three components—M_1, M_2 and H—must belong to one key, according to the definition of diatonic, the only types of counterpoint which can be diatonically harmonized are types I and II.

It is important for the composer to realize the versatility of relations which may exist among the modes of the three components. M_I may be written in any of the seven modes (d_0, d_1, d_2, d_3, d_4, d_5, d_6) of one scale; so may M_{II}, and so may the $H^{\rightarrow}$. The total number of these modal variations for one scale is: $7^3 = 343$. This, of course, includes all the identical as well as all the non-identical combinations; practically, however, this quantity must be somewhat diminished, if we want to preserve a consonant relation between the P.A.'s of M_I and M_{II}.

The number of seven-unit scales not containing identical units is 36; therefore, the total manifold of relations of M_I: M_{II}: $H^{\rightarrow}$ in diatonic counterpoint of types I and II is:

$$343 \cdot 36 = 12{,}348.$$

Any given combination may be modified into a new system of intonations, i.e., into a new scale, by simply readjusting the accidentals. All the above quantities, naturally, do not include the attack-relations which have to be established for the harmonization.

As the attacks of $\frac{M_I}{M_{II}}$ are fixed groups, the only relation that must be established concerns $H^{\rightarrow}$. The most refined form of harmonization results from assigning *each* harmonic consonance to one H. If counterpoint contains many delayed resolutions of one dissonance, then the number of attacks of M_I is quite great and the changes of H are not as frequent. On the other hand, *direct* resolutions produce *frequent* chord changes.

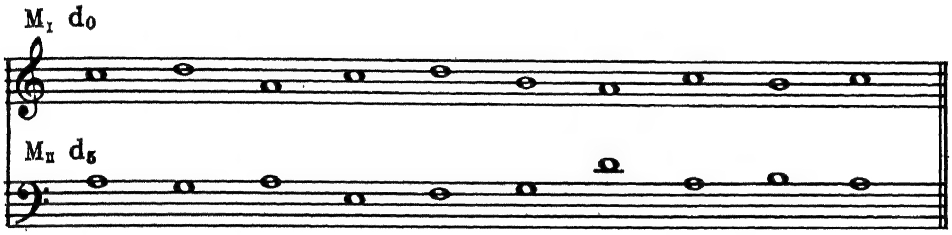
The assignment of two successive harmonic consonances to one H, amplifies the number of chords satisfying such a set, but at the same time neutralizes somewhat the character of the $H^{\rightarrow}$. This technique, however, permits a greater variety of attack-relations among the three components.

Let us see how we would harmonize counterpoint of type II, when $\frac{M_I}{M_{II}} = a$. In such a case, all the harmonic intervals are consonances. Therefore, we can have the following matching of attacks:

$$\begin{array}{lllllllll} \underline{M_I} & = & \underline{a} & \underline{M_I} & = & \underline{2a} & \underline{M_I} & = & \underline{3a} \\ \underline{M_{II}} & = & \underline{a} & \underline{M_{II}} & = & \underline{2a} & \underline{M_{II}} & = & \underline{3a}, \text{ etc.} \\ H^{\rightarrow} & = & a & H^{\rightarrow} & = & a & H^{\rightarrow} & = & a \end{array}$$

Examples of Diatonic Harmonization of Two-Part Counterpoint when $\frac{M_I}{M_{II}} = a$.

Theme:



Harmonization (1)

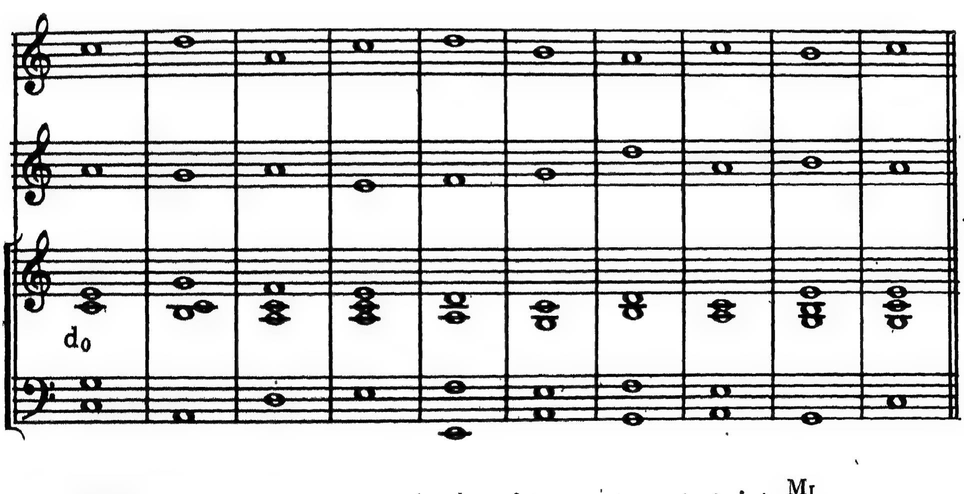
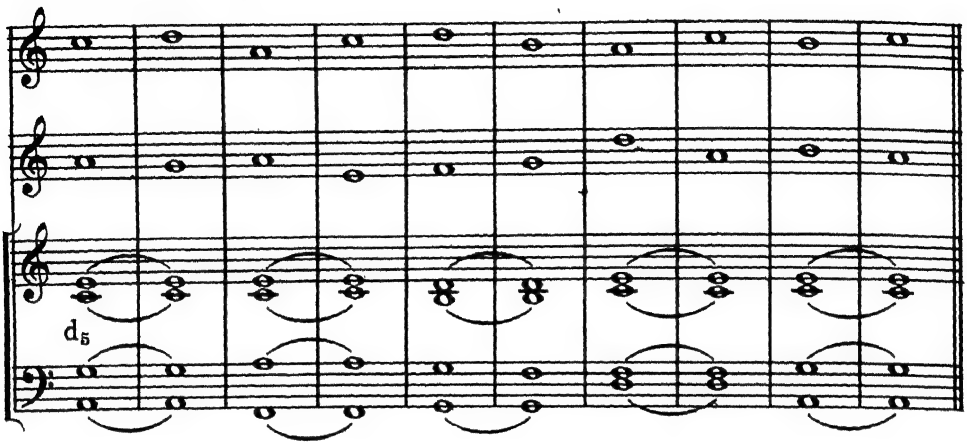


Figure 131. Diatonic harmonization of two-part counterpoint; $\frac{M_I}{M_{II}} = a$ *(continued).*

Harmonization (2)



Harmonization (3)

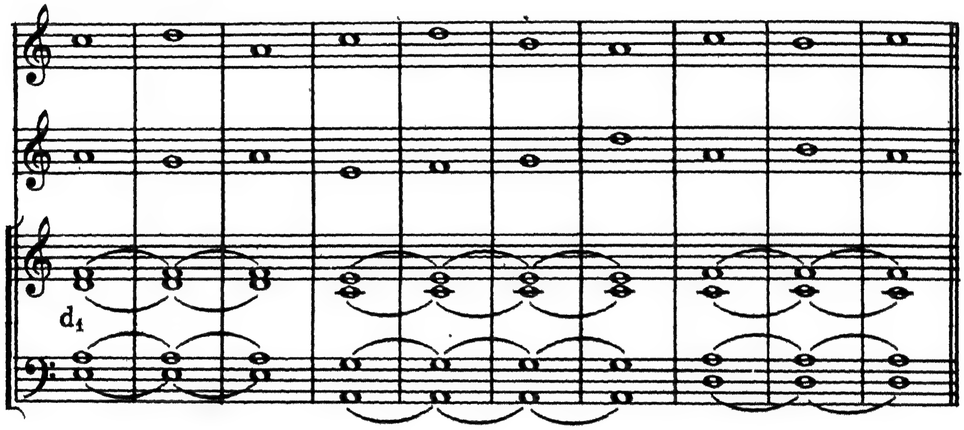


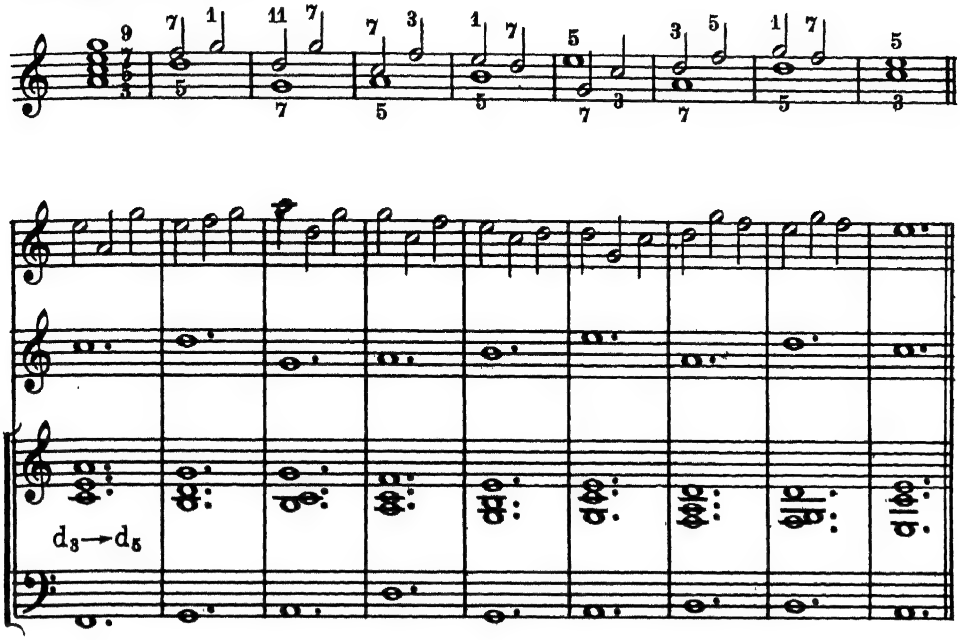
Figure 131. Diatonic harmonization of two-part counterpoint; $\frac{M_I}{M_{II}} = a$ *(concluded).*

Examples of Diatonic Harmonization of Two-Part Counterpoint when $\frac{M_I}{M_{II}} = \frac{3a}{a}$

Theme:

Figure 132. Diatonic harmonization of two-part counterpoint; $\frac{M_I}{M_{II}} = \frac{3a}{a}$ *(continued).*

Harmonization (1)



Harmonization (2)

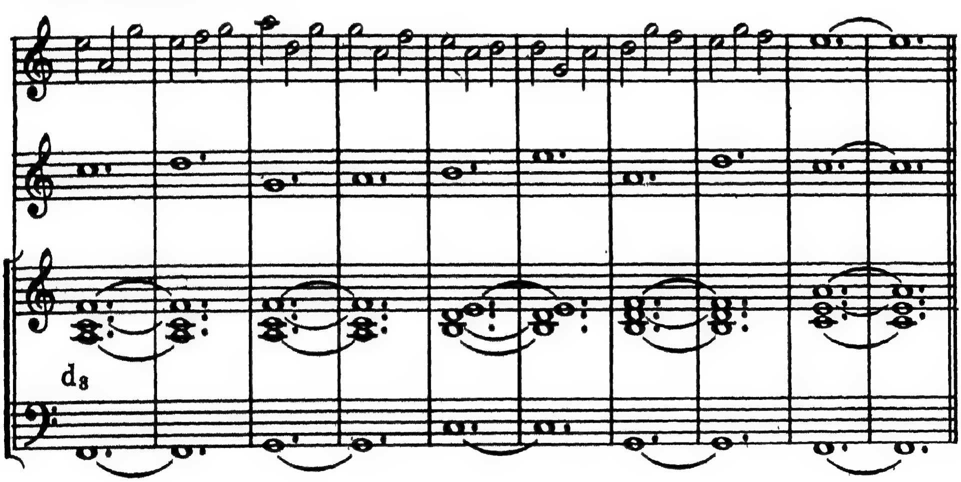
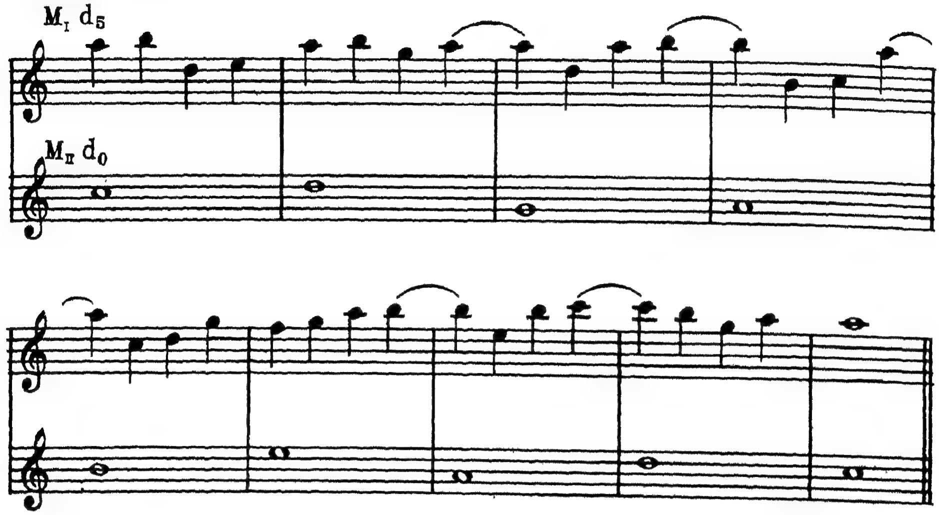


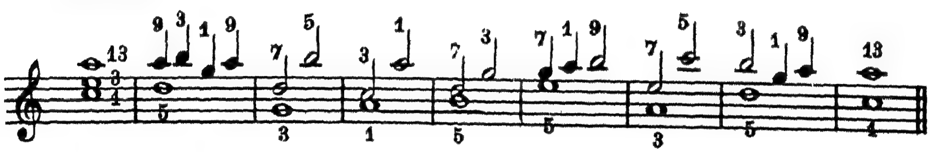
Figure 132. Diatonic harmonization of two-part counterpoint: $\frac{M_I}{M_{II}} = \frac{3a}{a}$ *(concluded).*

Examples of Diatonic Harmonization of Two-Part Counterpoint when $\frac{M_I}{M_{II}} = \frac{4a}{a}$ and $\frac{M_I}{M_{II}} = \frac{6a}{a}$

Theme: (1)



Harmonization:



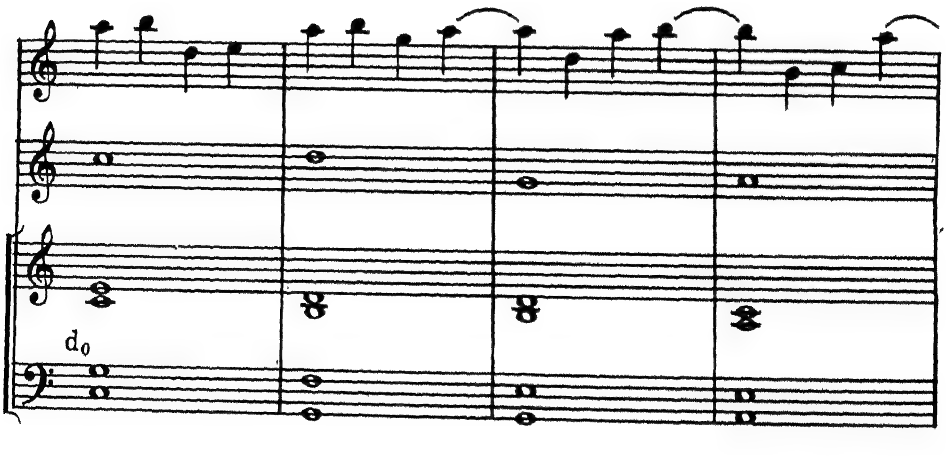


Figure 133. Diatonic harmonization of two-part counterpoint: $\frac{M_I}{M_{II}} = \frac{4a}{a}$ *(continued).*

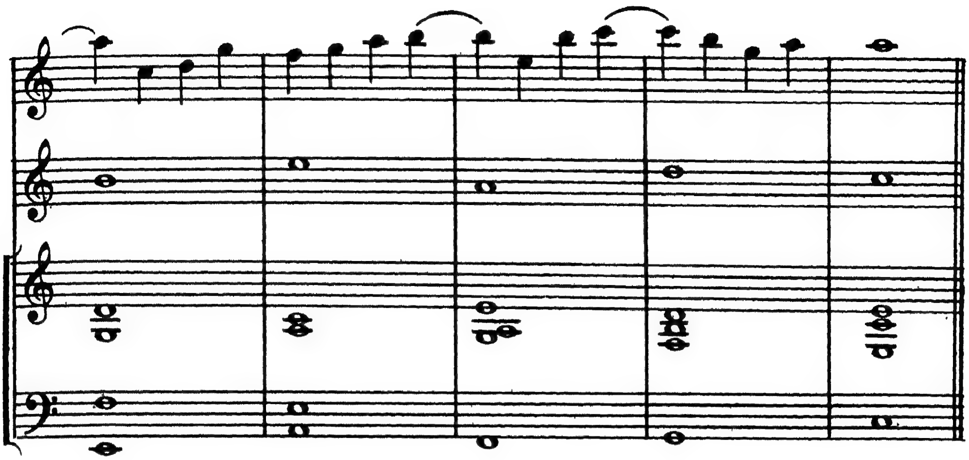


Figure 133. $\frac{M_{I}}{M_{II}} = \frac{4a}{a}$ *(concluded)*.

Theme: (2)

Harmonization:

Figure 134. $\frac{M_{I}}{M_{II}} = \frac{6a}{a}$ *(continued)*.

Figure 134. $\frac{M_I}{M_{II}} = \frac{6a}{a}$ (concluded).

B. Chromatization of Harmony Accompanying Two-Part Diatonic Counterpoint (Types I and II)

Chromatic variation of diatonic harmony accompanying two-part counterpoint may be obtained by means of auxiliary and passing chromatic tones. Of course, such altered tones must not conflict in any way with the two melodies.

For our example, we shall take the two-part counterpoint diatonically harmonized from Figure 133 (2).

Figure 135. Chromatization of harmonic counterpoint (continued).

Figure 135. Chromatization of harmonic counterpoint (concluded).

C. Diatonic Harmonization of Chromatic Counterpoint Whose Origin is Diatonic (Types I and II)

The principle of this form of harmonization is that of assigning the *diatonic* consonances as chordal functions; chromatic consonances, as well as all other forms of harmonic interval, are to be ignored so far as the $H^{\rightarrow}$ goes.

The number of successive consonances which should correspond to one H is optional; it is practical to make T, or 2T, or 3T correspond to one H.

When harmonizing a chromatic counterpoint whose diatonic original is known, one can assign chordal functions directly from the diatonic original; doing this obviously eliminates any possible confusion of the diatonic and the chromatic consonances.

We shall now harmonize a duet in which both parts are chromatic. The theme is taken from Figure 50 of Chapter 4. For clarity's sake, we shall write out both the original and the chromatized version. We shall choose the following relationship between $H^{\rightarrow}$ and $T^{\rightarrow}$:

$$H^{\rightarrow} T^{\rightarrow} = HT + H2T + HT + HT + HT + H2T + HT$$

which is a modified version of the $\underline{r_{3\div 2}}$, and which permits us to demonstrate diversified forms of attacks groups of M_{I} and M_{II} in relation to $H^{\rightarrow}$

Example of Diatonic Harmonization of Chromatic Counterpoint

Original

Chromatic variation

Figure 136. Chromatic counterpoint (continued).

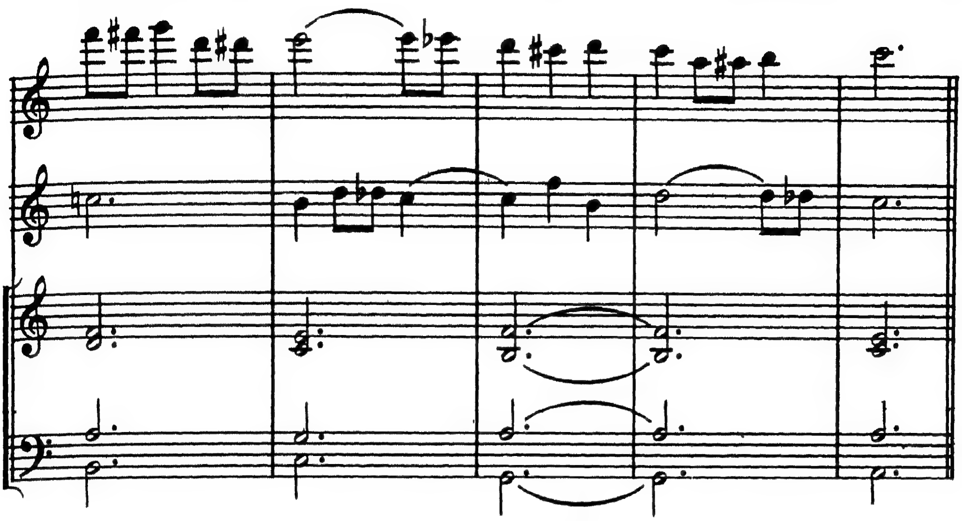


Figure 136. Chromatic counterpoint (concluded).

When the diatonic origin of the chromatic counterpoint is unknown, an analysis of diatonic consonances must precede the planning of the harmonization.

D. Symmetric Harmonization of Diatonic Two-Part Counterpoint (Types I, II, III and IV)

The principle of *symmetric* harmonization of two-part counterpoint is that of assigning *all* harmonic intervals to be chordal functions.

The fewer the attacks of M_I and M_{II} that correspond to one H, the easier it is to perform such harmonization by means of one $\Sigma 13$. But when a considerable number of attacks (even in only one of the two melodies) corresponds to one H, it becomes necessary to introduce two, and sometimes even three, $\Sigma 13$'s. The forms of the latter should vary only slightly, making sure that any change is for the purpose only of rectifying the particular non-corresponding pitch-unit. For instance, in using a $\Sigma 13$ XIII as Σ_1, a correction of the eleventh to f♮ gives a satisfactory solution for most cases; Σ_2 in this instance will differ from Σ_1 only with respect to the 11.

The selection of the original Σ 13 is a matter of harmonic character. For example, the use of Σ 13 XIII attributes to music a definitely "Ravelian" quality. However, harmonic quality still remains virgin territory awaiting the composer's exploration; most of the 36 forms of the Σ 13 have not been utilized at all.

The fact that counterpoint belongs to types I and II, or to types III and IV does not help us select any particular Σ 13. Whereas symmetric harmonization of counterpoint of types I and II is a luxury, it is an actual necessity for types III and IV, as the latter correlate two different *key*-axes.

The fact that *two different keys*, with identical or with non-identical scales, may be united by *one* chord is of particular importance. This is so because the quality of a selected Σ 13 can influence the two melodies. In our musical civiliza-

tion, our ears are so much conditioned by harmony that most of our listeners have lost any ability to enjoy melodic line as such. If the ear of an average music-lover can relate one diatonic melody to only one chord progression, the harmonic association of two melodies belonging to two different keys becomes impossible; the role of a harmonic master-structure (Σ 13 in this case) is that of *synthesizer*.

The simplest way to assign harmonic functions is to relate the latter first to consonances. The master-structure used in the following harmonization is Σ 13 XIII.*

Symmetric Harmonization of Diatonic
Two-Part Counterpoint of Types I and II

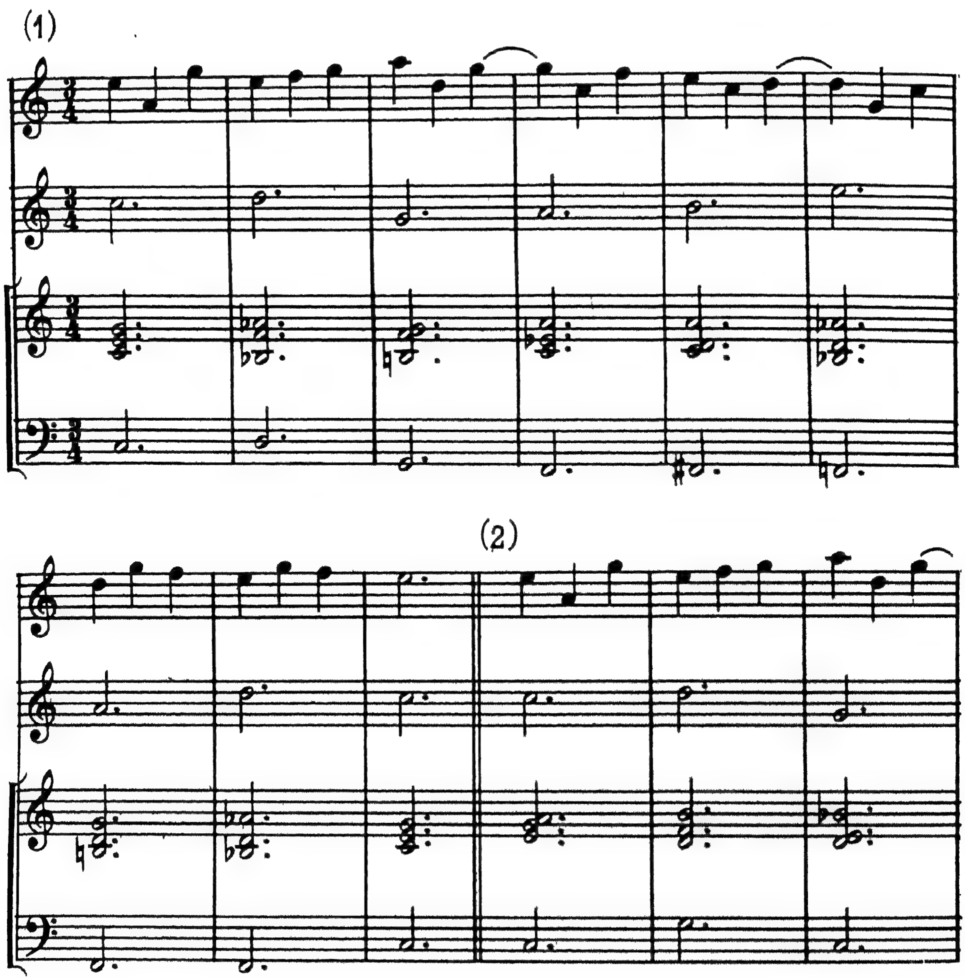


Figure 137. Symmetric harmonization of diatonic two-part counterpoint of types I and II (continued).

*Σ 13 (XIII) has the following form  The complete list of Σ 13's is presented in Book VI, Chapter 2. (Ed.)

(3) Chromatic variation of harmony (2)

Figure 137. Symmetric harmonization of diatonic two-part counterpoint of types I and II (concluded).

The chromatic variation of $H^{\rightarrow}$ in the foregoing example was obtained through the usual technique: the insertion of passing and auxiliary chromatic units.

Symmetric Harmonization of Diatonic Two-Part Counterpoint of Types III and IV

(1)

(2) Chromatic variation of harmony (1)

Figure 138. Symmetric harmonization of diatonic two-part counterpoint of types III and IV (continued).

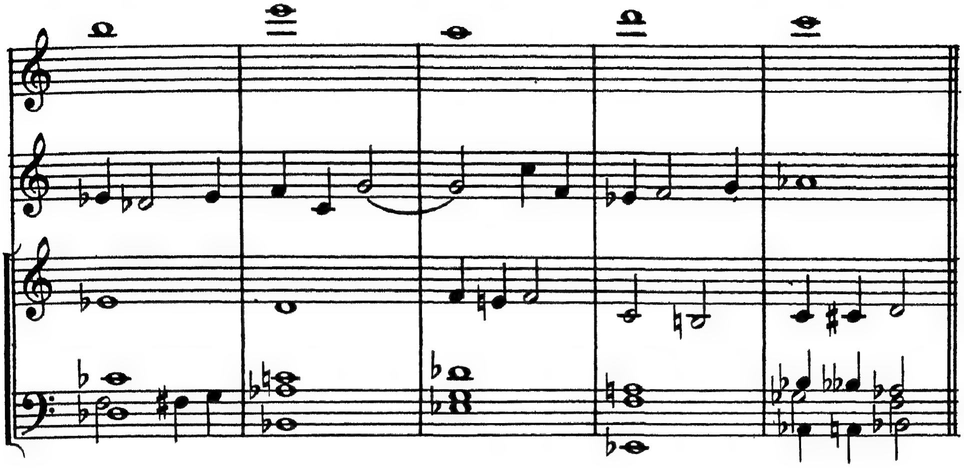


Figure 138. Symmetric harmonization of diatonic two-part counterpoint of types III and IV (concluded).

E. Symmetric Harmonization of Chromatic Two-Part Counterpoint Whose Origin Is Diatonic (Types I, II, III and IV)

The principle of symmetric harmonization of chromatic two-part counterpoint is that of assigning *all the diatonic pitch-units of both melodies* to be chordal functions of the master-structure (Σ 13)—but neglecting all the chromatic pitch-units as not belonging to the scale; it does not matter whether the chromatic units belong to the master-structure or not. When the diatonic original of the two-part counterpoint is unknown, then the diatonic units of both melodies should be isolated before proceeding.

Original (1)

Figure 139. Symmetric harmonization of chromatic two-part counterpoint (continued).

Chromatic variation harmonized (1)

Original (2)

Figure 139. Symmetric harmonization of chromatic two-part counterpoint (continued).

Chromatic variation harmonized (2)

Figure 139. Symmetric harmonization of chromatic two-part counterpoint (concluded).

Counterpoint executed in symmetric scales of the third and fourth groups may be harmonized by means of a symmetric master-structure. This master-structure is independent of the system of symmetry of the pitch-scales involved. As in previous cases, all units corresponding to one H must belong to one Σ 13.

After the harmonization is performed, it may be subjected, if desired, to chromatic variation.

F. Symmetric Harmonization of Symmetric Two-Part Counterpoint

Theme:

Harmonization

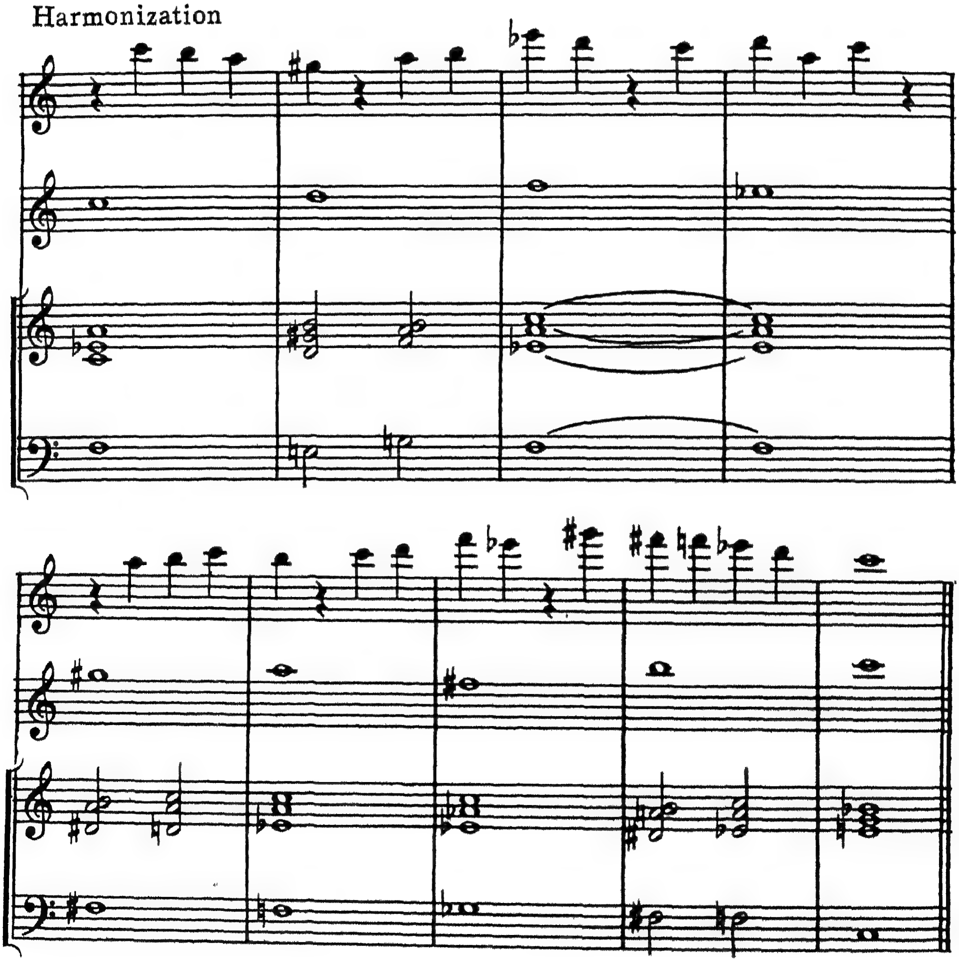


Figure 140. Symmetric harmonization of symmetric two-part counterpoint (cont.).

Chromatic variation of harmony

Figure 140. Symmetric harmonization of symmetric two-part counterpoint (concluded).

All forms of contrapuntal continuity and complete compositions in the form of canon and fugue may be harmonized by this technique. Any of the correspondences described above between counterpoint and harmony may be established by the composer. One should remember that *overloading* harmonic accompaniments is more a sin than a virtue; for this reason, the technique of *variable density* should receive the utmost consideration.

CHAPTER 12

MELODIC, HARMONIC, AND CONTRAPUNTAL OSTINATO

FORMS of *ostinato* or *ground motion* have been known since time immemorial. They appear in different folk and traditional music as a fundamental form of improvisation around a given theme. The characteristic of *ostinato* (literally: obstinate) is the continuous repetition of a certain thematic group—which may be either rhythm, melody, or harmony. As one example, the dance beat of 4/4 in a fox-trot is one of just such fundamental forms of *ostinato*. And, as a matter of fact, a *rhythmic ostinato* is ever-present in *all* the developments in classical symphonies! Take, for example, the first motif of Beethoven's *Fifth Symphony*, consisting of 4 notes, and follow it through the development (middle section of the first movement); the motif, rhythmically the same, changes its forms of intonation either melodically or in the form of accompanying harmony.

Repetitions of groups of chords, or repetitions of melodic fragments accompanied by continuously changing chords, are both forms of *ostinato*. Ostinato is one of the traditional forms of thematic growth and, as such, is very well known in the forms called chaconne (ciaconna) and passacaglia. In many Irish jigs, *ostinato* appears in the form of pedal point, as well as in repetitious melodic fragments. When portions of the same melody appear in succession, being harmonized every time anew, (which may be found even in such works as Chopin's mazurkas), we have still another case of ostinato.

A. Melodic Ostinato (Basso Ostinato)

Melodic *ostinato*, better known under the name of "ground bass," is a *harmonization* of an ever-repeating melody by continuously changing chords. *Ostinato* groups produce one uninterrupted continuity in which the recurrence of the bass form produces the unity and the accompanying harmony produces the variety. All forms of harmonization may be applied to the continuously repeating melody, regardless of whether it appears in the bass or in any of the middle voices, or in the upper voice (above the harmony).

As every harmonic setting of chords is subject to vertical permutations, a *basso ostinato* may be transformed into tenor, or alto, or soprano *ostinato*, i.e., it may appear in any desirable voice and in any desirable sequence after the harmonization has been completed.

In the following example, the *ostinato* of the theme is a melody in whole notes in the bass (the first four bars); later it repeats itself two more times. The form of harmonization is symmetric in this case, although it could have been diatonic or in any of the chromatic forms. This device may be used as a form of thematic *development*, —and in arranging it may be used with effect for the purpose of constructing introductions or transitions. Any characteristic melodic pattern may be converted into *basso ostinato* either with the preservation of its original rhythm or in an entirely new setting.*

*See Arensky's *Basso Ostinato* for piano. (J.S.)

Melodic Ostinato

Basso Ostinato (Ground Bass)

Symmetric Harmonization of the Bass.

Figure 141. Melodic ostinato.

B. Harmonic Ostinato

Harmonic *ostinato* might also be called, by analogy, "ground harmony." It consists of the repetition of a group of chords, in relation to which a continuously changing melody is evolved. This form of ostinato is the one which J. S. Bach employed in his D-minor chaconne for violin; it is also used in numerous other compositions—by Bach and other composers, too. Among my own students, George Gershwin used this device successfully in an exercise which later, at my suggestion, he put into the musical comedy, *Let 'Em Eat Cake,* as the song hit, *Mine.*

This form of *ostinato* may be applied to any type of harmonic progression. The technical procedure is exactly the opposite of the first one. In this case we deal with *melodization* of harmony. As in the previous case, the melody evolved against chords may be transferred to a different position in relation to the chord

by means of vertical permutation. Naturally, not every melody will be as good in the bass as in the soprano, for the chordal functions represented by melody are more advantageous for an upper part than for the lower, or *vice versa.*

In the following example, the harmonic theme of *ostinato* emphasizes four different chords (the first two bars), and is based on a Σ 13 [XIII]. The melody evolves through the principle of symmetric melodization forming its axis points in relation to the chord structure itself. The main resource by which variety is obtained against the uniformity of the *ostinato* is the manifold of melodic forms.

Harmonic Ostinato (Ground Harmony)
Symmetric Melodization of Harmony

Figure 142. Harmonic ostinato.

C. Contrapuntal Ostinato

The form, contrapuntal *ostinato*, is well known in the works of old masters. It was usually evolved against a melody, a *cantus firmus*. If a C.F. repeats itself continuously a number of times while the contrapuntal part or parts evolve in relation to it producing different relations with every appearance of the C.F., the result is a contrapuntal ostinato.

In the following example, the theme of the *ostinato* is taken from Figure 141 and the accompanying counterpoint is evolved through type II, adhering to a rhythmic *ostinato*, as well, except for a few intentional permutations. Naturally, both voices may be exchanged, or may be subjected to any of the variations through geometrical positions ⓐ, ⓑ, ⓒ, and ⓓ.

Contrapuntal Ostinato

Basso Ostinato (Ground Bass)

Figure 143. Contrapuntal ostinato.

Likewise, a counterpoint may be evolved to the soprano voice through the use of the same principle. In Figure 144, the same theme is employed, altered rhythmically; the counterpoint, in its rhythmic setting, produces a constant interference against the C.F., as it consists of a 3-bar group. The harmonic setting of this example is in type III: the C.F. is in natural C major, and the counterpoint is in natural A♭ major.

Soprano Ostinato (Ground Melody)

CP Type III

Figure 144. Soprano ostinato.

The latter two forms of *ostinato*—harmonic and contrapuntal—are extremely adaptable in all cases in which it is desirable to repeat one motif and yet introduce variety into the obligato. These characteristics make the devices extremely useful for introductions, transitions, and codas in arranging.